SOLUTIONS TO CONCEPTS
CHAPTER – 2

1. As shown in the figure,

   The angle between A and B = 110° – 20° = 90°

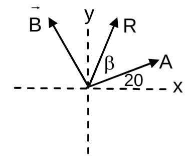
   | A | = 3 and | B | = 4m

   Resultant R = $\sqrt{A^2 + B^2 + 2AB\cos\theta}$ = 5 m

   Let β be the angle between R and A

   $\beta = \tan^{-1}\left(\dfrac{4\sin 90°}{3 + 4\cos 90°}\right) = \tan^{-1}(4/3) = 53°$

   ∴ Resultant vector makes angle (53° + 20°) = 73° with x-axis.



2. Angle between A and B is θ = 60° – 30° =30°

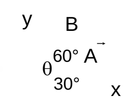
   | A | and | B | = 10 unit

   R = $10^2 + 10^2 + 2.10.10.\cos 30°$ = 19.3

   β be the angle between R and A

   $\beta = \tan^{-1}\left(\dfrac{10\sin 30°}{10 + 10\cos 30°}\right) = \tan^{-1}\left(\dfrac{1}{2+\sqrt{3}}\right) = \tan^{-1}(0.26795) = 15°$

   ∴ Resultant makes 15° + 30° = 45° angle with x-axis.



3. x component of A = 100 cos 45° = 100 / 2 unit

   x component of B = 100 cos 135° = 100 / 2

   x component of C = 100 cos 315° = 100 / 2

   Resultant x component = 100 / 2 – 100 / 2 + 100 / 2 = 100 / 2

   y component of A = 100 sin 45° = 100 / 2 unit

   y component of B = 100 sin 135° = 100 / 2

   y component of C = 100 sin 315° = – 100 / 2

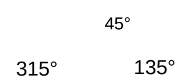
   Resultant y component = $100/\sqrt{2} + 100/\sqrt{2} - 100/\sqrt{2} = 100/\sqrt{2}$

   Resultant = 100

   Tan α = $\dfrac{\text{y component}}{\text{x component}} = 1$

   ⇒ α = $\tan^{-1}(1)$ = 45°

   The resultant is 100 unit at 45° with x-axis.



4. $\vec{a} = 4i + 3j,\ b = 3i + 4j$

   a) $|\vec{a}| = \sqrt{4^2 + 3^2} = 5$

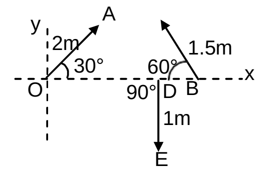
   b) $|b| = \sqrt{9 + 16} = 5$

   c) $|\vec{a} + b| = |7i + 7j| = 7\sqrt{2}$

   d) $\vec{a} - b = (-3 + 4)\hat{i} + (-4 + 3)\hat{j} = \hat{i} - \hat{j}$

   $|\vec{a} - \vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$.

5. x component of $\overrightarrow{OA}$ = 2cos30° = $\sqrt{3}$



x component of $\overrightarrow{BC}$ = 1.5 cos 120° = –0.75

x component of $\overrightarrow{DE}$ = 1 cos 270° = 0

y component of $\overrightarrow{OA}$ = 2 sin 30° = 1

y component of $\overrightarrow{BC}$ = 1.5 sin 120° = 1.3

y component of $\overrightarrow{DE}$ = 1 sin 270° = –1

R_x = x component of resultant = $\sqrt{3} - 0.75 + 0 = 0.98$ m

R_y = resultant y component = 1 + 1.3 – 1 = 1.3 m

So, R = Resultant = 1.6 m

If it makes and angle α with positive x-axis

Tan $\alpha = \frac{\text{y component}}{\text{x component}}$ = 1.32

$\Rightarrow \alpha = \tan^{-1} 1.32$

6. | a | = 3m | b | = 4

a) If R = 1 unit ⇒ $3^2 + 4^2 + 2.3.4.\cos\theta = 1$

θ = 180°

b) $3^2 + 4^2 + 2.3.4.\cos\theta = 5$

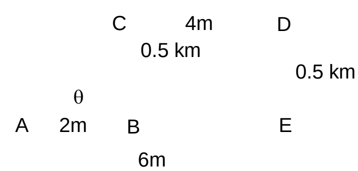
θ = 90°

c) $3^2 + 4^2 + 2.3.4.\cos\theta = 7$

θ = 0°

Angle between them is 0°.

7. $\overrightarrow{AD} = 2\hat{i} + 0.5\hat{J} + 4\hat{K} = 6\hat{i} + 0.5\hat{j}$



AD = $AE^2 + DE^2 = 6.02$ KM

Tan θ = DE / AE = 1/12

$\theta = \tan^{-1}$ (1/12)

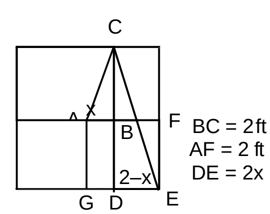
The displacement of the car is 6.02 km along the distance $\tan^{-1}$ (1/12) with positive x-axis.

8. In ΔABC, tanθ = x/2 and in ΔDCE, tanθ = (2 – x)/4 tan θ = (x/2) = (2 – x)/4 = 4x

⇒ 4 – 2x = 4x

⇒ 6x = 4 ⇒ x = 2/3 ft

a) In ΔABC, AC = $\sqrt{AB^2 + BC^2} = \frac{2}{3}\sqrt{10}$ ft

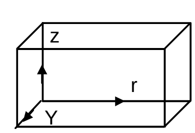


b) In ΔCDE, DE = 1 – (2/3) = 4/3 ft

CD = 4 ft. So, CE = $\sqrt{CD^2 + DE^2} = \frac{4}{3}\sqrt{10}$ ft

c) In ΔAGE, AE = $\sqrt{AG^2 + GE^2} = 2\sqrt{2}$ ft.

9. Here the displacement vector $\vec{r} = 7\hat{i} + 4\hat{j} + 3\hat{k}$



a) magnitude of displacement = $\sqrt{74}$ ft

b) the components of the displacement vector are 7 ft, 4 ft and 3 ft.

10. a is a vector of magnitude 4.5 unit due north.

a) $3|a| = 3 \times 4.5 = 13.5$

3 a is along north having magnitude 13.5 units.

b) $-4|a| = -4 \times 1.5 = -6$ unit

–4 a is a vector of magnitude 6 unit due south.

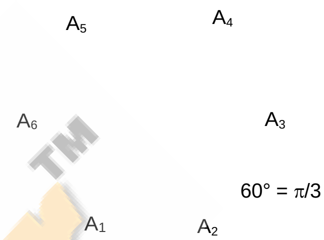
11. $|a| = 2$ m, $|b| = 3$ m

angle between them $\theta = 60°$

a) $\vec{a}\cdot b = |\vec{a}|\cdot|b|\cos 60° = 2 \times 3 \times 1/2 = 3\ m^2$

b) $|\vec{a}\times b| = |\vec{a}|\cdot|b|\sin 60° = 2 \times 3 \times \sqrt{3/2} = 3\sqrt{3}\ m^2$.

12. We know that according to polygon law of vector addition, the resultant of these six vectors is zero.



Here A = B = C = D = E = F (magnitude)

So, $R_x = A\cos\theta + A\cos\pi/3 + A\cos 2\pi/3 + A\cos 3\pi/3 + A\cos 4\pi/4 + A\cos 5\pi/5 = 0$

[As resultant is zero. X component of resultant $R_x = 0$]

$= \cos\theta + \cos\pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0$

Note : Similarly it can be proved that,

$\sin\theta + \sin\pi/3 + \sin 2\pi/3 + \sin 3\pi/3 + \sin 4\pi/3 + \sin 5\pi/3 = 0$

13. $\vec{a} = 2i + 3j + 4k;\ b = 3i + 4j + 5k$

$\vec{a}\cdot b = ab\cos\theta \Rightarrow \theta = \cos^{-1}\frac{\vec{a}\cdot b}{ab}$

$\Rightarrow \cos^{-1}\frac{2\times 3 + 3\times 4 + 4\times 5}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+4^2+5^2}} = \cos^{-1}\left(\frac{38}{\sqrt{1450}}\right)$

14. $A\cdot(A\times B) = 0$ (claim)

As, $A\times B = AB\sin\theta\hat{n}$

$AB\sin\theta\,\hat{n}$ is a vector which is perpendicular to the plane containing A and B, this implies that it is also perpendicular to A. As dot product of two perpendicular vector is zero.

Thus $A\cdot(A\times B) = 0$.

15. $A = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $B = 4\hat{i} + 3\hat{j} + 2\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 4 & 3 & 2\end{vmatrix} \Rightarrow \hat{i}(6-12) - \hat{j}(4-16) + \hat{k}(6-12) = -6\hat{i} + 12\hat{j} - 6\hat{k}.$$

16. Given that A, B and C are mutually perpendicular

A × B is a vector which direction is perpendicular to the plane containing A and B.

Also C is perpendicular to A and B

∴ Angle between C and A × B is 0° or 180° (fig.1)

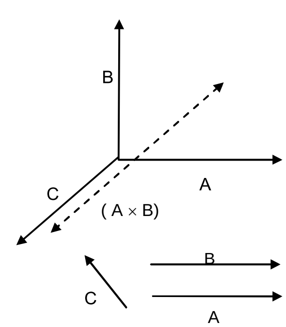
So, C × (A × B) = 0

The converse is not true.

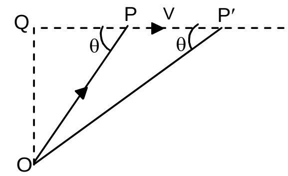
For example, if two of the vector are parallel, (fig.2), then also

C × (A × B) = 0

So, they need not be mutually perpendicular.



17. The particle moves on the straight line PP' at speed v.



From the figure,

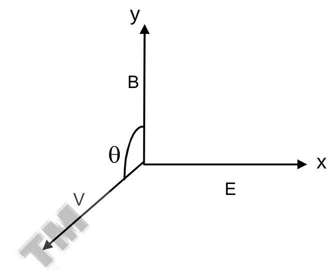
$$\overrightarrow{OP} \times v = (OP)v \sin\theta\hat{n} = v(OP)\sin\theta\ \hat{n} = v(OQ)\hat{n}$$

It can be seen from the figure, OQ = OP sin θ = OP' sin θ'

So, whatever may be the position of the particle, the magnitude and direction of $\overrightarrow{OP} \times \vec{v}$ remain constant.

$\therefore\ \overrightarrow{OP} \times \vec{v}$ is independent of the position P.

18. Give $F = qE + q(\vec{v} \times B) = 0$



$$\Rightarrow E = -(\vec{v} \times B)$$

So, the direction of $\vec{v} \times B$ should be opposite to the direction of E. Hence, v should be in the positive yz-plane.

Again, $E = vB\sin\theta \Rightarrow v = \dfrac{E}{B\sin\theta}$

For v to be minimum, θ = 90° and so $v_{min} = F/B$

So, the particle must be projected at a minimum speed of E/B along +ve z-axis (θ = 90°) as shown in the figure, so that the force is zero.

19. For example, as shown in the figure,

$A \perp B$	B along west
$B \perp C$	A along south
	C along north
$A \cdot B = 0$	$\therefore A \cdot B = B \cdot C$
$B \cdot C = 0$	But $B \neq C$

B C A B

20. The graph $y = 2x^2$ should be drawn by the student on a graph paper for exact results.

To find slope at any point, draw a tangent at the point and extend the line to meet x-axis. Then find tan θ as shown in the figure.

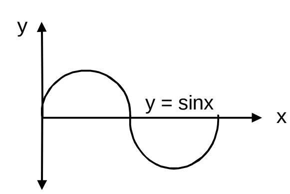
y=2x² Δy Δx θ

It can be checked that,

$$\text{Slope} = \tan\theta = \frac{dy}{dx} = \frac{d}{dx}(2x^2) = 4x$$

Where x = the x-coordinate of the point where the slope is to be measured.

21. y = sinx



So, y + Δy = sin (x + Δx)

Δy = sin (x + Δx) – sin x

$$= \left(\frac{\pi}{3} + \frac{\pi}{100}\right) - \sin\frac{\pi}{3} = 0.0157.$$

22. Given that, $i = i_0 e^{-t/RC}$

$$\therefore \text{Rate of change of current} = \frac{di}{dt} = \frac{d}{dt} i_0 e^{-i/RC} = i_0 \frac{d}{dt} e^{-t/RC} = \frac{-i_0}{RC} \times e^{-t/RC}$$

When a) $t = 0,\ \dfrac{di}{dt} = \dfrac{-i}{RC}$

b) when $t = RC,\ \dfrac{di}{dt} = \dfrac{-i}{RCe}$

c) when $t = 10\,RC,\ \dfrac{di}{dt} = \dfrac{-i_0}{RCe^{10}}$

23. Equation $i = i_0 e^{-t/RC}$

$i_0 = 2A$, $R = 6 \times 10^{-5}\ \Omega$, $C = 0.0500 \times 10^{-6}\ F = 5 \times 10^{-7}\ F$

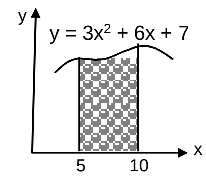
a) $i = 2 \times e^{\left(\frac{-0.3}{6\times10^{-5}\times5\times10^{-7}}\right)} = 2 \times e^{\left(\frac{-0.3}{0.3}\right)} = \frac{2}{e}$ amp.

b) $\frac{di}{dt} = \frac{-i_0}{RC} e^{-t/RC}$ when t = 0.3 sec ⇒ $\frac{di}{dt} = -\frac{2}{0.30} e^{(-0.3/0.3)} = \frac{-20}{3e}$ Amp / sec

c) At t = 0.31 sec, $i = 2e^{(-0.3/0.3)} = \frac{5.8}{3e}$ Amp .

24. $y = 3x^2 + 6x + 7$

∴ Area bounded by the curve, x axis with coordinates with x = 5 and x = 10 is given by,



Area = $\int_0^y dy = \int_5^{10} (3x^2 + 6x + 7)dx = 3\left[\frac{x^3}{3}\right]_5^{10} + 5\left[\frac{x^2}{3}\right]_5^{10} + 7x]_5^{10} = 1135$ sq.units.

25. Area = $\int_0^y dy = \int_0^{\pi} \sin x dx = -[\cos x]_0^{\pi} = 2$

y

y = sinx

x

26. The given function is $y = e^{-x}$

When x = 0, $y = e^{-0} = 1$

x increases, y value deceases and only at x = ∞, y = 0.

So, the required area can be found out by integrating the function from 0 to ∞.

So, Area = $\int_0^{\infty} e^{-x}dx = -[e^{-x}]_0^{\infty} = 1.$

y

x

27. $\rho = \frac{\text{mass}}{\text{length}} = a + bx$

a) S.I. unit of ‘a’ = kg/m and SI unit of ‘b’ = kg/m² (from principle of homogeneity of dimensions)

b) Let us consider a small element of length ‘dx’ at a distance x from the origin as shown in the figure.

∴ dm = mass of the element = ρ dx = (a + bx) dx

So, mass of the rod = m = $\int dm = \int_0^L (a + bx)dx = \left[ax + \frac{bx^2}{2}\right]_0^L = aL + \frac{bL^2}{2}$

y

O

x

x =1

28. $\frac{dp}{dt}$ = (10 N) + (2 N/S)t

momentum is zero at t = 0

∴ momentum at t = 10 sec will be

dp = [(10 N) + 2Ns t]dt

$\int_0^p dp = \int_0^{10} 10dt + \int_0^{10} (2tdt) = 10t]_0^{10} + 2\left[\frac{t^2}{2}\right]_0^{10}$ = 200 kg m/s.

29. The change in a function of y and the independent variable x are related as $\frac{dy}{dx} = x^2$.

$\Rightarrow dy = x^2\, dx$

Taking integration of both sides,

$\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + c$

$\therefore$ y as a function of x is represented by $y = \frac{x^3}{3} + c$.

30. The number significant digits

a) 1001 No.of significant digits = 4

b) 100.1 No.of significant digits = 4

c) 100.10 No.of significant digits = 5

d) 0.001001 No.of significant digits = 4

31. The metre scale is graduated at every millimeter.

1 m = 100 mm

The minimum no.of significant digit may be 1 (e.g. for measurements like 5 mm, 7 mm etc) and the maximum no.of significant digits may be 4 (e.g.1000 mm)

So, the no.of significant digits may be 1, 2, 3 or 4.

32. a) In the value 3472, after the digit 4, 7 is present. Its value is greater than 5.

So, the next two digits are neglected and the value of 4 is increased by 1.

$\therefore$ value becomes 3500

b) value = 84

c) 2.6

d) value is 28.

33. Given that, for the cylinder

Length = l = 4.54 cm, radius = r = 1.75 cm

Volume = $\pi r^2 l = \pi \times (4.54) \times (1.75)^2$

Since, the minimum no.of significant digits on a particular term is 3, the result should have 3 significant digits and others rounded off.

So, volume $V = \pi r^2 l = (3.14) \times (1.75) \times (1.75) \times (4.54) = 43.6577\ cm^3$

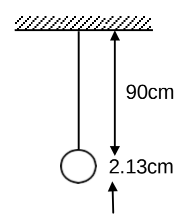
Since, it is to be rounded off to 3 significant digits, $V = 43.7\ cm^3$.

34. We know that,

Average thickness = $\frac{2.17 + 2.17 + 2.18}{3}$ = 2.1733 mm

Rounding off to 3 significant digits, average thickness = 2.17 mm.

35. As shown in the figure,



Actual effective length = (90.0 + 2.13) cm

But, in the measurement 90.0 cm, the no. of significant digits is only 2.

So, the addition must be done by considering only 2 significant digits of each measurement.

So, effective length = 90.0 + 2.1 = 92.1 cm.

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