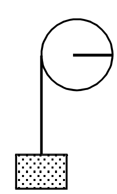
SOLUTIONS TO CONCEPTS
CHAPTER – 10

1. $\omega_0 = 0$; $\rho = 100$ rev/s ; $\omega = 2\pi$; $\rho = 200\ \pi$ rad/s
   $\Rightarrow \omega = \omega_0 = \alpha t$
   $\Rightarrow \omega = \alpha t$
   $\Rightarrow \alpha = (200\ \pi)/4 = 50\ \pi$ rad /s^2 or 25 rev/s^2
   $\therefore \theta = \omega_0 t + 1/2\ \alpha t^2 = 8 \times 50\ \pi = 400\ \pi$ rad
   $\therefore \alpha = 50\ \pi$ rad/s^2 or 25 rev/s^s
   $\theta = 400\ \pi$ rad.
2. $\theta = 100\ \pi$; t = 5 sec
   $\theta = 1/2\ \alpha t^2 \Rightarrow 100\pi = 1/2\ \alpha\ 25$
   $\Rightarrow \alpha = 8\pi \times 5 = 40\ \pi$ rad/s = 20 rev/s
   $\therefore \alpha = 8\pi$ rad/s^2 = 4 rev/s^2
   $\omega = 40\pi$ rad/s^2 = 20 rev/s^2.
3. Area under the curve will decide the total angle rotated
   $\therefore$ maximum angular velocity = 4 × 10 = 40 rad/s
   Therefore, area under the curve = 1/2 × 10 × 40 + 40 × 10 + 1/2 × 40 × 10
   = 800 rad
   $\therefore$ Total angle rotated = 800 rad.

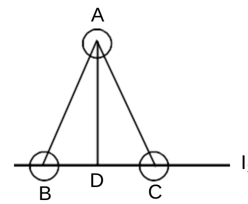
10ω

10 T 20

4. $\alpha = 1$ rad/s^2, $\omega_0 = 5$ rad/s; $\omega = 15$ rad/s
   $\therefore w = w_0 + \alpha t$
   $\Rightarrow t = (\omega - \omega_0)/\alpha = (15 - 5)/1 = 10$ sec
   Also, $\theta = \omega_0 t + 1/2\ \alpha t^2$
   = 5 ×10 + 1/2 × 1 × 100 = 100 rad.
5. $\theta = 5$ rev, $\alpha = 2$ rev/s^2, $\omega_0 = 0$; $\omega = ?$
   $\omega^2 = (2\ \alpha\ \theta)$
   $\Rightarrow \omega = \sqrt{2 \times 2 \times 5} = 2\sqrt{5}$ rev/s.
   or $\theta = 10\pi$ rad, $\alpha = 4\pi$ rad/s^2, $\omega_0 = 0$, $\omega = ?$
   $\omega = \sqrt{2\alpha\theta} = \sqrt{2 \times 4\pi \times 10\pi}$
   $= 4\pi\sqrt{5}$ rad/s = $2\sqrt{5}$ rev/s.
6. A disc of radius = 10 cm = 0.1 m
   Angular velocity = 20 rad/s
   $\therefore$ Linear velocity on the rim = ωr = 20 × 0.1 = 2 m/s
   $\therefore$ Linear velocity at the middle of radius = $\omega r/2$ = 20 × (0.1)/2 = 1 m/s.
7. t = 1 sec, r = 1 cm = 0.01 m
   $\alpha = 4$ rd/s^2
   Therefore $\omega = \alpha t = 4$ rad/s
   Therefore radial acceleration,
   $A_n = \omega^2 r = 0.16$ m/s^2 = 16 cm/s^2
   Therefore tangential acceleration, $a_r = \alpha r = 0.04$ m/s^2 = 4 cm/s^2.
8. The Block is moving the rim of the pulley
   The pulley is moving at a $\omega = 10$ rad/s
   Therefore the radius of the pulley = 20 cm
   Therefore linear velocity on the rim = tangential velocity = $r\omega$
   = 20 × 20 = 200 cm/s = 2 m/s.



9. Therefore, the $\perp$ distance from the axis (AD) = $\sqrt{3}/2 \times 10 = 5\sqrt{3}$ cm.



Therefore moment of inertia about the axis BC will be

$I = mr^2 = 200\ K\ (5\sqrt{3})^2 = 200 \times 25 \times 3$

$= 15000$ gm – cm^2 = 1.5×10^{-3} kg – m^2.

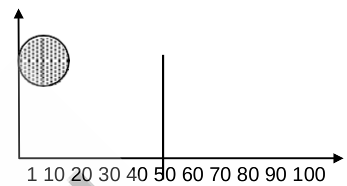
b) The axis of rotation let pass through A and $\perp$ to the plane of triangle

Therefore the torque will be produced by mass B and C

Therefore net moment of inertia = $I = mr^2 + mr^2$

$= 2 \times 200 \times 10^2 = 40000$ gm-cm^2 = 4×10^{-3} kg-m^2.

10. Masses of 1 gm, 2 gm ……100 gm are kept at the marks 1 cm, 2 cm, ……1000 cm on he x axis respectively. A perpendicular axis is passed at the 50th particle.



Therefore on the L.H.S. side of the axis there will be 49 particles and on the R.H.S. side there are 50 particles.

Consider the two particles at the position 49 cm and 51 cm.

Moment inertial due to these two particle will be =

$$49 \times 1^2 + 51 + 1^2 = 100 \text{ gm-cm}^2$$

Similarly if we consider 48th and 52nd term we will get 100×2^2 gm-cm^2

Therefore we will get 49 such set and one lone particle at 100 cm.

Therefore total moment of inertia =

$100\{1^2 + 2^2 + 3^2 + \ldots + 49^2\} + 100(50)^2$.

$= 100 \times (50 \times 51 \times 101)/6 = 4292500$ gm-cm^2

$= 0.429$ kg-m^2 = 0.43 kg-m^2.

11. The two bodies of mass m and radius r are moving along the common tangent.

Therefore moment of inertia of the first body about XY tangent.

$= mr^2 + 2/5\ mr^2$

– Moment of inertia of the second body XY tangent = $mr^2 + 2/5\ mr^2 = 7/5\ mr^2$

Therefore, net moment of inertia = $7/5\ mr^2 + 7/5\ mr^2 = 14/5\ mr^2$ units.

x

r r

1 2

y

12. Length of the rod = 1 m, mass of the rod = 0.5 kg

Let at a distance d from the center the rod is moving

Applying parallel axis theorem :

The moment of inertial about that point

$\Rightarrow (mL^2 / 12) + md^2 = 0.10$

$\Rightarrow (0.5 \times 1^2)/12 + 0.5 \times d^2 = 0.10$

$\Rightarrow d^2 = 0.2 - 0.082 = 0.118$

$\Rightarrow d = 0.342$ m from the centre.

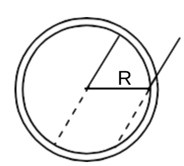
d

1m

$ml^2/12$ $(ml^2/12)+md^2$

13. Moment of inertia at the centre and perpendicular to the plane of the ring.

So, about a point on the rim of the ring and the axis $\perp$ to the plane of the ring, the moment of inertia

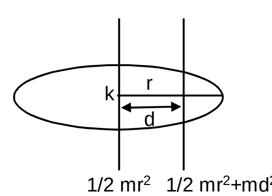


$= mR^2 + mR^2 = 2mR^2$ (parallel axis theorem)

$\Rightarrow mK^2 = 2mR^2$ (K = radius of the gyration)

$\Rightarrow K = \sqrt{2R^2} = \sqrt{2}\ R$.

14. The moment of inertia about the center and $\perp$ to the plane of the disc of radius r and mass m is = mr^2.



According to the question the radius of gyration of the disc about a point = radius of the disc.

Therefore $mk^2 = ½\ mr^2 + md^2$

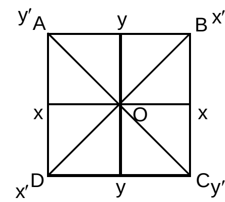
(K = radius of gyration about acceleration point, d = distance of that point from the centre)

$\Rightarrow K^2 = r^2/2 + d^2$

$\Rightarrow r^2 = r^2/2 + d^2$ ($\because$ K = r)

$\Rightarrow r^2/2 = d^2 \Rightarrow d = r/\sqrt{2}$.

15. Let a small cross sectional area is at a distance x from xx axis.
Therefore mass of that small section = $m/a^2 \times ax\,dx$
Therefore moment of inertia about xx axis



$$= I_{xx} = 2\int_0^{a/2} (m/a^2)\times(adx)\times x^2 = (2\times(m/a)(x^3/3)]_0^{a/2}$$

$= ma^2 / 12$
Therefore $I_{xx} = I_{xx} + I_{yy}$
$= 2 \times$ *$ma^2/12) = ma^2/6$
Since the two diagonals are $\perp$ to each other
Therefore $I_{zz} = I_{x'x'} + I_{y'y'}$
$\Rightarrow ma^2/6 = 2 \times I_{x'x'}$ (because $I_{x'x'} = I_{y'y'}$) $\Rightarrow I_{x'x'} = ma^2/12$

16. The surface density of a circular disc of radius a depends upon the distance from the centre as $P(r) = A + Br$
Therefore the mass of the ring of radius r will be
$\theta = (A + Br) \times 2\pi r\,dr \times r^2$
Therefore moment of inertia about the centre will be

r dx

$$= \int_0^a (A + Br)2\pi r \times dr = \int_0^a 2\pi Ar^3 dr + \int_0^a 2\pi Br^4 dr$$

$= 2\pi A\,(r^4/4) + 2\pi\,B(r^5/5)\,]_0^a = 2\pi a^4\,[(A/4) + (Ba/5)]$.

17. At the highest point total force acting on the particle id its weight acting downward.
Range of the particle = $u^2 \sin 2\pi / g$
Therefore force is at a $\perp$ distance, $\Rightarrow$ (total range)/2 = $(v^2 \sin 2\theta)/2g$
(From the initial point)
Therefore $\tau = F \times r$ (θ = angle of projection)
$= mg \times v^2 \sin 2\theta/2g$ (v = initial velocity)
$= mv^2 \sin 2\theta / 2 = mv^2 \sin\theta \cos\theta$.

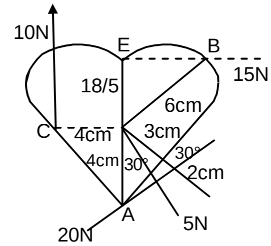
θ
$(v^2 \sin 2\theta)/2\theta$

18. A simple of pendulum of length l is suspended from a rigid support. A bob of weight W is hanging on the other point.
When the bob is at an angle θ with the vertical, then total torque acting on the point of suspension = i = F × r
$\Rightarrow W\,r \sin\theta = W\,l \sin\theta$
At the lowest point of suspension the torque will be zero as the force acting on the body passes through the point of suspension.

l θ
B A

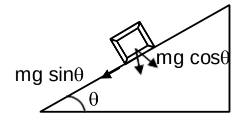
19. A force of 6 N acting at an angle of 30° is just able to loosen the wrench at a distance 8 cm from it.
Therefore total torque acting at A about the point 0
$= 6 \sin 30° \times (8/100)$
Therefore total torque required at B about the point 0
$= F \times 16/100 \Rightarrow F \times 16/100 = 6 \sin 30° \times 8/100$
$\Rightarrow F = (8 \times 3) / 16 = 1.5$ N.

20. Torque about a point = Total force × perpendicular distance from the point to that force.
Let anticlockwise torque = + ve
And clockwise acting torque = –ve
Force acting at the point B is 15 N
Therefore torque at O due to this force
$= 15 \times 6 \times 10^{-2} \times \sin 37°$
$= 15 \times 6 \times 10^{-2} \times 3/5 = 0.54$ N-m (anticlock wise)

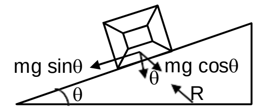


Force acting at the point C is 10 N
Therefore, torque at O due to this force
$= 10 \times 4 \times 10^{-2} = 0.4$ N-m (clockwise)
Force acting at the point A is 20 N
Therefore, Torque at O due to this force = $20 \times 4 \times 10^{-2} \times \sin 30°$
$= 20 \times 4 \times 10^{-2} \times 1/2 = 0.4$ N-m (anticlockwise)
Therefore resultant torque acting at ‘O’ = 0.54 – 0.4 + 0.4 = 0.54 N-m.

21. The force mg acting on the body has two components mg sin θ and mg cos θ and the body will exert a normal reaction. Let R =



Since R and mg cos θ pass through the centre of the cube, there will be no torque due to R and mg cos θ. The only torque will be produced by mg sin θ.



$\therefore$ i = F × r (r = a/2) (a = ages of the cube)

$\Rightarrow$ i = mg sin θ × a/2

= 1/2 mg a sin θ.

22. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis passing through its centre.

A force F is acting perpendicular to the rod at a distance L/4 from the centre.

Therefore torque about the centre due to this force

A′ t =sec

$i_i = F \times r = FL/4$.

This torque will produce a angular acceleration α.

Therefore $\tau_c = I_c \times \alpha$

A B

$\Rightarrow i_c = (mL^2 / 12) \times \alpha$ (I_c of a rod = $mL^2 / 12$)

1/4

F

$\Rightarrow$ F i/4 = $(mL^2 / 12) \times \alpha \Rightarrow \alpha = 3F/ml$

B′

Therefore $\theta = 1/2\ \alpha t^2$ (initially at rest)

$\Rightarrow \theta = 1/2 \times (3F / ml)t^2 = (3F/2ml)t^2$.

23. A square plate of mass 120 gm and edge 5 cm rotates about one of the edge.

Let take a small area of the square of width dx and length a which is at a distance x from the axis of rotation.

Therefore mass of that small area

$m/a^2 \times a\,dx$ (m = mass of the square ; a = side of the plate)

$$I = \int_0^a (m / a^2) \times ax^2 dx = (m / a)(x^3 / 3)]_0^a$$

x

dx

A

$= ma^2/3$

Therefore torque produced = I × α = $(ma^2/3) \times \alpha$

$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/3\}\ 0.2$

$= 0.2 \times 10^{-4} = 2 \times 10^{-5}$ N-m.

24. Moment of inertial of a square plate about its diagonal is $ma^2/12$ (m = mass of the square plate)

a = edges of the square

Therefore torque produced = $(ma^2/12) \times \alpha$

x′□x

$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/12 \times 0.2$

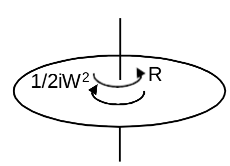
$= 0.5 \times 10^{-5}$ N-m.

25. A flywheel of moment of inertia 5 kg m is rotated at a speed of 60 rad/s. The flywheel comes to rest due to the friction at the axle after 5 minutes.

Therefore, the angular deceleration produced due to frictional force = $\omega = \omega_0 + \alpha t$

$\Rightarrow \omega_0 = -\alpha t$ (ω = 0+

$\Rightarrow \alpha = -(60/5 \times 60) = -1/5$ rad/s^2.



a) Therefore total workdone in stopping the wheel by frictional force

W = 1/2 iω^2 = 1/2 × 5 × (60 × 60) = 9000 Joule = 9 KJ.

b) Therefore torque produced by the frictional force (R) is

$I_R = I \times \alpha = 5 \times (-1/5)$ = IN – m opposite to the rotation of wheel.

c) Angular velocity after 4 minutes

$\Rightarrow \omega = \omega_0 + \alpha t = 60 - 240/5 = 12$ rad/s

Therefore angular momentum about the centre = 1 × ω = 5 × 12 = 60 kg-m^2/s.

26. The earth's angular speed decreases by 0.0016 rad/day in 100 years.
Therefore the torque produced by the ocean water in decreasing earth's angular velocity
$\tau = I\alpha$
$= 2/5\ mr^2 \times (\omega - \omega_0)/t$
$= 2/6 \times 6 \times 10^{24} \times 64^2 \times 10^{10} \times [0.0016 /(26400^2 \times 100 \times 365)]$ (1 year = 365 days= 365 × 56400 sec)
$= 5.678 \times 10^{20}$ N-m.

27. A wheel rotating at a speed of 600 rpm.
$\omega_0 = 600$ rpm = 10 revolutions per second.
T = 10 sec. (In 10 sec. it comes to rest)
$\omega = 0$
Therefore $\omega_0 = -\alpha t$
$\Rightarrow \alpha = -10/10 = -1\ rev/s^2$
$\Rightarrow \omega = \omega_0 + \alpha t = 10 - 1 \times 5 = 5$ rev/s.
Therefore angular deacceleration = 1 rev/s^2 and angular velocity of after 5 sec is 5 rev/s.

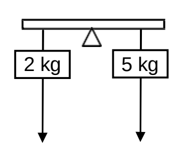
28. $\omega = 100$ rev/min = 5/8 rev/s = $10\pi/3$ rad/s
$\theta = 10$ rev = $20\ \pi$ rad, r = 0.2 m
After 10 revolutions the wheel will come to rest by a tangential force
Therefore the angular deacceleration produced by the force = $\alpha = \omega^2/2\theta$
Therefore the torque by which the wheel will come to an rest = $I_{cm} \times \alpha$
$\Rightarrow F \times r = I_{cm} \times \alpha \rightarrow F \times 0.2 = 1/2\ mr^2 \times [(10\pi/3)^2 / (2 \times 20\pi)]$
$\Rightarrow F = 1/2 \times 10 \times 0.2 \times 100\ \pi^2 / (9 \times 2 \times 20\pi)$
$= 5\pi / 18 = 15.7/18 = 0.87$ N.

29. A cylinder is moving with an angular velocity 50 rev/s brought in contact with another identical cylinder in rest. The first and second cylinder has common acceleration and deacceleration as 1 rad/s^2 respectively.
Let after t sec their angular velocity will be same 'ω'.
For the first cylinder $\omega = 50 - \alpha t$
$\Rightarrow t = (\omega - 50)/-1$
And for the 2nd cylinder $\omega = \alpha_2 t$
$\Rightarrow t = \omega/I$
So, $\omega = (\omega - 50)/-1$
$\Rightarrow 2\omega = 50 \Rightarrow \omega = 25$ rev/s.
$\Rightarrow t = 25/1$ sec = 25 sec.

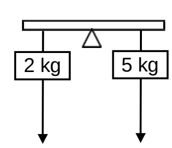
50 rev/s

30. Initial angular velocity = 20 rad/s
Therefore $\alpha = 2\ rad/s^2$
$\Rightarrow t_1 = \omega/\alpha_1 = 20/2 = 10$ sec
Therefore 10 sec it will come to rest.
Since the same torque is continues to act on the body it will produce same angular acceleration and since the initial kinetic energy = the kinetic energy at a instant.
So initial angular velocity = angular velocity at that instant
Therefore time require to come to that angular velocity,
$t_2 = \omega_2/\alpha_2 = 20/2 = 10$ sec
therefore time required = $t_1 + t_2 = 20$ sec.

31. $I_{net} = I_{net} \times \alpha$
$\Rightarrow F_1r_1 - F_2r_2 = (m_1 r_1^2 + m_2 r_2^2) \times \alpha - 2 \times 10 \times 0.5$
$\Rightarrow 5 \times 10 \times 0.5 = (5 \times (1/2)^2 + 2 \times (1/2)^2) \times \alpha$
$\Rightarrow 15 = 7/4\ \alpha$
$\Rightarrow \alpha = 60/7 = 8.57\ rad/s^2$.



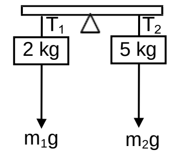
32. In this problem the rod has a mass 1 kg
a) $\tau_{net} = I_{net} \times \alpha$
$\Rightarrow 5 \times 10 \times 10.5 - 2 \times 10 \times 0.5$
$= (5 \times (1/2)^2 + 2 \times (1/2)^2 + 1/12) \times \alpha$



$\Rightarrow 15 = (1.75 + 0.084)\ \alpha$

$\Rightarrow \alpha = 1500/(175 + 8.4) = 1500/183.4 = 8.1\ rad/s^2$ (g = 10)

$= 8.01\ rad/s^2$ (if g = 9.8)



b) $T_1 - m_1 g = m_1 a$

$\Rightarrow T_1 = m_1 a + m_1 g = 2(a + g)$

$= 2(\alpha r + g) = 2(8 \times 0.5 + 9.8)$

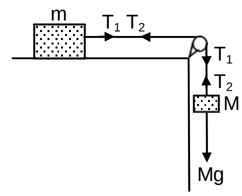
= 27.6 N on the first body.

In the second body

$\Rightarrow m_2 g - T_2 = m_2 a \Rightarrow T_2 = m_2 g - m_2 a$

$\Rightarrow T_2 = 5(g - a) = 5(9.8 - 8 \times 0.5) = 29$ N.

33. According to the question



$Mg - T_1 = Ma$ …(1)

$T_2 = ma$ …(2)

$(T_1 - T_2) = 1\ a/r^2$ …(3) [because $a = r\alpha$]…**[T.r =I(a/r)]**

If we add the equation 1 and 2 we will get

$Mg + (T_2 - T_1) = Ma + ma$ …(4)

$\Rightarrow Mg - Ia/r^2 = Ma + ma$

$\Rightarrow (M + m + I/r^2)a = Mg$

$\Rightarrow a = Mg/(M + m + I/r^2)$

34. $I = 0.20\ kg\text{-}m^2$ (Bigger pulley)

r = 10 cm = 0.1 m, smaller pulley is light

mass of the block, m = 2 kg

therefore $mg - T = ma$ …(1)

$\Rightarrow T = Ia/r^2$ …(2)

$\Rightarrow mg = (m + I/r^2)a \Rightarrow (2 \times 9.8) / [2 + (0.2/0.01)] = a$

$= 19.6 / 22 = 0.89\ m/s^2$

Therefore, acceleration of the block = $0.89\ m/s^2$.

T 10cm
T
2kg
mg

35. m = 2 kg, $i_1 = 0.10\ kg\text{-}m^2$, $r_1 = 5$ cm = 0.05 m

$i_2 = 0.20\ kg\text{-}m^2$, $r_2 = 10$ cm = 0.1 m

Therefore $mg - T_1 = ma$ …(1)

$(T_1 - T_2)r_1 = I_1\alpha$ …(2)

$T_2 r_2 = I_2\alpha$ …(3)

Substituting the value of T_2 in the equation (2), we get

$\Rightarrow (t_1 - I_2\ \alpha/r_1)r_2 = I_1\alpha$

$\Rightarrow (T_1 - I_2\ a/r_1^2) = I_1 a/r_2^2$

$\Rightarrow T_1 = [(I_1/r_1^2) + I_2/r_2^2)]a$

Substituting the value of T_1 in the equation (1), we get

$\Rightarrow mg - [(I_1/r_1^2) + I_2/r_2^2)]a = ma$

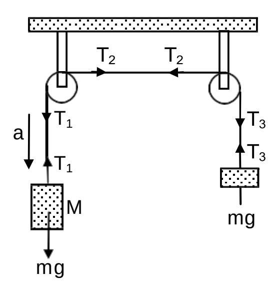
$$\Rightarrow \frac{mg}{[(I_1/r_1^2) + (I_2/r_2^2)] + m} = a$$

$$\Rightarrow a = \frac{2 \times 9.8}{(0.1/0.0025) + (0.2/0.01) + 2} = 0.316\ m/s^2$$

$$\Rightarrow T_2 = I_2 a/r_2^2 = \frac{0.20 \times 0.316}{0.01} = 6.32\ N.$$

T2 T T2 I2
T1 r2
T1
2kg
mg

36. According to the question



$Mg - T_1 = Ma$ …(1)

$(T_2 - T_1)R = Ia/R_2 \Rightarrow (T_2 - T_1) = Ia/R^2$ …(2)

$(T_2 - T_3)R = Ia/R$ …(3)

$\Rightarrow T_3 - mg = ma$ …(4)

By adding equation (2) and (3) we will get,

$\Rightarrow (T_1 - T_3) = 2\ Ia/R^2$ …(5)

By adding equation (1) and (4) we will get

$-mg + Mg + (T_3 - T_1) = Ma + ma$ ...(6)

Substituting the value for $T_3 - T_1$ we will get

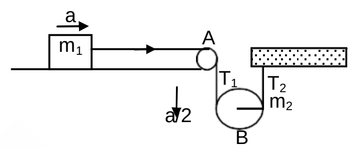
$\Rightarrow Mg - mg = Ma + ma + 2Ia/R^2$

$\Rightarrow a = \dfrac{(M-m)G}{(M+m+2I/R^2)}$

37. A is light pulley and B is the descending pulley having I = 0.20 kg – m^2 and r = 0.2 m

Mass of the block = 1 kg

According to the equation



$T_1 = m_1a$ ...(1)

$(T_2 - T_1)r = I\alpha$ ...(2)

$m_2g - m_2\ a/2 = T_1 + T_2$ ...(3)

$T_2 - T_1 = Ia/2R^2 = 5a/2$ and $T_1 = a$ (because $\alpha = a/2R$)

$\Rightarrow T_2 = 7/2\ a$

$\Rightarrow m_2g = m_2a/2 + 7/2\ a + a$

$\Rightarrow 2I / r^2g = 2I/r^2\ a/2 + 9/2\ a$ $(1/2\ mr^2 = I)$

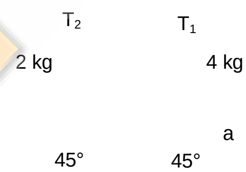
$\Rightarrow 98 = 5a + 4.5\ a$

$\Rightarrow a = 98/9.5 = 10.3\ ms^2$

38. $m_1g \sin\theta - T_1 = m_1a$ ...(1)

$(T_1 - T_2) = Ia/r^2$ ...(2)

$T_2 - m_2g \sin\theta = m_2a$ ...(3)



Adding the equations (1) and (3) we will get

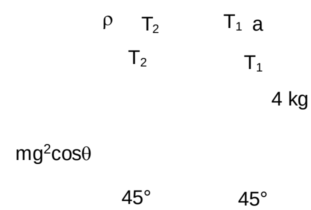
$m_1g \sin\theta + (T_2 - T_1) - m_2g \sin\theta = (m_1 + m_2)a$

$\Rightarrow (m_1 - m_2)g \sin\theta = (m_1 + m_2 + 1/r^2)a$

$\Rightarrow a = \dfrac{(m_1 - m_2)g\sin\theta}{(m_1 + m_2 + 1/r^2)} = 0.248 = 0.25\ ms^{-2}$.

39. $m_1 = 4$ kg, $m_2 = 2$ kg

Frictional co-efficient between 2 kg block and surface = 0.5



R = 10 cm = 0.1 m

I = 0.5 kg – m^2

$m_1g \sin\theta - T_1 = m_1a$ ...(1)

$T_2 - (m_2g \sin\theta + \mu m_2g \cos\theta) = m_2a$ ...(2)

$(T_1 - T_2) = Ia/r^2$

Adding equation (1) and (2) we will get

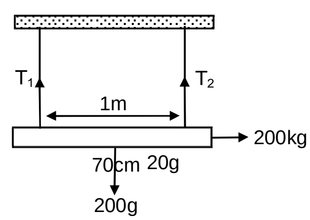
$m_1g \sin\theta - (m_2g \sin\theta + \mu m_2g \cos\theta) + (T_2 - T_1) = m_1a + m_2a$

$\Rightarrow 4 \times 9.8 \times (1/\sqrt{2}) - \{(2 \times 9.8 \times (1/\sqrt{2}) + 0.5 \times 2 \times 9.8 \times (1/\sqrt{2})\} = (4 + 2 + 0.5/0.01)a$

$\Rightarrow 27.80 - (13.90 + 6.95) = 65\ a \Rightarrow a = 0.125\ ms^{-2}$.

40. According to the question

$m_1 = 200$ g, I = 1 m, $m_2 = 20$ g



Therefore, $(T_1 \times r_1) - (T_2 \times r_2) - (m_1f \times r_3g) = 0$

$\Rightarrow T_1 \times 0.7 - T_2 \times 0.3 - 2 \times 0.2 \times g = 0$

$\Rightarrow 7T_1 - 3T_2 = 3.92$ ...(1)

$T_1 + T_2 = 0.2 \times 9.8 + 0.02 \times 9.8 = 2.156$ ...(2)

From the equation (1) and (2) we will get

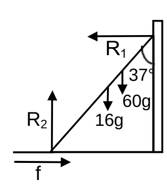
$10\ T_1 = 10.3$

$\Rightarrow T_1 = 1.038$ N = 1.04 N

Therefore $T_2 = 2.156 - 1.038 = 1.118 = 1.12$ N.

41. $R_1 = \mu R_2$, $R_2 = 16g + 60\ g = 745$ N

$R_1 \times 10 \cos 37° = 16g \times 5 \sin 37° + 60\ g \times 8 \times \sin 37°$

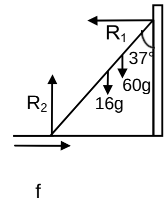


$\Rightarrow 8R_1 = 48g + 288\ g$

$\Rightarrow R_1 = 336g/8 = 412\ N = f$

Therefore $\mu = R_1 / R_2 = 412/745 = 0.553$.

42. $\mu = 0.54$, $R_2 = 16g + mg$; $R_1 = \mu R_2$



$\Rightarrow R_1 \times 10 \cos 37° = 16g \times 5 \sin 37° + mg \times 8 \times \sin 37°$

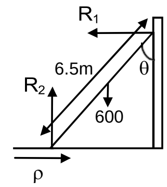
$\Rightarrow 8R_1 = 48g + 24/5\ mg$

$\Rightarrow R_2 = \dfrac{48g + 24/5\ mg}{8 \times 0.54}$

$\Rightarrow 16g + mg = \dfrac{24.0g + 24mg}{5 \times 8 \times 0.54} \Rightarrow 16 + m = \dfrac{240 + 24m}{40 \times 0.54}$

$\Rightarrow m = 44$ kg.

43. m = 60 kg, ladder length = 6.5 m, height of the wall = 6 m



Therefore torque due to the weight of the body

a) $\tau = 600 \times 6.5 / 2 \sin \theta = i$

$\Rightarrow \tau = 600 \times 6.5 / 2 \times \sqrt{[1-(6/6.5)^2]}$

$\Rightarrow \tau = 735$ N-m.

b) $R_2 = mg = 60 \times 9.8$

$R_1 = \mu R_2 \Rightarrow 6.5\ R_1 \cos \theta = 60g \sin \theta \times 6.5/2$

$\Rightarrow R_1 = 60\ g \tan \theta = 60\ g \times (2.5/12)$ [because $\tan \theta = 2.5/6$]

$\Rightarrow R_1 = (25/2)\ g = 122.5$ N.

44. According to the question

$8g = F_1 + F_2$; $N_1 = N_2$

F1 R1 A N1 R2 B 8g N2

Since, $R_1 = R_2$

Therefore $F_1 = F_2$

$\Rightarrow 2F_1 = 8\ g \Rightarrow F_1 = 40$

Let us take torque about the point B, we will get $N_1 \times 4 = 8\ g \times 0.75$.

$\Rightarrow N_1 = (80 \times 3) / (4 \times 4) = 15$ N

Therefore $\sqrt{F_1^2 + N_1^2} = R_1 = \sqrt{40^2 + 15^2} = 42.72 = 43$ N.

45. Rod has a length = L

It makes an angle θ with the floor

The vertical wall has a height = h

R1 R1cosθ θ R1sinθ R2 h mg θ ρ

$R_2 = mg - R_1 \cos \theta$ ...(1)

$R_1 \sin \theta = \mu R_2$ ...(2)

$R_1 \cos \theta \times (h/\tan \theta) + R_1 \sin \theta \times h = mg \times 1/2 \cos \theta$

$\Rightarrow R_1 (\cos^2 \theta / \sin \theta)h + R_1 \sin \theta\ h = mg \times 1/2 \cos \theta$

$\Rightarrow R_1 = \dfrac{mg \times L/2 \cos \theta}{\{(\cos^2 \theta / \sin \theta)h + \sin \theta h\}}$

$\Rightarrow R_1 \cos \theta = \dfrac{mgL / 2 \cos^2 \theta \sin \theta}{\{(\cos^2 \theta / \sin \theta)h + \sin \theta h\}}$

$\Rightarrow \mu = R_1 \sin \theta / R_2 = \dfrac{mg\ L / 2 \cos \theta . \sin \theta}{\{(\cos^2 \theta / \sin \theta)h + \sin \theta h)\}mg - mg\ 1/2 \cos^2 \theta}$

$\Rightarrow \mu = \dfrac{L / 2 \cos \theta . \sin \theta \times 2 \sin \theta}{2(\cos^2 \theta h + \sin^2 \theta h) - L \cos^2 \theta \sin \theta}$

$\Rightarrow \mu = \dfrac{L \cos \theta \sin^2 \theta}{2h - L \cos^2 \theta \sin \theta}$.

46. A uniform rod of mass 300 grams and length 50 cm rotates with an uniform angular velocity = 2 rad/s about an axis perpendicular to the rod through an end.

a) $L = I\omega$

I at the end $= mL^2/3 = (0.3 \times 0.5^2)/3 = 0.025$ kg-m^2

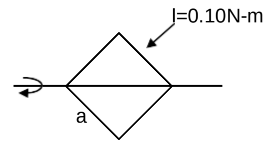
$= 0.025 \times 2 = 0.05$ kg – m^2/s

b) Speed of the centre of the rod

$V = \omega r = w \times (50/2) = 50$ cm/s $= 0.5$ m/s.

c) Its kinetic energy $= 1/2\ I\omega^2 = (1/2) \times 0.025 \times 2^2 = 0.05$ Joule.

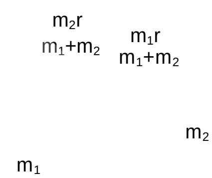
47. I = 0.10 N-m; a = 10 cm = 0.1 m; m = 2 kg



Therefore $(ma^2/12) \times \alpha = 0.10$ N-m
$\Rightarrow \alpha = 60$ rad/s
Therefore $\omega = \omega_0 + \alpha t$
$\Rightarrow \omega = 60 \times 5 = 300$ rad/s
Therefore angular momentum = $I\omega = (0.10 / 60) \times 300 = 0.50$ kg-m^2/s
And 0 kinetic energy = $1/2\ I\omega^2 = 1/2 \times (0.10 / 60) \times 300^2 = 75$ Joules.

48. Angular momentum of the earth about its axis is
$= 2/5\ mr^2 \times (2\pi / 85400)$ (because, $I = 2/5\ mr^2$)
Angular momentum of the earth about sun's axis
$= mR^2 \times (2\pi / 86400 \times 365)$ (because, $I = mR^2$)
Therefore, ratio of the angular momentum $= \dfrac{2/5mr^2 \times (2\pi / 86400)}{mR^2 \times 2\pi/(86400 \times 365)}$
$\Rightarrow (2r^2 \times 365) / 5R^2$
$\Rightarrow (2.990 \times 10^{10}) / (1.125 \times 10^{17}) = 2.65 \times 10^{-7}$.

49. Angular momentum due to the mass m_1 at the centre of system is $= m_1 r^{12}$.

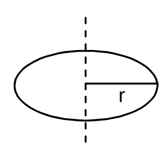


$= m1\left(\dfrac{m_2}{m_1+m_2}\right)^2\omega = \dfrac{m_1 m_2^2 r^2}{(m_1+m_2)^2}\omega$ ...(1)
Similarly the angular momentum due to the mass m_2 at the centre of system is m_2 $r^{112}\omega$
$= m_2\left(\dfrac{m_1 r}{m_1 m_2}\right)^2\omega = \dfrac{m_2 m_1^2}{(m_1+m_2)^2}\omega$ ...(2)
Therefore net angular momentum $= \dfrac{m_1 m_2^2 r^2\omega}{(m_1+m_2)^2} + \dfrac{m_2 m_1^2 r^2\omega}{(m_1+m_2)^2}$
$\Rightarrow \dfrac{m_1 m_2 (m_1+m_2) r^2\omega}{(m_1+m_2)^2} = \dfrac{m_1 m_2}{(m_1+m_2)} r^2\omega = \mu r^2\omega$ (proved)

50. $\tau = I\alpha$
$\Rightarrow F \times r = (mr^2 + mr^2)\alpha \Rightarrow 5 \times 0.25 = 2mr^2 \times \alpha$
$\Rightarrow \alpha = \dfrac{1.25}{2 \times 0.5 \times 0.025 \times 0.25} = 20$

$\omega_0 = 10$ rad/s, t = 0.10 sec, $\omega = \omega_0 + \alpha t$
$\Rightarrow \omega = 10 + 010 \times 230 = 10 + 2 = 12$ rad/s.

51. A wheel has
$I = 0.500$ Kg-m^2, r = 0.2 m, $\omega = 20$ rad/s
Stationary particle = 0.2 kg

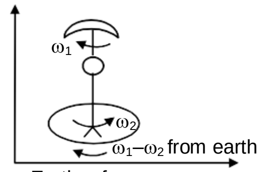


Therefore $I_1\omega_1 = I_2\omega_2$ (since external torque = 0)
$\Rightarrow 0.5 \times 10 = (0.5 + 0.2 \times 0.2^2)\omega_2$
$\Rightarrow 10/0.508 = \omega_2 = 19.69 = 19.7$ rad/s

52. $I_1 = 6$ kg-m^2, $\omega_1 = 2$ rad/s , $I_2 = 5$ kg-m^2
Since external torque = 0
Therefore $I_1\omega_1 = I_2\omega_2$
$\Rightarrow \omega_2 = (6 \times 2) / 5 = 2.4$ rad/s

53. $\omega_1 = 120$ rpm $= 120 \times (2\pi / 60) = 4\pi$ rad /s.
$I_1 = 6$ kg – m^2, $I_2 = 2$ kgm^2
Since two balls are inside the system
Therefore, total external torque = 0
Therefore, $I_1\omega_1 = I_2\omega_2$
$\Rightarrow 6 \times 4\pi = 2\omega_2$
$\Rightarrow \omega_2 = 12\ \pi$ rad/s = 6 rev/s = 360 rev/minute.

54. $I_1 = 2 \times 10^{-3}$ kg-m^2 ; $I_2 = 3 \times 10^{-3}$ kg-m^2 ; $\omega_1 = 2$ rad/s



From the earth reference the umbrella has a angular velocity $(\omega_1 - \omega_2)$

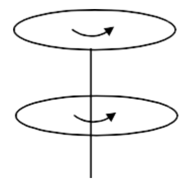
And the angular velocity of the man will be ω_2

Therefore $I_1(\omega_1 - \omega_2) = I_2\omega_2$

$\Rightarrow 2 \times 10^{-3} (2 - \omega_2) = 3 \times 10^{-3} \times \omega_2$

$\Rightarrow 5\omega_2 = 4 \Rightarrow \omega_2 = 0.8$ rad/s.

55. Wheel (1) has



$I_1 = 0.10$ kg-m^2, $\omega_1 = 160$ rev/min

Wheel (2) has

$I_2 = ?$; $\omega_2 = 300$ rev/min

Given that after they are coupled, $\omega = 200$ rev/min

Therefore if we take the two wheels to bean isolated system

Total external torque = 0

Therefore, $I_1\omega_1 + I_1\omega_2 = (I_1 + I_1)\omega$

$\Rightarrow 0.10 \times 160 + I_2 \times 300 = (0.10 + I_2) \times 200$

$\Rightarrow 5I_2 = 1 - 0.8 \Rightarrow I_2 = 0.04$ kg-m^2.

56. A kid of mass M stands at the edge of a platform of radius R which has a moment of inertia I. A ball of m thrown to him and horizontal velocity of the ball v when he catches it.

Therefore if we take the total bodies as a system

Therefore $mvR = \{I + (M + m)R^2\}\omega$

(The moment of inertia of the kid and ball about the axis = $(M + m)R^2$)

$\Rightarrow \omega = \dfrac{mvR}{1 + (M + m)R^2}$.

57. Initial angular momentum = Final angular momentum

(the total external torque = 0)

Initial angular momentum = mvR (m = mass of the ball, v = velocity of the ball, R = radius of platform)

Therefore angular momentum = $I\omega + MR^2\omega$

Therefore $mVR = I\omega + MR^2\omega$

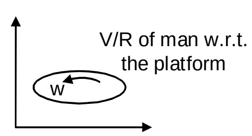
$\Rightarrow \omega = \dfrac{mVR}{(1 + MR^2)}$.

58. From a inertial frame of reference when we see the (man wheel) system, we can find that the wheel moving at a speed of ω and the man with $(\omega + V/R)$ after the man has started walking.

(ω' = angular velocity after walking, ω = angular velocity of the wheel before walking.

Since $\Sigma I = 0$

Extended torque = 0



Therefore $(1 + MR^2)\omega = I\omega' + mR^2(\omega' + V/R)$

$\Rightarrow (I + mR^2)\omega + I\omega' + mR^2\omega' + mVR$

$\Rightarrow \omega' = \omega - \dfrac{mVR}{(1 + mR^2)}$.

59. A uniform rod of mass m length ℓ is struck at an end by a force F. $\perp$ to the rod for a short timet

a) Speed of the centre of mass

$mv = Ft \Rightarrow v = \dfrac{Ft}{m}$

b) The angular speed of the rod about the centre of mass

$\ell\omega - r \times p$

$\Rightarrow (m\ell^2 / 12) \times \omega = (1/2) \times mv$

$\Rightarrow m\ell^2 / 12 \times \omega = (1/2)\, \ell\omega^2$

$\Rightarrow \omega = 6Ft / m\ell$

c) K.E. $= (1/2)\, mv^2 + (1/2)\, \ell\omega^2$

$= (1/2) \times m(Ft / m)^2\, (1/2)\, \ell\omega^2$

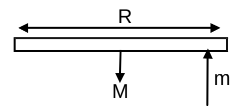
$= (1/2) \times m \times (F^2t^2/m^2) + (1/2) \times (m\ell^2/12)\, (36 \times (F^2t^2/m^2\ell^2))$

$= F^2 t^2 / 2m + 3/2 (F^2t^2) / m = 2 F^2 t^2 / m$

d) Angular momentum about the centre of mass:-

$L = mvr = m \times Ft / m \times (1/2) = F \ell t / 2$

60. Let the mass of the particle = m & the mass of the rod = M



Let the particle strikes the rod with a velocity V.

If we take the two body to be a system,

Therefore the net external torque & net external force = 0

Therefore Applying laws of conservation of linear momentum

$MV' = mV$ (V' = velocity of the rod after striking)

$\Rightarrow V' / V = m / M$

Again applying laws of conservation of angular momentum

$\Rightarrow \frac{mVR}{2} = \ell\omega$

$\Rightarrow \frac{mVR}{2} = \frac{MR^2}{12} \times \frac{\pi}{2t} \Rightarrow t = \frac{MR\pi}{m12 \times V}$

Therefore distance travelled :-

$V' t = V'\left(\frac{MR\pi}{m12\pi}\right) = \frac{m}{M} \times \frac{M}{m} \times \frac{R\pi}{12} = \frac{R\pi}{12}$

61. a) If we take the two bodies as a system therefore total external force = 0

Applying L.C.L.M :-

$mV = (M + m) v'$

$\Rightarrow v' = \frac{mv}{M + m}$

v′

b) Let the velocity of the particle w.r.t. the centre of mass = V′

M m, v

$\Rightarrow v' = \frac{m \times 0 + Mv}{M + m} \Rightarrow v' = \frac{Mv}{M + m}$

c) If the body moves towards the rod with a velocity of v, i.e. the rod is moving with a velocity – v towards the particle.

Therefore the velocity of the rod w.r.t. the centre of mass = V^-

$\Rightarrow V^- = \frac{M \times O = m \times v}{M + m} = \frac{-mv}{M + m}$

d) The distance of the centre of mass from the particle

$= \frac{M \times l / 2 + m \times O}{(M + m)} = \frac{M \times l / 2}{(M + m)}$

Therefore angular momentum of the particle before the collision

$= I\,\omega = Mr^2 \text{ cm } \omega$

$= m\{m\, l/2) / (M + m)\}^2 \times V/ (l/2)$

$= (mM^2vl) / 2(M + m)$

Distance of the centre of mass from the centre of mass of the rod =

$R^1_{cm} = \frac{M \times 0 + m \times (l / 2)}{(M + m)} = \frac{(ml / 2)}{(M + m)}$

Therefore angular momentum of the rod about the centre of mass

$= MV_{cm} R^1_{cm}$

$= M \times \{(-mv) / (M + m)\} \{(ml/2) / (M + m)\}$

$= \left|\frac{-\ Mm^2lv}{2(M + m)^2}\right| = \frac{Mm^2lv}{2(M + m)^2}$ (If we consider the magnitude only)

e) Moment of inertia of the system = M.I. due to rod + M.I. due to particle

$$= \frac{Ml^2}{12} + \frac{M(ml/2)^2}{(M+m)^2} + \frac{m(Ml/s)^2}{(M+m)^2}$$

$$= \frac{Ml^2(M+4m)}{12(M+m)}$$

f) Velocity of the centre of mass $V_m = \frac{M\times 0 + mV}{(M+m)} = \frac{mV}{(M+m)}$

(Velocity of centre of mass of the system before the collision = Velocity of centre of mass of the system after the collision)
(Because External force = 0)
Angular velocity of the system about the centre of mass,
$P_{cm} = I_{cm}\,\omega$

$$\Rightarrow MV_M \times \vec{r}_m + m\vec{v}_m \times \vec{r}_m = I_{cm}\omega$$

$$\Rightarrow M\times \frac{mv}{(M+m)} \times \frac{ml}{2(M+m)} + m\times \frac{Mv}{(M+m)} \times \frac{Ml}{2(M+m)} = \frac{Ml^2(M+4m)}{12(M+m)} \times \omega$$

$$\Rightarrow \frac{Mm^2vl + mM^2vl}{2(M+m)^2} = \frac{Ml^2(M+4m)}{12(M+m)} \times \omega$$

$$\Rightarrow \frac{Mm/(M+m)}{2(M+m)^2} = \frac{Ml^2(M+m)}{12(M+m)} \times \omega$$

$$\Rightarrow \frac{6mv}{(M+4m)l} = \omega$$

62. Since external torque = 0
Therefore $I_1\omega_1 = I_2\omega_2$

$$I_1 = \frac{ml^2}{4} + \frac{ml^2}{4} = \frac{ml^2}{2}$$

$\omega_1 = \omega$

$$I_2 = \frac{2ml^2}{4} + \frac{ml^2}{4} = \frac{3ml^2}{4}$$

L

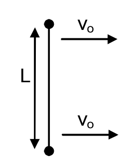
m m m

Therefore $\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\left(\frac{ml^2}{2}\right)\times\omega}{\frac{3ml^2}{4}} = \frac{2\omega}{3}$

63. Two balls A & B, each of mass m are joined rigidly to the ends of a light of rod of length L. The system moves in a velocity v_0 in a direction $\perp$ to the rod. A particle P of mass m kept at rest on the surface sticks to the ball A as the ball collides with it.

a) The light rod will exert a force on the ball B only along its length. So collision will not affect its velocity.

B has a velocity = v_0
If we consider the three bodies to be a system
Applying L.C.L.M.



Therefore $mv_0 = 2mv' \Rightarrow v' = \frac{v_0}{2}$

Therefore A has velocity = $\frac{v_0}{2}$

b) if we consider the three bodies to be a system
Therefore, net external force = 0

Therefore $V_{cm} = \frac{m\times v_0 + 2m\left(\frac{v_0}{2}\right)}{m+2m} = \frac{mv_0 + mv_0}{3m} = \frac{2v_0}{3}$ (along the initial velocity as before collision)

c) The velocity of (A + P) w.r.t. the centre of mass $= \frac{2v_0}{3} - \frac{v_0}{2} = \frac{v_0}{6}$ &

The velocity of B w.r.t. the centre of mass $v_0 - \frac{2v_0}{3} = \frac{v_0}{3}$

[Only magnitude has been taken]
Distance of the (A + P) from centre of mass = l/3 & for B it is 2 l/3.
Therefore $P_{cm} = I_{cm} \times \omega$

$$\Rightarrow\ 2m \times \frac{v_0}{6} \times \frac{1}{3} + m \times \frac{v_0}{3} \times \frac{2\,l}{3} = 2m\left(\frac{1}{3}\right)^2 + m\left(\frac{2l}{3}\right)^2 \times \omega$$

$$\Rightarrow\ \frac{6mv_0 l}{18} = \frac{6ml}{9} \times \omega \Rightarrow \omega = \frac{v_0}{2\,l}$$

64. The system is kept rest in the horizontal position and a particle P falls from a height h and collides with B and sticks to it.

Therefore, the velocity of the particle 'ρ' before collision = $\sqrt{2gh}$

If we consider the two bodies P and B to be a system. Net external torque and force = 0

Therefore, $m\ \ 2gh = 2m \times v$

p,o

x v

$\Rightarrow v' = \ (2gh)/2$

Therefore angular momentum of the rod just after the collision

A B

y

$\Rightarrow 2m\ (v' \times r) = 2m \times \ (2gh)/2 \times l/2 \Rightarrow ml\ \ (2gh)/2$

$$\omega = \frac{L}{I} = \frac{ml\ 2gh}{2(ml^2/4 + 2ml^2/4)} = \frac{2\ gh}{3l} = \frac{8gh}{3l}$$

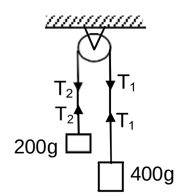
b) When the mass 2m will at the top most position and the mass m at the lowest point, they will automatically rotate. In this position the total gain in potential energy = 2 mg × (l/2) – mg (l/2) = mg(l/2)
Therefore $\Rightarrow$ mg l/2 = l/2 $I\omega^2$
$\Rightarrow$ mg l/2 = (1/2 × 3ml²) / 4 × (8gh / 9gl²)
$\Rightarrow$ h = 3l/2.

2m

65. According to the question
$0.4g - T_1 = 0.4\ a$ …(1)
$T_2 - 0.2g = 0.2\ a$ …(2)
$(T_1 - T_2)r = Ia/r$ …(3)
From equation 1, 2 and 3

m

$$\Rightarrow a = \frac{(0.4 - 0.2)g}{(0.4 + 0.2 + 1.6/0.4)} = g/5$$

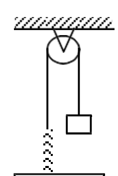


Therefore (b) $V = \sqrt{2ah} = \sqrt{(2 \times gl^5 \times 0.5)}$

$\Rightarrow \sqrt{(g/5)} = \sqrt{(9.8/5)}$ = 1.4 m/s.

a) Total kinetic energy of the system
= $1/2\ m_1V^2 + 1/2\ m_2V^2 + 1/2\ I8^2$
= $(1/2 \times 0.4 \times 1.4^2) + (1/2 \times 0.2 \times 1.4^2) + (1/2 \times (1.6/4) \times 1.4^2)$ = 0.98 Joule.

66. $I = 0.2$ kg-m², r = 0.2 m, K = 50 N/m,
m = 1 kg, g = 10 ms², h = 0.1 m
Therefore applying laws of conservation of energy
$mgh = 1/2\ mv^2 + 1/2\ kx^2$
$\Rightarrow 1 = 1/2 \times 1 \times V^2 + 1/2 \times 0.2 \times V^2/0.04 + (1/2) \times 50 \times 0.01\ (x = h)$
$\Rightarrow 1 = 0.5\ v^2 + 2.5\ v^2 + 1/4$
$\Rightarrow 3v^2 = 3/4$
$\Rightarrow v = 1/2 = 0.5$ m/s



67. Let the mass of the rod = m

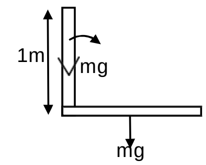
Therefore applying laws of conservation of energy

$1/2\ I\omega^2 = mg\ l/2$

$\Rightarrow 1/2 \times M\ l^2/3 \times \omega^2 = mg\ 1/2$

$\Rightarrow \omega^2 = 3g / l$

$\Rightarrow \omega = \sqrt{3g/l} = 5.42$ rad/s.



68. $1/2\ I\omega^2 - 0 = 0.1 \times 10 \times 1$

$\Rightarrow \omega = \sqrt{20}$

For collision

$0.1 \times 1^2 \times \sqrt{20} + 0 = [(0.24/3)\times 1^2 + (0.1)^2\ 1^2]\omega'$

$\Rightarrow \omega' = \sqrt{20}\ /[10.(0.18)]$

$\Rightarrow 0 - 1/2\ \omega'^2 = -m_1 g\ l\ (1 - \cos\theta) - m_2 g\ l/2\ (1 - \cos\theta)$

$= 0.1 \times 10\ (1 - \cos\theta) = 0.24 \times 10 \times 0.5\ (1 - \cos\theta)$

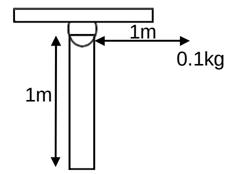
$\Rightarrow 1/2 \times 0.18 \times (20/3.24) = 2.2(1 - \cos\theta)$

$\Rightarrow (1 - \cos\theta) = 1/(2.2 \times 1.8)$

$\Rightarrow 1 - \cos\theta = 0.252$

$\Rightarrow \cos\theta = 1 - 0.252 = 0.748$

$\Rightarrow \omega = \cos^{-1}(0.748) = 41°$.



69. Let l = length of the rod, and m = mass of the rod.

Applying energy principle

$(1/2)\ I\omega^2 - O = mg\ (1/2)\ (\cos 37° - \cos 60°)$

37°

60°

$\Rightarrow \frac{1}{2} \times \frac{ml^2}{3}\ \omega^2 = mg \times \frac{1}{2}\left(\frac{4}{5} - \frac{1}{2}\right)l$

$\Rightarrow \omega^2 = \frac{9g}{10\ l} = 0.9\left(\frac{g}{l}\right)$

Again $\left(\frac{ml^2}{3}\right)\alpha = mg\left(\frac{1}{2}\right)\sin 37° = mgl \times \frac{3}{5}$

$\therefore\ \alpha = 0.9\left(\frac{g}{l}\right)$ = angular acceleration.

So, to find out the force on the particle at the tip of the rod

F_i = centrifugal force = (dm) ω^2 l = 0.9 (dm) g

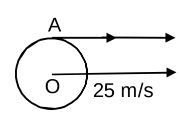
F_t = tangential force = (dm) α l = 0.9 (dm) g

So, total force F = $\sqrt{(F_i^2 + F_t^2)} = 0.9\sqrt{2}$ (dm) g

70. A cylinder rolls in a horizontal plane having centre velocity 25 m/s.

At its age the velocity is due to its rotation as well as due to its leniar motion & this two velocities are same and acts in the same direction ($v = r\,\omega$)

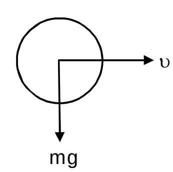
Therefore Net velocity at A = 25 m/s + 25 m/s = 50 m/s



71. A sphere having mass m rolls on a plane surface. Let its radius R. Its centre moves with a velocity v

Therefore Kinetic energy = $(1/2)\ I\omega^2 + (1/2)\ mv^2$

$= \frac{1}{2} \times \frac{2}{5} mR^2 \times \frac{v^2}{R^2} + \frac{1}{2} mv^2 = \frac{2}{10} mv^2 + \frac{1}{2} mv^2 = \frac{2+5}{10} mv^2 = \frac{7}{10} mv^2$



72. Let the radius of the disc = R

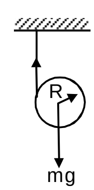
Therefore according to the question & figure

Mg – T = ma …(1)

& the torque about the centre

= T × R = I × α

$\Rightarrow$ TR = (1/2) mR^2 ×a/R



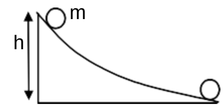
$\Rightarrow T = (1/2)\ ma$

Putting this value in the equation (1) we get

$\Rightarrow mg - (1/2)\ ma = ma$

$\Rightarrow mg = 3/2\ ma \Rightarrow a = 2g/3$

73. A small spherical ball is released from a point at a height on a rough track & the sphere does not slip. Therefore potential energy it has gained w.r.t the surface will be converted to angular kinetic energy about the centre & linear kinetic energy.



Therefore $mgh = (1/2)\ I\omega^2 + (1/2)\ mv^2$

$\Rightarrow mgh = \frac{1}{2} \times \frac{2}{5}\ mR^2\ \omega^2\ +\ \frac{1}{2} mv^2$

$\Rightarrow gh = \frac{1}{5} v^2 +\ \frac{1}{2} v^2$

$\Rightarrow v^2 = \frac{10}{7} gh \Rightarrow v = \sqrt{\frac{10}{7} gh}$

74. A disc is set rolling with a velocity V from right to left. Let it has attained a height h.

m

h

Therefore $(1/2)\ mV^2 + (1/2)\ I\omega^2 = mgh$

$\Rightarrow (1/2)\ mV^2 + (1/2) \times (1/2)\ mR^2\ \omega^2 = mgh$

$\Rightarrow (1/2)\ V^2 + 1/4\ V^2 = gh \Rightarrow (3/4)\ V^2 = gh$

$\Rightarrow h = \frac{3}{4} \times \frac{V^2}{g}$

75. A sphere is rolling in inclined plane with inclination θ

Therefore according to the principle

$MgI \sin\theta = (1/2)\ I\omega^2 + (1/2)\ mv^2$

$\Rightarrow mgl \sin\theta = 1/5\ mv^2 + (1/2)\ mv^2$

$Gl \sin\theta = 7/10\ \omega^2$

R

L sinθ L R

$\Rightarrow v = \frac{10}{7} gl \sin\theta$

76. A hollow sphere is released from a top of an inclined plane of inclination θ.

To prevent sliding, the body will make only perfect rolling. In this condition,

$mg \sin\theta - f = ma$ …(1)

& torque about the centre

$f \times R = \frac{2}{3} mR^2 \times \frac{a}{R}$

$\Rightarrow f = \frac{2}{3} ma$ …(2)

R m

mg cosθ mg sinθ

R θ

Putting this value in equation (1) we get

$\Rightarrow mg \sin\theta - \frac{2}{3} ma = ma \Rightarrow a = \frac{3}{5}\ g \sin\theta$

$\Rightarrow mg \sin\theta - f = \frac{3}{5}\ mg \sin\theta \Rightarrow f = \frac{2}{5} mg \sin\theta$

$\Rightarrow \mu\ mg \cos\theta = \frac{2}{5}\ mg \sin\theta \Rightarrow \mu = \frac{2}{5} \tan\theta$

b) $\frac{1}{5} \tan\theta\ (mg \cos\theta)\ R = \frac{2}{3} mR^2\ \alpha$

$\Rightarrow \alpha = \frac{3}{10} \times \frac{g \sin\theta}{R}$

$a_c = g \sin\theta - \frac{g}{5} \sin\theta = \frac{4}{5} \sin\theta$

$\Rightarrow t^2 = \frac{2s}{a_c} = \frac{2l}{\left(\frac{4g\sin\theta}{5}\right)} = \frac{5l}{2g\sin\theta}$

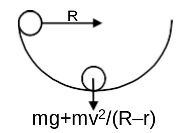
Again, $\omega = \alpha t$

K.E. = $(1/2)\ mv^2 + (1/2)\ I\omega^2 = (1/2)\ m(2as) + (1/2)\ I\ (\alpha^2 t^2)$

$= \frac{1}{2} m \times \frac{4g\sin\theta}{5} \times 2 \times l + \frac{1}{2} \times \frac{2}{3} mR^2 \times \frac{9}{100}\ \frac{g^2 \sin^2\theta}{R} \times \frac{5l}{2g\sin\theta}$

$= \frac{4mgl\sin\theta}{5} + \frac{3mgl\sin\theta}{40} = \frac{7}{8} mgl\sin\theta$

77. Total normal force = $mg + \frac{mv^2}{R-r}$



$\Rightarrow mg\ (R-r) = (1/2)\ I\omega^2 + (1/2)\ mv^2$

$\Rightarrow mg\ (R-r) = \frac{1}{2} \times \frac{2}{5} mv^2 + \frac{1}{2} mv^2$

$\Rightarrow \frac{7}{10} mv^2 = mg(R-r) \Rightarrow v^2 = \frac{10}{7} g(R-r)$

Therefore total normal force = $mg + \frac{m\left(\frac{10}{7}\right)g(R-r)}{R-r} = mg + mg\left(\frac{10}{7}\right) = \frac{17}{7} mg$

78. At the top most point

$\frac{mv^2}{R-r} = mg \Rightarrow v^2 = g(R-r)$

Let the sphere is thrown with a velocity v'

Therefore applying laws of conservation of energy

$\Rightarrow (1/2)\ mv'^2 + (1/2)\ I\omega'^2 = mg\ 2\ (R-r) + (1/2)\ mv^2 + (1/2)\ I\omega^2$

$\Rightarrow \frac{7}{10} v'^2 = g\ 2(R-r) + \frac{7}{10} v^2$

$\Rightarrow v'^2 = \frac{20}{7} g\ (R-r) + g\ (R-r)$

$\Rightarrow v' = \frac{27}{7} g(R-r)$

79. a) Total kinetic energy y = $(1/2)\ mv^2 + (1/2)\ I\omega^2$

Therefore according to the question

$mg\ H = (1/2)\ mv^2 + (1/2)\ I\omega^2 + mg\ R\ (1 + \cos\theta)$

$\Rightarrow mg\ H - mg\ R\ (1 + \cos\theta) = (1/2)\ mv^2 + (1/2)\ I\omega^2$

$\Rightarrow (1/2)\ mv^2 + (1/2)\ I\omega^2 = mg\ (H - R - R\sin\theta)$

b) to find the acceleration components

$\Rightarrow (1/2)\ mv^2 + (1/2)\ I\omega^2 = mg\ (H - R - R\sin\theta)$

$\Rightarrow 7/10\ mv^2 = mg\ (H - R - R\sin\theta)$

$\frac{v^2}{R} = \frac{10}{7} g \left[\left(\frac{H}{R}\right) - 1 - \sin\theta\right] \rightarrow$ radical acceleration

$\Rightarrow v^2 = \frac{10}{7} g\ (H - R) - R\sin\theta$

$\Rightarrow 2v \frac{dv}{dt} = -\frac{10}{7} g\ R\cos\theta \frac{d\theta}{dt}$

$\Rightarrow \omega\ R \frac{dv}{dt} = -\frac{5}{7}\ g\ R\cos\theta\ \frac{d\theta}{dt}$

$\Rightarrow \frac{dv}{dt} = -\frac{5}{7} g\cos\theta \rightarrow$ tangential acceleration

c) Normal force at $\theta = 0$

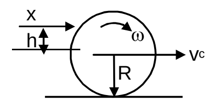
$$\Rightarrow \frac{mv^2}{R} = \frac{70}{1000} \times \frac{10}{7} \times 10\left(\frac{0.6 - 0.1}{0.1}\right) = 5N$$

Frictional force :-

$$f = mg - ma = m(g - a) = m\left(10 - \frac{5}{7} \times 10\right) = 0.07\left(\frac{70 - 50}{7}\right) = \frac{1}{100} \times 20 = 0.2N$$

80. Let the cue strikes at a height 'h' above the centre, for pure rolling, $V_c = R\omega$

Applying law of conservation of angular momentum at a point A,



$mv_ch - \ell\omega = 0$

$$mv_ch = \frac{2}{3} mR^2 \times \left(\frac{v_c}{R}\right)$$

$$h = \frac{2R}{3}$$

81. A uniform wheel of radius R is set into rotation about its axis (case-I) at an angular speed ω

This rotating wheel is now placed on a rough horizontal. Because of its friction at contact, the wheel accelerates forward and its rotation decelerates. As the rotation decelerates the frictional force will act backward.

If we consider the net moment at A then it is zero.

Therefore the net angular momentum before pure rolling & after pure rolling remains constant

Before rolling the wheel was only rotating around its axis.

Therefore Angular momentum = $\ell\,\omega = (1/2)\, MR^2\, \omega$ …(1)

After pure rolling the velocity of the wheel let v

Therefore angular momentum = $\ell_{cm}\,\omega + m(V \times R)$

$= (1/2)\, mR^2\, (V/R) + mVR = 3/2\, mVR$ …(2)

Because, Eq(1) and (2) are equal

Therefore, $3/2\, mVR = ½\, mR^2\, \omega$

$\Rightarrow V = \omega R/3$

ω
(1st case)
ω v
mg
ρ
(2nd case)

82. The shell will move with a velocity nearly equal to v due to this motion a frictional force well act in the background direction, for which after some time the shell attains a pure rolling. If we consider moment about A, then it will be zero. Therefore, Net angular momentum about A before pure rolling = net angular momentum after pure rolling.

v R v
ρ
A
(1st case)

Now, angular momentum before pure rolling about A = M (V × R) and angular momentum after pure rolling :-

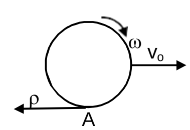
$(2/3)\, MR^2 \times (V_0 / R) + M V_0 R$

(V_0 = velocity after pure rolling)

$\Rightarrow MVR = 2/3\, MV_0R + MV_0R$

$\Rightarrow (5/3)\, V_0 = V$

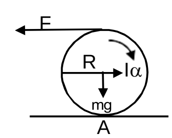
$\Rightarrow V_0 = 3V/5$



(2nd case)

83. Taking moment about the centre of hollow sphere we will get

$$F \times R = \frac{2}{3} MR^2\, \alpha$$



$$\Rightarrow \alpha = \frac{3F}{2MR}$$

Again, $2\pi = (1/2)\, \alpha t^2$ (From $\theta = \omega_0 t + (1/2)\, \alpha t^2$)

$$\Rightarrow t^2 = \frac{8\pi MR}{3F}$$

$$\Rightarrow a_c = \frac{F}{m}$$

$$\Rightarrow X = (1/2)\, a_c t^2 = (1/2) = \frac{4\pi R}{3}$$

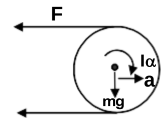
84. If we take moment about the centre, then

$F \times R = \ell\alpha \times f \times R$

$\Rightarrow F = 2/5\ mR\alpha + \mu mg \qquad \ldots(1)$

Again, $F = ma_c - \mu\, mg \qquad \ldots(2)$

$\Rightarrow a_c = \dfrac{F + \mu\, mg}{m}$



Putting the value a_c in eq(1) we get

$\Rightarrow \dfrac{2}{5} \times m \times \left(\dfrac{F + \mu\, mg}{m}\right) + \mu\, mg$

$\Rightarrow 2/5\ (F + \mu\, mg) + \mu\, mg$

$\Rightarrow F = \dfrac{2}{5} F + \dfrac{2}{5} \times 0.5 \times 10 + \dfrac{2}{7} \times 0.5 \times 10$

$\Rightarrow \dfrac{3F}{5} = \dfrac{4}{7} + \dfrac{10}{7} = 2$

$\Rightarrow F = \dfrac{5 \times 2}{3} = \dfrac{10}{3} = 3.33$ N

85. a) if we take moment at A then external torque will be zero

Therefore, the initial angular momentum = the angular momentum after rotation stops (i.e. only leniar velocity exits)

$MV \times R - \ell\, \omega = MV_O \times R$

$\Rightarrow MVR - 2/5 \times MR^2\ V / R = MV_O\ R$

$\Rightarrow V_O = 3V/5$

w=V/R v A

b) Again, after some time pure rolling starts

therefore $\Rightarrow M \times v_o \times R = (2/5)\ MR^2 \times (V'/R) + MV'R$

$\Rightarrow m \times (3V/5) \times R = (2/5)\ MV'R + MV'R$

$\Rightarrow V' = 3V/7$

86. When the solid sphere collides with the wall, it rebounds with velocity 'v' towards left but it continues to rotate in the clockwise direction.

So, the angular momentum = $mvR - (2/5)\ mR^2 \times v/R$

After rebounding, when pure rolling starts let the velocity be v′

and the corresponding angular velocity is v′ / R

v v V/R

Therefore angular momentum = $mv'R + (2/5)\ mR^2\ (v'/R)$

So, $mvR - (2/5)\ mR^2,\ v/R = mvR + (2/5)\ mR^2 (v'/R)$

$mvR \times (3/5) = mvR \times (7/5)$

$v' = 3v/7$

So, the sphere will move with velocity 3v/7.

* * * *