## SOLUTIONS TO CONCEPTS

## CHAPTER 6

1. Let $\mathrm{m}=$ mass of the block

From the freebody diagram,
$R-\mathrm{mg}=0 \Rightarrow R=\mathrm{mg}$
Again ma $-\mu \mathrm{R}=0 \Rightarrow \mathrm{ma}=\mu \mathrm{R}=\mu \mathrm{mg}$ (from (1))
$\Rightarrow \mathrm{a}=\mu \mathrm{g} \Rightarrow 4=\mu \mathrm{g} \Rightarrow \mu=4 / \mathrm{g}=4 / 10=0.4$
The co-efficient of kinetic friction between the block and the plane is 0.4
2. Due to friction the body will decelerate

Let the deceleration be ' $a$ '
$\mathrm{R}-\mathrm{mg}=0 \Rightarrow \mathrm{R}=\mathrm{mg}$
$\mathrm{ma}-\mu \mathrm{R}=0 \Rightarrow \mathrm{ma}=\mu \mathrm{R}=\mu \mathrm{mg}$ (from (1))
$\Rightarrow a=\mu \mathrm{g}=0.1 \times 10=1 \mathrm{~m} / \mathrm{s}^{2}$.
Initial velocity $u=10 \mathrm{~m} / \mathrm{s}$
Final velocity $v=0 \mathrm{~m} / \mathrm{s}$

$\mathrm{a}=-1 \mathrm{~m} / \mathrm{s}^{2}$ (deceleration)

$S=\frac{v^{2}-u^{2}}{2 a}=\frac{0-10^{2}}{2(-1)}=\frac{100}{2}=50 \mathrm{~m}$
It will travel 50 m before coming to rest.
3. Body is kept on the horizontal table.

If no force is applied, no frictional force will be there
$f \rightarrow$ frictional force
F $\rightarrow$ Applied force
From grap it can be seen that when applied force is zero,
frictional force is zero.

4. From the free body diagram,
$R-m g \cos \theta=0 \Rightarrow R=m g \cos \theta$..(1)
For the block
$U=0, \quad s=8 m, t=2 \mathrm{sec}$.

$\therefore s=u t+1 / 2 a^{2} \Rightarrow 8=0+1 / 2 a 2^{2} \Rightarrow a=4 \mathrm{~m} / \mathrm{s}^{2}$
Again, $\mu \mathrm{R}+\mathrm{ma}-\mathrm{mg} \sin \theta=0$
$\Rightarrow \mu \mathrm{mg} \cos \theta+\mathrm{ma}-\mathrm{mg} \sin \theta=0$
[from (1)]
$\Rightarrow \mathrm{m}(\mu \mathrm{g} \cos \theta+\mathrm{a}-\mathrm{g} \sin \theta)=0$
$\Rightarrow \mu \times 10 \times \cos 30^{\circ}=g \sin 30^{\circ}-a$
$\Rightarrow \mu \times 10 \times \sqrt{(3 / 3)}=10 \times(1 / 2)-4$
$\Rightarrow(5 / \sqrt{3}) \mu=1 \Rightarrow \mu=1 /(5 / \sqrt{3})=0.11$
$\therefore$ Co-efficient of kinetic friction between the two is 0.11 .
5. From the free body diagram
$4-4 a-\mu R+4 g \sin 30^{\circ}=0$

$R-4 g \cos 30^{\circ}=0$

$\Rightarrow R=4 \mathrm{~g} \cos 30^{\circ}$
Putting the values of $R$ is \& in equn. (1)
$4-4 \mathrm{a}-0.11 \times 4 \mathrm{~g} \cos 30^{\circ}+4 \mathrm{~g} \sin 30^{\circ}=0$
$\Rightarrow 4-4 \mathrm{a}-0.11 \times 4 \times 10 \times(3 / 2)+4 \times 10 \times(1 / 2)=0$
$\Rightarrow 4-4 \mathrm{a}-3.81+20=0 \Rightarrow \mathrm{a} \approx 5 \mathrm{~m} / \mathrm{s}^{2}$
For the block $u=0, t=2 \mathrm{sec}, \quad a=5 \mathrm{~m} / \mathrm{s}^{2}$


Distance $s=u t+1 / 2 a t^{2} \Rightarrow s=0+(1 / 2) 5 \times 2^{2}=10 \mathrm{~m}$
The block will move 10 m .
6. To make the block move up the incline, the force should be equal and opposite to the net force acting down the incline $=\mu \mathrm{R}+2 \mathrm{~g} \sin 30^{\circ}$
$=0.2 \times(9.8) \sqrt{3}+2 \mathrm{I} 9.8 \times(1 / 2) \quad$ [from (1)]
$=3.39+9.8=13 \mathrm{~N}$
With this minimum force the body move up the incline with a constant velocity as net force on it is zero.
b) Net force acting down the incline is given by,
$F=2 \mathrm{~g} \sin 30^{\circ}-\mu \mathrm{R}$
$=2 \times 9.8 \times(1 / 2)-3.39=6.41 \mathrm{~N}$
Due to $F=6.41 \mathrm{~N}$ the body will move down the incline with acceleration.
No external force is required.
$\therefore$ Force required is zero.
7. From the free body diagram
$\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}, \quad \mathrm{~m}=2 \mathrm{~kg}, \quad \theta=30^{\circ}, \quad \mu=0.2$
$R-m g \cos \theta-F \sin \theta=0$
$\Rightarrow R=m g \cos \theta+F \sin \theta$
And $m g \sin \theta+\mu R-F \cos \theta=0$
$\Rightarrow m g \sin \theta+\mu(m g \cos \theta+F \sin \theta)-F \cos \theta=0$
$\Rightarrow m g \sin \theta+\mu m g \cos \theta+\mu F \sin \theta-F \cos \theta=0$
$\Rightarrow F=\frac{(m g \sin \theta-\mu m g \cos \theta)}{(\mu \sin \theta-\cos \theta)}$
$\Rightarrow F=\frac{2 \times 10 \times(1 / 2)+0.2 \times 2 \times 10 \times(\sqrt{3} / 2)}{0.2 \times(1 / 2)-\sqrt{3} / 2)}=\frac{13.464}{0.76}=17.7 \mathrm{~N} \approx 17.5 \mathrm{~N}$

8. $\mathrm{m} \rightarrow$ mass of child
$R-m g \cos 45^{\circ}=0$
$\Rightarrow R=m g \cos 45^{\circ}=m g / v^{2}$
Net force acting on the boy due to which it slides down is $\mathrm{mg} \sin 45^{\circ}-\mu \mathrm{R}$
$=m g \sin 45^{\circ}-\mu \mathrm{mg} \cos 45^{\circ}$

$=\mathrm{m} \times 10(1 / \sqrt{2})-0.6 \times \mathrm{m} \times 10 \times(1 / \sqrt{2})$
$=m[(5 / \sqrt{2})-0.6 \times(5 / \sqrt{2})]$
$=m(2 \sqrt{2})$

acceleration $=\frac{\text { Force }}{\text { mass }}=\frac{m(2 \sqrt{2})}{m}=2 \sqrt{2} \mathrm{~m} / \mathrm{s}^{2}$
9. Suppose, the body is accelerating down with acceleration 'a'.

From the free body diagram
$R-m g \cos \theta=0$
$\Rightarrow R=m g \cos \theta$
$m a+m g \sin \theta-\mu R=0$
$\Rightarrow a=\frac{m g(\sin \theta-\mu \cos \theta)}{m}=g(\sin \theta-\mu \cos \theta)$


For the first half $\mathrm{mt} . \mathrm{u}=0, \mathrm{~s}=0.5 \mathrm{~m}, \quad \mathrm{t}=0.5 \mathrm{sec}$.
So, $v=u+a t=0+(0.5) 4=2 \mathrm{~m} / \mathrm{s}$

$$
S=u t+1 / 2 a t^{2} \Rightarrow 0.5=0+1 / 2 a(0 / 5)^{2} \Rightarrow a=4 \mathrm{~m} / \mathrm{s}^{2} \ldots(2)
$$

For the next half metre
$u^{\prime}=2 \mathrm{~m} / \mathrm{s}, \quad \mathrm{a}=4 \mathrm{~m} / \mathrm{s}^{2}, \quad \mathrm{~s}=0.5$.
$\Rightarrow 0.5=2 \mathrm{t}+(1 / 2) 4 \mathrm{t}^{2} \Rightarrow 2 \mathrm{t}^{2}+2 \mathrm{t}-0.5=0$
$\Rightarrow 4 \mathrm{t}^{2}+4 \mathrm{t}-1=0$
$\therefore=\frac{-4 \pm \sqrt{6+16}}{2 \times 4}=\frac{1.656}{8}=0.207 \mathrm{sec}$
Time taken to cover next half meter is 0.21 sec .
10. $f \rightarrow$ applied force
$\mathrm{F}_{\mathrm{i}} \rightarrow$ contact force
$\mathrm{F} \rightarrow$ frictional force
$R \rightarrow$ normal reaction
$\mu=\tan \lambda=F / R$


When $F=\mu R, F$ is the limiting friction (max friction). When applied force increase, force of friction increase upto limiting friction ( $\mu \mathrm{R}$ )
Before reaching limiting friction
F $<\mu R$
$\therefore \tan \lambda=\frac{\mathrm{F}}{\mathrm{R}} \leq \frac{\mu \mathrm{R}}{\mathrm{R}} \Rightarrow \tan \lambda \leq \mu \Rightarrow \lambda \leq \tan ^{-1} \mu$
11. From the free body diagram

$$
\begin{align*}
& T+0.5 a-0.5 g=0  \tag{1}\\
& \mu R+1 a+T_{1}-T=0  \tag{2}\\
& \mu R+1 a-T_{1}=0 \\
& \mu R+1 a=T_{1}
\end{align*}
$$

From (2) \& (3) $\Rightarrow \mu \mathrm{R}+\mathrm{a}=\mathrm{T}-\mathrm{T}_{1}$
$\therefore \mathrm{T}-\mathrm{T}_{1}=\mathrm{T}_{1}$
$\Rightarrow \mathrm{T}=2 \mathrm{~T}_{1}$
Equation (2) becomes $\mu \mathrm{R}+\mathrm{a}+\mathrm{T}_{1}-2 \mathrm{~T}_{1}=0$
$\Rightarrow \mu R+a-T_{1}=0$
$\Rightarrow T_{1}=\mu R+a=0.2 g+a$


Equation (1) becomes $2 \mathrm{~T}_{1}+0 / 5 \mathrm{a}-0.5 \mathrm{~g}=0$
$\Rightarrow \mathrm{T}_{1}=\frac{0.5 \mathrm{~g}-0.5 \mathrm{a}}{2}=0.25 \mathrm{~g}-0.25 \mathrm{a}$
From (4) \& (5) $0.2 g+a=0.25 g-0.25 a$
$\Rightarrow \mathrm{a}=\frac{0.05}{1.25} \times 10=0.04 \mathrm{I} 10=0.4 \mathrm{~m} / \mathrm{s}^{2}$
a) Accln of 1 kg blocks each is $0.4 \mathrm{~m} / \mathrm{s}^{2}$
b) Tension $\mathrm{T}_{1}=0.2 \mathrm{~g}+\mathrm{a}+0.4=2.4 \mathrm{~N}$
c) $\mathrm{T}=0.5 \mathrm{~g}-0.5 \mathrm{a}=0.5 \times 10-0.5 \times 0.4=4.8 \mathrm{~N}$
12. From the free body diagram
$\mu_{1} R+1-16=0$
$\Rightarrow \mu_{1}(2 \mathrm{~g})+(-15)=0$
$\Rightarrow \mu_{1}=15 / 20=0.75$


$\mu_{2} R_{1}+4 \times 0.5+16-4 g \sin 30^{\circ}=0$
$\Rightarrow \mu_{2}(20 \sqrt{3})+2+16-20=0$
$\Rightarrow \mu_{2}=\frac{2}{20 \sqrt{3}}=\frac{1}{17.32}=0.057 \approx 0.06$
$\therefore$ Co-efficient of friction $\mu_{1}=0.75 \& \mu_{2}=0.06$

13.


From the free body diagram

$$
\begin{array}{lll}
T+15 a-15 g=0 & T-\left(T_{1}+5 a+\mu R\right)=0 & T_{1}-5 g-5 a=0 \\
\Rightarrow T=15 g-15 a & \Rightarrow(i) & \Rightarrow T-(5 g+5 a+5 a+\mu R)=0
\end{array} \quad \Rightarrow T_{1}=5 g+5 a \ldots .
$$

From (i) \& (ii) $15 g-15 a=5 g+10 a+0.2(5 g)$
$\Rightarrow 25 \mathrm{a}=90 \Rightarrow \mathrm{a}=3.6 \mathrm{~m} / \mathrm{s}^{2}$
Equation (ii) $\Rightarrow \mathrm{T}=5 \times 10+10 \times 3.6+0.2 \times 5 \times 10$
$\Rightarrow 96 \mathrm{~N}$ in the left string
Equation (iii) $\mathrm{T}_{1}=5 \mathrm{~g}+5 \mathrm{a}=5 \times 10+5 \times 3.6=68 \mathrm{~N}$ in the right string.
14. $\mathrm{s}=5 \mathrm{~m}, \quad \mu=4 / 3, \quad \mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}$
$u=36 \mathrm{~km} / \mathrm{h}=10 \mathrm{~m} / \mathrm{s}, \quad \mathrm{v}=0$,
$a=\frac{v^{2}-u^{2}}{2 s}=\frac{0-10^{2}}{2 \times 5}=-10 \mathrm{~m} / \mathrm{s}^{2}$
From the freebody diagrams,
$R-m g \cos \theta=0 ; g=10 \mathrm{~m} / \mathrm{s}^{2}$
$\Rightarrow R=m g \cos \theta \ldots$. (i) ; $\mu=4 / 3$.


Again, $m a+m g \sin \theta-\mu R=0$
$\Rightarrow \mathrm{ma}+\mathrm{mg} \sin \theta-\mu \mathrm{mg} \cos \theta=0$
$\Rightarrow \mathrm{a}+\mathrm{g} \sin \theta-\mathrm{mg} \cos \theta=0$
$\Rightarrow 10+10 \sin \theta-(4 / 3) \times 10 \cos \theta=0$
$\Rightarrow 30+30 \sin \theta-40 \cos \theta=0$
$\Rightarrow 3+3 \sin \theta-4 \cos \theta=0$
$\Rightarrow 4 \cos \theta-3 \sin \theta=3$
$\Rightarrow 4 \sqrt{1-\sin ^{2} \theta}=3+3 \sin \theta$
$\Rightarrow 16\left(1-\sin ^{2} \theta\right)=9+9 \sin ^{2} \theta+18 \sin \theta$
$\sin \theta=\frac{-18 \pm \sqrt{18^{2}-4(25)(-7)}}{2 \times 25}=\frac{-18 \pm 32}{50}=\frac{14}{50}=0.28$ [Taking +ve sign only]
$\Rightarrow \theta=\sin ^{-1}(0.28)=16^{\circ}$
Maximum incline is $\theta=16^{\circ}$
15. to reach in minimum time, he has to move with maximum possible acceleration. Let, the maximum acceleration is ' $a$ '
$\therefore \mathrm{ma}-\mu \mathrm{R}=0 \Rightarrow \mathrm{ma}=\mu \mathrm{mg}$
$\Rightarrow \mathrm{a}=\mu \mathrm{g}=0.9 \times 10=9 \mathrm{~m} / \mathrm{s}^{2}$
a) Initial velocity $u=0, t=$ ?
$\mathrm{a}=9 \mathrm{~m} / \mathrm{s}^{2}, \mathrm{~s}=50 \mathrm{~m}$
$s=u t+1 / 2 a t^{2} \Rightarrow 50=0+(1 / 2) 9 t^{2} \Rightarrow t=\sqrt{\frac{100}{9}}=\frac{10}{3} \mathrm{sec}$.
b) After overing 50 m , velocity of the athelete is
$\mathrm{V}=\mathrm{u}+\mathrm{at}=0+9 \times(10 / 3)=30 \mathrm{~m} / \mathrm{s}$
He has to stop in minimum time. So deceleration ia $-\mathrm{a}=-9 \mathrm{~m} / \mathrm{s}^{2}(\mathrm{max})$


$$
\left.\begin{array}{l}
{\left[\begin{array}{l}
R=m a \\
m a=\mu R(\text { max frictional force })
\end{array}\right]} \\
L a=\mu g=9 \mathrm{~m} / \mathrm{s}^{2} \text { (Deceleration) }
\end{array}\right] . \quad \begin{aligned}
& u^{1}=30 \mathrm{~m} / \mathrm{s}, \\
& \mathrm{t}=\frac{v^{1}-\mathrm{u}^{1}}{\mathrm{a}}=\frac{0-30}{\mathrm{a}}=\frac{-30}{-\mathrm{a}}=\frac{10}{3} \mathrm{sec} .
\end{aligned}
$$

16. Hardest brake means maximum force of friction is developed between car's type \& road.

Max frictional force $=\mu R$
From the free body diagram
$R-m g \cos \theta=0$
$\Rightarrow R=m g \cos \theta$
and $\mu R+m a-m g \sin )=0$
$\Rightarrow \mu \mathrm{mg} \cos \theta+\mathrm{ma}-\mathrm{mg} \sin \theta=0$
$\Rightarrow \mu \mathrm{g} \cos \theta+\mathrm{a}-10 \times(1 / 2)=0$
$\Rightarrow \mathrm{a}=5-\{1-(2 \sqrt{3})\} \times 10(\sqrt{3} / 2)=2.5 \mathrm{~m} / \mathrm{s}^{2}$


When, hardest brake is applied the car move with acceleration $2.5 \mathrm{~m} / \mathrm{s}^{2}$
$S=12.8 \mathrm{~m}, \mathrm{u}=6 \mathrm{~m} / \mathrm{s}$
S0, velocity at the end of incline
$V=\sqrt{\mathrm{u}^{2}+2 \mathrm{as}}=\sqrt{6^{2}+2(2.5)(12.8)}=\sqrt{36+64}=10 \mathrm{~m} / \mathrm{s}=36 \mathrm{~km} / \mathrm{h}$
Hence how hard the driver applies the brakes, that car reaches the bottom with least velocity $36 \mathrm{~km} / \mathrm{h}$.
17. Let, , a maximum acceleration produced in car.
$\therefore \mathrm{ma}=\mu \mathrm{R}$ [For more acceleration, the tyres will slip]
$\Rightarrow \mathrm{ma}=\mu \mathrm{mg} \Rightarrow \mathrm{a}=\mu \mathrm{g}=1 \times 10=10 \mathrm{~m} / \mathrm{s}^{2}$
For crossing the bridge in minimum time, it has to travel with maximum acceleration
$\mathrm{u}=0, \quad \mathrm{~s}=500 \mathrm{~m}, \quad \mathrm{a}=10 \mathrm{~m} / \mathrm{s}^{2}$
$s=u t+1 / 2 a t^{2}$
$\Rightarrow 500=0+(1 / 2) 10 t^{2} \Rightarrow t=10 \mathrm{sec}$.


If acceleration is less than $10 \mathrm{~m} / \mathrm{s}^{2}$, time will be more than 10 sec . So one can't drive through the bridge in less than 10 sec .
18. From the free body diagram
$R=4 g \cos 30^{\circ}=4 \times 10 \times \sqrt{3} / 2=20 \sqrt{3}$
$\mu_{2} R+4 a-P-4 g \sin 30^{\circ}=0 \Rightarrow 0.3$ (40) $\cos 30^{\circ}+4 a-P-40 \sin 20^{\circ}=0$
$P+2 a+\mu_{1} R_{1}-2 g \sin 30^{\circ}=0$
$R_{1}=2 g \cos 30^{\circ}=2 \times 10 \times \sqrt{3} / 2=10 \sqrt{3}$
Equn. (ii) $6 \sqrt{3}+4 \mathrm{a}-\mathrm{P}-20=0$
Equn (iv) $P+2 a+2 \sqrt{3}-10=0$
From Equn (ii) \& (iv) $6 \sqrt{3}+6 a-30+2 \sqrt{3}=0$
$\Rightarrow 6 \mathrm{a}=30-8 \sqrt{3}=30-13.85=16.15$
$\Rightarrow \mathrm{a}=\frac{16.15}{6}=2.69=2.7 \mathrm{~m} / \mathrm{s}^{2}$

b) can be solved. In this case, the 4 kg block will travel with more acceleration because, coefficient of friction is less than that of 2 kg . So, they will move separately. Drawing the free body diagram of 2 kg mass only, it can be found that, $a=2.4 \mathrm{~m} / \mathrm{s}^{2}$.
19. From the free body diagram

$R_{1}=M_{1} g \cos \theta$
$R_{2}=M_{2} g \cos \theta$
$T+M_{1} g \sin \theta-m_{1} a-\mu R_{1}=0$
$T-M_{2}-M_{2} a+\mu R_{2}=0$
Equn (iii) $\Rightarrow T+M_{1} g \sin \theta-M_{1} a-\mu M_{1} g \cos \theta=0$
Equn (iv) $\Rightarrow T-M_{2} g \sin \theta+M_{2} a+\mu M_{2} g \cos \theta=0$
Equn (iv) \& (v) $\Rightarrow g \sin \theta\left(M_{1}+M_{2}\right)-a\left(M_{1}+M_{2}\right)-\mu g \cos \theta\left(M_{1}+M_{2}\right)=0$
$\Rightarrow a\left(M_{1}+M_{2}\right)=g \sin \theta\left(M_{1}+M_{2}\right)-\mu g \cos \theta\left(M_{1}+M_{2}\right)$
$\Rightarrow a=g(\sin \theta-\mu \cos \theta)$
$\therefore$ The blocks (system has acceleration $g(\sin \theta-\mu \cos \theta)$
The force exerted by the rod on one of the blocks is tension.
Tension $T=-M_{1} g \sin \theta+M_{1} a+\mu M_{1} g \sin \theta$
$\Rightarrow T=-M_{1} g \sin \theta+M_{1}(g \sin \theta-\mu g \cos \theta)+\mu M_{1} g \cos \theta$
$\Rightarrow \mathrm{T}=0$
20. Let ' $p$ ' be the force applied to at an angle $\theta$

From the free body diagram
$R+P \sin \theta-m g=0$
$\Rightarrow R=-P \sin \theta+m g$
$\mu R-p \cos \theta$
Equn. (i) is $\mu(m g-P \sin \theta)-P \cos \theta=0$
$\Rightarrow \mu \mathrm{mg}=\mu \rho \sin \theta-\mathrm{P} \cos \theta \Rightarrow \rho=\frac{\mu \mathrm{mg}}{\mu \sin \theta+\cos \theta}$


Applied force P should be minimum, when $\mu \sin \theta+\cos \theta$ is maximum.
Again, $\mu \sin \theta+\cos \theta$ is maximum when its derivative is zero.
$\therefore \mathrm{d} / \mathrm{d} \theta(\mu \sin \theta+\cos \theta)=0$
$\Rightarrow \mu \cos \theta-\sin \theta=0 \Rightarrow \theta=\tan ^{-1} \mu$
So, $P=\frac{\mu \mathrm{mg}}{\mu \sin \theta+\cos \theta}=\frac{\mu \mathrm{mg} / \cos \theta}{\frac{\mu \sin \theta}{\cos \theta}+\frac{\cos \theta}{\cos \theta}}=\frac{\mu \mathrm{mg} \sec \theta}{1+\mu \tan \theta}=\frac{\mu \mathrm{mg} \mathrm{sec} \theta}{1+\tan ^{2} \theta}$

$$
=\frac{\mu \mathrm{mg}}{\sec \theta}=\frac{\mu \mathrm{mg}}{\sqrt{\left(1+\tan ^{2} \theta\right.}}=\frac{\mu \mathrm{mg}}{\sqrt{1+\mu^{2}}}
$$

Minimum force is $\frac{\mu \mathrm{mg}}{\sqrt{1+\mu^{2}}}$ at an angle $\theta=\tan ^{-1} \mu$.
21. Let, the max force exerted by the man is $T$.

From the free body diagram
$R+T-M g=0$
$\Rightarrow R=M g-T$
$R_{1}-R-m g=0$
$\Rightarrow R_{1}=R+m g$
And $T-\mu R_{1}=0$

$\Rightarrow \mathrm{T}-\mu(\mathrm{R}+\mathrm{mg})=0 \quad$ [From equn. (ii)]
$\Rightarrow \mathrm{T}-\mu \mathrm{R}-\mu \mathrm{mg}=0$
$\Rightarrow \mathrm{T}-\mu(\mathrm{Mg}+\mathrm{T})-\mu \mathrm{mg}=0$
[from (i)]
$\Rightarrow T(1+\mu)=\mu M g+\mu \mathrm{mg}$
$\Rightarrow T=\frac{\mu(M+m) g}{1+\mu}$
Maximum force exerted by manis $\frac{\mu(M+m) g}{1+\mu}$
22.

$R_{1}-2 g=0$
$\Rightarrow R_{1}=2 \times 10=20$
$2 \mathrm{a}+0.2 \mathrm{R}_{1}-12=0$
$4 a_{1}-\mu R_{1}=0$
$\Rightarrow 2 a+0.2(20)=12$
$\Rightarrow 4 a_{1}=\mu R_{1}=0.2(20)$
$\Rightarrow 2 \mathrm{a}=12-4=8$

$$
\Rightarrow 4 \mathrm{a}_{1}=4
$$

$\Rightarrow a=4 \mathrm{~m} / \mathrm{s}^{2}$

$$
\Rightarrow \mathrm{a}_{1}=1 \mathrm{~m} / \mathrm{s}^{2}
$$

2 kg block has acceleration $4 \mathrm{~m} / \mathrm{s}^{2}$ \& that of 4 kg is $1 \mathrm{~m} / \mathrm{s}^{2}$


(ii) $\mathrm{R}_{1}=2 g=20$
$\mathrm{Ma}-\mu \mathrm{R}_{1}=0$
$\Rightarrow 2 \mathrm{a}=0.2(20)=4$
$\Rightarrow a=2 \mathrm{~m} / \mathrm{s}^{2}$

$$
\begin{aligned}
& 4 \mathrm{a}+0.2 \times 2 \times 10-12=0 \\
& \Rightarrow 4 \mathrm{a}+4=12 \\
& \Rightarrow 4 \mathrm{a}=8 \\
& \Rightarrow \mathrm{a}=2 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

23. 


a) When the 10 N force applied on 2 kg block, it experiences maximum frictional force $\mu \mathrm{R}_{1}=\mu \times 2 \mathrm{~kg}=(0.2) \times 20=4 \mathrm{~N}$ from the 3 kg block.
So, the 2 kg block experiences a net force of $10-4=6 \mathrm{~N}$
So, $a_{1}=6 / 2=3 \mathrm{~m} / \mathrm{s}^{2}$
But for the 3 kg block, (fig-3) the frictional force from 2 kg block ( 4 N ) becomes the driving force and the maximum frictional force between 3 kg and 7 kg block is
$\mu_{2} R_{2}=(0.3) \times 5 \mathrm{~kg}=15 \mathrm{~N}$
So, the 3 kg block cannot move relative to the 7 kg block. The 3 kg block and 7 kg block both will have same acceleration $\left(a_{2}=a_{3}\right)$ which will be due to the 4 N force because there is no friction from the floor.
$\therefore \mathrm{a}_{2}=\mathrm{a}_{3}=4 / 10=0.4 \mathrm{~m} / \mathrm{s}^{2}$

b) When the 10 N force is applied to the 3 kg block, it can experience maximum frictional force of $15+4$ $=19 \mathrm{~N}$ from the 2 kg block \& 7 kg block.
So, it can not move with respect to them.
As the floor is frictionless, all the three bodies will move together
$\therefore \mathrm{a}_{1}=\mathrm{a}_{2}=\mathrm{a}_{3}=10 / 12=(5 / 6) \mathrm{m} / \mathrm{s}^{2}$
c) Similarly, it can be proved that when the 10 N force is applied to the 7 kg block, all the three blocks will move together.
Again $\mathrm{a}_{1}=\mathrm{a}_{2}=\mathrm{a}_{3}=(5 / 6) \mathrm{m} / \mathrm{s}^{2}$
24. Both upper block \& lower block will have acceleration $2 \mathrm{~m} / \mathrm{s}^{2}$

$R_{1}=m g$
$\mathrm{F}-\mu \mathrm{R}_{1}-\mathrm{T}=0 \Rightarrow \mathrm{~F}-\mu \mathrm{mg}-\mathrm{T}=0$

$$
\begin{equation*}
T-\mu R_{1}=0 \tag{i}
\end{equation*}
$$

$\Rightarrow \mathrm{T}=\mu \mathrm{mg}$
$\therefore \mathrm{F}=\mu \mathrm{mg}+\mu \mathrm{mg}=2 \mu \mathrm{mg}$
[putting $\mathrm{T}=\mu \mathrm{mg}$ ]

b) $2 \mathrm{~F}-\mathrm{T}-\mu \mathrm{mg}-\mathrm{ma}=0$

$$
\begin{align*}
& \mathrm{T}-\mathrm{Ma}-\mu \mathrm{mg}=0 \quad\left[\therefore \mathrm{R}_{1}=\mathrm{mg}\right]  \tag{i}\\
& \Rightarrow \mathrm{T}=\mathrm{Ma}+\mu \mathrm{mg}
\end{align*}
$$

Putting value of $T$ in (i)
$2 f-M a-\mu \mathrm{mg}-\mu \mathrm{mg}-\mathrm{ma}=0$
$\Rightarrow 2(2 \mu \mathrm{mg})-2 \mu \mathrm{mg}=\mathrm{a}(\mathrm{M}+\mathrm{m}) \quad$ [Putting $\mathrm{F}=2 \mu \mathrm{mg}$ ]
$\Rightarrow 4 \mu \mathrm{mg}-2 \mu \mathrm{mg}=\mathrm{a}(\mathrm{M}+\mathrm{m}) \quad \Rightarrow \mathrm{a}=\frac{2 \mu \mathrm{mg}}{\mathrm{M}+\mathrm{m}}$
Both blocks move with this acceleration 'a' in opposite direction.
25.

$R_{1}+m a-m g=0$
$\Rightarrow R_{1}=m(g-a)=m g-m a \ldots$ (i)
$T-\mu R_{1}=0 \Rightarrow T=m(m g-m a)$
Again, $\mathrm{F}-\mathrm{T}-\mu \mathrm{R}_{1}=0$
$\Rightarrow F-\{\mu(m g-m a)\}-u(m g-m a)=0$
$\Rightarrow F-\mu \mathrm{mg}+\mu \mathrm{ma}-\mu \mathrm{mg}+\mu \mathrm{ma}=0$
$\Rightarrow \mathrm{F}=2 \mu \mathrm{mg}-2 \mu \mathrm{ma} \Rightarrow \mathrm{F}=2 \mu \mathrm{~m}(\mathrm{~g}-\mathrm{a})$
b) Acceleration of the block be $\mathrm{a}_{1}$

$R_{1}=m g-m a$
$2 F-T-\mu R_{1}-m a_{1}=0$
$\Rightarrow 2 F-t-\mu m g+\mu a-m a_{1}=0$

$T-\mu R_{1}-M a_{1}=0$
$\Rightarrow T=\mu R_{1}+M a_{1}$
$\Rightarrow T=\mu(m g-m a)+M a_{1}$
$\Rightarrow \mathrm{T}=\mu \mathrm{mg}-\mu \mathrm{ma}+\mathrm{M} \mathrm{a}_{1}$

Subtracting values of $F \& T$, we get
$2(2 \mu \mathrm{~m}(\mathrm{~g}-\mathrm{a}))-2\left(\mu \mathrm{mg}-\mu \mathrm{ma}+M \mathrm{a}_{1}\right)-\mu \mathrm{mg}+\mu \mathrm{ma}-\mu \mathrm{a}_{1}=0$
$\Rightarrow 4 \mu \mathrm{mg}-4 \mu \mathrm{ma}-2 \mu \mathrm{mg}+2 \mu \mathrm{ma}=\mathrm{ma}_{1}+\mathrm{M} \mathrm{a}_{1}$

$$
\Rightarrow a_{1}=\frac{2 \mu m(g-a)}{M+m}
$$

Both blocks move with this acceleration but in opposite directions.
26. $R_{1}+Q E-m g=0$
$R_{1}=m g-Q E$
$\mathrm{F}-\mathrm{T}-\mu \mathrm{R}_{1}=0$
$\Rightarrow \mathrm{F}-\mathrm{T} \mu(\mathrm{mg}-\mathrm{QE})=0$
$\Rightarrow F-T-\mu \mathrm{mg}+\mu \mathrm{QE}=0$.

$\mathrm{T}-\mu \mathrm{R}_{1}=0$
$\Rightarrow T=\mu R_{1}=\mu(m g-Q E)=\mu \mathrm{mg}-\mu \mathrm{QE}$
Now equation (ii) is $F-m g+\mu Q E-\mu \mathrm{mg}+\mu \mathrm{QE}=0$
$\Rightarrow F-2 \mu \mathrm{mg}+2 \mu \mathrm{QE}=0$
$\Rightarrow F=2 \mu \mathrm{mg}-2 \mu \mathrm{QE}$
$\Rightarrow F=2 \mu(m g-Q E)$
Maximum horizontal force that can be applied is $2 \mu(\mathrm{mg}-\mathrm{QE})$.
27. Because the block slips on the table, maximum frictional force acts on it.

From the free body diagram
$R=m g$

$\therefore F-\mu R=0 \Rightarrow F=\mu R=\mu \mathrm{mg}$
But the table is at rest. So, frictional force at the legs of the table is not $\mu R_{1}$. Let be $f$, so form the free body diagram.
$f_{0}-\mu \mathrm{R}=0 \Rightarrow f_{0}=\mu \mathrm{R}=\mu \mathrm{mg}$.
Total frictional force on table by floor is $\mu \mathrm{mg}$.


Let the acceleration of block $M$ is ' $a$ ' towards right. So, the block ' $m$ ' must go down with an acceleration ' $2 a$ '.


As the block ' $m$ ' is in contact with the block ' $M$ ', it will also have acceleration ' $a$ ' towards right. So, it will experience two inertia forces as shown in the free body diagram-1.
From free body diagram -1
$R_{1}-m a=0 \Rightarrow R_{1}=m a$
Again, $2 m a+T-m g+\mu_{1} R_{1}=0$
$\Rightarrow \mathrm{T}=\mathrm{mg}-\left(2-\mu_{1}\right) \mathrm{ma}$
From free body diagram-2
$T+\mu_{1} R_{1}+m g-R_{2}=0$
$\Rightarrow \mathrm{R}_{2}=\mathrm{T}+\mu_{1} \mathrm{ma}+\mathrm{Mg}$
[Putting the value of $\mathrm{R}_{1}$ from (i)]
$=\left(\mathrm{mg}-2 \mathrm{ma}-\mu_{1} \mathrm{ma}\right)+\mu_{1} \mathrm{ma}+\mathrm{Mg} \quad$ [Putting the value of T from (ii)]
$\therefore \mathrm{R}_{2}=\mathrm{Mg}+\mathrm{mg}-2 \mathrm{ma}$
Again, form the free body diagram -2
$\mathrm{T}+\mathrm{T}-\mathrm{R}-\mathrm{Ma}-\mu_{2} \mathrm{R}_{2}=0$
$\Rightarrow 2 T-M A-m A-\mu_{2}(M g+m g-2 m a)=0 \quad$ [Putting the values of $R_{1}$ and $R_{2}$ from (i) and (iii)]
$\Rightarrow 2 \mathrm{~T}=(\mathrm{M}+\mathrm{m})+\mu_{2}(\mathrm{Mg}+\mathrm{mg}-2 \mathrm{ma})$
From equation (ii) and (iv)
$2 \mathrm{~T}=2 \mathrm{mg}-2\left(2+\mu_{1}\right) \mathrm{mg}=(\mathrm{M}+\mathrm{m}) \mathrm{a}+\mu_{2}(\mathrm{Mg}+\mathrm{mg}-2 \mathrm{ma})$
$\Rightarrow 2 m g-\mu_{2}(M+m) g=a\left(M+m-2 \mu_{2} m+4 m+2 \mu_{1} m\right)$
$\Rightarrow a=\frac{\left[2 m-\mu_{2}(M+m)\right] g}{M+m\left[5+2\left(\mu_{1}-\mu_{2}\right)\right]}$
29. Net force $={ }^{*}(202+(15) 2-(0.5) \times 40=25-20=5 N$
$\therefore \tan \theta=20 / 15=4 / 3 \Rightarrow \mu=\tan ^{-1}(4 / 3)=53^{\circ}$
So, the block will move at an angle $53^{\circ}$ with an 15 N force
30. a) Mass of man $=50 \mathrm{~kg} . \mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$

Frictional force developed between hands, legs \& back side with the wall the wt of man. So he remains in equilibrium.
He gives equal force on both the walls so gets equal reaction $R$ from both the walls. If he applies unequal forces $R$ should be different he can't rest between the walls. Frictional force $2 \mu \mathrm{R}$ balance his wt.


From the free body diagram
$\mu R+\mu R=40 \mathrm{~g} \Rightarrow 2 \mu R=40 \times 10 \Rightarrow R=\frac{40 \times 10}{2 \times 0.8}=250 \mathrm{~N}$
b) The normal force is 250 N .
31. Let $a_{1}$ and $a_{2}$ be the accelerations of ma and $M$ respectively.

Here, $a_{1}>a_{2}$ so that moves on $M$
Suppose, after time ' $t$ ' $m$ separate from $M$.
In this time, $m$ covers $v t+1 / 2 a_{1} t^{2}$ and $S_{M}=v t+1 / 2 a_{2} t^{2}$


For ' $m$ ' to $m$ to ' $m$ ' separate from $M$. $v t+1 / 2 a_{1} t^{2}=v t+1 / 2 a_{2} t^{2}+\ell$
Again from free body diagram
$M a_{1}+\mu / 2 R=0$
$\Rightarrow \mathrm{ma}_{1}=-(\mu / 2) \mathrm{mg}=-(\mu / 2) \mathrm{m} \times 10 \Rightarrow \mathrm{a}_{1}=-5 \mu$
Again,
$\mathrm{Ma}_{2}+\mu(\mathrm{M}+\mathrm{m}) \mathrm{g}-(\mu / 2) \mathrm{mg}=0$
$\Rightarrow 2 \mathrm{Ma}_{2}+2 \mu(\mathrm{M}+\mathrm{m}) \mathrm{g}-\mu \mathrm{mg}=0$

$\Rightarrow 2 \mathrm{M} \mathrm{a}_{2}=\mu \mathrm{mg}-2 \mu \mathrm{Mg}-2 \mu \mathrm{mg}$
$\Rightarrow \mathrm{a}_{2} \frac{-\mu \mathrm{mg}-2 \mu \mathrm{Mg}}{2 \mathrm{M}}$
Putting values of $a_{1} \& a_{2}$ in equation (1) we can find that
$T=\sqrt{\left(\frac{4 m l}{(M+m) \mu g}\right)}$

