SURA'S

MATHEMATICS

10th Standard

Based on the Updated New Textbook

FREE Self Evaluation

Salient Features

- Complete Solutions to Textbook Exercises.
- Exhaustive Additional Question in all Chapters.
- Chapter-wise Unit Tests with Answers.
- Model Question Papers 1 to 6 (PTA) : Questions are incorporated in the appropriate sections.
- Govt. Model Question Paper-2019 [Govt. MQP-2019],Quarterly Exam - 2019 [QY-2019], Half yearly Exam - 2019 [HY-2019] , Supplementary Exam September - 2020, 2021 & August - 2022 [Sep.-2020 ; 2021 & Aug. - 2022], First & Second Revision Test - 2022 [FRT & SRT - 2022] and Public Examination May - 2022 [May - 2022] questions are incorporated at appropriate sections.
- Instant Supplementary Exam August - 2022 Question Paper is given with answers.

SURA PUBLICATIONS
Chennai

CONTENTS

SYLLABUS

MONTH	S.No.	CHAPTER NAME	UNITS
JUNE	1	Relations and Functions	1.1 - 1.10
	2	Numbers and Sequences	2.1 - 2.6
JULY	2	Numbers and Sequences Algebra	2.7 - 2.11
	3	Algebra	3.1 - 3.3
	4	Geometry	4.1 - 4.2
I MID TERM TEST			
AUGUST	3	Algebra	3.4 - 3.6
	4	Geometry	4.3
	5	Coordinate Geometry	5.1 - 5.6
SEPTEMBER	3	Algebra	3.7
	6	Trigonometry	6.1 - 6.2
QUARTERLY EXAMINATION			
OCTOBER	3	Algebra	3.8 - 3.9
	4	Geometry	4.5 - 4.6
NOVEMBER	6	Trigonometry	6.3
	7	Mensuration	7.1 - 7.5
II MID TERM TEST			
DECEMBER	8	Statistics and Probability	8.1 - 8.6
HALF YEARLY EXAMINATION			
JANUARY		**FIRST REVISION TEST**	
FEBRUARY		**SECOND REVISION TEST**	
MARCH		**THIRD REVISION TEST**	

Chapter 1

RELATIONS AND FUNCTIONS

FORMULAE TO REMEMBER

- ☐ **Vertical line test :**

  A curve drawn in a graph represents a functions, if every vertical line intersects the curve in at most one point.

- ☐ **Horizontal line test :**

  A function represented in a graph is one - one, if every horizontal line intersect the curve in at most one point.

- ☐ Linear functions has applications in Cryptography as well as in several branches of Science and Technology.

EXERCISE 1.1

1. Find A× B, A×A and B ×A
(i) A = {2,−2,3} and B ={1,−4} (ii) A= B ={p,q}
(iii) A= {m,n} ; B = ϕ **[PTA - 1; FRT - 2022]**

Sol. (i) $A = \{2, -2, 3\}, B = \{1, -4\}$

$A \times B = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$

$A \times A = \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$

$B \times A = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$

(ii) $A = B = \{(p,q)\}$

$A \times B = \{(p, p), (p, q), (q, p), (q, q)\}$

$A \times A = \{(p, p), (p, q), (q, p), (q, q)\}$

$B \times A = \{(p, p), (p, q), (q, p), (q, q)\}$

(iii) $A = \{m,n\}$, $B = \phi$

$A \times B = \{ \}$

$A \times A = \{(m,m), (m,n), (n, m), (n, n)\}$

$B \times A = \{ \}$

2. Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A× B and B × A. **[May - 2022]**

Sol. $A = \{1, 2, 3\}, B = \{2, 3, 5, 7\}$

$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$

3. If B × A={(−2, 3),(−2, 4),(0, 3),(0, 4),(3, 3), (3, 4)} find A and B. **[Qy. - 2019]**

Sol. Given $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$

Here $B = \{ -2, 0, 3\}$

[All the first elements of the order pair]

and $A = \{3, 4\}$

[All the second elements of the order pair]

4. If A ={5, 6}, B = {4, 5, 6} , C ={5, 6, 7}, Show that A×A = (B × B)∩(C ×C). **[FRT & Aug. - 2022]**

Sol. $A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$

$A \times A = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(1)

$B \times B = \{(4, 4), (4, 5), (4, 6), (5, 4), \underline{(5, 5)}, \underline{(5, 6)}, (6, 4), \underline{(6, 5)}, \underline{(6, 6)}\}$ ...(2)

$C \times C = \{\underline{(5, 5)}, \underline{(5, 6)}, (5, 7), \underline{(6, 5)}, \underline{(6, 6)}, (6, 7), (7, 5), (7, 6), (7, 7)\}$ ...(3)

$(B \times B) \cap (C \times C) = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(4)

(1) = (4)

$A \times A = (B \times B) \cap (C \times C)$. It is proved.

5. Given A ={1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if (A∩C) × (B∩D) = (A×B)∩(C × D) is true? **[Qy. - 2019]**

Sol. LHS $= \{(A \cap C) \times (B \cap D)$

$A \cap C = \{3\}$

$B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D) = \{(3, 3) , (3, 5)\}$ ...(1)

RHS $= (A \times B) \cap (C \times D)$

$A \times B = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), \underline{(3, 3)}, \underline{(3, 5)}\}$

$C \times D = \{(3, 1), \underline{(3, 3)}, \underline{(3, 5)}, (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ...(2)

∴ (1) = (2) ∴ It is true.

6. Let A={$x \in \mathbb{W}$ |$x < 2$}, B = {$x \in \mathbb{N}$ |$1 < x \leq 4$} and C = {3, 5} . Verify that

(i) A×(B∪C)=(A×B)∪(A×C) **[PTA-2; FRT-2022]**

(ii) A×(B∩C) = (A × B)∩(A × C) **[PTA - 5; Sep. - 2021]**

(iii) (A∪B) × C = (A×C)∪(B × C)

(i) A × (B ∪ C) = (A × B) ∪ (A × C)

Sol. $A = \{x \in \mathbb{W} | x < 2\} = \{0, 1\}$

[Whole numbers less than 2]

$B = \{x \in \mathbb{N} | 1 < x \leq 4\} = \{2, 3, 4\}$

[Natural numbers from 2 to 4]

$C = \{3, 5\}$

LHS $= A \times (B \cup C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$= \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(1)

RHS $= (A \times B) \cup (A \times C)$

$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1,5)\}$

$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(2)

(1) = (2), LHS = RHS Hence it is proved.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$\begin{aligned}
\text{LHS} &= A \times (B \cap C)\\
(B \cap C) &= \{3\}\\
A \times (B \cap C) &= \{(0, 3), (1, 3)\} \quad ...(1)\\
\text{RHS} &= (A \times B) \cap (A \times C)\\
(A \times B) &= \{(0, 2), (\underline{0, 3}),(0, 4), (1, 2),\\
&\quad (\underline{1, 3}), (1, 4)\}\\
(A \times C) &= \{(\underline{0, 3}), (0, 5), (\underline{1, 3}), (1,5)\}\\
(A \times B) \cap (A \times C) &= \{(0, 3), (1, 3)\} \quad ...(2)\\
(1) = (2) \Rightarrow \text{LHS} &= \text{RHS.}
\end{aligned}$$

Hence it is verified.

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$$\begin{aligned}
\text{LHS} &= (A \cup B) \times C\\
A \cup B &= \{0, 1, 2, 3, 4\}\\
(A \cup B) \times C &= \{(0, 3), (0, 5), (1, 3), (1, 5),\\
&\quad (2, 3), (2, 5), (3, 3), (3, 5),\\
&\quad (4, 3), (4, 5)\} \quad ...(1)\\
\text{RHS} &= (A \times C) \cup (B \times C)\\
(A \times C) &= \{(0, 3), (0, 5), (1, 3), (1, 5)\}\\
(B \times C) &= \{(2, 3), (2, 5), (3, 3), (3, 5),\\
&\quad (4, 3), (4, 5)\}\\
(A \times C) \cup (B \times C) &= \{(0, 3), (0, 5), (1, 3), (1, 5),\\
&\quad (2, 3), (2, 5), (3, 3), (3, 5),\\
&\quad (4, 3), (4, 5)\} ...(2)
\end{aligned}$$

(1) = (2)

$\therefore$ LHS = RHS. Hence it is verified.

7. **Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that**

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ **[Sep. - 2020]**

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ **[PTA - 1]**

$$\begin{aligned}
A &= \{1, 2, 3, 4, 5, 6, 7\} \quad \textbf{[May - 2022]}\\
B &= \{2, 3, 5, 7\}\\
C &= \{2\}
\end{aligned}$$

[$\because$ 2 is the only even prime number]

Sol. **(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$**

$$\begin{aligned}
\text{LHS} &= (A \cap B) \times C\\
A \cap B &= \{2, 3, 5, 7\}\\
(A \cap B) \times C &= \{(2, 2), (3, 2), (5, 2), (7, 2)\}\\
&\quad ...(1)\\
\text{RHS} &= (A \times C) \cap (B \times C)\\
(A \times C) &= \{(1, 2), (\underline{2, 2}), (\underline{3, 2}), (4, 2), (\underline{5, 2}),\\
&\quad (6, 2), (\underline{7, 2})\}\\
(B \times C) &= \{(\underline{2, 2}), (\underline{3, 2}), (\underline{5, 2}), (\underline{7, 2})\}\\
(A \times C) \cap (B \times C) &= \{(2, 2), (3, 2), (5, 2), (7, 2)\}\\
&\quad ...(2)
\end{aligned}$$

(1) = (2)

$\therefore$ LHS = RHS. Hence it is verified.

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

$$\begin{aligned}
\text{LHS} &= A \times (B - C)\\
(B - C) &= \{3, 5, 7\}\\
A \times (B - C) &= \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5),\\
&\quad (2, 7), (3, 3), (3, 5), (3, 7), (4, 3),\\
&\quad (4, 5), (4, 7), (5, 3), (5, 5), (5, 7),\\
&\quad (6, 3), (6, 5), (6, 7), (7, 3), (7, 5),\\
&\quad (7, 7)\} \quad ...(1)\\
\text{RHS} &= (A \times B) - (A \times C)\\
(A \times B) &= \{(1, 2), (1, 3), (1, 5), (1, 7),\\
&\quad (2, 2), (2, 3), (2, 5), (2, 7),\\
&\quad (3, 2), (3, 3), (3, 5), (3, 7),\\
&\quad (4, 2), (4, 3), (4, 5), (4, 7),\\
&\quad (5, 2), (5, 3), (5, 5), (5, 7),\\
&\quad (6, 2), (6, 3), (6, 5), (6, 7),\\
&\quad (7, 2), (7, 3), (7, 5), (7,7)\}\\
(A \times C) &= \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}\\
(A \times B) - (A \times C) &= \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5),\\
&\quad (2, 7), (3, 3), (3, 5), (3, 7), (4, 3),\\
&\quad (4, 5), (4, 7), (5, 3), (5, 5), (5, 7),\\
&\quad (6, 3), (6, 5), (6, 7), (7, 3), (7, 5),\\
&\quad (7, 7)\} \quad ...(2)
\end{aligned}$$

(1) = (2) $\Rightarrow$ LHS = RHS. Hence it is verified.

EXERCISE 1.2

1. **Let A = {1,2,3,7} and B = {3,0,–1,7}, which of the following are relation from A to B ?**

(i) $\mathbb{R}_1 = \{(2,1), (7,1)\}$

(ii) $\mathbb{R}_2 = \{(-1,1)\}$

(iii) $\mathbb{R}_3 = \{(2,-1), (7,7), (1,3)\}$ **[FRT - 2022]**

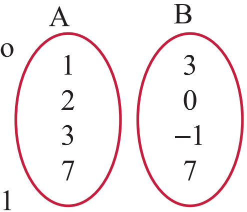
(iv) $\mathbb{R}_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

Sol. Given A = {1, 2, 3, 7} and B = {3, 0, –1, 7}

(i) $R_1 = \{(2, 1), (7, 1)\}$

2 and 7 cannot be related to 1 since $1 \notin B$

$\therefore$ R_1 is not a relation.

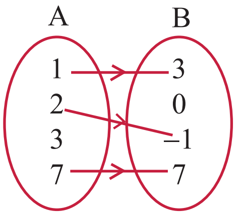


(ii) $R_2 = \{(-1, 1)\}$

–1 cannot be related to 1 since $-1 \notin A$ and $1 \notin B$

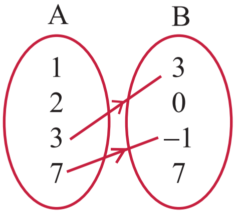
$\therefore$ R_2 is not a relation.

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$



R_3 is a relation since 2 is related to –1, 7 is related to 7 and 1 is related to 3.

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$



7 is related to –1
3 is related to 3
Since $0 \notin A$, 0 cannot be related to 3 and 7.
$\therefore R_4$ is not a relation.

2. Let A={1, 2, 3, 4,...,45} and R be the relation defined as "is square of a number" on A. Write ℝ as a subset of A × A. Also, find the domain and range of ℝ. **[Sep. - 2021]**

Sol. Given $A = \{1, 2, 3, 4, \ldots 45\}$

$\therefore A \times A = \{(1, 1)\ (1, 2)\ (1,3) \ldots (1, 45)$
$(2, 1)\ (2, 2) \ldots (2, 45)\ (45, 1)\ (45, 2)$
$(45, 3) \ldots (45, 45)\}$ ... (1)

R is defined as "is square of "

$\therefore R = \{(1,1)\ (2,4)\ (3,9)\ (4,16)\ (5, 25)\ (6,36)\}$... (2)
[$\because$ 1 is the square of 1, 2 is the square of 4 and so on]

From (1) and (2), R is the subset of A × A

$\therefore R \subset A \times A$

Domain of R = {1, 2, 3, 4, 5, 6}
[All the first elements of the order pair in (2)]

Range of R = {1, 4, 9, 16, 25, 36}
[All the second elements of the order pair in (2)]

3. A Relation ℝ is given by the set $\{(x, y)\ /y = x + 3,$ $x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range. **[PTA - 5]**

Sol. Given $\mathbb{R} = \{(x, y)\ /y = x + 3\}$ and $x \in \{0, 1, 2, 3, 4, 5\}$

When $x = 0$, $y = 0 + 3 = 3$ [$\because y = x + 3$]
When $x = 1$, $y = 1 + 3 = 4$
When $x = 2$, $y = 2 + 3 = 5$
When $x = 3$, $y = 3 + 3 = 6$
When $x = 4$, $y = 4 + 3 = 7$
When $x = 5$, $y = 5 + 3 = 8$

$\therefore \mathbb{R} = \{(0, 3),(1, 4),(2, 5),(3, 6),(4, 7), (5, 8)\}$

$\therefore$ Domain of $\mathbb{R}$ = {0, 1, 2, 3, 4, 5}
[All the first element in ℝ]

Range of $\mathbb{R}$ = {3, 4, 5, 6, 7, 8}
[All the second element in ℝ]

4. Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y)|x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

(ii) $\{(x, y)|y = x + 3, x, y$ are natural numbers $< 10\}$ **[Aug. - 2022]**

Sol. (i) $R = \{(x, y)|\ x = 2y, x \in \{2, 3, 4, 5\}$ and $y \in \{1, 2, 3, 4\}\}$

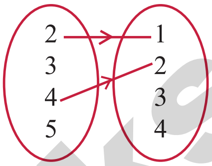
When $x = 2$, $y = \frac{x}{2} = \frac{2}{2} = 1$
[$\because x = 2y \Rightarrow y = \frac{x}{2}$]

When $x = 3$, $y = \frac{3}{2}$

When $x = 4$, $y = \frac{4}{2} = 2$

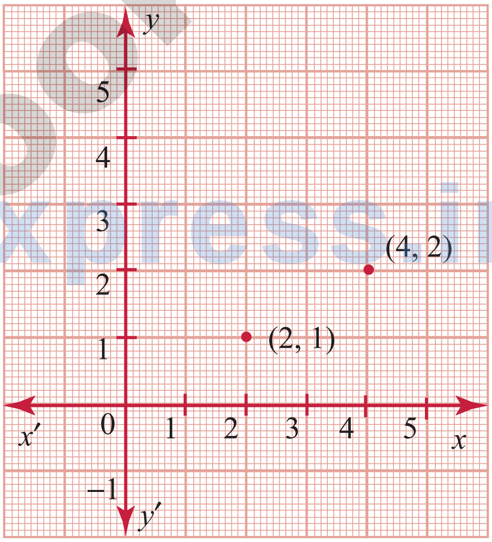
When $x = 5$, $y = \frac{5}{2}$

(a) an arrow diagram



3 cannot be related to $\frac{3}{2}$ and 5 cannot be related to $\frac{5}{2}$.

(b) a graph



(c) Roster form : R = {(2, 1), (4, 2)}

(ii) $R = \{(x, y)|y = x + 3,$
x and y are natural numbers <10}
$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9$
$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
[$\because x$ and y are natural numbers less than 10]

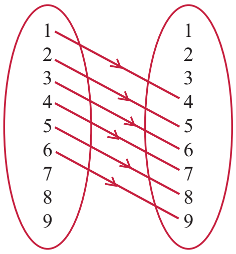
Given $y = x + 3$

When $x = 1$, $y = 1 + 3 = 4$
When $x = 2$, $y = 2 + 3 = 5$
When $x = 3$, $y = 3 + 3 = 6$
When $x = 4$, $y = 4 + 3 = 7$
When $x = 5$, $y = 5 + 3 = 8$
When $x = 6$, $y = 6 + 3 = 9$

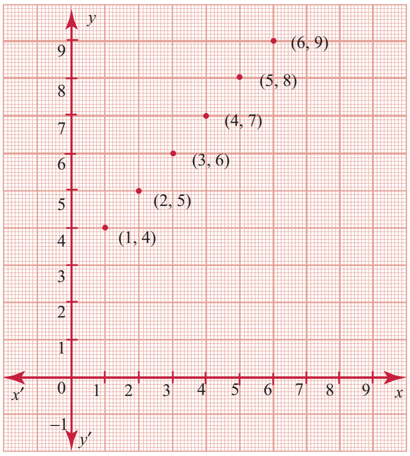
$$\left.\begin{array}{ll}\text{When } x = 7, & y = 7 + 3 = 10\\ \text{When } x = 8, & y = 8 + 3 = 11\\ \text{When } x = 9, & y = 9 + 3 = 12\end{array}\right\}[10, 11, 12 \notin y]$$

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

(a) an arrow diagram



(b) a graph



(c) Roster form :

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

5. **A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation ℝ is defined by $x\mathbb{R}y$, where x is the salary given to person y, express the relation ℝ through an ordered pair and an arrow diagram.** **[FRT - 2022]**

Sol.

A – Assistants → A_1, A_2, A_3, A_4, A_5

C – Clerks → C_1, C_2, C_3, C_4

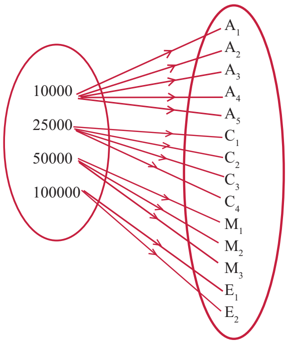
M – Managers → M_1, M_2, M_3

E – Executive officer → E_1, E_2

xRy is defined as x is the salary for assistants is ₹10,000, clerks is ₹25,000, Manger is ₹50,000 and for the executing officer ₹1,00,000.

(a) ∴R = {(10,000, A_1), (10,000, A_2), (10,000, A_3), (10,000, A_4), (10,000, A_5), (25,000, C_1), (25,000, C_2), (25,000,C_3), (25,000, C_4) (50,000, M_1), (50,000, M_2), (50,000, M_3), (1,00,000, E_1), (1,00,000, E_2)}

(b)



EXERCISE 1.3

1. **Let $f = \{(x, y) | x, y \in \mathbb{N}$ and $y = 2x\}$ be a relation on ℕ. Find the domain, co-domain and range. Is this relation a function?**

Sol. Given $f = \{(x, y) \,|x, y \in \mathbb{N}$ and $y = 2x\}$

When $x = 1$, $y = 2(1) = 2$

When $x = 2$, $y = 2(2) = 4$

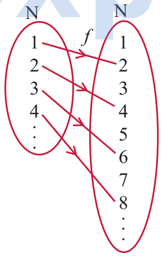
When $x = 3$, $y = 2(3) = 6$

When $x = 4$, $y = 2(4) = 8$ and so on.

R = {(1, 2), (2, 4), (3, 6), (4, 8), (5, 10),...}

Domain of R = {1, 2, 3, 4,...},

Range of R = {2, 4, 6, 8,...}



Since all the elements of domain are related to some elements of co-domain, this relation f is a function.

2. **Let X = {3, 4, 6, 8}. Determine whether the relation ℝ ={(x, f(x)) |x ∈ X, f(x) = x^2 + 1} is a function from X to ℕ ?**

Sol. $x = \{3, 4, 6, 8\}$

R = $((x, f(x))|x \in X, f(x) = x^2 + 1\}$

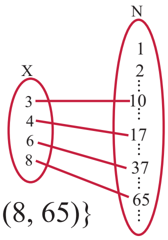
$f(x) = x^2 + 1$

$f(3) = 3^2 + 1 = 10$

$f(4) = 4^2 + 1 = 17$

$f(6) = 6^2 + 1 = 37$

$f(8) = 8^2 + 1 = 65$



R = {(3, 10), (4, 17), (6, 37), (8, 65)}

Yes, R is a function from X to ℕ.

Since all the elements of X are related to some elements of ℕ.

3. **Given the function $f : x \to x^2 - 5x + 6$, evaluate**

(i) $f(-1)$ **(ii) $f(2a)$**

(iii) $f(2)$ **(iv) $f(x-1)$**

Unit Test

Time : 45 Minutes **Marks : 25**

Section - A $5 \times 1 = 5$

1. If n (A × B) = 6 and A = {1, 3} then n (B) is
(A) 1 (B) 2 (C) 3 (D) 6

2. A={a, b, p}, B = {2, 3}, C = {p, q, r, s} then n[(A∪C) × B] is
(A) 8 (B) 20 (C) 12 (D) 16

3. Let A = {1,2,3,4} and B = {4,8,9,10}. A function f:A→ B given by f = {(1, 4),(2, 8),(3, 9),(4, 10)} is a
(A) Many-one function
(B) Identity function
(C) One-to-one function
(D) Into function

4. If g = {(1, 1),(2, 3),(3, 5),(4, 7)} is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
(A) (–1, 2) (B) (2, –1)
(C) (–1, –2) (D) (1,2)

5. The range of the relation R = {(x, x^2) |x is a prime number less than 13} is
(A) {2,3,5,7} (B) {2,3,5,7,11}
(C) {4,9,25,49,121} (D) {1,4,9,25,49,121}

Section - B $5 \times 2 = 10$

1. Let A={1, 2, 3, 4,...,45} and R be the relation defined as "is square of" on A. Write ℝ as a subset of A × A. Also, find the domain and range of ℝ.

2. Let f = {(x, y)|x, y ∈ ℕ and $y = 2x$} be a relation on ℕ. Find the domain, co-domain and range. Is this relation a function?

3. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

4. Let A,B,C ⊆ ℕ and a function f : A→ B be defined by $f(x) = 2x + 1$ and g : B → C be defined by $g(x) = x^2$. Find the range of fog and gof.

5. Let A = {−1, 1}and B ={0, 2}. If the function f :A → B defined by $f(x) = ax + b$ is an onto function? Find a and b.

Section - C $2 \times 5 = 10$

1. Show that the function f : ℕ → ℕ defined by $f(x) = 2x - 1$ is one - one but not onto.

2. Represent the function f ={(1, 2),(2, 2),(3, 2),(4,3), (5,4)} through
(i) an arrow diagram
(ii) a table form
(iii) a graph

Answers

SECTION - A

1. (C) 3
2. (C) 12
3. (C) One-to-one function
4. (B) (2, –1)
5. (C) {4,9,25,49,121}

Section - B

1. Refer Sura's Guide Exercise 1.2; Q.No.2
2. Refer Sura's Guide Exercise 1.3; Q. No.1
3. Refer Sura's Guide Exercise 1.3; Q. No.8
4. Refer Sura's Guide Exercise 1.5; Q. No.5
5. Refer Sura's Guide Exercise 1.4; Q. No.8

Section - C

1. Refer Sura's Guide Exercise 1.4; Q.No.4
2. Refer Sura's Guide Exercise 1.4; Q.No.3

Chapter 2

NUMBERS AND SEQUENCES

FORMULAE TO REMEMBER

- ❑ Euclid's Division Lemma Let a and b ($a > b$) be any two positive integers. Then, there exist unique integers q and r such that $a = bq + r$, $0 \leq r < b$.
- ❑ If a, b are two positive integers with $a > b$ then G.C.D of (a, b) = G.C.D of $(a - b, b)$.
- ❑ Number of terms in A.P., $n = \frac{l-a}{d} + 1$
- ❑ If the sum of three consecutive terms of an A.P is given, then they can be taken as $a - d$, a and $a + d$.
- ❑ If a prime number p divides ab then either p divides a or p divides b, that is p divides atleast one of them.
- ❑ Two integers a and b are congruence modulo n if they differ by an integer multiple of n. That is $b - a = kn$ for some integer k. This can also be written as $a \equiv b \pmod{n}$.
- ❑ A real valued sequence is a function defined on the set of natural numbers and taking real values.
- ❑ Let a and d be real numbers. Then the numbers of the form a, $a + d$, $a+ 2d$, $a + 3d$, $a + 4d$,... is said to form Arithmetic progression.

EXERCISE 2.1

1. Find all positive integers, when divided by 3 leaves remainder 2.

Sol. The positive integers when divided by 3 leaves remainder 2.

By Euclid's division lemma

$$a = bq + r, 0 \le r < b.$$

Here $a = 3q + 2$, where $0 \le q < 3$.

When $q = 0$, $a = 3(0) + 2 = 2$

When $q = 1$, $a = 3(1) + 2 = 5$

When $q = 2$, $a = 3(2) + 2 = 8$

When $q = 3$, $a = 3(3) + 2 = 11$

The required positive numbers are 2, 5, 8, 11,...

2. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.

Sol. By Euclid's division algorithm,

$a = bq + r, \ 0 \le r < b$.

Here $532 = 21q + r$ ... (1)

$\Rightarrow 532 = 21(25) + 7$ ... (2)

$\therefore q = 25$, and $r = 7$

[Comparing (1) and (2)]

$$\therefore \begin{array}{r} 25 \\ 21\overline{)532} \\ 42 \\ \hline 112 \\ 105 \\ \hline 7 \\ \hline \end{array}$$

$\therefore$ Number of completed rows = 25 and the leftover flower pots = 7.

3. Prove that the product of two consecutive positive integers is divisible by 2.

Sol. Let $n - 1$ and n be two consecutive positive integers. Then their product is $(n - 1)n$.

$(n - 1)(n) = n^2 - n$.

We know that any positive integer is of the form $2q$ or $2q + 1$ for some integer q. So, following cases arise.

Case I. When $n = 2q$. In this case, we have

$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q - 1)$

$\Rightarrow n^2 - n = 2r$, where $r = q(2q - 1)$

$\Rightarrow n^2 - n$ is divisible by 2.

Case II : When $n = 2q + 1$. In this case, we have

$n^2 - n = (2q + 1)^2 - (2q + 1)$

$= (2q + 1)(2q + 1 - 1) = 2q(2q + 1)$

$\Rightarrow n^2 - n = 2r$, where $r = q(2q + 1)$.

$\Rightarrow n^2 - n$ is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

Hence it is proved.

4. When the positive integers a, b and c are divided by 13, the respective remainders are 9, 7 and 10. Show that $a + b + c$ is divisible by 13.

Sol. When a is divided by 13, the remainder is 9.

By Euclid's lemma, $a = bq + r, 0 \le r < b$

$\Rightarrow a = 13q + 9$ ... (1)

Similarly when the positive integers b and c are divided by 13, the remainders are 7 and 10.

$\therefore b = 13q + 7$ ... (2)

and $c = 13q + 10$ ... (3)

Adding (1), (2) and (3) we get,

$a + b + c = 13q + 9 + 13q + 7 + 13q + 10$

$= 39q + 26 = 13(3q + 2)$

Which is divisible by 13.

$\therefore a + b + c$ is divisible by 13.

5. Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.

Sol. Let x be any integer.

The square of x is x^2.

Case (i) : Let x be an even integer.

$\Rightarrow x = 2q$ [$\because x$ is even]

Where q is some integer

$\Rightarrow x^2 = (2q)^2 = 4q^2$

$\Rightarrow x^2 = 4(q^2)$

$\Rightarrow x^2$ is divisible by 4.

$\therefore$ When x is an even integer, x^2 is divisible by 4.

$\Rightarrow x^2$ leaves the remainder 0 when divided by 4.

Case (ii) : Let x be an odd integer.

$\therefore x = 2k + 1$ for some integer k.

$\therefore x^2 = (2k + 1)^2 = 4k^2 + 4k + 1$

$= 4k(k + 1) + 1$

$= 4q + 1$ where $q = k(k + 1)$

$\Rightarrow x^2 = 4q + 1$ leaves the remainder 1 when divided by 4.

From Case (i) and Case (ii), the square of any integer leaves the remainder either 0 or 1 when divided by 4.

6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of
(i) 340 and 412 (ii) 867 and 255
(iii) 10224 and 9648
(iv) 84, 90 and 120 **[FRT - 2022]**

Sol. (i) To find the HCF of 340 and 412. Using Euclid's division algorithm.

We get $412 = 340 \times 1 + 72$

The remainder $72 \ne 0$

Again applying Euclid's division algorithm

$340 = 72 \times 4 + 52$

The remainder $52 \neq 0$.

Again applying Euclid's division algorithm

$$72 = 52 \times 1 + 20$$

The remainder $20 \neq 0$.

Again applying Euclid's division algorithm,

$$52 = 20 \times 2 + 12$$

The remainder $12 \neq 0$.

Again applying Euclid's division algorithm.

$$20 = 12 \times 1 + 8$$

The remainder $8 \neq 0$.

Again applying Euclid's division algorithm

$$12 = 8 \times 1 + 4$$

The remainder $4 \neq 0$.

Again applying Euclid's division algorithm

$$8 = 4 \times 2 + 0$$

The remainder is zero.

Therefore HCF of 340 and 412 is 4.

(ii) To find the HCF of 867 and 255, using Euclid's division algorithm.

$$867 = 255 \times 3 + 102$$

The remainder $102 \neq 0$.

Again using Euclid's division algorithm

$$255 = 102 \times 2 + 51$$

The remainder $51 \neq 0$.

Again using Euclid's division algorithm

$$102 = 51 \times 2 + 0$$

The remainder is zero.

Therefore the HCF of 867 and 255 is 51.

(iii) To find HCF 10224 and 9648. Using Euclid's division algorithm.

$$10224 = 9648 \times 1 + 576$$

The remainder $576 \neq 0$.

Again using Euclid's division algorithm

$$9648 = 576 \times 16 + 432$$

Remainder $432 \neq 0$.

Again applying Euclid's division algorithm

$$576 = 432 \times 1 + 144$$

Remainder $144 \neq 0$.

Again using Euclid's division algorithm

$$432 = 144 \times 3 + 0$$

The remainder is zero.

There HCF of 10224 and 9648 is 144.

(iv) To find HCF of 84, 90 and 120.

Using Euclid's division algorithm

$$90 = 84 \times 1 + 6$$

The remainder $6 \neq 0$.

Again using Euclid's division algorithm

$$84 = 6 \times 14 + 0$$

The remainder is zero.

$\therefore$ The HCF of 84 and 90 is 6. To find the HCF of 6 and 120 using Euclid's division algorithm.

$$120 = 6 \times 20 + 0$$

The remainder is zero.

Therefore HCF of 120 and 6 is 6

$\therefore$ HCF of 84, 90 and 120 is 6.

7. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

[Aug. - 2022]

Sol. The required number is the HCF of the numbers.

$$1230 - 12 = 1218,$$
$$1926 - 12 = 1914$$

First we find the HCF of 1218 & 1914 by Euclid's division algorithm.

$$1914 = 1218 \times 1 + 696$$

The remainder $696 \neq 0$.

Again using Euclid's algorithm

$$1218 = 696 \times 1 + 522$$

The remainder $522 \neq 0$.

Again using Euclid's algorithm.

$$696 = 522 \times 1 + 174$$

The remainder $174 \neq 0$.

Again by Euclid's algorithm

$$522 = 174 \times 3 + 0$$

The remainder is zero.

$\therefore$ The HCF of 1218 and 1914 is 174.

$\therefore$ The required number is 174.

8. If d is the Highest Common Factor of 32 and 60, find x and y satisfying $d = 32x + 60y$.

Sol. Applying Euclid's division lemma to 32 and 60, we get

$$60 = 32 \times 1 + 28 \quad \text{...(i)}$$

The remainder is $28 \neq 0$.

Again applying division lemma

$$32 = 28 \times 1 + 4 \quad \text{...(ii)}$$

The remainder $4 \neq 0$.

Again applying division lemma

$$28 = 4 \times 7 + 0 \quad \text{...(iii)}$$

The remainder zero.

$\therefore$ HCF of 32 and 60 is 4.

From (ii), we get

$$32 = 28 \times 1 + 4$$
$$\Rightarrow \quad 4 = 32 - 28 \times 1$$
$$\Rightarrow \quad 4 = 32 - (60 - 32 \times 1) \times 1$$
$$[\because 28 = (60 - 32) \times 1]$$
$$\Rightarrow \quad 4 = 32 - 60 + 32$$
$$\Rightarrow \quad 4 = 32 \times \underline{2} + (-1) \times \underline{60}$$
$$\therefore \quad x = 2 \text{ and } y = -1$$

9. A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?

Sol. Let the positive integer be x.

$$x = 88 \times y + 61$$
$$61 = 11 \times 5 + 6$$
$$(\because 88 \text{ is multiple of } 11)$$

$\therefore$ 6 is the remainder. (When the number is divided by 88 giving the remainder 61 and when divided by 11 giving the remainder 6).

10. Prove that two consecutive positive integers are always coprime.

Sol. Let the two consecutive integers be n and $n + 1$.

Suppose HCF $(n, n + 1) = p$

$\Rightarrow p$ divides n ... (1)

and p divides $(n + 1)$ ... (2)

$\Rightarrow p$ divides $(n + 1 – n)$ [From (2) – (1)]

$\Rightarrow p$ divides 1

There is no number which divides 1 except 1

$\Rightarrow p = 1$ $\therefore$ HCF $(n, n + 1) = 1$.

$\Rightarrow n$ and $(n + 1)$ are Coprime.

EXERCISE 2.2

1. For what values of natural number n, 4^n can end with the digit 6?

Sol. $4^n = (2 \times 2)^n = 2^n \times 2^n$

2 is a factor of 4^n.

So, 4^n is always even and end with 4 and 6.

When n is an even number say 2, 4, 6, 8 then 4^n can end with the digit 6.

Example:

$4^2 = 16$	$4^3 = 64$
$4^4 = 256$	$4^5 = 1,024$
$4^6 = 4,096$	$4^7 = 16,384$
$4^8 = 65,536$	$4^9 = 262,144$

2. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?

Sol. $2^n \times 5^m$

2^n is always even for all values of n.

5^m is always odd and ends with 5 for all values of m.

But $2^n \times 5^m$ is always even and ends in 0.

[$\because$ even number X odd number = even number]

$\therefore$ $2^n \times 5^m$ cannot end with the digit 5 for any values of m. No value of m will satisfy $2^n \times 5^m$ ends in 5.

3. Find the HCF of 252525 and 363636.

Sol. To find the HCF of 252525 and 363636

Using Euclid's Division algorithm

$$363636 = 252525 \times 1 + 111111$$

The remainder $111111 \neq 0$.

$\therefore$ Again by division algorithm

$$252525 = 111111 \times 2 + 30303$$

The remainder $30303 \neq 0$.

$\therefore$ Again by division algorithm.

$$111111 = 30303 \times 3 + 20202$$

The remainder $20202 \neq 0$.

$\therefore$ Again by division algorithm

$$30303 = 20202 \times 1 + 10101$$

The remainder $10101 \neq 0$.

$\therefore$ Again using division algorithm

$$20202 = 10101 \times 2 + 0$$

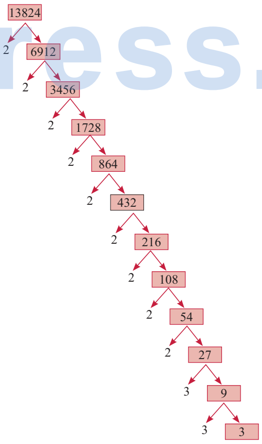
The remainder is 0.

Therefore HCF. of 252525 and 363636 is 10101.

4. If $13824 = 2^a \times 3^b$ then find a and b. **[May - 2022]**

Sol. If $13824 = 2^a \times 3^b$

Using the prime factorisation tree



$$13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= 2^9 \times 3^3 = 2^a \times 3^b$$
$$\therefore \quad a = 9, b = 3.$$

5. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4. **[FRT - 2022]**

Sol. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

p_1, p_2, p_3, p_4 are primes in ascending order, x_1, x_2, x_3, x_4 are integers.

Using prime factorisation tree.

EXERCISE 2.5

1. **Check whether the following sequences are in A.P.**
(i) $a-3, a-5, a-7, ...$ **(ii)** $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...$
(iii) 9, 13, 17, 21, 25,... **(iv)** $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, ...$
(v) 1, –1, 1, –1, 1, –1,...

Sol. To prove it is an A.P, we have to show $d = t_2 - t_1 = t_3 - t_2$.

(i) $\underset{t_1}{a-3}, \underset{t_2}{a-5}, \underset{t_3}{a-7}, ...$

$$d = t_2 - t_1 = a-5-(a-3) = \cancel{a}-5-\cancel{a}+3 = -2$$
$$d = t_3 - t_2 = a-7-(a-5) = \cancel{a}-7-\cancel{a}+5 = -2$$
$$\therefore d = -2$$

$\therefore a-3, a-5, a-7, ...$ is an A.P.

(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...$

$$d = t_2 - t_1 \Rightarrow \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$$
$$d = t_3 - t_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = \frac{-1}{12}$$
$$\frac{-1}{6} \neq \frac{-1}{12}$$
$$\Rightarrow t_2 - t_1 \neq t_3 - t_2$$

$\therefore$ The given sequence is not an A.P.

(iii) 9, 13, 17, 21, 25, ...

$$d = t_2 - t_1 = 13 - 9 = 4$$
$$d = t_3 - t_2 = 17 - 13 = 4$$
$$4 = 4$$

$\therefore$ The given sequence is an A.P.

(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, ...$

$$d = t_2 - t_1 = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$$
$$d = t_3 - t_2 = \frac{1}{3} - 0 = \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3}$$

$\therefore \frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, ...$ is an A.P.

(v) 1, –1, 1, –1, 1, –1, ...

$$d = t_2 - t_1 = -1-1 = -2$$
$$d = t_3 - t_2 = 1-(-1) = 2$$
$$-2 \neq 2$$

$\therefore$ 1, –1, 1, –1, ... is not an A.P.

2. **First term a and common difference d are given below. Find the corresponding A.P.**
(i) $a = 5, d = 6$ **(ii)** $a = 7, d = -5$
(iii) $a = \frac{3}{4}, d = \frac{1}{2}$

Sol. (i) $a = 5, d = 6$

A.P is $a, a+d, a+2d, ...$
$= 5, 5+6, 5+2\times 6, ...$
$= 5, 11, 17, ...$

(ii) $a = 7, d = -5$

A.P is $a, a+d, a+2d,...$
$= 7, 7+(-5), 7+2(-5), ...$
$= 7, 2, -3, ...$

(iii) $a = \frac{3}{4}, d = \frac{1}{2}$

A.P is $a, a+d, a+2d, ...$

$$= \frac{3}{4}, \frac{3}{4}+\frac{1}{2}, \frac{3}{4}+\cancel{2}\left(\frac{1}{\cancel{2}}\right), ... = \frac{3}{4}, \frac{3+2}{4}, \frac{3+4}{4}, ...$$

A.P is $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, ...$

3. **Find the first term and common difference of the Arithmetic Progressions whose n^{th} terms are given below**
(i) $t_n = -3+2n$ **(ii)** $t_n = 4-7n$

Sol. (i)

$$a = t_1 = -3+2(1) = -3+2 = -1$$
$$\text{Here } t_2 = -3+2(2) = -3+4 = 1$$
$$\therefore d = t_2 - t_1 = 1-(-1) = 2$$

First term = –1, common difference = 2

(ii)

$$a = t_1 = 4-7(1) = 4-7 = -3$$
$$d = t_2 - t_1$$
$$\text{Here } t_2 = 4-7(2) = 4-14 = -10$$
$$\therefore d = t_2 - t_1 = -10-(-3) = -7$$

First term = – 3 , common difference = – 7

4. **Find the 19th term of an A.P. –11, –15, –19,...**

Sol. Given A.P is –11, –15, –19, ... **[FRT & Aug. - 2022]**

$$\text{Here } a = -11$$
$$d = t_2 - t_1 = -15-(-11)$$
$$= -15+11 = -4$$
$$n = 19$$
$$\therefore t_n = a+(n-1)d$$

$$\begin{aligned} t_{19} &= -11 + (19-1)(-4) \\ &= -11 + 18 \times -4 = -11 - 72 \\ &= -83 \end{aligned}$$

Therefore 19$^{\text{th}}$ term is – 83.

5. Which term of an A.P. 16, 11, 6, 1,... is –54 ?

Sol. Given A.P is 16, 11, 6, 1, ... **[Qy. - 2019; May - 2022]**

It is given that

$$\begin{aligned} t_n &= -54 \\ \text{Here } a = 16,\ d &= t_2 - t_1 = 11 - 16 = -5 \\ \therefore t_n &= a + (n-1)d \\ -54 &= 16 + (n-1)(-5) \\ -54 &= 16 - 5n + 5 \\ 21 - 5n &= -54 \\ -5n &= -54 - 21 \\ -5n &= -75 \\ n &= \frac{75}{5} = 15 \end{aligned}$$

∴ 15th term is –54.

6. Find the middle term(s) of an A.P. 9, 15, 21, 27,…,183. **[PTA - 1]**

Sol. Given A.P is 9, 15, 21, 27,..., 183

No. of terms in an A.P. is

$$n = \frac{l-a}{d} + 1$$

$$\begin{aligned} \text{Here } a = 9, l = 183, d &= 15 - 9 = 6 \\ \therefore n &= \frac{183-9}{6} + 1 = \frac{174}{6} + 1 \\ &= 29 + 1 = 30 \end{aligned}$$

∴ No. of terms = 30. The middle must be 15th term and 16th term.

[∵ $n = 30$ is even, the middle terms are $t_{\frac{30}{2}}$ and $t_{\frac{30}{2}} + 1$]

$$\begin{aligned} \therefore t_{15} &= a + (n-1)d \\ &= 9 + 14 \times 6 \\ &= 9 + 84 = 93 \\ t_{16} &= a + 15d \\ &= 9 + 15 \times 6 = 9 + 90 = 99 \end{aligned}$$

∴ The middle terms are 93, 99.

7. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero. **[Aug. - 2022]**

Sol. Let a and d be the first term and common difference of the A.P.

$$\begin{aligned} \text{Given that } 9t_9 &= 15t_{15} \\ \text{Since } t_n &= a + (n-1)d, \text{ we get} \\ 9(a + 8d) &= 15(a + 14d) \\ 9a + 72d &= 15a + 210d \\ 9a + 72d - 15a - 210d &= 0 \\ \Rightarrow -6a - 138d &= 0 \\ \Rightarrow 6a + 138d &= 0 \quad [\text{Dividing by } -1] \quad \dots (1) \end{aligned}$$

$$\begin{aligned} \text{To prove that } 6t_{24} &= 0 \\ \text{Consider } 6t_{24} &= 6(a + 23d) \\ &= 6a + 138d = 0 \quad [\text{by (1)}] \\ \Rightarrow 6t_{24} &= 0 \end{aligned}$$

⇒ 6 times 24$^{\text{th}}$ term is 0. Hence proved.

8. If 3 + *k*, 18 – *k*, 5*k* + 1 are in A.P. then find *k*. **[PTA - 3 & 5; Sep. - 2021]**

Sol. Given 3 + k, 18 – k, 5k + 1 are in A.P

⇒ Common difference is same

$$\begin{aligned} \Rightarrow t_2 - t_1 &= t_3 - t_2 \\ \Rightarrow 18 - k - (3 + k) &= 5k + 1 - (18 - k) \\ \Rightarrow 18 - k - 3 - k &= 5k + 1 - 18 + k \\ \Rightarrow 15 - 2k &= 6k - 17 \\ \Rightarrow 15 + 17 &= 6k + 2k \Rightarrow 32 = 8k \\ \Rightarrow k &= \frac{32}{8} = 4 \qquad \therefore k = 4 \end{aligned}$$

9. Find *x*, *y* and *z*, given that the numbers *x*, 10, *y*, 24, *z* are in A.P.

Sol. Given A.P is x, 10, y, 24, z, ...

⇒ Common difference is same.

$$\begin{aligned} d &= t_2 - t_1 = 10 - x & \dots(1) \\ &= t_3 - t_2 = y - 10 & \dots(2) \\ &= t_4 - t_3 = 24 - y & \dots(3) \\ &= t_5 - t_4 = z - 24 & \dots(4) \end{aligned}$$

(2) and (3)

$$\begin{aligned} \Rightarrow y - 10 &= 24 - y \\ 2y &= 24 + 10 = 34 \\ y &= \frac{34}{2} = 17 \end{aligned}$$

(1) and (2)

$$\begin{aligned} \Rightarrow 10 - x &= y - 10 \\ 10 - x &= 17 - 10 = 7 \\ -x &= 7 - 10 \\ \cancel{-}x &= \cancel{-}3 \Rightarrow x = 3. \end{aligned}$$

From (3) and (4)

$$\begin{aligned} 24 - y &= z - 24 \\ 24 - 17 &= z - 24 \\ 7 &= z - 24 \\ \therefore z &= 7 + 24 = 31 \end{aligned}$$

∴ The required values are

$$\begin{aligned} x &= 3 \\ y &= 17 \\ z &= 31 \end{aligned}$$

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Sol. Since there are 20 seats in the first row, **[PTA - 4]**

$$\text{let } a = 20 = t_1$$

Each successive row contains 2 additional seats than its previous row.

$$t_2 = t_1 + 2 = 20 + 2 = 22$$

$$\therefore \quad t_3 = t_2 + 2 = 22 + 2 = 24$$

and so on

$\therefore$ The A.P is 20, 22, 24, 26, ...

Since there are 30 rows, $n = 30$

$$\begin{aligned} \text{Here } a &= 20, d = t_2 - t_1 \\ &= 22 - 20 = 2 \\ \therefore t_{30} &= 20 + 29(2) \\ & \quad [\because t_n = a + (n - 1)\, d] \\ &= 20 + 58 = 78 \end{aligned}$$

$\therefore$ There will be 78 seats in the last row.

11. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms. **[Sep. - 2021]**

Sol. Let the three consecutive terms be $a - d, a, a + d$

$$\begin{aligned} \text{Their sum} &= a - \cancel{d} + a + a + \cancel{d} = 27 \\ 3a &= 27 \\ a &= \frac{27}{3} = 9 \\ \text{Their product} &= (a - d)(a)(a + d) = 288 \\ &= a(a^2 - d^2) \\ \Rightarrow \quad 9(9^2 - d^2) &= 288 \\ \Rightarrow \quad \cancel{9}(81 - d^2) &= \cancel{288}^{32} \\ 81 - d^2 &= 32 \\ -d^2 &= 32 - 81 \\ -d^2 &= -49 \\ d^2 &= 49 \quad \Rightarrow d = \pm 7 \end{aligned}$$

$\therefore$ The three terms are if $a = 9, d = 7$

$$\begin{aligned} a - d, a, a + d &= 9 - 7, 9 + 7 \\ \text{A.P} &= 2, 9, 16 \\ \text{if } a &= 9, d = -7, \\ \text{A.P} &= 9 - (-7), 9, 9 + (-7) \\ &= 16, 9, 2 \end{aligned}$$

12. The ratio of 6th and 8th term of an A.P is 7:9. Find the ratio of 9th term to 13th term. **[May - 2022]**

Sol. **[FRT - 2022]**

$$\frac{t_6}{t_8} = \frac{7}{9}$$

$$\frac{a + 5d}{a + 7d} = \frac{7}{9} \quad [\because t_n = a + (n - 1)d]$$

$$\begin{aligned} 9a + 45d &= 7a + 49d \\ 9a + 45d - 7a - 49d &= 0 \\ 2a - 4d &= 0 \Rightarrow \quad 2a = 4d \\ a &= 2d \end{aligned}$$

Substitute $a = 2d$ in

$$\begin{aligned} \frac{t_9}{t_{13}} &= \frac{a + 8d}{a + 12d} = \frac{2d + 8d}{2d + 12d} \\ &= \frac{10\cancel{d}}{14\cancel{d}} = \frac{5}{7} \\ \therefore t_9 : t_{13} &= 5:7. \end{aligned}$$

13. In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0° C and the sum of the temperatures from Wednesday to Friday is 18° C. Find the temperature on each of the five days.

Sol. Let the five days temperature be $(a - d), a, a + d, a + 2d, a + 3d.$

The three days sum $= a - \cancel{d} + a + a + \cancel{d} = 0$

$\Rightarrow 3a = 0 \Rightarrow a = 0.$ (given)

$a + d + a + 2d + a + 3d = 18$

$[\because$ Sum of the last 3 days = 18° C]

$$\begin{aligned} 3a + 6d &= 18 \\ 3(0) + 6d &= 18 \\ 6d &= 18 \\ d &= \frac{18}{6} = 3 \end{aligned}$$

$\therefore$ The temperature of each five days is $a - d, a, a + d, a + 2d, a + 3d$

$0 - 3, 0, 0 + 3, 0 + 2(3), 0 + 3(3)$

$= -3°C, 0°C, 3°C, 6°C, 9°C$

14. Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.

Sol.

	Yearly Salary	Yearly expenses	Yearly savings
1st year	15000	13000	2000
2nd year	16500	13900	2600
3rd year	18000	14800	3200

We find that the yearly savings is in A.P with $a_1 = 2000$ and $d = 600.$

We are required to find how many years are required to save 20,000 a year .....

$$\text{Given } a_n = 20{,}000$$

Time : 45 Minutes

Unit Test

Marks : 25

SECTION - A (5 × 1 = 5)

1. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are

   (A) 0, 1, 8 (B) 1, 4, 8

   (C) 0, 1, 3 (D) 1, 3, 5

2. The sum of the exponents of the prime factors in the prime factorization of 1729 is

   (A) 1 (B) 2 (C) 3 (D) 4

3. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P

   (A) 4551 (B) 10091

   (C) 7881 (D) 13531

4. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120?

   (A) 6 (B) 7 (C) 8 (D) 9

5. The value of $(1^3 + 2^3 + 3^3 + ... + 15^3) - (1 + 2 + 3 + ... + 15)$ is

   (A) 14400 (B) 14200

   (C) 14280 (D) 14520

SECTION - B (5 × 2 = 10)

1. Find all positive integers which when divided by 3 leaves remainder 2.
2. For what values of natural number n, $4n$ can end with the digit 6?
3. What is the time 100 hours after 7 a.m.?
4. Find the 19th term of an A.P. –11, –15, –19,...
5. The sum of the squares of the first n natural numbers is 285, while the sum of their cubes is 2025. Find the value of n.

SECTION - C (2 × 5 = 10)

1. Sivamani is attending an interview for a job and the company gave two offers to him.

   Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.

   Offer B: ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.

   What is his salary in the 4^{th} year with respect to the offers A and B?

2. Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs ₹2 to mail one letter, find the amount spent on postage when 8th set of letters is mailed.

Answers

SECTION - A

1. (A) 0, 1, 8
2. (A) 1
3. (C) 7881
4. (C) 8
5. (C) 14280

SECTION - B

1. Refer Sura's Guide Exercise 2.1, Q.No.1
2. Refer Sura's Guide Exercise 2.1, Q.No.1
3. Refer Sura's Guide Exercise 2.3, Q.No.5
4. Refer Sura's Guide Exercise 2.5 Q.No.4
5. Refer Sura's Guide Exercise 2.9 Q.No.

SECTION - C

1. Refer Sura's Guide Exercise No.2.7, Q.No.11
2. Refer Sura's Guide Exercise 2.8 Q.No.8

* * *

Chapter

3

ALGEBRA

FORMULAE TO REMEMBER

- ☐ $f(x) \times g(x) = \text{LCM } [f(x), g(x)] \times \text{GCD } [f(x), g(x)]$
- ☐ Roots of the quadratic equation, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- ☐ General Form of an equation : x^2 – (sum of the roots)x + product of the roots = 0

EXERCISE 3.1

1. Solve the following system of linear equations in three variables

(i) $x + y + z = 5$; $2x - y + z = 9$; $x - 2y + 3z = 16$

[PTA - 5; Hy. - 2019; Sep. - 2021]

(ii) $\frac{1}{x} - \frac{2}{y} + 4 = 0$; $\frac{1}{y} - \frac{1}{z} + 1 = 0$; $\frac{2}{z} + \frac{3}{x} = 14$

(iii) $x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$

Sol. Let

(i)

$$\begin{aligned} x + y + z &= 5 & \text{...(1)} \\ 2x - y + z &= 9 & \text{...(2)} \\ x - 2y + 3z &= 16 & \text{...(3)} \end{aligned}$$

$$\begin{aligned} (1) + (2) \Rightarrow \quad x + \cancel{y} + z &= 5 \\ 2x - \cancel{y} + z &= 5 \\ \hline \text{Adding, } 3x + 2z &= 14 & \text{...(4)} \end{aligned}$$

$$\begin{aligned} (2) \times 2 \quad \Rightarrow 4x - \cancel{2y} + 2z &= 18 \\ (3) \quad \Rightarrow \underset{(-)}{x} \underset{(+)}{- \cancel{2y}} \underset{(-)}{+ 3z} &= \underset{(-)}{16} \\ \hline \text{Subtracting} \quad 3x - z &= 2 & \text{...(5)} \end{aligned}$$

$$\begin{aligned} (4) - (5) \Rightarrow \quad 3x + 2z &= 14 \\ \Rightarrow \quad \underset{(-)}{3x} \underset{(+)}{-z} &= \underset{(-)}{2} \\ \hline \text{Subtracting,} \quad 3z &= 12 \\ z &= 4 \end{aligned}$$

Substitute $z = 4$ in (4)

$$\begin{aligned} 3x + 2(4) &= 14 \\ 3x + 8 &= 14 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

Substitute $x = 2, z = 4$ in (1)

$$2 + y + 4 = 5 \Rightarrow y = -1$$

$$\boxed{x = 2, \; y = -1, \quad z = 4}$$

(ii)

$$\begin{aligned} \frac{1}{x} - \frac{2}{y} + 4 &= 0 & \text{...(1)} \\ \frac{1}{y} - \frac{1}{z} + 1 &= 0 & \text{...(2)} \\ \frac{2}{z} + \frac{3}{x} &= 14 & \text{...(3)} \end{aligned}$$

$$\begin{aligned} \text{Put } \frac{1}{x} &= a \\ \frac{1}{y} &= b \\ \frac{1}{z} &= c \text{ in (1), (2) \& (3)} \end{aligned}$$

$$\begin{aligned} a - 2b + 4 = 0 \; &\Rightarrow a - 2b = -4 & \text{...(1)} \\ b - c + 1 \;\; = 0 \; &\Rightarrow b - c \;\; = -1 & \text{...(2)} \\ 2c + 3a \;\; = 14 &\Rightarrow 2c + 3a = 14 & \text{...(3)} \end{aligned}$$

$$\begin{aligned} (1) \Rightarrow \quad a - \cancel{2b} &= -4 \\ (2) \times 2 \Rightarrow -2c + \cancel{2b} &= -2 \\ \hline a - 2c &= -6 & \text{...(4)} \\ \text{Adding, } 3a + 2c &= 14 \\ \hline (4) + (3) \Rightarrow \quad 4a &= 8 \\ a &= 2 \end{aligned}$$

Substitute $a = 2$ in (1), we get

$$\begin{aligned} 2 - 2b &= -4 \\ -2b &= -6 \\ b &= 3 \end{aligned}$$

Substitute $b = 3$ in (2), we get

$$\begin{aligned} 3 - c &= -1 \\ -c &= -4 \Rightarrow c = 4 \end{aligned}$$

$$\begin{aligned} a = \frac{1}{x} = 2 \Rightarrow x &= \frac{1}{2} \\ b = \frac{1}{y} = 3 \Rightarrow y &= \frac{1}{3} \\ c = \frac{1}{z} = 4 \Rightarrow z &= \frac{1}{4} \end{aligned}$$

$$\boxed{x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{4}} \quad \text{Solution set } \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$$

(iii) Given equations are

I II III IV

$$x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$$

$$\begin{aligned} \text{Consider } x + 20 &= \frac{3y}{2} + 10 \quad \text{[From I \& II]} \\ \Rightarrow \qquad x &= \frac{3y}{2} + 10 - 20 \\ \Rightarrow \qquad x &= \frac{3y}{2} - 10 \end{aligned}$$

Multiply by 2, we get, $2x = 3y - 20$

$$\Rightarrow \qquad 2x - 3y = -20$$

From [I and III]

$$\begin{aligned} \text{Now, } x + 20 &= 2z + 5 \\ \Rightarrow \qquad x - 2z &= 5 - 20 \\ \Rightarrow \qquad x - 2z &= -15 & \text{... (2)} \end{aligned}$$

From [I and IV]

$$\begin{aligned} \text{Also} \quad x + 20 &= 110 - (y + z) \\ \text{Also} \quad x + 20 &= 110 - y - z \\ \Rightarrow \qquad x + y + z &= 110 - 20 \\ \Rightarrow \qquad x + y + z &= 90 & \text{... (3)} \\ 2 \times (3) \Rightarrow \quad 2x - 2y + 2z &= 180 \\ (2) \Rightarrow \qquad x - 2z &= -15 \\ \hline \text{Adding} \qquad 3x + 2y &= 165 & \text{... (4)} \end{aligned}$$

Consider equations (1) and (4)

$$\begin{aligned}
(1)\times 3 \Rightarrow \quad 6x - 9y &= -60\\
(4)\times 2 \Rightarrow \quad \overset{(-)}{6x} + \overset{(-)}{4y} &\overset{(-)}{=} 330
\end{aligned}$$

$$\text{Subtracting,} \quad -13y = -390$$

$$\Rightarrow \quad y = \frac{-390}{-13} = 30$$

$$\Rightarrow \quad \boxed{y = 30}$$

Substituting $y = 30$ in (1), we get

$$\begin{aligned}
\Rightarrow \quad 2x - 3\,(30) &= -20\\
2x - 90 &= -20\\
\Rightarrow \quad 2x &= -20 + 90 = 70\\
\Rightarrow \quad x &= \frac{70}{2} = 35
\end{aligned}$$

$$\boxed{\therefore x = 35}$$

Substituting $x = 35$ in (2) we get,

$$\begin{aligned}
35 - 2z &= -15\\
\Rightarrow \quad 35 + 15 &= 2z\\
\Rightarrow \quad 50 &= 2z\\
\Rightarrow \quad z &= \frac{50}{2}
\end{aligned}$$

$$\boxed{\therefore z = 25}$$

Solution set is {35, 30, 25}

Hence, the system has unique solution

2. Discuss the nature of solutions of the following system of equations

(i) $x + 2y - z = 6$; $-3x - 2y + 5z = -12$; $x - 2z = 3$

(ii) $2y + z = 3(-x + 1)$; $-x + 3y - z = -4$

$3x + 2y + z = -\frac{1}{2}$

(iii) $\frac{y+z}{4} = \frac{z+x}{3} = \frac{x+y}{2}$; $x + y + z = 27$

Sol. (i)

$$\begin{aligned}
x + 2y - z &= 6 && \text{...(1)}\\
-3x - 2y + 5z &= -12 && \text{...(2)}\\
x - 2z &= 3 && \text{...(3)}
\end{aligned}$$

Consider (1) and (2)

$$\begin{aligned}
x + \cancel{2y} - z &= 6 && \text{...(1)}\\
-3x - \cancel{2y} + 5z &= -12 && \text{...(2)}\\
\text{Adding,} \quad -2x + 4z &= -6\\
-x + 2z &= -3 && \text{[Divided by 2]}\\
x - \cancel{2z} &= 3\\
(3) \quad \underset{(-)}{x} - \underset{(+)}{\cancel{2z}} &= \underset{(-)}{3}\\
0 &= 0
\end{aligned}$$

We see that the system has an infinite number of solutions.

(ii) Given equations are $2y + z = 3\,(-x + 1)$; $-x + 3y - z = -4$, $3x + 2y + z = -\frac{1}{2}$

$$\begin{aligned}
\text{Consider} \quad 2y + z &= 3(-x + 1)\\
\Rightarrow \quad 2y + z &= -3x + 3\\
\Rightarrow \quad 3x + 2y + z &= 3 && \text{... (1)}\\
\text{Now,} \quad -x + 3y - z &= -4 && \text{... (2)}\\
\text{Also,} \quad 3x + 2y + z &= -\frac{1}{2}
\end{aligned}$$

Multiplying by 2 we get,

$$6x + 4y + 2z = -1 \quad \text{... (3)}$$

Consider (1) and (2)

$$\begin{aligned}
(1) \Rightarrow \quad 3x + 2y + z &= 3\\
(2)\times 3 \Rightarrow \quad -3x + 9y - 3z &= -12\\
\text{Adding} \quad 11y - 2z &= -9 && \text{... (4)}
\end{aligned}$$

Consider (2) and (3)

$$\begin{aligned}
(2)\times 6 \Rightarrow -6x + 18y - 6z &= -24\\
(3) \quad \Rightarrow \quad 6x + 4y + 2z &= -1\\
\text{Adding} \quad 22y - 4z &= -25 && \text{... (5)}
\end{aligned}$$

$$\begin{aligned}
\text{Now } (4)\times 2 \Rightarrow \quad 22y - 4z &= -18\\
(5) \quad \Rightarrow \quad \underset{(-)}{22y} - \underset{(+)}{4z} &= \underset{(+)}{-25}\\
\text{Subtracting} \quad 0 &= -7
\end{aligned}$$

Since we have got a false equation, the system is inconsistent and has no solution.

(iii) Given equation are

$$\underset{\text{I}}{\frac{y+z}{4}} = \underset{\text{II}}{\frac{z+x}{3}} = \underset{\text{III}}{\frac{x+y}{2}} \text{ and } x + y + z = 27$$

$$\text{Let } x + y + z = 27 \quad \text{.... (1)}$$

From I and II, we get

$$\frac{y+z}{4} = \frac{z+x}{3}$$

Cross multiplying we get, $3\,(y + z) = 4\,(z + x)$

$$\begin{aligned}
\Rightarrow \quad 3y + 3z &= 4z + 4x\\
\Rightarrow \quad 3y + 3z - 4z - 4x &= 0\\
\Rightarrow \quad -4x + 3y - z &= 0 && \text{... (2)}
\end{aligned}$$

From II and II, we get,

$$\frac{z+x}{3} = \frac{x+y}{2}$$

Cross multiplying we get, $2\,(z + x) = 3\,(x + y)$

$$\begin{aligned}
\Rightarrow \quad 2z + 2x &= 3x + 3y\\
\Rightarrow \quad 2z + 2x - 3x - 3y &= 0\\
\Rightarrow \quad -x - 3y + 2z &= 0 && \text{... (3)}
\end{aligned}$$

Consider (1) and (3),

$$(1) \Rightarrow \quad x + y + z = 27$$
$$(3) \Rightarrow \quad -x - 3y + 2z = 0$$

$$\text{Adding,} \quad -2y + 3z = 27 \qquad \ldots (4)$$

Consider (1) and (2)

$$(1) \times 4 \Rightarrow 4x + 4y + 4z = 108$$
$$(2) \Rightarrow \quad -4x + 3y - z = 0$$

$$\text{Adding,} \quad 7y + 3z = 108 \qquad \ldots (5)$$

Now, Consider (4) and (5)

$$(4) \Rightarrow \quad -2y + 3z = 27$$
$$(5) \Rightarrow \quad \overset{(-)}{7y} + \overset{(-)}{3z} = \overset{(-)}{108}$$

$$\text{Subtracting,} \quad -9y = -81 \quad \Rightarrow y = \frac{-81}{-9} = 9$$

$$\boxed{\therefore y = 9}$$

Substituting $y = 9$ in (4) we get,

$$-2(9) + 3z = 27$$
$$\Rightarrow \quad -18 + 3z = 27$$
$$\Rightarrow \quad 3z = 27 + 18 = 45$$
$$\Rightarrow \quad z = \frac{45}{3} = 15$$
$$\Rightarrow \quad \boxed{z = 15}$$

Substituting y and z in (1) we get,

$$x + 9 + 15 = 27$$
$$\Rightarrow \quad x + 24 = 27$$
$$\Rightarrow \quad x = 27 - 24 = 3$$
$$\boxed{\therefore x = 3}$$

Solution set is {3, 9, 15}

Hence th system has unique solution.

3. Vani, her father and her grand father have an average age of 53. One-half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now ? [PTA - 2]

Sol. Let Vani's age be x, her father's age be y and her grand father's age be z.

Given average age of them is 53

$$\Rightarrow \quad \frac{x + y + z}{3} = 53$$
$$\Rightarrow \quad x + y + z = 3(53) = 159 \qquad \ldots(1)$$

$$\text{Also,} \quad \frac{z}{2} + \frac{y}{3} + \frac{x}{4} = 65$$

$$\frac{6z + 4y + 3x}{12} = 65$$

[∵ LCM of (2, 3, 4) is 12]

$$\Rightarrow \quad 3x + 4y + 6z = 65(12) = 780 \qquad \ldots(2)$$

Four years ago, grand father's age was $(z - 4)$ and Vani's age was $(x - 4)$.

$$\therefore z - 4 = 4(x - 4)$$
$$\Rightarrow \quad z - 4 = 4x - 16$$
$$\Rightarrow \quad 4x - z = -4 + 16$$
$$\Rightarrow \quad 4x - z = 12 \qquad \ldots (3)$$

Consider (1) and (2)

$$(1) \times 4 \quad \Rightarrow 4x + 4y + 4z = 636$$
$$(2) \quad \Rightarrow \overset{(-)}{3x} + \overset{(-)}{4y} + \overset{(-)}{6z} = \overset{(-)}{780}$$

$$\text{Subtracting,} \quad x - \cancel{2z} = -144 \qquad \ldots(4)$$

$$(3) \times 2 \quad \Rightarrow \overset{(-)}{8x} - \overset{(+)}{\cancel{2z}} = \overset{(-)}{24}$$

$$\text{Subtracting,} \quad -7x = -168$$

$$\Rightarrow \quad x = \frac{-168}{-7} = 24$$

Substituting $x = 24$ in (3) we get,

$$4(24) - z = 12$$
$$\Rightarrow \quad 96 - 12 = z$$
$$\Rightarrow \quad \boxed{z = 84}$$

Substituting the values of x and z in (1) we get,

$$\Rightarrow \quad 24 + y + 84 = 159$$
$$\Rightarrow \quad 108 + y = 159$$
$$\Rightarrow \quad y = 159 - 108 = 51$$

Thus, Vani's age is 24, her father's age is 51 and his grand father's age is 84.

4. The sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to the units digit, then find the original three digit number?

Sol. Let the number be $100x + 10y + z$.

[$z \rightarrow$ unit place
$y \rightarrow$ Tens place
$x \rightarrow$ hundred place]

Reversed number be $100z + 10y + x$.

$$x + y + z = 11 \qquad \ldots(1)$$
$$100z + 10y + x = 5(100x + 10y + z) + 46$$
$$100z + 10y + x = 500x + 50y + 5z + 46$$
$$499x + 40y - 95z = -46 \qquad \ldots(2)$$
$$x + 2y = z$$
$$x + 2y - z = 0 \qquad \ldots(3)$$
$$x + y + z = 11 \qquad \ldots(1)$$
$$x + 2y - z = 0 \qquad \ldots(3)$$

(1) + (3)

$$\Rightarrow \quad 2x + 3y = 11 \qquad \ldots(4)$$

$$\begin{array}{lrcl}
(2) \Rightarrow & 499x + 40y - 95z & = & -46 \\
 & \overset{(-)}{} \quad \overset{(-)}{} \qquad \overset{(+)}{} & & \\
(3) \times 95 \Rightarrow & 95x + 190y - 95z & = & 0 \\ \hline
 & 404x - 150y & = & -46 \quad \ldots(5) \\
(4) \times 50 \Rightarrow & 100x + 150y & = & 550 \\
(5) \Rightarrow & 404x - 150y & = & -46 \\ \hline
 & 504x & = & 504 \\
 & x & = & 1
\end{array}$$

Sub. $x = 1$ in (4)

$$\begin{aligned} 2 \times 1 + 3y &= 11 \\ 3y &= 9 \\ y &= 3 \end{aligned}$$

Sub. $x = 1, y = 3$ in (1)

$$\begin{aligned} 1 + 3 + z &= 11 \\ z &= 7 \end{aligned}$$

$\therefore$ The number is $xyz = 137$

5. There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹105. When first 2 sorts are interchanged in their numbers its value will be increased by ₹20. Find the number of currencies in each sort. **[FRT - 2022]**

Sol. Let x, y and z be number of currency pieces of 5, 10, 20 rupees respectively.

$$x + y + z = 12 \quad \ldots(1)$$

[$\because$ There are 12 pieces]

$$5x + 10y + 20z = 105 \quad \ldots(2)$$

Since x and y are inter changed

$$10x + 5y + 20z = 105 + 20 = 125 \quad \ldots(3)$$

$$\begin{array}{lrcl}
(1) \times 5 \Rightarrow & 5x + 5y + 5z & = & 60 \\
 & (-) \quad (-) \quad (-) & & (-) \\
(2) \Rightarrow & 5x + 10y + 20z & = & 105 \\ \hline
 & -5y - 15z & = & -45 \quad \ldots(4) \\
(2) \times 2 \Rightarrow & 10x + 20y + 40z & = & 210 \\
 & (-) \quad (-) \quad (-) & & (-) \\
(3) \Rightarrow & 10x + 5y + 20z & = & 125 \\ \hline
 & 15y + 20z & = & 85 \quad \ldots(5) \\
(4) \times 3 \Rightarrow & -\cancel{15y} - 45z & = & -135 \\
(5) & \cancel{15y} + 20z & = & 85 \\ \hline
 & -25z & = & -50 \\
 & z & = & 2
\end{array}$$

Sub, $z = 2$ in (5), we get

$$\begin{aligned} 15y + 20 \times 2 &= 85 \\ 15y &= 45 \\ y &= 3 \end{aligned}$$

Sub, $y = 3, z = 2$ in (1)

$$\begin{aligned} x + y + z &= 12 \\ x &= 7 \end{aligned}$$

$\therefore$ Number of 5 rupees currency = 7
Number of 10 rupees currency = 3
Number of 20 rupees currency = 2

EXERCISE 3.2

1. Find the GCD of the given polynomials

(i) $x^4 + 3x^3 - x - 3, x^3 + x^2 - 5x + 3$ **[Sep. - 2020; FRT - 2022]**

(ii) $x^4 - 1, x^3 - 11x^2 + x - 11$

(iii) $3x^4 + 6x^3 - 12x^2 - 24x, 4x^4 + 14x^3 + 8x^2 - 8x$ **[Qy. - 2019]**

(iv) $3x^3 + 3x^2 + 3x + 3, 6x^3 + 12x^2 + 6x + 12$

Sol. **(i) $x^4 + 3x^3 - x - 3, x^3 + x^2 - 5x + 3$**

Let $f(x) = x^4 + 3x^3 - x - 3$

$g(x) = x^3 + x^2 - 5x + 3$

$$\begin{array}{r|l}
 & x + 2 \\ \hline
x^3 + x^2 - 5x + 3 & \cancel{x^4} + 3x^3 + 0x^2 - 1x - 3 \\
 & \cancel{x^4} + x^3 - 5x^2 + 3x \\
 & (-) \quad (-) \quad (+) \quad (-) \\ \hline
 & \cancel{2x^3} + 5x^2 - 4x - 3 \\
 & \cancel{2x^3} + 2x^2 - 10x + 6 \\
 & (-) \quad (-) \quad (+) \quad (-) \\ \hline
 & 3x^2 + 6x - 9
\end{array}$$

$= 3(x^2 + 2x - 3) \neq 0$

Note that 3 is not a divisor of $g(x)$. Now dividing $g(x) = x^3 + x^2 - 5x + 3$ by the new remainder $x^2 + 2x - 3$ (leaving the constant factor 3) we get

$$\begin{array}{r|l}
 & x - 1 \\ \hline
x^2 + 2x - 3 & x^3 + x^2 - 5x + 3 \\
 & x^3 + 2x^2 - 3x \\
 & (-) \quad (-) \quad (+) \\ \hline
 & \cancel{-x^2} - \cancel{2x} \cancel{+3} \\
 & \cancel{-x^2} - \cancel{2x} \cancel{+3} \\
 & (+) \quad (+) \quad (-) \\ \hline
 & 0
\end{array}$$

Here we get zero remainder.

G.C.D of $(x^4 + 3x^3 - x - 3), (x^3 + x^2 - 5x + 3)$ is $(x^2 + 2x - 3)$

(ii) $x^4 - 1, x^3 - 11x^2 + x - 11$

$$\begin{array}{r|l}
 & x + 11 \\ \hline
x^3 - 11x^2 + x - 11 & \cancel{x^4} + 0x^3 + 0x^2 + 0x - 1 \\
 & \cancel{x^4} - 11x^3 + x^2 - 11x \\
 & (-) \quad (+) \quad (-) \quad (+) \\ \hline
 & \cancel{11x^3} - x^2 + 11x - 1 \\
 & \cancel{11x^3} - 121x^2 + 11x - 121 \\
 & (-) \quad (+) \quad (-) \quad (+) \\ \hline
 & 120x^2 + 120 \neq 0
\end{array}$$

$= 120(x^2 + 0x + 1)$

$$\begin{array}{r|l}
 & x \\ \hline
x^2 + 0x + 1 & \cancel{x^3} - 11x^2 \cancel{+x} - 11 \\
 & \cancel{x^3} + 0x^2 \cancel{+x} \\
 & (-) \quad (-) \quad (-) \\ \hline
 & -11x^2 - 11 \neq 0 \\ \hline
 & -11(x^2 + 1) \neq 0
\end{array}$$

$$\begin{array}{r|l} & \quad 1 \\ \hline x^2+1 & \cancel{x^2}+\cancel{0x}+\cancel{1} \\ & \cancel{x^2}+\cancel{0x}+\cancel{1} \\ & (-)\quad(-)\quad(-) \\ \hline & \quad 0 \end{array}$$

0 Remainder Thus,

G.C.D. of $x^4 - 1, x^2 - 11x^2 + x - 11$ is $(x^2 + 1)$

(iii) $3x^4 + 6x^3 - 12x^2 - 24x, 4x^4 + 14x^3 + 8x^2 - 8x$

$4x^4 + 14x^3 + 8x^2 - 8x = 2\,(2x^4 + 7x^3 + 4x^2 - 4x)$

[Taking 2 as common]

$3x^4 + 6x^3 - 12x^2 - 24x = 3(x^4 + 2x^3 - 4x^2 - 8x)$

Let us divide $2x^4 + 7x^3 + 4x^2 - 4x$ by $x^4 + 2x^3 - 4x^2 - 8x$

$$\begin{array}{r|l} & \quad 2 \\ \hline x^4+2x^3-4x^2-8x & 2x^4+7x^3+4x^2-4x \\ & 2x^4+4x^3-8x^2-16x \\ & (-)\quad(-)\quad(+)\quad(+) \\ \hline & \quad 3x^3+12x^2+12x \div 3 \end{array}$$

$= 3\,(x^2 + 4x^2 + 4x)$

$(x^3 + 4x^2 + 4x) \neq 0$

Now let us divide $x^4 + 2x^3 - 4x^2 - 8x$ by $x^3 + 4x^2 + 4x$

$$\begin{array}{r|l} & \quad x-2 \\ \hline x^3+4x^2+4x & x^4+2x^3-4x^2-8x \\ & x^4+4x^3+4x^2 \\ & (-)\quad(-)\quad(-) \\ \hline & -2x^3-8x^2-8x \\ & -2x^3-8x-8x \\ & (+)\quad(+)\quad(+) \\ \hline & \quad 0 \end{array}$$

$\therefore x^3 + 4x^2 + 4x = x\,(x^2 + 4x + 4)$is the G.C.D of $3x^4 + 6x^3 - 12x^2 - 24x, 4x^4 + 14x^3 + 8x^2 - 8x$

(iv) Given polynomials are

Let $f(x) = 3x^3 + 3x^2 + 3x + 3$ and

$g(x) = 6x^3 + 12x^2 + 6x + 12$

Let us divide $g(x)$ by $f(x)$.

Step 1 :

$$\begin{array}{r|l} & \quad 2 \\ \hline 3x^3+3x^2+3x+3 & 6x^3+12x^2+6x+12 \\ & 6x^3+\ 6x^2+6x+6 \\ & (-)\quad(-)\quad(-)\quad(-) \\ \hline & 6x^2+6 = 6\,(x^2+1) \neq 0 \end{array}$$

Step 2 :

$$\begin{array}{r|l} & \quad 3x+3 \\ \hline x^2+1 & 3x^3+3x^2+3x+3 \\ & 3x^3+0\quad+3x \\ & (-)\quad(-)\quad(-) \\ \hline & 3x^2+3 \\ & 3x^2+3 \\ & (-)\quad(-) \\ \hline & \quad 0 \end{array}$$

$\therefore$ G.C.D. is $3\,(x^2 + 1)$

2. Find the LCM of the given expressions.

(i) $4x^2y, 8x^3y^2$ (ii) $9a^3b^2, 12a^2b^2c$ [FRT - 2022]

(iii) $16m, 12m^2n^2, 8n^2$

(iv) $p^2 - 3p + 2, p^2 - 4$

(v) $2x^2 - 5x - 3, 4x^2 - 36$

(vi) $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

Sol. (i) $4x^2y, 8x^3y^2$

$$\begin{aligned} \text{Let } f(x) &= 4x^2y = 2^2\,x^2y \\ g(x) &= 8x^3\,y^2 = 2^3x^3y^2 \\ \text{LCM} &= 2^3\,x^3\,y^2 \end{aligned}$$

[$\because$ max $(2^2, 2^3) = 2^3$
max $(x^2, x^3) = x^3$
max $(y, y^2) = y^2$]

$= 8x^3\,y^2$

(ii)
$$\begin{aligned} \text{Let } f(x) &= 9a^3b^2 = 3 \times 3a^3b^2 \\ g(a) &= 12a^2b^2c = 3 \times 2 \times 2a^2b^2c \\ \text{LCM} &= 36\,a^3b^2c \end{aligned}$$

[$\because$LCM of 9, 12 is 36 $\Rightarrow$
max $(a^3, a^2) = a^3$
max $(b^2, b^2) = b^2$
max $(1, c) = c$]

$$\begin{array}{r|l} 3 & 9,\ 12 \\ \hline 3 & 3,\ 4 \\ \hline & 1,\ 4 \end{array}$$

LCM = 3 × 3 × 4
= 36

(iii) $16m, 12m^2n^2, 8n^2$

$\therefore$LCM of 16, 12, 8 is

$$\begin{aligned} 2 \times 2 \times 2 \times 2 \times 3 &= 48 \\ \max\,(m, m^2) &= m^2 \\ \max\,(1, n^2, n^2) &= n^2 \end{aligned}$$

$\therefore$LCM is 48 m^2n^2

$$\begin{array}{r|l} 2 & 16,\ 12,\ 8 \\ \hline 2 & 8,\ 6,\ 4 \\ \hline 2 & 4,\ 3,\ 2 \\ \hline 2 & 2,\ 3,\ 1 \\ \hline 3 & 1,\ 3,\ 1 \\ \hline & 1,\ 1,\ 1 \end{array}$$

(iv) $p^2 - 3p + 2, p^2 - 4$

$$\begin{aligned} \text{Let } f(p) &= p^2 - 3p + 2 \\ &= (p-2)\,(p-1) \\ \text{and } g(p) &= p^2 - 4 = p^2 - 2^2 \\ &= (p+2)\,(p-2) \end{aligned}$$

Hint :
2
–3
–2 –1

$\therefore$ L.C.M is $(p-2)(p-1)(p+2)$

[max $((p-2),(p-2))$ is $p - 2$
max $((p-1),1)$ is $p - 1$
max $(1, p+2)$ is $p + 2$]

(v) $2x^2 - 5x - 3, 4x^2 - 36$

$$\begin{aligned} \text{Let } f(x) &= 2x^2 - 5x - 3 \\ &= (x-3)(2x+1) \quad \ldots (1) \\ \text{and } g(x) &= 4(x^2 - 9) = 4(x^2 - 3^2) \\ &= 4(x+3)(x-3) \quad \ldots (2) \end{aligned}$$

From (1) and (2), LCM of 1, 4 is

$\therefore$ LCM is $4\,(x-3)\,(2x+1)\,(x+3)$

[$\because$ LCM of polynomial factors is writing all the factors without repeating the factors]

(vi) $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

$$\begin{aligned} \text{Consider } (2x^2 - 3xy)^2 &= [x\,(2x - 3y)]^2 \\ &= x^2\,(2x - 3y)^2 \quad \ldots (1) \\ (4x - 6y)^3 &= [2\,(2x - 3y)^3] \end{aligned}$$

$$= 2^3\,(2x-3y)^3$$
$$= 8\,(2x-3y)^3 \qquad \ldots (2)$$

and $8x^3 - 27y^3 = (2x)^3 - (3y)^3$

$[\because a^3 - b^3 = (a-b)\,(a^2+ab+b^2)]$

$$= (2x-3y)\,(4x^2+6xy+9y^2) \ldots (3)$$

From (1), (2) and (3),

LCM is $8x^2\,(2x-3y)^3\,(4x^2+6xy+9y^2)$

$[\because$ max $(x^2, 1, 1) = x^2$ max $((2x-3y)^2, (2x-3y)^3, (2x-3y))$ is $(2x-3y)^3$ and max $(1, 1, 4x^2+6xy+9y^2) = 4x^2+6xy+9y^2]$

EXERCISE 3.3

1. Find the LCM and GCD for the following and verify that $f(x) \times g(x)$ = LCM × GCD.

(i) $21x^2y,\ 35xy^2$

(ii) $(x^3-1)(x+1),\ (x^3+1)$ **[FRT - 2022]**

(iii) $(x^2y+xy^2),\ (x^2+xy)$

Sol. (i) Let $f(x) = 21x^2y$ and $g(x) = 35xy^2$

LCM of 21, 35 is 105

$[\because$ max $(x^2, x) = x^2$
max $(y, y^2) = y^2]$

7 | 21, 35
3 | 3, 5
5 | 1, 5
  1, 1
= 7 × 3 × 5
= 105

$\therefore$ LCM is $105\,x^2\,y^2$

GCD is $7xy$

$$[\because\ f(x) = 3 \times \boxed{7} \times \boxed{x} \times x \times \boxed{y}$$
$$g(x) = 5 \times 7 \times \boxed{x} \times y \times \boxed{y}$$

Consider $f(x).\,g(x) = (21x^2y)\,(35xy^2) = 735\,x^3\,y^3 \quad \ldots (1)$

Also LCM × GCD $= (105x^2y^2)\,(7xy) = 735\,x^3\,y^3 \quad \ldots (2)$

From (1) and (2), LCM × GCD $= f(x).\,g(x)$

(ii) Let $f(x) = (x^3-1)\,(x+1)$ and $g(x) = x^3+1$

Now, $f(x) = (x^3-1^3)\,(x+1)$
$= (x-1)\,(x^2+x+1)\,(x+1)$

and $g(x) = (x+1)\,(x^2-x+1)$

LCM $= (x-1)\,(x^2+x+1)\,(x+1)(x^2-x+1)$

[Write all the factors of $f(x)$ and $g(x)$ without repeating any factor]

GCD $= x+1$

[Common factors of $f(x)$ and $g(x)$]

Now, LCM × GCD=

$$(x-1)\,(x^2+x+1)\,(x+1)\,(x^2-x+1)(x+1)$$
$$= (x^3-1)\,(x^3+1)\,(x+1)$$
$$= (x^6-1)\,(x+1) \qquad \ldots (1)$$
$$[\because (a+b)\,(a-b) = a^2-b^2]$$

Also $f(x).g(x) = (x^3-1)\,(x+1)\,(x^3+1)$
$= (x^6-1)\,(x+1) \quad \ldots (2)$

From (1) and (2),

LCM × GCD $= f(x).\,g(x)$

(iii) Let $f(x) = x^2y + xy^2$ and $g(x) = x^2 + xy$

Now, $f(x) = xy\,(x+y)$ [Taking xy as common]

and $g(x) = x(x+y)$ [Taking x as common]

LCM $= x\,y\,(x+y)$

[Write all the factors of $f(x)$ and $g(x)$, without repeating any factors]

GCD $= x\,(x+y)$

[Common factors of $f(x)$ and $g(x)$]

$\therefore$ LCM × GCD $= [x\,y\,(x+y)].[x\,(x+y)]$
$= x^2y\,(x+y)^2 \quad \ldots(1)$

and $f(x).g(x) = [x\,y\,(x+y)][x\,(x+y)]$
$= x^2y\,(x+y)^2 \quad \ldots(2)$

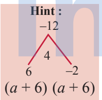
From (1) and (2), LCM × GCD $= f(x) \times g(x)$

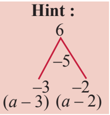
2. Find the LCM of each pair of the following polynomials

(i) $a^2+4a-12,\ a^2-5a+6$ whose GCD is $a-2$ **[PTA - 6]**

(ii) $x^4-27a^3x,\ (x-3a)^2$ whose GCD is $(x-3a)$

Sol. (i) Let $f(a) = a^2+4a-12$
$g(a) = a^2-5a+6$
GCD $= (a-2)$





Since $f(a) \times g(a)$ = LCM × GCD

$$\text{LCM} = \frac{f(a) \times g(a)}{\text{GCD}}$$
$$= \frac{(a^2+4a-12)(a^2-5a+6)}{(a-2)}$$
$$= \frac{(a+6)(a-2)(a-3)\,\cancel{(a-2)}}{\cancel{(a-2)}}$$
$$= (a+6)\,(a-2)(a-3)$$

(ii) $f(x) = x^4 - 27a^3x$ and $g(x) = (x-3a)^2$

GCD $= (x-3a)$

$f(x) = x(x^3-27a^3) = x\,(x^3-(3a^3))$
$= x\,(x-3a)\,(x^2+3ax+9a^2)$

$[\because a^3 - b^3 = (a-b)\,(a^2+ab+b^2)$
Here $a = x$ and $b = 3a]$

and $g(x) = (x-3a)^2$

Since $f(x).g(x)$ = LCM × GCD

$$\text{LCM} = \frac{f(x).g(x)}{\text{GCD}}$$
$$\therefore \text{LCM} = \frac{x(x-\cancel{3a})(x^2+3ax+9a^2).(x-3a)^2}{(x-\cancel{3a})}$$
$$\text{LCM} = x(x-3a)^2\,(x^2+3ax+9a^2)$$

3. **Find the GCD of each pair of the following polynomials**

(i) **$12(x^4 - x^3)$, $8(x^4 - 3x^3 + 2x^2)$ whose LCM is $24x^3(x-1)(x-2)$**

(ii) **$(x^3 + y^3)$, $(x^4 + x^2y^2 + y^4)$ whose LCM is $(x^3 + y^3)(x^2 + xy + y^2)$**

Sol. (i) Let $f(x) = 12(x^4 - x^3)$

$g(x) = 8(x^4 - 3x^3 + 2x^2)$ and

LCM is $24x^3(x-1)(x-2)$

Consider $f(x) = 12(x^4 - x^3)$

$= 12x^3(x-1)$

[Taking x^3 as common]

$g(x) = 8(x^4 - 3x^3 + 2x^2)$

$= 8x^2(x^2 - 3x + 2)$

[Taking x^2 as common]

$= 8x^2(x-2)(x-1)$

Since

$f(x).\,g(x) = \text{LCM}.\,\text{GCD}$

We get,

$$\text{GCD} = \frac{f(x)\cdot g(x)}{\text{LCM}}$$

$$\text{GCD} = \frac{\cancel{12}\,\cancel{x^3}(x-1)\cdot \overset{4}{\cancel{8}}\,x^2\,\cancel{(x-2)}\,\cancel{(x-1)}}{\cancel{24}\,\cancel{x^3}\,\cancel{(x-1)}\,\cancel{(x-2)}}$$

$= 4x^2(x-1)$

(ii) Let $f(x) = x^3 + y^3$, $g(x) = x^4 + x^2y^2 + y^4$

and LCM $= (x^3 + y^3)(x^2 + xy + y^2)$

Since $f(x).\,g(x) = \text{LCM}.\,\text{GCD}$,

$$\text{We get GCD} = \frac{f(x)\cdot g(x)}{\text{LCM}}$$

$$= \frac{\cancel{(x^3+y^3)}(x^4+x^2y^2+y^4)}{\cancel{(x^3+y^3)}(x^2+xy+y^2)}$$

$$= \frac{x^4+x^2y^2+y^4}{x^2+xy+y^2}$$

$$\begin{array}{r|l}
 & x^2 - xy + y^2 \\ \hline
x^2+xy+y^2 & x^4 + x^2y^2 + \quad y^4 \\
 & x^4 + x^3y + x^2y^2 \\
 & {\scriptstyle(-)} \quad {\scriptstyle(-)} \quad {\scriptstyle(-)} \\ \hline
 & -x^3y + y^4 \\
 & -x^3y - x^2y^2 - xy^3 \\
 & {\scriptstyle(+)} \quad {\scriptstyle(+)} \quad {\scriptstyle(+)} \\ \hline
 & x^2y^2 + xy^3 + y^4 \\
 & x^2y^2 + xy^3 + y^4 \\
 & {\scriptstyle(-)} \quad {\scriptstyle(-)} \quad {\scriptstyle(-)} \\ \hline
 & 0 \\ \hline
\end{array}$$

$\therefore$ GCD $= x^2 - xy + y^2$

4. **Given the LCM and GCD of the two polynomials $p(x)$ and $q(x)$ find the unknown polynomial in the following table**

S. No	LCM	GCD	$p(x)$	$q(x)$
(i)	$a^3 - 10a^2 + 11a + 70$	$a - 7$	$a^2 - 12a + 35$	
(ii)	$(x^4 - y^4)(x^4 + x^2y^2 + y^4)$	$(x^2 - y^2)$		$(x^4 - y^4)(x^2 + y^2 - xy)$

Sol. (i) L.C.M $= a^3 - 10a^2 + 11a + 70$

G.C.D $= a - 7$

$p(x) = a^2 - 12a + 35$

Hint:

$$\begin{array}{r|l}
7 & 1, -10, 11, 70 \\
 & 0, 7, -21, -70 \\ \hline
 & 1, -3, -10, \underline{|0} \\ \hline
\end{array}$$

$$q(x) = \frac{\text{L.C.M.}\times\text{G.C.D}}{p(x)}$$

$$= \frac{(a^3 - 10a^2 + 11a + 70)(a-7)}{(a^2 - 12a + 35)}$$

Hint:

35

–5 –7

$$= \frac{(a^2 - 3a - 10)(a-7)\,\cancel{(a-7)}}{(a-5)\,\cancel{(a-7)}}$$

$$= \frac{(a+2)\,\cancel{(a-5)}\,(a-7)}{\cancel{(a-5)}}$$

$\therefore\ q(x) = (a+2)(a-7)$

(ii) L.C.M $= (x^4 - y^4)(x^4 + x^2y^2 + y^4)$

G.C.D. $= (x^2 - y^2)$

$q(x) = (x^4 - y^4)(x^2 + y^2 - xy)$

$$p(x) = \frac{\text{L.C.M}\times\text{G.C.D}}{q(x)}$$

$$= \frac{(x^4 - y^4)(x^4 + x^2y^2 + y^4)(x^2 - y^2)}{(x^4 - y^4)(x^2 + y^2 - xy)}$$

$$= \frac{\cancel{(x^4 - y^4)}(x^2 + xy + y^2)\cancel{(x^2 - xy + y^2)}(x^2 - y^2)}{\cancel{(x^4 - y^4)}\,\cancel{(x^2 + y^2 - xy)}}$$

$= (x^2 + xy + y^2)(x^2 - y^2)$

EXERCISE 3.4

1. **Reduce each of the following rational expressions to its lowest form.**

(i) $\dfrac{x^2 - 1}{x^2 + x}$ **(ii)** $\dfrac{x^2 - 11x + 18}{x^2 - 4x + 4}$

(iii) $\dfrac{9x^2 + 81x}{x^3 + 8x^2 - 9x}$ **(iv)** $\dfrac{p^2 - 3p - 40}{2p^3 - 24p^2 + 64p}$

Sol. (i) $\dfrac{x^2 - 1}{x^2 + x} = \dfrac{(x+1)(x-1)}{x(x+1)} = \dfrac{x-1}{x}$

(ii) $\dfrac{x^2-11x+18}{x^2-4x+4} = \dfrac{(x-2)(x-9)}{(x-2)(x-2)}$

$= \dfrac{x-9}{x-2}$

Hint:
18: −11; −2, −9
4: −4; −2, −2

(iii) $\dfrac{9x^2+81x}{x^3+8x^2-9x} = \dfrac{9x(x+9)}{x(x^2+8x-9)}$

$= \dfrac{9(x+9)}{(x+9)(x-1)} = \dfrac{9}{x-1}$

Hint:
−9: 8; 9, −1

(iv) $\dfrac{p^2-3p-40}{2p^3-24p^2+64p} = \dfrac{(p-8)(p+5)}{2p\left(p^2-12p+32\right)}$

$= \dfrac{\cancel{(p-8)}\,(p+5)}{2p(p-4)\,\cancel{(p-8)}}$

$= \dfrac{p+5}{2p(p-4)}$

Hint:
−40: −3; −8, 5
32: −62; −4, −8

2. **Find the excluded values, if any of the following expressions.**

(i) $\dfrac{y}{y^2-25}$ **(ii)** $\dfrac{t}{t^2-5t+6}$

(iii) $\dfrac{x^2+6x+8}{x^2+x-2}$ **(iv)** $\dfrac{x^3-27}{x^3+x^2-6x}$

Sol.
(i) $\dfrac{y}{y^2-25} = \dfrac{y}{(y+5)(y-5)}$ is undefined when $(y + 5)(y - 5) = 0$ that is $y = -5, 5$.

$[\because a^2 - b^2 = (a + b)\,(a - b)]$

∴ The excluded values are −5, 5.

(ii) $\dfrac{t}{t^2-5t+6}$ is undefined when $t^2 - 5t + 6 = 0$

i.e. $(t - 3)(t - 2) = 0.$ $\Rightarrow t = 3, 2$

∴ The excluded values are 3, 2.

Hint:
6: −5; −3, −2

(iii) $\dfrac{x^2+6x+8}{x^2+x-2}$

$\dfrac{x^2+6x+8}{x^2+x-2} = \dfrac{(x+2)(x+4)}{(x-1)(x+2)} = \dfrac{x+4}{x-1}$

When $x = 1$, the expression $\dfrac{x^2+6x+8}{x^2+x-2}$ is not defined.

∴ The excluded value of the given expression is $x = 1$.

(iv) $\dfrac{x^3-27}{x^3+x^2-6x}$ is undefined when $x^3+x^2-6x=0$, i.e.

$x(x^2 + x - 6) = 0$

$x(x + 3)(x - 2) = 0$

∴ The excluded values are 0, −3, 2.

Hint:
−6: 1; 3, −2

EXERCISE 3.5

1. **Simplify**

(i) $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4}$ **[Govt. MQP - 2019]**

(ii) $\dfrac{p^2-10p+21}{p-7} \times \dfrac{p^2+p-12}{(p-3)^2}$

(iii) $\dfrac{5t^3}{4t-8} \times \dfrac{6t-12}{10t}$

Sol.
(i) $\dfrac{\overset{2}{\cancel{4}}\,x^2y}{\cancel{2}z^2} \times \dfrac{\overset{3}{\cancel{6}}\,xz^3}{\underset{\underset{5}{\cancel{10}}}{\cancel{20}}\,y^4} = \dfrac{3x^3yz^3}{5y^3z^2} = \dfrac{3x^3z}{5y^3}$

(ii) $\dfrac{p^2-10p+21}{p-7} \times \dfrac{p^2+p-12}{(p-3)^2}$

$= \dfrac{\cancel{(p-7)}\,\cancel{(p-3)}}{\cancel{(p-7)}} \times \dfrac{(p+4)\,\cancel{(p-3)}}{\cancel{(p-3)}\,\cancel{(p-3)}} = p+4$

(iii) $\dfrac{5t^3}{4t-8} \times \dfrac{6t-12}{10t} = \dfrac{\cancel{5}t^3 \times \cancel{6}^3\,\cancel{(t-2)}}{4\,\cancel{(t-2)} \times \underset{2}{\cancel{10}}\,t} = \dfrac{3t^2}{4}$

2. **Simplify**

(i) $\dfrac{x+4}{3x+4y} \times \dfrac{9x^2-16y^2}{2x^2+3x-20}$

(ii) $\dfrac{x^3-y^3}{3x^2+9xy+6y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

Sol.
(i) $\dfrac{x+4}{3x+4y} \times \dfrac{9x^2-16y^2}{2x^2+3x-20}$

$= \dfrac{(x+4)[(3x)^2-(4y)^2]}{(3x+4y)(x+4)(2x-5)}$

$= \dfrac{\cancel{(x+4)}\,\cancel{(3x+4y)}\,(3x-4y)}{\cancel{(3x+4y)}\,\cancel{(x+4)}\,(2x-5)} = \dfrac{3x-4y}{2x-5}$

Hint :
−40: 3; 8, −5
$\dfrac{8}{2}$, $\dfrac{-5}{2}$
4, $\dfrac{-5}{2}$
(x + 4) (2x – 5)

(ii) $\dfrac{x^3-y^3}{3x^2+9xy+6y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

Consider $x^3 - y^3 = (x - y)\,(x^2 + xy + y^2)$

$x^2 + 2xy + y^2 = (x + y)^2 = (x + y)\,(x + y)$

$3x^2 + 9xy + 6y^2 = 3[x^2 + 3xy + 2y^2]$

$= 3\,[(x + 2y)\,(x + y)]$

$x^2 - y^2 = (x + y)\,(x - y)$

∴ $\dfrac{x^3-y^3}{3x^2+9xy+6y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

Hint:
2: 3; 2, 1
(x + 2y) (x + y)

$$= \frac{\cancel{(x-y)}(x^2+xy+y^2)}{3(x+2y)\cancel{(x+y)}} \times \frac{\cancel{(x+y)}\ \cancel{(x+y)}}{\cancel{(x+y)}\ \cancel{(x-y)}}$$

$$= \frac{(x^2+xy+y^2)}{3(x+2y)}$$

3. Simplify

(i) $\dfrac{2a^2+5a+3}{2a^2+7a+6} \div \dfrac{a^2+6a+5}{-5a^2-35a-50}$

(ii) $\dfrac{b^2+3b-28}{b^2+4b+4} \div \dfrac{b^2-49}{b^2-5b-14}$

[FRT & Aug. - 2022]

(iii) $\dfrac{x+2}{4y} \div \dfrac{x^2-x-6}{12y^2}$

(iv) $\dfrac{12t^2-22t+8}{3t} \div \dfrac{3t^2+2t-8}{2t^2+4t}$

Sol. (i) $\dfrac{2a^2+5a+3}{2a^2+7a+6} \div \dfrac{a^2+6a+5}{-5a^2-35a-50}$

$$= \frac{2a^2+5a+3}{2a^2+7a+6} \times \frac{-5a^2-35a-50}{a^2+6a+5}$$

Hint:
6
5
$\frac{3}{2}$ $\frac{2}{2}$
$= \frac{3}{2}$ 1
$= (2a+3)(a+1)$

Consider $2a^2 + 5a + 3 = (2a+3)(a+1)$

$2a^2 + 7a + 6 = (a+2)(2a+3)$

$-5a^2 - 35a - 50 = -5[a^2 + 7a + 10]$

$= -5(a+5)(a+2)$

$a^2 + 6a + 5 = (a+5)(a+1)$

Hint:
12
7
$\frac{4}{2}$ $\frac{3}{2}$
$= 2$ $\frac{3}{2}$

Hint:
10
7
5 2

$$\therefore \frac{2a^2+5a+3}{2a^2+7a+6} \times \frac{-5a^2-35a-50}{a^2+6a+5}$$

$$= \frac{\cancel{(2a+3)}\ \cancel{(a+1)}}{\cancel{(a+2)}\ \cancel{(2a+3)}} \times \frac{-5\cancel{(a+5)}\ \cancel{(a+2)}}{\cancel{(a+5)}\ \cancel{(a+1)}}$$

$= -5$

Hint:
5
6
5 1

(ii) $\dfrac{b^2+3b-28}{b^2+4b+4} \div \dfrac{b^2-49}{b^2-5b-14}$

$b^2 + 3b - 28 = (b+7)(b-4)$

$b^2 + 4b + 4 = (b+2)(b+2)$

$b^2 - 5b - 14 = (b-7)(b+2)$

and $b^2 - 49 = b^2 - 7^2$

$= (b+7)(b-7)$

Hint:
–28
3
7 –4
4
4
2 2
–14
–5
–7 +2

$$\therefore \frac{b^2+3b-28}{b^2+4b+4} \times \frac{b^2-5b-14}{b^2-49}$$

$$= \frac{\cancel{(b+7)}(b-4)}{(b+2)\cancel{(b+2)}} \times \frac{\cancel{(b-7)}\ \cancel{(b+2)}}{\cancel{(b+7)}\ \cancel{(b-7)}} = \frac{b-4}{b+2}$$

(iii) $\dfrac{x+2}{4y} \div \dfrac{x^2-x-6}{12y^2} = \dfrac{x+2}{\cancel{4y}} \times \dfrac{{}^{3}\cancel{12y^2}}{x^2-x-6}$

$$= \frac{\cancel{(x+2)}}{1} \times \frac{3y^2}{(x-3)\cancel{(x+2)}} = \frac{3y}{x-3}$$

Hint:
–6
–1
–3 2

$[\because x^2 - x - 6 = (x-3)(x+2)]$

(iv) $\dfrac{12t^2-22t+8}{3t} \div \dfrac{3t^2+2t-8}{2t^2+4t}$

$$= \frac{12t^2-22t+8}{3t} \times \frac{2t^2+4t}{3t^2+2t-8}$$

Hint:
24
–11
$\frac{-8}{6}$ $\frac{-3}{6}$
$\frac{-4}{3}$ $-\frac{1}{2}$

Consider

$12t^2 - 22t + 8 = 2(6t^2 - 11t + 4)$

$= 2(3t-4)(2t-1)$

$2t^2 + 4t = 2t(t+2)$ and

$3t + 2t - 8 = (t+2)(3t-4)$

Hint:
–24
2
$\frac{6}{3}$ $\frac{-4}{3}$
2 $\frac{-4}{3}$

$$\therefore \frac{12t^2-22t+8}{3t} \times \frac{2t^2+4t}{3t^2+2t-8}$$

$$= \frac{2\cancel{(3t-4)}(2t-1)}{3\cancel{t}} \times \frac{2\cancel{t}\cancel{(t+2)}}{\cancel{(t+2)}\ \cancel{(3t-4)}} = \frac{4(2t-1)}{3}$$

4. If $x = \dfrac{a^2+3a-4}{3a^2-3}$ and $y = \dfrac{a^2+2a-8}{2a^2-2a-4}$ find the value of x^2y^{-2}. **[PTA - 3]**

Sol. Given $x = \dfrac{a^2+3a-4}{3a^2-3}$ and $y = \dfrac{a^2+2a-8}{2a^2-2a-4}$

$a^2 + 3a - 4 = (a+4)(a-1)$

$3a^2 - 3 = 3(a^2 - 1)$

$= 3(a+1)(a-1)$

Hint:
–4
3
4 –1

$$\therefore x = \frac{(a+4)\cancel{(a-1)}}{3(a+1)\cancel{(a-1)}} = \frac{a+4}{3(a+1)}$$

$$x^2 = \left[\frac{a+4}{3(a+1)}\right]^2 = \frac{(a+4)(a+4)}{9(a+1)(a+1)} \quad ...(1)$$

Now, $y = \dfrac{a^2+2a-8}{2a^2-2a-4}$

$a^2 + 2a - 8 = (a+4)(a-2)$

$2a^2 - 2a - 4 = 2(a^2 - a - 2)$

$= 2(a-2)(a+1)$

Hint:
–8
2
4 –2

$$\therefore y = \frac{(a+4)\cancel{(a-2)}}{2\cancel{(a-2)}(a+1)}$$

$$= \frac{a+4}{2(a+1)}$$

Hint:
–2
–1
–2 1

$$\therefore y^2 = \frac{(a+4)(a+4)}{4(a+1)(a+1)} \quad \ldots (2)$$

Now, $$x^2y^{-2} = \frac{x^2}{y^2} = \frac{(a+4)(a+4)}{9(a+1)(a+1)} \div \frac{(a+4)(a+4)}{4(a+1)(a+1)}$$

$$= \frac{\cancel{(a+4)}\ \cancel{(a+4)}}{9\,\cancel{(a+1)}\ \cancel{(a+1)}} \times \frac{4\,\cancel{(a+1)}\ \cancel{(a+1)}}{\cancel{(a+4)}\ \cancel{(a+4)}} = \frac{4}{9}$$

5. If a polynomial $p(x) = x^2 - 5x - 14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$ find $q(x)$. **[PTA - 2]**

Sol. Given $p(x) = x^2 - 5x - 14 = (x - 7)\ (x + 2)$

Hint:
–14
–5
–7 2

$= (x - 7)\ (x + 2)$

Also, it is given that

$$P(x) \div q\,(x) = \frac{x-7}{x+2}$$

$$(x^2 - 5x - 14) \times \frac{1}{q(x)} = \frac{x-7}{x+2}$$

$$\Rightarrow (x-7)(x+2) \times \frac{1}{q(x)} = \frac{x-7}{x+2}$$

$$\Rightarrow \frac{\cancel{(x-7)}\,(x+2) \times (x+2)}{\cancel{x-7}} = q\,(x)$$

[By cross multiplication]

$\Rightarrow q(x) = (x + 2)\ (x + 2) = x^2 + 4x + 4$

EXERCISE 3.6

1. Simplify (i) $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$

(ii) $\frac{x+2}{x+3} + \frac{x-1}{x-2}$ (iii) $\frac{x^3}{x-y} + \frac{y^3}{y-x}$ **[FRT - 2022]**

Sol. (i) $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$

LCM of $(x - 2), (x - 2)$ is $x - 2$

$$\therefore \frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2} = \frac{x(x+1)+x(1-x)}{(x-2)}$$

$$= \frac{\cancel{x^2} + x + x - \cancel{x^2}}{x-2} = \frac{x+x}{x-2} = \frac{2x}{x-2}$$

(ii) $\frac{x+2}{x+3} + \frac{x-1}{x-2}$

LCM of $(x + 3), (x - 2)$ is $(x + 3)(x - 2)$

$$\therefore \frac{x+2}{x+3} + \frac{x-1}{x-2} = \frac{(x+2)(x-2)+(x-1)(x+3)}{(x+3)(x-2)}$$

$$= \frac{x^2-4+x^2+2x-3}{(x+3)(x-2)} = \frac{2x^2+2x-7}{(x+3)(x-2)}$$

[Simplifying the like terms]

(iii) $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

$$\frac{x^3}{x-y} + \frac{y^3}{y-x} = \frac{x^3}{x-y} - \frac{y^3}{x-y}$$

[Taking (–1) common from the second term]

LCM of $(x - y), (x - y)$ is $x - y$

$$= \frac{x^3 - y^3}{(x-y)} = \frac{\cancel{(x-y)}\,(x^2+xy+y^2)}{\cancel{(x-y)}}$$

$[\therefore a^3 - b^3 = (a - b)\ (a^2 + ab + b^2)]$

$= x^2 + xy + y^2$

2. Simplify (i) $\frac{(2x+1)(x-2)}{x-4} - \frac{(2x^2-5x+2)}{x-4}$

(ii) $\frac{4x}{x^2-1} - \frac{x+1}{x-1}$

Sol. (i) LCM of $(x - 4)\ (x - 4)$ is $x - 4$

$$= \frac{(2x+1)(x-2)-(2x^2-5x+2)}{x-4}$$

$$= \frac{\cancel{2x^2} - 4x + x - 2 - \cancel{2x^2} + 5x - 2}{x-4} = \frac{2x-4}{x-4}$$

[Simplifying the like terms]

$$= \frac{2(x-2)}{x-4}$$

[Taking 2 common from the numerator]

(ii) $\frac{4x}{(x+1)(x-1)} - \frac{(x+1)}{x-1}$

LCM of $(x + 1)\ (x - 1), (x - 1)$ is $(x + 1)\ (x - 1)$

$$\therefore \frac{4x}{x^2-1} - \frac{x+1}{x-1} = \frac{4x-(x+1)(x+1)}{(x+1)(x-1)}$$

$$= \frac{4x-(x^2+2x+1)}{(x+1)(x-1)} = \frac{4x-x^2-2x-1}{(x+1)(x-1)}$$

$$= \frac{-x^2+2x-1}{(x+1)(x-1)} = \frac{-(x^2-2x+1)}{(x+1)(x-1)}$$

[Taking (–1) common from the numerator]

$$= \frac{-(x-1)\,\cancel{(x-1)}}{(x+1)\,\cancel{(x-1)}} = \frac{-(x-1)}{(x+1)} = \frac{1-x}{1+x}$$

3. **Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$.**

Sol. Find the value of $\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}$

LCM of $(x^2 + 2)^2$, $(x^2 + 2)$ is $(x^2 + 2)^2$

$$\therefore \frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2} = \frac{2x^3+x^2+3-(x^2+2)}{(x^2+2)^2}$$

$$= \frac{2x^3+\cancel{x^2}+3-\cancel{x^2}-2}{(x^2+2)^2} = \frac{2x^3+1}{(x^2+2)^2}$$

4. **Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ to get $\frac{3}{x^2-2x+4}$.** **[PTA - 4]**

Sol. Let the required rational expression be $q(x)$

Given, $\frac{x^2+6x+8}{x^3+8}-q(x) = \frac{3}{x^2-2x+4}$

$$q(x) = \frac{x^2+6x+8}{x^3+8}-\frac{3}{x^2-2x+4}$$

$$= \frac{(x+4)\cancel{(x+2)}}{\cancel{(x+2)}(x^2-2x+4)}-\frac{3}{x^2-2x+4}$$

Hint:
8
4 6 2
$(a^2+b^3)=(a+b)$
(a^2-ab+b^2)
Here $a = x, b = 2$

$$= \frac{x+4}{x^2-2x+4}-\frac{3}{x^2-2x+4}$$

LCM of $x^2 - 2x + 4$, $x^2 - 2x + 4$ is $x^2 - 2x + 4$

$$\therefore q(x) = \frac{x+4-3}{x^2-2x+4} = \frac{x+1}{x^2-2x+4}$$

5. **If A $=\frac{2x+1}{2x-1}$, B $=\frac{2x-1}{2x+1}$ find $\frac{1}{A-B}-\frac{2B}{A^2-B^2}$**

Sol. Given A $=\frac{2x+1}{2x-1}$, B $=\frac{2x-1}{2x+1}$

Find $\frac{1}{A-B}-\frac{2B}{(A+B)(A-B)} = \frac{A+B-2B}{(A+B)(A-B)}$

[∵ LCM of (A – B), (A+B) (A–B) is (A+B) (A–B)]

$$= \frac{\cancel{A-B}}{(A+B)\cancel{(A-B)}} = \frac{1}{A+B}$$

Consider $A+B = \frac{2x+1}{2x-1}+\frac{2x-1}{2x+1}$

[∵ LCM of $(2x - 1)$, $(2x + 1)$ is $(2x - 1)(2x + 1)$]

$$\Rightarrow A+B = \frac{(2x+1)(2x+1)+(2x-1)(2x-1)}{(2x-1)(2x+1)}$$

$$\Rightarrow A+B = \frac{4x^2+\cancel{4x}+1+4x^2-\cancel{4x}+1}{4x^2-1}$$

$$= \frac{8x^2+2}{4x^2-1} = \frac{2(4x^2+1)}{4x^2-1}$$

[∵ $(a + b)(a - b) = a^2 - b^2$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$]

$$\therefore \frac{1}{A+B} = \frac{4x^2-1}{2(4x^2+1)}$$

6. **If A $=\frac{x}{x+1}$, B $=\frac{1}{x+1}$, prove that $\frac{(A+B)^2+(A-B)^2}{A\div B} = \frac{2(x^2+1)}{x(x+1)^2}$.**

Sol. Given A $=\frac{x}{x+1}$, B $=\frac{1}{x+1}$

P.T $\frac{(A+B)^2+(A-B)^2}{A\div B} = \frac{2(x^2+1)}{x(x+1)^2}$

Consider Numerator = $(A + B^2) + (A - B^2)$

$= A^2 + \cancel{2AB} + B^2 + A^2 - \cancel{2AB} + B^2$

$= 2A^2 + 2B^2 = 2(A^2 + B^2)$

$$= 2\left[\left(\frac{x}{x+1}\right)^2+\frac{1}{(x+1)^2}\right] = 2\left[\frac{x^2}{(x+1)^2}+\frac{1}{(x+1)^2}\right]$$

LCM of $(x + 1)^2$, $(x + 1)^2$ is $(x + 1)^2$

$$\therefore \text{Numerator} = 2\left[\frac{x^2+1}{(x+1)^2}\right] \quad \ldots (1)$$

$$\text{Denominator} = A \div B = \frac{x}{x+1} \div \frac{1}{x+1}$$

$$= \frac{x}{\cancel{x+1}} \times \frac{\cancel{x+1}}{1} = x \quad \ldots (2)$$

From (1) and (2),

$$\frac{(A+B)^2+(A-B)^2}{A\div B} = \frac{2(x^2+1)}{(x+1)^2\times x} = \frac{2(x^2+1)}{x(x+1)^2}$$

Hence, $\frac{(A+B)^2+(A-B)^2}{A\div B} = \frac{2(x^2+1)}{x(x+1)^2}$

7. **Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?** **[Govt. MQP - 2019]**

Sol. Pari: time required to complete the work = 4 hrs.

∴ In 1 hr. he will complete = $\frac{1}{4}$ of the work.

$= \frac{1}{4}\ w.$

Yuvan: Time required to complete the work = 6 hrs.

∴ In 1 hr. he will complete the $\frac{1}{6}$ of the work

$= \frac{1}{6}\ w.$

Working together, in 1 hr. they will complete $\frac{w}{4}+\frac{w}{6}$ of the work $= \frac{6w+4w}{24} = \frac{10w}{24} = \frac{5w}{12}\ w$

LCM of (4,6) is 24

∴ To complete the total work time taken $= \dfrac{w}{\frac{5}{12}w}$

$= \frac{12}{5} = 2.4$ hrs. [∵ (.4) hrs = .4 × 60 = 24 min.]

= 2 hrs 24 minutes.

8. Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought ₹ 1800 worth of apples and ₹ 600 worth bananas, then how many kgs of each fruit did she buy?

Sol. Let the weight of apples be x kg and the weight of bananas be y kg.

Given $x + y = 50$ ... (1)

The cost of 1 apple is twice as that of banana, and Iniya bought ₹1800 worth apples and ₹600 worth bananas.

$\therefore\ 2x : y = 1800 : 600$

$\Rightarrow\ 600\,(2x) = y\,(1800)$

[∵ In a ratio, product of the extremes = product of the means]

$\Rightarrow\ 1200x - 1800y = 0$

$\Rightarrow\ 2x - 3y = 0$ ... (2)

[∵ Divided by 600]

$(1) \times 2 \Rightarrow\ 2x + 2y = 100$

$(2) \Rightarrow\ \overset{(-)}{2x} \overset{(+)}{-} \overset{(+)}{3y} = \overset{(-)}{0}$

$5y = -100 \Rightarrow y = \frac{100}{5} = 20$

Substituting $y = 20$ in (1) we get,

$x + 20 = 50 \Rightarrow x = 30$

Hence, Iniya bought 30 kg of apples and 20 kg of bananas.

EXERCISE 3.7

1. Find the square root of the following rational expressions.

(i) $\dfrac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$ **[Aug. - 2022]**

(ii) $\dfrac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$

(iii) $\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

Sol. (i) $\sqrt{\dfrac{400x^4y^{12}z^{16}}{100x^8y^4z^4}} = \dfrac{20x^2y^6z^8}{10x^4y^2z^2} = \dfrac{2|y^4z^6|}{|x^2|}$

(ii) $\sqrt{\dfrac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}} = \sqrt{\dfrac{(\sqrt{7}x+\sqrt{2})(\sqrt{7}x+\sqrt{2})}{\left(x-\frac{1}{4}\right)\left(x-\frac{1}{4}\right)}}$

$= \dfrac{\sqrt{7}x+\sqrt{2}}{x-\frac{1}{4}} = \dfrac{\sqrt{7}x+\sqrt{2}}{\frac{4x-1}{4}}$

$= 4\left|\dfrac{(\sqrt{7}x+\sqrt{2})}{4x-1}\right|$

Hint:

14

$\sqrt{14}$ $\sqrt{14}$

$= \dfrac{\sqrt{2\times7}}{\sqrt{7}\times\sqrt{7}}$ $\dfrac{\sqrt{2\times7}}{\sqrt{7}\times\sqrt{7}}$

$= (\sqrt{7}x+\sqrt{2})(\sqrt{7}x+\sqrt{2})$

(iii) $\sqrt{\dfrac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}}$

$= \dfrac{11(a+b)^4(x+y)^4\cancel{(b-c)^4}}{9\cancel{(b-c)^2}(a-b)^6\cancel{(b-c)^2}} = \dfrac{11}{9}\left|\dfrac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$

2. Find the square root of the following

(i) $4x^2 + 20x + 25$

(ii) $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$

(iii) $(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$

(iv) $\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

Sol. (i) Consider $4x^2 + 20x + 5$

$= (2x)^2 + 2(2x)(5) + (5)^2$

$= (2x + 5)^2$ [∵ $a^2 + 2ab + b^2 = (a + b)^2$

Here $a = 2x$, $b = 5$]

$\therefore \sqrt{4x^2+20x+25} = \sqrt{(2x+5)^2} = |2x + 5|$

(ii) Consider $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$

$= 9x^2 + 16y^2 + 25z^2 - 24xy - 40yz + 30xz$

$= (3x)^2 + (-4y)^2 + (5z)^2 + 2(3x)(-4y) + 2(-4y)(5z) + 2(5z)(3x)$

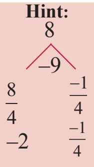
$= (3x - 4y + 5z)^2$ [$\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$
Here $a = 3x$, $b = -4y$, $c = 5z$]

$$\therefore \sqrt{9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2}$$

$$= \sqrt{(3x - 4y + 5z)^2} = |3x - 4y + 5z|$$

(iii) $(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$

Consider $4x^2 - 9x + 2 = (x - 2)(4x - 1)$
$7x^2 - 13x - 2 = (x - 2)(7x + 1)$
$28x^2 - 3x - 1 = (4x - 1)(7x + 1)$

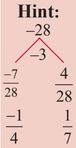


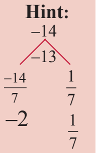
$\therefore (4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$

$$= \sqrt{(x-2)(4x-1)(x-2)(7x+1)(4x-1)(7x+1)}$$

$$= \sqrt{(x-2)^2(4x-1)^2(7x+1)^2}$$

$$= |(x - 2)(4x - 1)(7x + 1)|$$



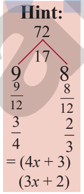


(iv) $\sqrt{\left(2x^2 + \frac{17}{6}x + 1\right)\left(\frac{3}{2}x^2 + 4x + 2\right)\left(\frac{4}{3}x^2 + \frac{11}{3}x + 2\right)}$

Consider $2x^2 + \frac{17x}{6} + 1 = \frac{12x^2 + 17x + 6}{6}$

[Multiplied by 6]

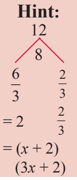
$$= \frac{1}{6}[(4x + 3)(3x + 2)]$$



Consider $\frac{3}{2}x^2 + 4x + 2 = \frac{3x^2 + 8x + 4}{2}$

[Multiplied by 2]

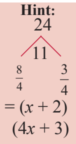
$$= \frac{1}{2}[(x + 2)(3x + 2)]$$



Consider $\frac{4}{3}x^2 + \frac{11}{3}x + 2$

$= \frac{1}{3}[4x^2 + 11x + 6]$ [Multiplied by 3]

$$\therefore \frac{1}{3}[(x + 2)(4x + 3)]$$



$$\therefore \sqrt{\left(2x^2 + \frac{17x}{6} + 1\right)\left(\frac{3}{2}x^2 + 4x + 2\right)\left(\frac{4}{3}x^2 + \frac{11x}{3} + 2\right)}$$

$$= \sqrt{\frac{1}{6}(4x+3)(3x+2) \cdot \frac{1}{2}(x+2)(3x+2) \cdot \frac{1}{3}(x+2)(4x+3)}$$

$$= \sqrt{\frac{(4x+3)^2(3x+2)^2(x+2)^2}{36}}$$

$$= \left|\frac{(4x+3)(3x+2)(x+2)}{6}\right|$$

$$= \frac{1}{6}|(4x + 3)(3x + 2)(x + 2)|$$

EXERCISE 3.8

1. Find the square root of the following polynomials by division method

(i) $x^4 - 12x^3 + 42x^2 - 36x + 9$ **[Aug. - 2022]**
(ii) $37x^2 - 28x^3 + 4x^4 + 42x + 9$
(iii) $16x^4 + 8x^2 + 1$
(iv) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

Sol. The long division method in finding the square root of a polynomial is useful when the degrees of a polynomial is higher.

(i)

$$\begin{array}{r|l}
 & x^2 - 6x + 3 \\ \hline
x^2 & \cancel{x^4} - 12x^3 + 42x^2 - 36x + 9 \\
 & \cancel{x^4} \\ \hline
2x^2 - 6x & -\cancel{12x^3} + 42x^2 \\
 & -\cancel{12x^3} + 36x^2 \\
 & \quad (+) \qquad\quad (-) \\ \hline
2x^2 - 12x + 3 & \cancel{6x^2} - \cancel{36x} + \cancel{9} \\
 & \cancel{6x^2} - \cancel{36x} + \cancel{9} \\
 & (-) \qquad (+) \qquad (-) \\ \hline
 & 0
\end{array}$$

$$\therefore \sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = |x^2 - 6x + 3|$$

(ii) $37x^2 - 28x^3 + 4x^4 + 42x + 9 = ?$

$$\begin{array}{r|l}
 & 2x^2 - 7x - 3 \\ \hline
2x^2 & \cancel{4x^4} - 28x^3 + 37x^2 + 42x + 9 \\
 & \cancel{4x^4} \\ \hline
4x^2 - 7x & -\cancel{28x^3} + 37x^2 \\
 & -\cancel{28x^3} + 49x^2 \\ \hline
4x^2 - 14x - 3 & -\cancel{12x^2} + \cancel{42x} + \cancel{9} \\
 & -\cancel{12x^2} + \cancel{42x} + \cancel{9} \\ \hline
 & 0
\end{array}$$

$$\therefore \sqrt{37x^2 - 28x^3 + 4x^4 + 42x + 9} = |2x^2 - 7x - 3|$$

(iii) $16x^4 + 8x^2 + 1$

$$\begin{array}{r|l}
 & 4x^2 + 0x + 1 \\ \hline
4x^2 & \cancel{16x^4} + 0x^3 + 8x^2 + 0x + 1 \\
 & \cancel{16x^4} \\
 & \ (-) \\ \hline
8x^2 + 0x & 0x^3 + 8x^2 \\
 & 0x^3 + 0x^2 \\ \hline
8x^2 + 0x + 1 & \cancel{8x^2} + \cancel{0x} + \cancel{1} \\
 & \cancel{8x^2} + \cancel{0x} + \cancel{1} \\
 & (-) \quad (-) \quad (-) \\ \hline
 & 0
\end{array}$$

$$\therefore \sqrt{16x^4 + 8x^2 + 1} = |4x^2 + 1|$$

(iv) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

$$\begin{array}{r|l}
 & 11x^2 - 9x - 12 \\ \cline{2-2}
11x^2 & \cancel{121x^4} - 198x^3 - 183x^2 + 216x + 144 \\
 & \underset{(-)}{\cancel{121x^4}} \\ \cline{2-2}
22x^2 - 9x & -\cancel{198x^3} - 183x^2 \\
 & -\underset{(+)}{\cancel{198x^3}} + \underset{(-)}{81x^2} \\ \cline{2-2}
22x^2 - 18x - 12 & -\cancel{264x^2} + \cancel{216x} + \cancel{144} \\
 & \underset{(+)}{-\cancel{264x^2}} + \underset{(-)}{\cancel{216x}} + \underset{(-)}{\cancel{144}} \\ \cline{2-2}
 & 0
\end{array}$$

$\therefore \sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144}$

$= |11x^2 - 9x - 12|$

2. Find the values of *a* and *b* if the following polynomials are perfect squares

(i) $4x^4 - 12x^3 + 37x^2 + bx + a$ [PTA - 4]

(ii) $ax^4 + bx^3 + 361x^2 + 220x + 100$

Sol. (i)

$$\begin{array}{r|l}
 & 2x^2 - 3x + 7 \\ \cline{2-2}
2x^2 & \cancel{4x^4} - 12x^3 + 37x^2 + bx + a \\
 & \underset{(-)}{\cancel{4x^4}} \\ \cline{2-2}
4x^2 - 3x & -\cancel{12x^3} + 37x^2 \\
 & -\underset{(+)}{\cancel{12x^3}} + \underset{(-)}{9x^2} \\ \cline{2-2}
4x^2 - 6x + 7 & \cancel{28x^2} + bx + a \\
 & \underset{(-)}{\cancel{28x^2}} - \underset{(+)}{42x} + \underset{(-)}{49} \\ \cline{2-2}
 & (b + 42)x + (a - 49)
\end{array}$$

Since it is a perfect square.

Remainder = 0 $\Rightarrow b + 42 = 0, a - 49 = 0$

$b = -42, a = 49$

(ii) $ax^4 + bx^3 + 36x^2 + 220x + 100$

$$\begin{array}{r|l}
 & 10 + 11x + 12x^2 \\ \cline{2-2}
10 & \cancel{100} + 220x + 361x^2 + bx^3 + ax^4 \\
 & \underset{(-)}{\cancel{100}} \\ \cline{2-2}
20 + 11x & \cancel{220x} + 361x^2 \\
 & \underset{(-)}{\cancel{220x}} + \underset{(-)}{121x^2} \\ \cline{2-2}
20 + 22x + 12x^2 & \cancel{240x^2} + bx^3 + ax^4 \\
 & \underset{(-)}{\cancel{240x^2}} + \underset{(-)}{264bx^3} + \underset{(-)}{144x^4} \\ \cline{2-2}
 & (b - 264)x^3 + (a-144)x^4
\end{array}$$

Since remainder is 0

$a = 144$

$b = 264$

3. Find the values of *m* and *n* if the following polynomials are perfect squares

(i) $36x^4 - 60x^3 + 61x^2 - mx + n$ [May - 2022]

(ii) $x^4 - 8x^3 + mx^2 + nx + 16$ [FRT - 2022]

Sol. (i)

$$\begin{array}{r|l}
 & 6x^2 - 5x + 3 \\ \cline{2-2}
6x^2 & \cancel{36x^4} - 60x^3 + 61x^2 - mx + n \\
 & \underset{(-)}{\cancel{36x^4}} \\ \cline{2-2}
12x^2 - 5x & -\cancel{60x^3} + 61x^2 \\
 & -\underset{(+)}{\cancel{60x^3}} + \underset{(-)}{25x^2} \\ \cline{2-2}
12x^2 - 10x + 3 & \cancel{36x^2} - mx + n \\
 & \underset{(-)}{\cancel{36x^2}} - \underset{(+)}{30x} + \underset{(-)}{9} \\ \cline{2-2}
 & (-m + 30)x + (n - 9) = 0
\end{array}$$

Since remainder is 0

$\Rightarrow m = 30, n = 9$

(ii)

$$\begin{array}{r|l}
 & x^2 - 4x + 4 \\ \cline{2-2}
x^2 & \cancel{x^4} - 8x^3 + mx^2 + nx + 16 \\
 & \underset{(-)}{\cancel{x^4}} \\ \cline{2-2}
2x^2 - 4x & -\cancel{8x^3} + mx^2 \\
 & -\underset{(+)}{\cancel{8x^3}} + \underset{(-)}{16x^2} \\ \cline{2-2}
2x^2 - 8x + 4 & (m - 16)x^2 + nx + \cancel{16} \\
 & \underset{(-)}{8x^2} - \underset{(+)}{32x} + \underset{(-)}{\cancel{16}} \\ \cline{2-2}
 & (m - 24)x^2 + (n + 32) = 0
\end{array}$$

Since remainder is 0,

$m = 24, n = -32$

EXERCISE 3.9

1. Determine the quadratic equations, whose sum and product of roots are [Qy. - 2019]

(i) $-9, 20$ [Sep. - 2021] **(ii) $\frac{5}{3}, 4$**

(iii) $\frac{-3}{2}, -1$ [PTA - 4]

(iv) $-(2 - a)^2, (a + 5)^2$

Sol. If the roots are given, general form of the quadratic equation is x^2 – (sum of the roots) x + product of the roots = 0.

(i)

Sum of the roots = -9

Product of the roots = 20

The equation = $x^2 - (-9x) + 20 = 0$

$\Rightarrow x^2 + 9x + 20 = 0$

(ii)

Sum of the roots = $\frac{5}{3}$

Product of the roots = 4

Required equation = x^2 – (sum of the roots) x + product of the roots = 0

$$\Rightarrow \quad x^2 - \frac{5}{3}x + 4 = 0$$

$$\Rightarrow \quad 3x^2 - 5x + 12 = 0 \quad \text{[Multiplied by 3]}$$

(iii) Sum of the roots $= \left(\frac{-3}{2}\right)$

$$(\alpha + \beta) = \frac{-3}{2}$$

Product of the roots $(\alpha\beta) = (-1)$

Required equation $= x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$x^2 - \left(\frac{-3}{2}\right)x - 1 = 0$$

$$2x^2 + 3x - 2 = 0 \quad \text{[Multiplied by 2]}$$

(iv) $\alpha + \beta = -(2 - a)^2$

$$\alpha\beta = (a + 5)^2$$

Required equation $= x^2 - (\alpha + \beta)x - \alpha\beta = 0$

$\Rightarrow x^2 - (-(2 - a)^2)x + (a + 5)^2 = 0$

$\Rightarrow x^2 + (2 - a)^2x + (a + 5)^2 = 0$

2. Find the sum and product of the roots for each of the following quadratic equations

(i) $x^2 + 3x - 28 = 0$ **(ii) $x^2 + 3x = 0$** **[Qy. - 2019]**

(iii) $3 + \frac{1}{a} = \frac{10}{a^2}$ **(iv) $3y^2 - y - 4 = 0$**

Sol. (i) $x^2 - (-3)x + (-28) = 0$.

Comparing this with $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

$(\alpha + \beta) =$ Sum of the roots $= -3$

$\alpha\beta =$ product of the roots $= -28$

(ii) $x^2 + 3x = 0 = x^2 - (-3)x + 0 = 0$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$\therefore$ Sum of the roots $\alpha + \beta = -3$

Products of the roots $\alpha\beta = 0$

(iii) Given equation is $3 + \frac{1}{a} = \frac{10}{a^2}$

LCM of a, a^2 is a^2

Multiplying by a^2 throughout we get,

$$3a^2 + \frac{1}{a}(a^2) = \frac{10}{a^2}(a^2)$$

$$\Rightarrow \quad 3a^2 + a = 10$$

$$\Rightarrow \quad 3a^2 + a - 10 = 0$$

Dividing by 3 we get, $a^2 + \frac{1}{3} = 9 - \frac{10}{3} = 0$

$$a^2 - \left(-\frac{1}{3}\right)a + \left(\frac{-10}{3}\right) = 0$$

Comparing this with $x^2 - x$ (Sum of the roots) + product of the roots = 0, we get

Sum of the roots $= \frac{-1}{3}$ and

Product of the roots $= \frac{-10}{3}$

(iv) Given equation is $3y^2 - y - 4 = 0$

Dividing by 3 we get,

$$y^2 - \frac{1}{3}y - \frac{4}{3} = 0$$

$$y^2 - y\left(\frac{1}{3}\right) - \frac{4}{3} = 0$$

Equating this with $x^2 - x$ (Sum of the roots) + Product of the roots = 0

We get, sum of the roots $= \frac{1}{3}$ and

Product of the roots $= \frac{-4}{3}$

EXERCISE 3.10

1. Solve the following quadratic equations by factorization method

(i) $4x^2 - 7x - 2 = 0$

(ii) $3(p^2 - 6) = p(p + 5)$

(iii) $\sqrt{a(a - 7)} = 3\sqrt{2}$

(iv) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

(v) $2x^2 - x + \frac{1}{8} = 0$

Sol. (i) Given equation is $4x^2 - 7x - 2 = 0$

On factorization we get,

$$(x - 2)(4x + 1) = 0$$

$$\Rightarrow \quad x - 2 = 0 \text{ or}$$

$$4x + 1 = 0$$

$$x = 2 \text{ or}$$

$$4x = -1$$

$$x = 2 \text{ or}$$

$$x = -\frac{1}{4}$$

$\therefore$ Solution set is $\left\{2, \frac{-1}{4}\right\}$

Hint:

-8

-7

$-8 \quad 1$

$= \frac{-8}{4} \quad \frac{1}{4}$

$= -2 \quad \frac{1}{4}$

$= (x - 2)(4x + 1)$

(ii) Given equation is $3(p^2 - 6) = p(p + 5)$

$$\Rightarrow \quad 3p^2 - 18 = p^2 + 5p$$

$$\Rightarrow \quad 3p^2 - 18 - p^2 - 5p = 0$$

$$\Rightarrow \quad 2p^2 - 5p - 18 = 0$$

On factorization we get,

$$(2p - 9)(p + 2) = 0$$

Hint:

-36

-5

$-9 \quad 4$

$= (2p - 9)(p + 2)$

$\Rightarrow \quad 2p - 9 = 0$

or $\quad p + 2 = 0$

$\Rightarrow \quad 2p = 9 \quad$ or $\quad p = -2$

$\Rightarrow \quad p = \frac{9}{2} \quad$ or $\quad p = -2$

$\therefore$ Solution set is $\left\{\frac{9}{2}, -2\right\}$

(iii) Given equation is $\sqrt{a(a-7)} = 3\sqrt{2}$

Squaring on both sides we get,

$$\left[\sqrt{a(a-7)}\right]^2 = (3\sqrt{2})^2$$

$\Rightarrow \quad a(a-7) = 9\ (2)$

$a^2 - 7a = 18$

$a^2 - 7a - 18 = 0$

On factorization we get,

$(x-9)(x+2) = 0$

$x - 9 = 0$ or $x + 2 = 0$

$x = 9$ or -2

Hint:
-18
-7
$-9 \quad 2$

$\therefore$ Solution set is $\{-2, 9\}$

(iv) Given equation is $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

On factorization we get,

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow \quad \sqrt{2}x + 5 = 0$ or

$x + \sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x = -5$ or

$x = -\sqrt{2}$

$\Rightarrow \quad x = \frac{-5}{\sqrt{2}}$ or

$x = -\sqrt{2}$

Hint:
10
-7
$\frac{5}{\sqrt{2}} \quad \frac{2}{\sqrt{2}}$

$= \frac{5}{\sqrt{2}} \quad \frac{\sqrt{2}.\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$= \frac{5}{\sqrt{2}} \quad \frac{\sqrt{2}}{1}$

$= (\sqrt{2}x + 5)$
$(x + \sqrt{2})$

$\therefore$ Solution set is $\left\{\frac{-5}{\sqrt{2}}, -\sqrt{2}\right\}$

(v) Given equation is $2x^2 - x + \frac{1}{8} = 0$

Multiplying by 8 we get, $16x^2 - 8x + 1 = 0$

On factorization we get $(4x - 1)(4x - 1) = 0$

$\Rightarrow \quad 4x - 1 = 0$ or $\quad 4x - 1 = 0$

$\Rightarrow \quad 4x = 1$ or $\quad 4x = 1$

$\Rightarrow \quad x = \frac{1}{4}$ or $\quad 4x = \frac{1}{4}$

$\therefore$ Solution set is $\left\{\frac{1}{4}, \frac{1}{4}\right\}$

Hint:
16
-8
$\frac{-4}{16} \quad \frac{-4}{16}$
$\frac{-1}{4} \quad \frac{-1}{4}$
$(4x-1)\ (4x-1)$

2. The number of volleyball games that must be scheduled in a league with n teams is given by $G(n) = \frac{n^2 - n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

Sol.

$$G(n) = \frac{n^2 - n}{2} \Rightarrow 15 = \frac{n^2 - n}{2}$$

$\Rightarrow \quad 30 = n^2 - n$

$n^2 - n - 30 = 0$

$\Rightarrow n^2 - 6n + 5n - 30 = 0$

$n(n-6) + 5(n-6) = 0$

$(n-6)(n+5) = 0$

$\Rightarrow \quad n = 6, -5$

As n cannot be (–ve), $n = 6$.

$\therefore$ There are 6 teams in the league.

Hint:
-30
-1
$-6 \quad 5$
$(n-6)\ (n+5)$

EXERCISE 3.11

1. Solve the following quadratic equations by completing the square method

(i) $9x^2 - 12x + 4 = 0$

(ii) $\frac{5x+7}{x-1} = 3x + 2$ **[PTA - 3]**

Sol. (i) Given equation is

$9x^2 - 12x + 4 = 0$

To make the co-efficient of x^2 as 1, divide by 9

$\therefore x^2 - \frac{12x}{9} + \frac{4}{9} = 0$

$\Rightarrow \quad x^2 - \frac{4x}{3} + \frac{4}{9} = 0$

$\Rightarrow \quad x^2 - \frac{4x}{3} = -\frac{4}{9}$

[Adding $\left[\frac{1}{2}(\text{co - efficient of } x)\right]^2$

$= \left[\frac{1}{2}\left(\frac{-4}{3}\right)\right]^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$ to both sides]

Hint:

$$\left(\frac{-b}{2a}\right)^2 = \left(\frac{-2}{3}\right)^2$$

$$\left[\frac{4}{2(3)}\right]^2 = \left(\frac{2}{3}\right)^2$$

$\Rightarrow \quad x^2 - \frac{4x}{3} + \frac{4}{9} = \frac{-4}{9} + \frac{4}{9} \Rightarrow \left(x - \frac{2}{3}\right)^2 = 0$

$x - \frac{2}{3} = 0$ or $x - \frac{2}{3} = 0$

$x = \frac{2}{3}, \frac{2}{3}$

$[a^2 - 2ab + b^2 = (a-b)^2$ Here $a = x,\ b = +\frac{2}{3}]$

$\therefore$ Solution set is $\left\{\frac{2}{3}, \frac{2}{3}\right\}$

(ii) Given equation is $\frac{5x+7}{x-1} = 3x+2$

Cross multiplying we get,

$$\begin{aligned}
5x+7 &= (x-1)(3x+2)\\
\Rightarrow\quad 5x+7 &= 3x^2-3x+2x-2\\
\Rightarrow\quad 5x+7 &= 3x^2-x+2\\
\Rightarrow\quad 3x^2-x-2-5x-7 &= 0\\
\Rightarrow\quad 3x^2-6x-9 &= 0\\
\Rightarrow\quad x^2-2x-3 &= 0 \qquad \text{[Divided by 3]}\\
\Rightarrow\quad x^2-2x &= 3
\end{aligned}$$

[Adding $\left[\frac{1}{2}(\text{co-efficient of } x)\right]^2$

$$\left[\frac{1}{2}-(2)\right]^2 = (-1)^2 = 1 \Big]$$

$$\begin{aligned}
\text{We get, } x^2-2x+1 &= 3+1\\
\Rightarrow\quad (x-1)^2 &= 4\\
\Rightarrow\quad (x-1)^2 &= 2^2\\
\Rightarrow\quad x-1 &= \pm 2
\end{aligned}$$

[Taking square root both sides]

$$\begin{aligned}
\Rightarrow\quad x-1 &= 2 \text{ or } x-1 = -2\\
\Rightarrow\quad x &= 3 \text{ or } x = -2+1 = -1
\end{aligned}$$

$\therefore$ Solution set is $\{3, -1\}$

2. Solve the following quadratic equations by formula method

(i) $2x^2 - 5x + 2 = 0$

(ii) $\sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$

(iii) $3y^2 - 20y - 23 = 0$

(iv) $36y^2 - 12ay + (a^2 - b^2) = 0$

Sol. (i) $2x^2 - 5x + 2 = 0$

The formula for finding roots of a quadratic equation $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$2x^2 - 5x + 2 = 0$ [Here $a = 2$, $b = -5$, $c = 2$]

$$\therefore x = \frac{-(-5)\pm\sqrt{(-5)^2-4\times2\times2}}{2\times2}$$

$$= \frac{5\pm\sqrt{25-16}}{4} = \frac{5\pm\sqrt{9}}{4}$$

$$x = \frac{5\pm\sqrt{9}}{4}$$

$$x = \frac{5+3}{4} \text{ or } \frac{5-3}{4} = \frac{8}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 2, \frac{1}{2}$$

(ii) $\sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$

[Here $a = \sqrt{2}$, $b = -6$, $c = 3\sqrt{2}$]

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\text{Here } f = \frac{-(-6)\pm\sqrt{(-6)^2-4\times\sqrt{2}\times3\sqrt{2}}}{2\times\sqrt{2}}$$

$$= \frac{6\pm\sqrt{36-24}}{2\sqrt{2}} = \frac{6\pm\sqrt{12}}{2\sqrt{2}}$$

$$= \frac{\cancel{2}\left(3\pm\sqrt{3}\right)}{\cancel{2}\sqrt{2}} \Rightarrow \frac{3+\sqrt{3}}{\sqrt{2}}, \frac{3-\sqrt{3}}{\sqrt{2}}$$

$\therefore$ Solution set is $\left\{\frac{3+\sqrt{3}}{\sqrt{2}}, \frac{3-\sqrt{3}}{\sqrt{2}}\right\}$

(iii) $3y^2 - 20y - 23 = 0$

[Here $a = 3$, $b = -20$, $c = -23$]

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\text{Here } y = \frac{-(-20)\pm\sqrt{(-20)^2-4\times3\times-23}}{2\times3}$$

$$= \frac{20\pm\sqrt{400+276}}{6}$$

$$= \frac{20\pm\sqrt{676}}{6} = \frac{20\pm26}{6} = \frac{46}{6} \text{ or } \frac{-6}{6}$$

$$y = \frac{23}{3} \text{ or } -1$$

$\therefore$ Solution set is $\left\{\frac{23}{3}, -1\right\}$

(iv) $36y^2 - 12ay + (a^2 - b^2) = 0$

[Here $a = 36$, $b = -12a$, $c = a^2 - b^2$]

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\text{Here } y = \frac{-(-12a)\pm\sqrt{(-12a)^2-4\times36\times(a^2-b^2)}}{2\times36}$$

$$= \frac{12a\pm\sqrt{144a^2-144a^2+144b^2}}{72}$$

$$= \frac{12a\pm12b}{72} \Rightarrow \frac{a\pm b}{6} = \frac{a+b}{6}, \frac{a-b}{6}$$

$\therefore$ Solution set is $\left\{\frac{a+b}{6}, \frac{a-b}{6}\right\}$

3. **A ball rolls down a slope and travels a distance $d = t^2 - 0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet.**

Sol. Distance $d = t^2 - 0.75\, t$,

Given that $d = 11.25 = t^2 - 0.75\, t$.

$t^2 - 0.75\, t - 11.25 = 0$

[Here $a = 1$, $b = -0.75$, $c = -11.25$]

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(+0.75) \pm \sqrt{(-0.75)^2 - 4 \times 1 \times -11.25}}{2 \times 1}$$

$$= \frac{+0.75 \pm \sqrt{0.5625 + 45}}{2}$$

$$= \frac{+0.75 \pm \sqrt{45.5625}}{2} = \frac{+0.75 \pm 6.75}{2}$$

$$= \frac{7.50}{2} \text{ or } \frac{-6}{2} = 3.75 \text{ or } -3 \text{ is not possible.}$$

$\therefore$ $t = 3.75$ sec.

$\therefore$ The required time is 3.75 sec.

EXERCISE 3.12

1. **If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.** **[PTA - 6]**

Sol. Let the required number be x and hence its reciprocal is $\frac{1}{x}$.

$$\text{Given } \frac{x}{1} - \frac{1}{x} = \frac{24}{5}$$

LCM of 1, x is x

$$\therefore \frac{x^2 - 1}{x} = \frac{24}{5}$$

$$\Rightarrow 5(x^2 - 1) = 24x$$

[By cross multiplying]

$$\Rightarrow 5x^2 - 5 = 24x$$

$$5x^2 - 24x - 5 = 0$$

On factorization we get,

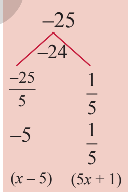
$$(x - 5)(5x + 1) = 0$$

$$\Rightarrow x - 5 = 0 \text{ or } 5x + 1 = 0$$

$$\Rightarrow x = 5 \text{ or } 5x = -1$$

$$\Rightarrow x = 5 \text{ or } x = \frac{-1}{5}$$

Hint:



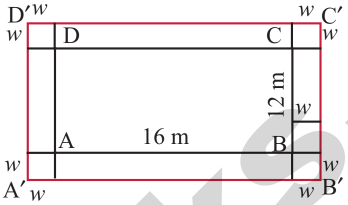
Hence, the required number may be 5 or $\frac{-1}{5}$.

$$x = \frac{-1}{5}, 5$$

$\therefore$ The number is $\frac{-1}{5}$ or 5.

2. **A garden measuring 12m by 16m is to have a pedestrian pathway that is 'w' meters wide installed all the way around so that it increases the total area to 285 m². What is the width of the pathway?**

Sol.



Length and breadth of rectangle ABCD are 16 m and 12m.

Area of ABCD = Length × breadth

$= 12 \times 16 = 192$ m²

Given Area of rectangle A′ B′ C′ D′ = 285 m²

Length of rectangle A′ B′ C′ D′

$= 16 + w + w = 16 + 2w$

and Breadth of rectangle A′ B′ C′ D′

$= 12 + w + w = 12 + 2w$

$\therefore$ $(16 + 2w)(12 + 2w) = 285$ [Given]

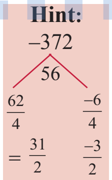
$\Rightarrow$ $192 + 32w + 24w + 4w^2 = 285$

$\Rightarrow$ $4w^2 + 56w + 192 - 285 = 0$

[Simplifying the like terms]

$\Rightarrow$ $4w^2 + 56w - 93 = 0$

Hint:



On factorization we get, $(2w + 31)(2w - 3) = 0$

$\Rightarrow$ $2w + 31 = 0$ or $2w - 3 = 0$

$\Rightarrow$ $2w = -31$ or $2w = 3$

$$\Rightarrow w = \frac{-31}{2} \text{ or } w = \frac{3}{2} = 1.5$$

Since the width of the pedestrian pathway cannot be negative $w = 1.5$.

$\therefore$ Width of the pathway = 1.5 m.

3. **A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.**

Sol. Let x km/hr be the constant speed of the bus.

The time taken to cover 90 km = $\frac{90}{x}$ hrs.

When the speed is increased by 15 km/hr, the time taken

$$= \frac{90}{x + 15} \text{ hrs.}$$

$$\text{Given that } \frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{90(x+15)-90x}{x(x+15)} = \frac{1}{2}$$

[∴LCM of x, $(x + 15)$ is $x\,(x + 15)$]

$$\Rightarrow \quad \frac{\cancel{90x}+1350-\cancel{90x}}{x+(x+15)} = \frac{1}{2}$$

$$\Rightarrow \quad 2(1350) = x\,(x + 15)$$

[By cross multiplication]

$$\Rightarrow \quad 2700 = x^2 + 15x$$

$$\Rightarrow \quad x^2 + 15x - 2700 = 0$$

Hint:
–2700
15
60 –45

On factorizing we get,

$$(x - 45)\,(x + 60) = 0$$

$$\Rightarrow \quad x - 45 = 0 \text{ or } x + 60 = 0$$

$$\Rightarrow \quad x = 45 \text{ or } x = -60$$

Since the speed of the bus cannot be negative,

$$x = 45$$

Hence, original speed of the bus is 45 km/hr.

4. **A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.** **[PTA - 4]**

Sol. Let the present age of the girl be $2x$ and her sisters age be x years.

[∵ the girl is twice as old as her sister]

Five years hence their ages will be $(2x + 5)$ and $(x + 5)$.

$$\text{Given that } (2x + 5)\,(x + 5) = 375$$

$$\Rightarrow \quad 2x^2 + 10x + 5x + 25 = 375$$

$$\Rightarrow \quad 2x^2 + 15x + 25 - 375 = 0$$

$$\Rightarrow \quad 2x^2 + 15x - 350 = 0$$

Hint:
–700
15
$\frac{-20}{2}$ $\frac{35}{2}$
–10 $\frac{35}{2}$
$(x – 10)$ $(2x + 35)$

On factorizing we get,

$$(x - 10)\,(2x + 35) = 0$$

$$\Rightarrow \quad x = 10 \text{ or } 2x = -35$$

$$\Rightarrow \quad x = 10 \text{ or } x = \frac{-35}{2}$$

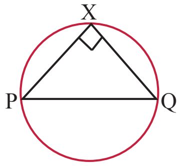
Since the age of the girl cannot be negative,

$$x = 10$$

Hence, the age of the girl is $2x = 2(10) = 20$ years and the age of her sister is $x = 10$ years.

5. **A pole has to be erected at a point on the boundary of a circular ground of diameter 20 m in such a way that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4 m. Is it possible to do so? If answer is yes at what distance from the two gates should the pole be erected?**

Sol.



Let X be the position of the pole and P, Q be the position of the fixed gates on a circular ground.

Given PQ = 20 m

and PX – XQ = 4m ... (1)

Squaring both sides we get,

$$(PX - XQ)^2 = 4^2$$

$$\Rightarrow PX^2 + XQ^2 - 2(PX)\,(XQ) = 16$$

[∵ $(a - b)^2 = a^2 - 2ab + b^2$]

$$\Rightarrow \quad PQ^2 - 2(PX)\,(XQ) = 16$$

[∵ In right angled ΔPXQ, ∠PXQ = 90°
[angle in a semi circle in 90°]
and $PX^2 + XQ^2 = PQ^2$ by pythagoras theorem]

$$\Rightarrow \quad 20^2 - 2(PX)(XQ) = 16$$

[∵ PQ = 20]

$$\Rightarrow \quad 400 - 16 = 2\,(PX)\,(XQ)$$

$$\Rightarrow \quad 384 = 2\,(PX)\,(XQ)$$

$$\Rightarrow \quad 192 = (PX)\,(XQ) \quad \text{... (2)}$$

Consider $(PX + QX)^2 = PX^2 + QX^2 + 2\,(PX)\,(QX)$

$= PQ^2 + 2\,(PX)\,(QX)$

$= 20^2 + 2\,(192)$

[∵ PQ = 20 and from (2)]

$= 400 + 384$

$= 784 = 28^2$

$$\Rightarrow \quad (PX + QX)^2 = 28^2$$

Taking square root both sides we get,

$$PX + QX = 28 \quad \text{... (3)}$$

(1) ⇒ $PX - XQ = 4$

(3) ⇒ $PX + XQ = 28$

Adding, $2PX = 32$

$$\Rightarrow \quad PX = \frac{32}{2} = 16$$

Substituting PX =16 in (3) we get,

$$16 + XQ = 28$$

$$\Rightarrow \quad XQ = 28 - 16 = 12$$

∴ The distance from the two gates to the pole are 12m and 16m and such a pole can be erected.

6. **From a group of $2x^2$ black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total?**

Sol. Given that the number of black bees = $2x^2$ ... (1)
Number of bees went to a tree

$$= \sqrt{\frac{1}{\cancel{2}} \times \cancel{2}x^2} = \sqrt{x^2} = x \quad ... (2)$$

Second batch of bees that went to the same tree

$$= \frac{8}{9} \times 2x^2 = \frac{16}{9}x^2 \quad ... (3)$$

The number of remaining bees = 2 ... (4)
From (1), (2), (3) and (4),

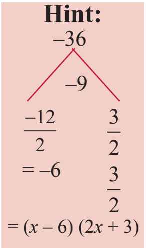
$$2x^2 - x - \frac{16}{9}x^2 = 2$$

Multiplying by 9 we get,

$$18x^2 - 9x - 16x^2 = 18$$
$$\Rightarrow \quad 2x^2 - 9x = 18$$
$$\Rightarrow \quad 2x^2 - 9x - 18 = 0$$

On factorizing we get, $(x-6)(2x+3) = 0$

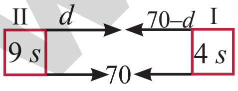
$$\Rightarrow \quad x = 6, \text{ or } 2x + 3 = 0$$
$$\Rightarrow \quad x = 6, \text{ or } 2x = -3$$
$$\Rightarrow \quad x = 6, \text{ or } x = \frac{-3}{2}$$



Since number of bees cannot be negative $x = 6$
Hence, total number of bees = $2x^2 = 2(6)^2$
$= 2(36) = 72$

7. **Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70 m. Where should a person stand for hearing the same intensity of the singers voice? (Hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances).**

Sol. Let the person stand at a distance 'd' from 2nd gallery having 9 singers.



Given that ratio of sound intensity is equal to the square of the ratio of their corresponding distance.

$$\therefore \quad \frac{9}{4} = \frac{d^2}{(70-d)^2}$$
$$4d^2 = 9(70-d)^2$$
$$4d^2 = 9(70^2 - 140d + d^2)$$
$$4d^2 = 9 \times 70^2 - 9 \times 140d + 9d^2$$
$$\therefore 5d^2 - 9 \times 140d + 9 \times 70^2 = 0$$
$$5d^2 = 1260d + 44100 = 0$$
$$d^2 - 252d + 8820 = 0 \quad \text{[Divided by 5]}$$

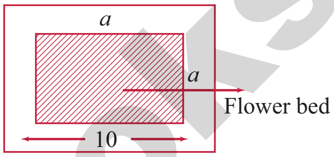
On factorisation,

$$(d-210)(d-42) = 0$$
$$d = 210 \text{ or } d = 42$$
$$d = 210 \text{ is not possible}$$
$$\therefore d = 42$$

$\therefore$ The person stands at a distance of 42 m from second gallery will hear the singers voice with same intensity.

8. **There is a square field whose side is 10 m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹3 and ₹4 per square metre respectively is ₹364. Find the width of the gravel path.**

Sol.



Let a be the side of the square flower bed

$\therefore$ Area of the flower bed $= a \times a = a^2$

Total area of the square garden = 10 × 10 = 100

$\therefore$ Area of the gravel path = Area of the garden – area of the flower bed
$= 100 - a^2$

Cost of making flower bed for 1 sq.m = ₹ 3

$\therefore$ Cost of making flower bed for a^2 sq.m = ₹ $3a^2$ ... (1)

Cost of gravel path for 1 sq.m = ₹ 4

$\therefore$ Cost of making gravel path for $(100 - a^2)$ sq.m = $4(100 - a^2)$ ... (2)

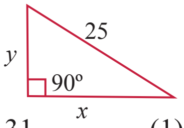
Given that the cost of flower bed + Cost of gravelling = ₹ 364

$$3a^2 + 4(100 - a^2) = 364$$
[From (1) and (2)]
$$3a^2 + 400 - 4a^2 = 364$$
$$\therefore a^2 = 400 - 364$$
$$a^2 = 36 \Rightarrow a = 6$$

$$\therefore \text{ Width of gravel path } = \frac{10-6}{2} = \frac{4}{2} = 2 \text{ m}$$

9. **The hypotenuse of a right angled triangle is 25 cm and its perimeter 56 cm. Find the length of the smallest side.**

Sol. Let x, y and 25 be the length of the right angled triangle.



Given perimeter = 56 cm

$$\therefore x + y + 25 = 56$$
$$\Rightarrow \quad x + y = 56 - 25 = 31 \quad ... (1)$$

Also, $x^2 + y^2 = 25^2$ [By Pythagoras theorem] ... (2)

Since $x^2 + y^2 = (x + y)^2 - 2xy$

$$\Rightarrow 25^2 = 31^2 - 2\,xy$$
$$\Rightarrow 625 = 961 - 2\,xy$$
$$\Rightarrow 2\,xy = 961 - 625 = 336$$
$$\Rightarrow xy = \frac{336}{3} = 168 \quad ... (3)$$

From (1) and (3), sum of the roots = 31 and product of the roots = 168.

∴The quadratic equation is $x^2 - x$ (Sum of the roots) + Product of the roots = 0.

$$\Rightarrow x^2 - 31x + 168 = 0$$

On factorising we get,

$$(x - 24)(x - 7) = 0$$
$$\Rightarrow x = 24, \text{ or } x = 7$$

Hint:
168
–31
–24 –7

∴ Length of the smallest side is 7 cm.

EXERCISE 3.13

1. Determine the nature of the roots for the following quadratic equations

(i) $15x^2 + 11x + 2 = 0$ [Sep. - 2021]

(ii) $x^2 - x - 1 = 0$ [FRT - 2022]

(iii) $\sqrt{2}\,t^2 - 3t + 3\sqrt{2} = 0$

(iv) $9y^2 - 6\sqrt{2}\,y + 2 = 0$

(v) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$
$a \neq 0, b \neq 0$

Sol.

(i) $15x^2 + 11x + 2 = 0$ comparing with $ax^2 + bx + c = 0$. Here $a = 15$, $b = 11$, $c = 2$.

$$\Delta = b^2 - 4ac$$
$$= 11^2 - 4 \times 15 \times 2$$
$$= 121 - 120$$
$$= 1 > 0.$$

∴ The roots are real and unequal.

(ii) $x^2 - x - 1 = 0$, Here $a = 1$, $b = -1$, $c = -1$.

$$\Delta = b^2 - 4ac = (-1)^2 - 4 \times 1 \times -1$$
$$= 1 + 4 = 5 > 0.$$

∴ The roots are real and unequal.

(iii) $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$

$a = \sqrt{2}$, $b = -3$, $c = 3\sqrt{2}$

$$\Delta = b^2 - 4ac$$
$$= (-3)^2 - 4 \times \sqrt{2} \times 3\sqrt{2}$$
$$= 9 - 24$$
$$= -15 < 0.$$

∴ The roots are not real.

(iv) $9y^2 - 6\sqrt{2}\,y + 2 = 0$

$a = 9$, $b = -6\sqrt{2}$, $c = 2$

$$\Delta = b^2 - 4ac$$
$$= (-6\sqrt{2})^2 - 4 \times 9 \times 2$$
$$= 36 \times 2 - 72 = 72 - 72 = 0$$

∴ The roots are real and equal.

(v) $\underset{a}{9a^2b^2}x^2 - \underset{b}{24abcd}x + \underset{c}{16c^2d^2} = 0$

$$\Delta = b^2 - 4ac$$
$$= (-24abcd)^2 - 4 \times 9a^2b^2 \times 16c^2d^2$$
$$= 576a^2b^2c^2d^2 - 576a^2b^2c^2d^2 = 0$$

∴ The roots are real and equal.

2. Find the value(s) of 'k' for which the roots of the following equations are real and equal.

(i) $(5k - 6)x^2 + 2kx + 1 = 0$

(ii) $kx^2 + (6k + 2)x + 16 = 0$

Sol.

(i) $\underset{a}{(5k - 6)}x^2 + \underset{b}{2k}x + \underset{c}{1} = 0$

$$\Delta = b^2 - 4ac$$
$$= (2k)^2 - 4\,(5k - 6)(1)$$
$$\Rightarrow 4k^2 - 20k + 24 = 0$$
$$\Rightarrow k^2 - 5k + 6 = 0$$
$$\Rightarrow (k - 3)(k - 2) = 0$$
$$k = 3, 2$$

∵ the roots are real & equal.

(ii) $\underset{a}{k}x^2 + (6k + \underset{b}{2})x + \underset{c}{16} = 0$

$$\Delta = b^2 - 4ac = 0$$
$$\Rightarrow (6k + 2)^2 - 4 \times k \times 16 = 0$$
$$\Rightarrow 36k^2 + 24k + 4 - 64k = 0$$
$$\Rightarrow 36k^2 - 40k + 4 = 0$$

Dividing by 4 we get,

$$\Rightarrow 9k^2 - 10k + 1 = 0$$
$$\Rightarrow (k - 1)\ (9k - 1) = 0$$
$$\Rightarrow k = 1 \text{ or } k = \frac{1}{9}$$

Hint:
9
–10
$\frac{-9}{9}$ $\frac{-1}{9}$
–1 $\frac{-1}{9}$

3. If the roots of $(a - b)x^2 + (b - c)x + (c - a) = 0$ are real and equal, then prove that b, a, c are in arithmetic progression. [Hy. - 2019; FRT - 2022]

Sol. $\underset{A}{(a - b)}x^2 + \underset{B}{(b - c)}x + \underset{C}{(c - a)} = 0$

$$\Delta = B^2 - 4AC = 0$$
$$\Rightarrow (b - c)^2 - 4(a - b)(c - a)$$
$$\Rightarrow b^2 - 2bc + c^2 - 4\,(ac - bc - a^2 + ab)$$
$$\Rightarrow b^2 - 2bc + c^2 - 4ac + 4bc + 4a^2 - 4ab = 0$$
$$\Rightarrow 4a^2 + b^2 + c^2 + 2bc - 4ac - 4ab = 0$$
$$\Rightarrow (-2a + b + c)^2 = 0 \ [\because (a + b + c)^2 = a^2+b^2+c^2 + 2ab + 2bc + 2ca)]$$

5. If one root of the equation $2y^2 - ay + 64 = 0$ is twice the other then find the values of a.

Sol. Given equation is $2y^2 - ay + 64 = 0$

$$\Rightarrow \quad y^2 - \frac{a}{2}y + 32 = 0 \qquad \text{[Dividing by 2]}$$

Since one root is twice the other, let the roots be α and 2α

$$\therefore \text{Sum of the roots} = \alpha + 2\alpha = + \frac{a}{2}$$

$$\Rightarrow \quad 3\alpha = \frac{a}{2} \Rightarrow \alpha = \frac{a}{6} \qquad \dots (1)$$

and product of the roots = $\alpha\,(2\alpha) = 32$

$$\Rightarrow \quad 2\alpha^2 = 32 \Rightarrow \alpha^2 = \frac{32}{2} = 16 = 4^2$$

$$\Rightarrow \quad \alpha = \pm 4$$

Substituting $\alpha = 4$ in (1) we get $4 = \frac{a}{6} \Rightarrow a = 24$

Substituting $\alpha = -4$ in (1) we get, $-4 = \frac{a}{6}$

$\Rightarrow a = -24$

$\therefore$ The values of a are 24, – 24.

6. If one root of the equation $3x^2 + kx + 81 = 0$ (having real roots) is the square of the other then find k. **[PTA - 3]**

Sol. $3x^2 + kx + 81 = 0 \Rightarrow x^2 + \frac{k}{3}x + 27 = 0$

$$\Rightarrow x^2 - \left(\frac{-k}{3}\right)x + 27 = 0$$

Let the roots be α and α^2

$$\alpha + \alpha^2 = \frac{-k}{3} \qquad \dots(1)$$

$$\text{and } \alpha \times \alpha^2 = \frac{81}{3}$$

$$\Rightarrow \quad \alpha^3 = 27 \quad \Rightarrow \alpha = 3 \qquad \dots(2)$$

Sub (2) in (1) we get

$$3 + 3^2 = \frac{-k}{3} \Rightarrow (3 + 9) = \frac{-k}{3}$$

$$\Rightarrow \quad k = -36.$$

EXERCISE 3.15

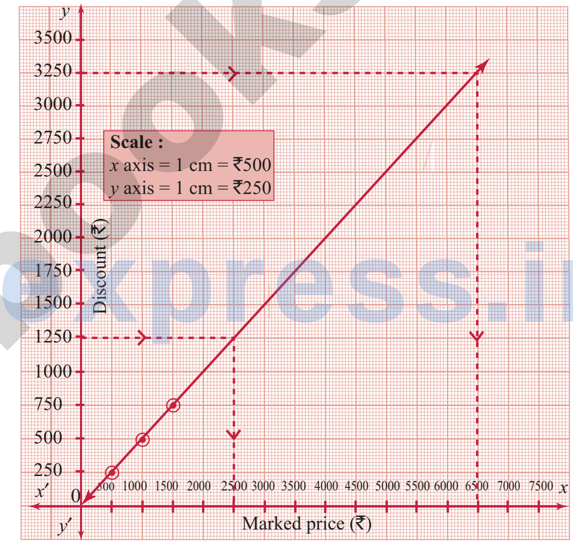
1. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find

(i) the marked price when a customer gets a discount of ₹3250 (from graph)

(ii) the discount when the marked price is ₹2500

Sol. Let x be the marked price and y be the discount.

Marked price x	500	1000	1500
Discount y	250	500	750



(i) From the graph the marked price for a discount of ₹3250 is ₹6500

(ii) the discount for marked price ₹2500 is ₹1250.

2. Draw the graph of $xy = 24$, $x, y > 0$. Using the graph find, (i) y when $x = 3$ and (ii) x when $y = 6$.

Sol.

x	1	2	3	4	6
y	24	12	8	6	4

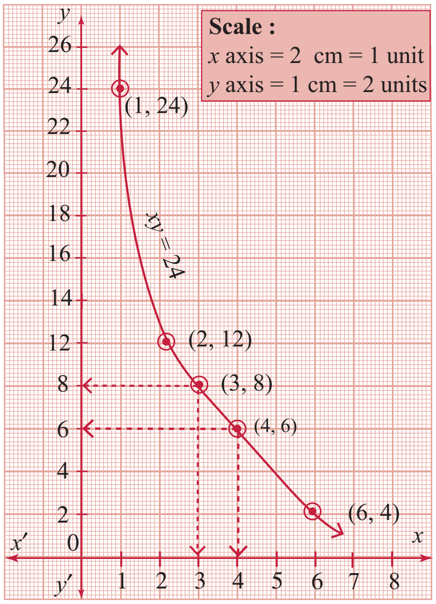
From the table we observe that as x increases y decreases. This type of variation is called indirect variation.

$y \;\alpha\; \frac{1}{x}$ or $xy = k$ where k is a constant of proportionality. Also from the table we find that,

$1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6 = 6 \times 4 = 24 = k.$

$\therefore$ We get $k = 24$

Plot the points (1, 24), (2, 12), (3, 8), (4, 6) and (6, 4) and join them.



∴ The relation $xy = 24$ is a rectangular hyperbola as exhibited in the graph. From the graph, we find

(i) when $x = 3$, $y = 8$ (ii) when $y = 6$, $x = 4$

3. **Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also (i) find y when $x = 9$ (ii) find x when $y = 7.5$.**

Sol.

x	2	4	6	8
$y = \frac{1}{2}x$	1	2	3	4

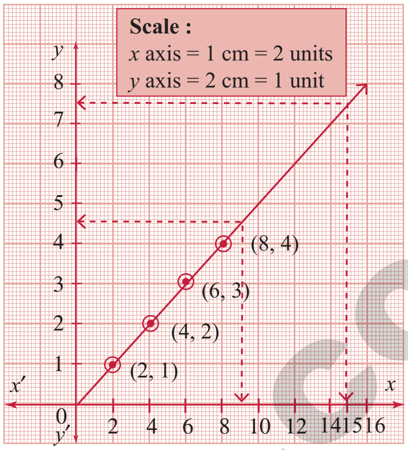
From the table above table we observe that as x increases y also increase. Therefore it is in direct variation.

We get $y \propto x$ (i.e.) $y = kx \Rightarrow \frac{y}{x} = k$, where k is a constant of proportionality.

From the table we find $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{1}{2} = k$.

∴ We get $k = \frac{1}{2}$.

Plot the points (2, 1), (4, 2), (6, 3) and (8, 4) and join them.



∴ The relation $y = \frac{1}{2}x$ forms a straight line is exhibited in the graph.

From the graph we find (i) when $x = 9$, $y = 4.5$ (ii) when $y = 7.5$, $x = 15$

4. **The following table shows the data about the number of pipes and the time taken to fill the same tank.**

No. of pipes (x)	**2**	**3**	**6**	**9**
Time Taken (in min) (y)	**45**	**30**	**15**	**10**

Draw the graph for the above data and hence

(i) find the time taken to fill the tank when five pipes are used

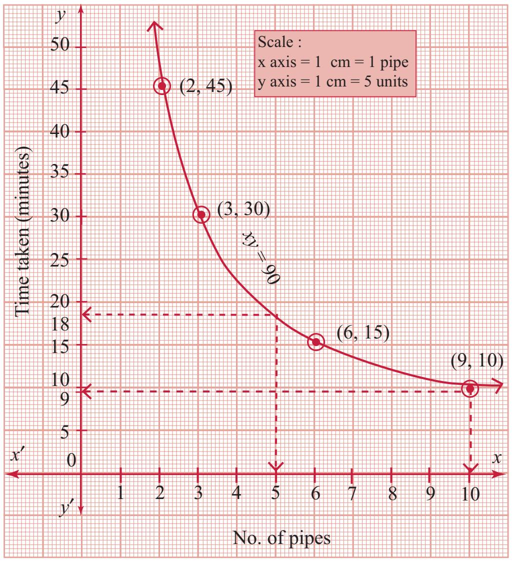
(ii) Find the number of pipes when the time is 9 minutes.

Sol. From the table we observe that as x increases y decreases.

This type of variation is called indirect variation.

We get $y \propto \frac{1}{x}$ or $xy = k$ where k is a constant of proportionality. Also from the table we find, $2 \times 45 = 3 \times 30 = 6 \times 15 = 9 \times 10 = 90 = k$.

Plot the points (2, 45), (3, 30), (6, 15) and (9, 10) and join them.



∴ The relation $xy = 90$ is a rectangular hyperbola as exhibited in the graph.

From the graph we find

(i) The time taken when five pipes are used is 18 minutes.

(ii) Number of pipes used is 10 when the time is 9 minutes.

5. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of participants(x)	**2**	**4**	**6**	**8**	**10**
Amount for each participants in ₹(y)	**180**	**90**	**60**	**45**	**36**

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

Sol. (i) From the table we observe that as x increases y decreases.

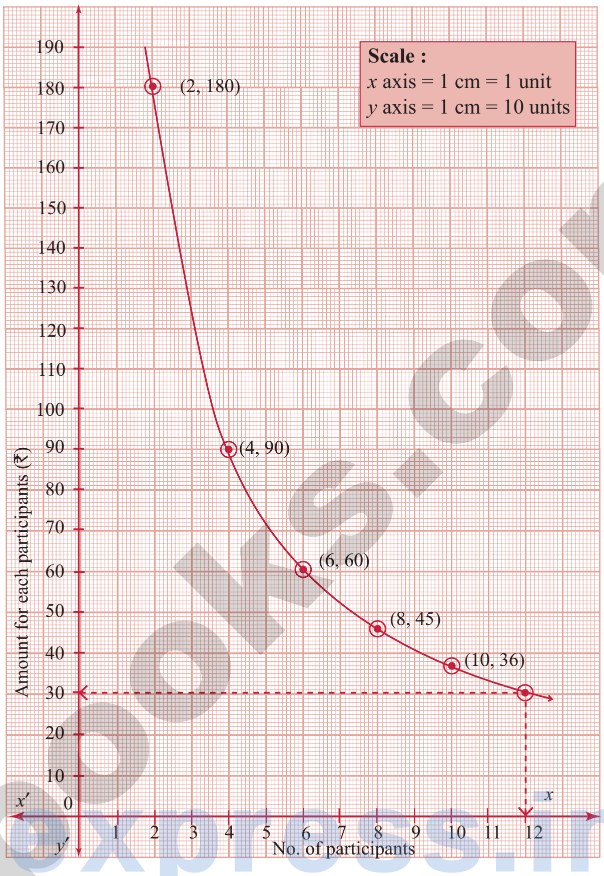
This type of variation is called indirect variation.

∴ We get $y \propto \frac{1}{x}$ or $xy = k$ where k is a constant of proportionality.

Also from the table we find,

$2 \times 180 = 4 \times 90 = 6 \times 60 = 8 \times 45 = 10 \times 36 = 360 = k$. Hence constant of variation is 360.

(ii) Plot the points (2, 180), (4, 90), (6, 60), (8, 45) and (10, 36) and join them.



If the number of participant is 12, each participant will get ₹30.

6. A two wheeler parking zone near bus stand charges as below.

Time (in hours) (x)	**4**	**8**	**12**	**24**
Amount ₹ (y)	**60**	**120**	**180**	**360**

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 hr; (ii) find the parking duration when the amount paid is ₹150.

Sol. From the table we observe that as x increases, y also increases. Therefore it is indirect variation.

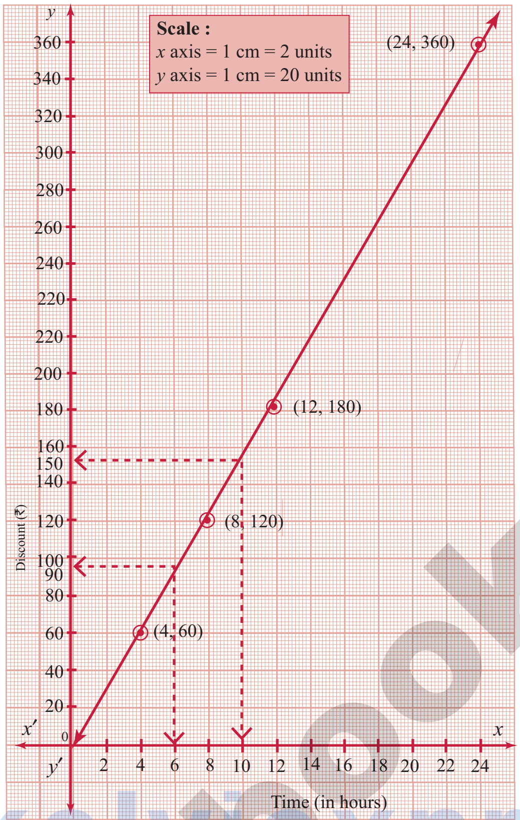
(i.e) we get $y \propto x$ (i.e.) $y = kx$ where k is constant of proportionality.

Since $\frac{y}{x} = k$, From the table we find

$$\frac{60}{4} = \frac{120}{8} = \frac{180}{12} = \frac{360}{24}$$

∴ We get $k = 15$.

Plot the points (4, 60), (8, 120), (12, 180) and (24, 360) and join them.



(i) The amount to be paid for parking time 6 hr is ₹90.

(ii) The parking duration is 10 hr when the amount paid is ₹150.

EXERCISE 3.16

1. Graph the following quadratic equations and state their nature of solutions.

(i) $x^2 - 9x + 20 = 0$ **[Hy. - 2019; Aug. - 2022]**

Sol.

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-9x$	+36	27	18	9	0	–9	–18	–27	–36
20	20	20	20	20	20	20	20	20	20
	72	56	42	30	20	12	6	2	0

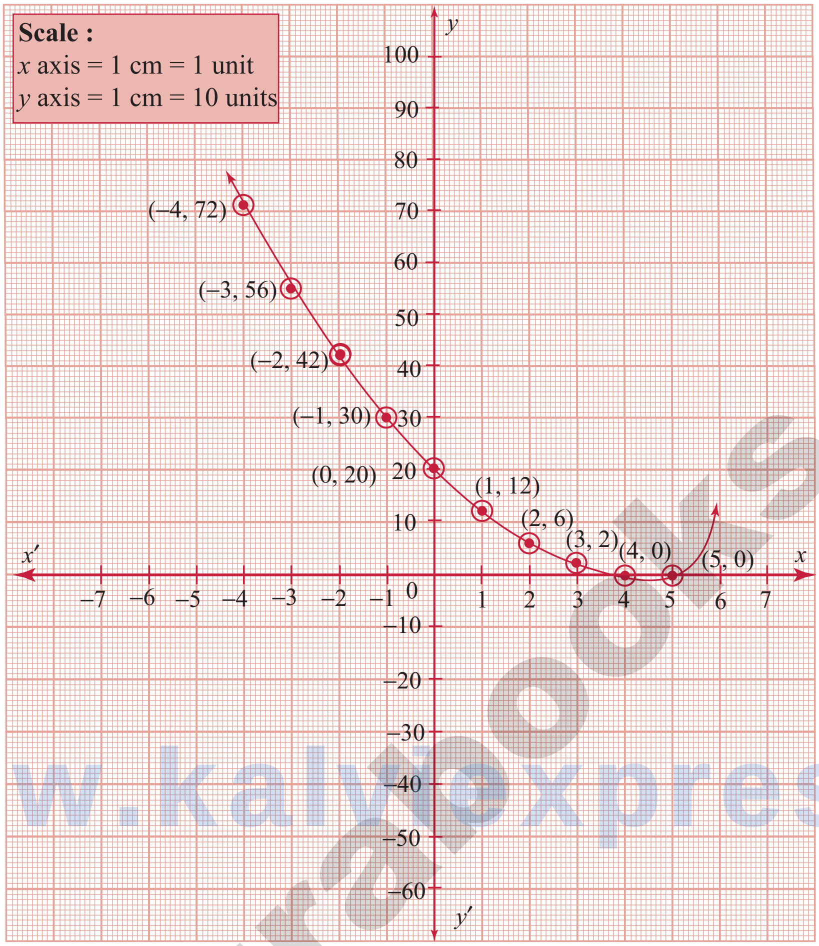
Step 1:

Points to be plotted : (–4, 72), (–3, 56), (–2, 42), (–1, 30), (0, 20), (1, 12), (2, 6), (3, 2), (4, 0)

Step 2:

The point of intersection of the curve with x axis is (4, 0) (5,0)

Step 3:



The roots are real & unequal

∴ Solution {4, 5}

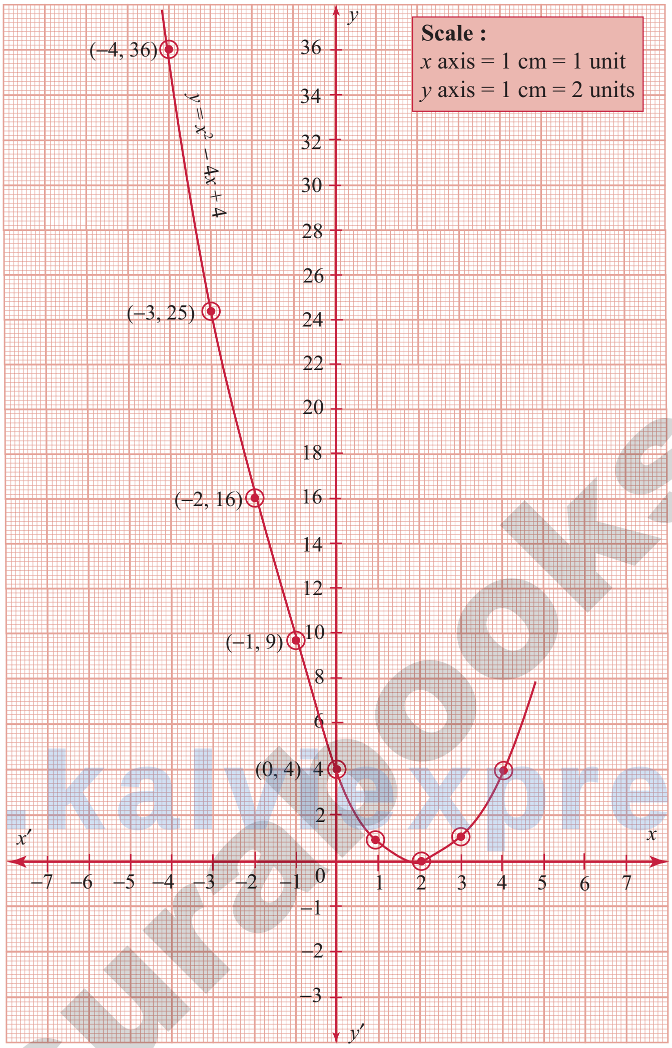
(ii) $x^2 - 4x + 4 = 0$ **[May - 2022]**

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	–4	–8	–12	–16
4	4	4	4	4	4	4	4	4	4
$y = x^2 - 4x + 4$	36	25	16	9	4	1	0	1	4

Step 1: Points to be plotted : (–4, 36), (–3, 25), (–2, 16), (–1, 9), (0, 4), (1, 1), (2, 0), (3, 1), (4, 4)

Step 2: The point of intersection of the curve with x axis is (2, 0)

Step 3:



Since there is only one point of intersection with x axis, the quadratic equation $x^2 - 4x + 4 = 0$ has real and equal roots.

$\therefore$ Solution $\{2, 2\}$

(iii) $\boldsymbol{x^2 + x + 7 = 0}$

Let $y = x^2 + x + 7$

Step 1:

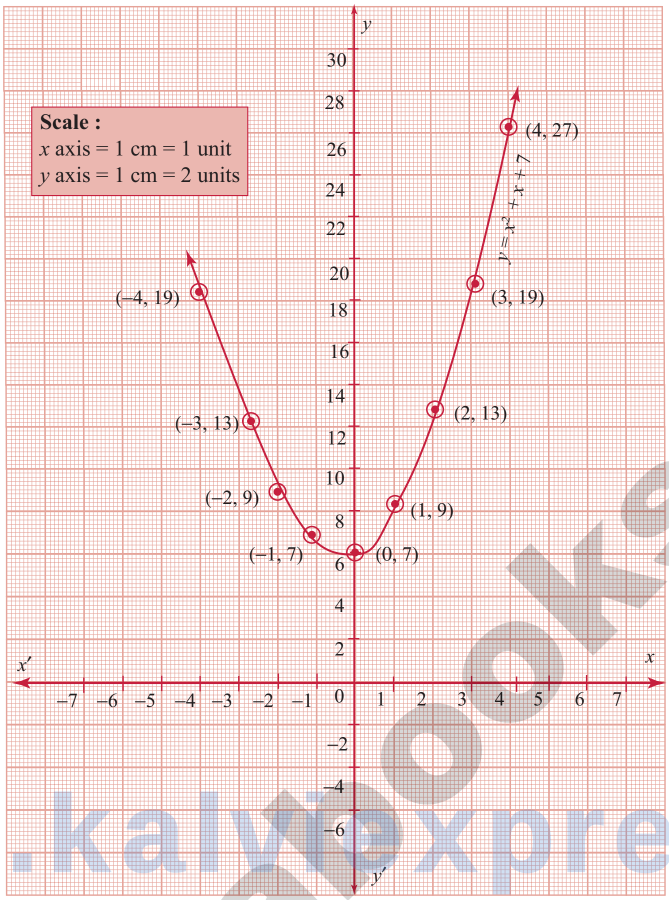
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
7	7	7	7	7	7	7	7	7	7
$y = x^2 + x + 7$	19	13	9	7	7	9	13	19	27

Step 2:

Points to be plotted: (–4, 19), (–3, 13), (–2, 9), (–1, 7), (0, 7), (1, 9), (2, 13), (3, 19), (4, 27)

Step 3:

Draw the parabola and mark the co-ordinates of the parabola which intersect with the x-axis.



Step 4:

The roots of the equation are the points of intersection of the parabola with the x axis. Here the parabola does not intersect the x axis at any point.

So, we conclude that there is no real roots for the given quadratic equation.

(iv) $x^2 - 9 = 0$

Let $y = x^2 - 9$

Step 1:

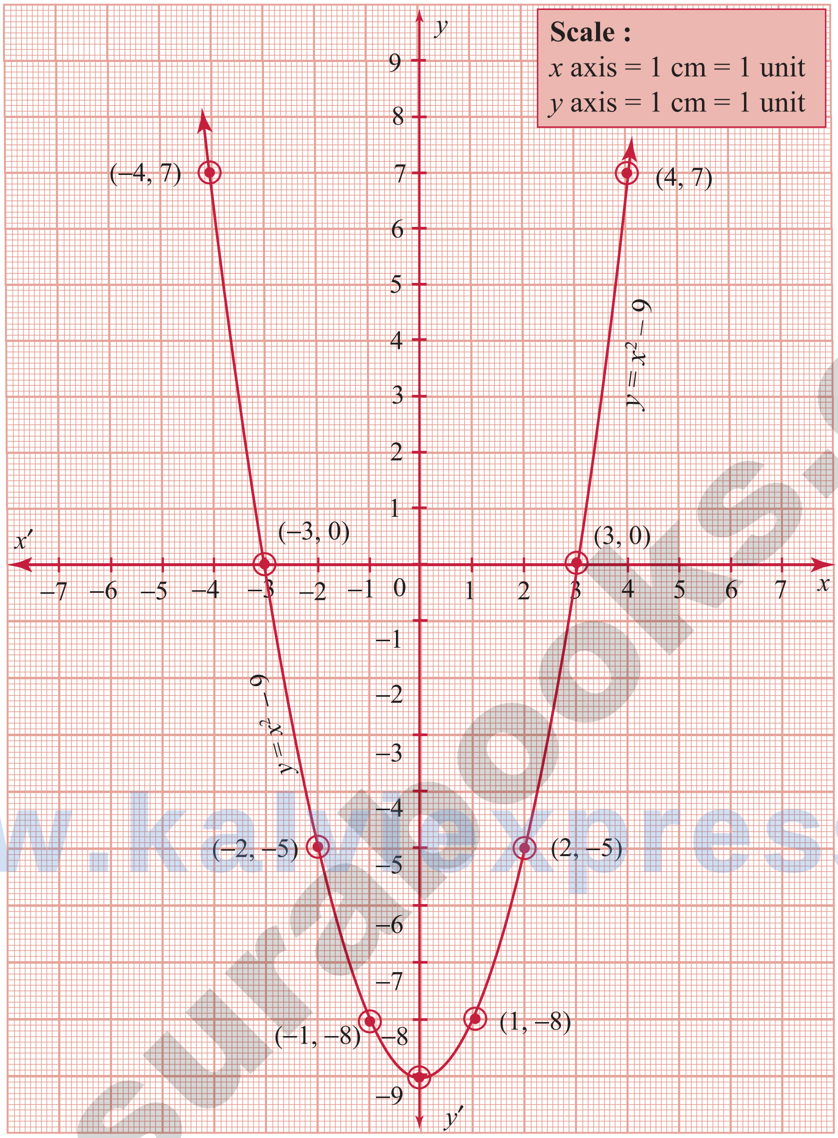
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–9	–9	–9	–9	–9	–9	–9	–9	–9	–9
$y = x^2 - 9$	7	0	–5	–8	–9	–8	–5	0	7

Step 2:

The points to be plotted: (–4, 7), (–3, 0), (–2, –5), (–1, –8), (0, –9), (1, –8), (2, –5), (3, 0), (4, 7)

Step 3:

Draw the parabola and mark the co-ordinates of the parabola which intersect the x-axis.



Step 4:
The roots of the equation are the co-ordinates of the intersecting points (–3, 0) and (3, 0) of the parabola with the x-axis which are –3 and 3 respectively.

Step 5:
Since there are two points of intersection with the x axis, the quadratic equation has real and unequal roots.

∴ Solution $\{-3, 3\}$

(v) $x^2 - 6x + 9 = 0$

Let $y = x^2 - 6x + 9$

Step 1:

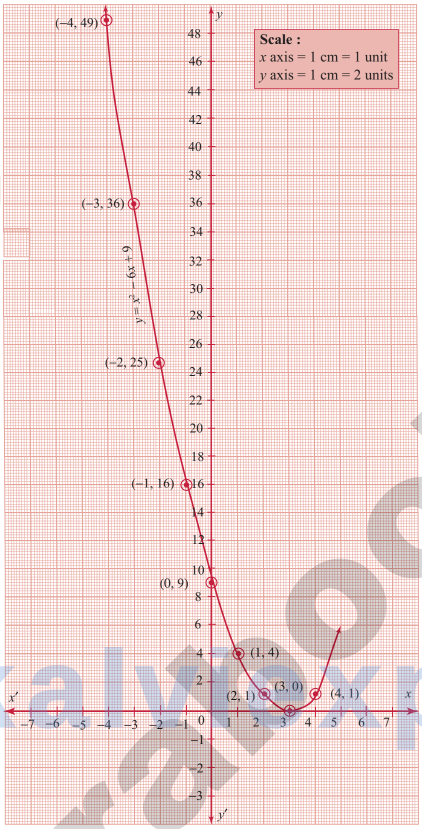
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-6x$	24	18	12	6	0	–6	–12	–18	–24
9	9	9	9	9	9	9	9	9	9
$y = x^2 - 6x + 9$	49	36	25	16	9	4	1	0	1

Step 2:

Points to be plotted: (–4, 49), (–3, 36), (–2, 25), (–1, 16), (0, 9), (1, 4), (2, 1), (3, 0), (4, 1)

Step 3:

Draw the parabola and mark the co-ordinates of the intersecting points.



Step 4:

The point of intersection of the parabola with x axis is (3, 0)

Since there is only one point of intersection with the x-axis, the quadratic equation has real and equal roots.

∴ Solution (3, 3)

(vi) **$(2x - 3)(x + 2) = 0$**

$2x^2 - 3x + 4x - 6 = 0; 2x^2 + 1x - 6 = 0$

Let $y = 2x^2 + x - 6 = 0$

Step 1:

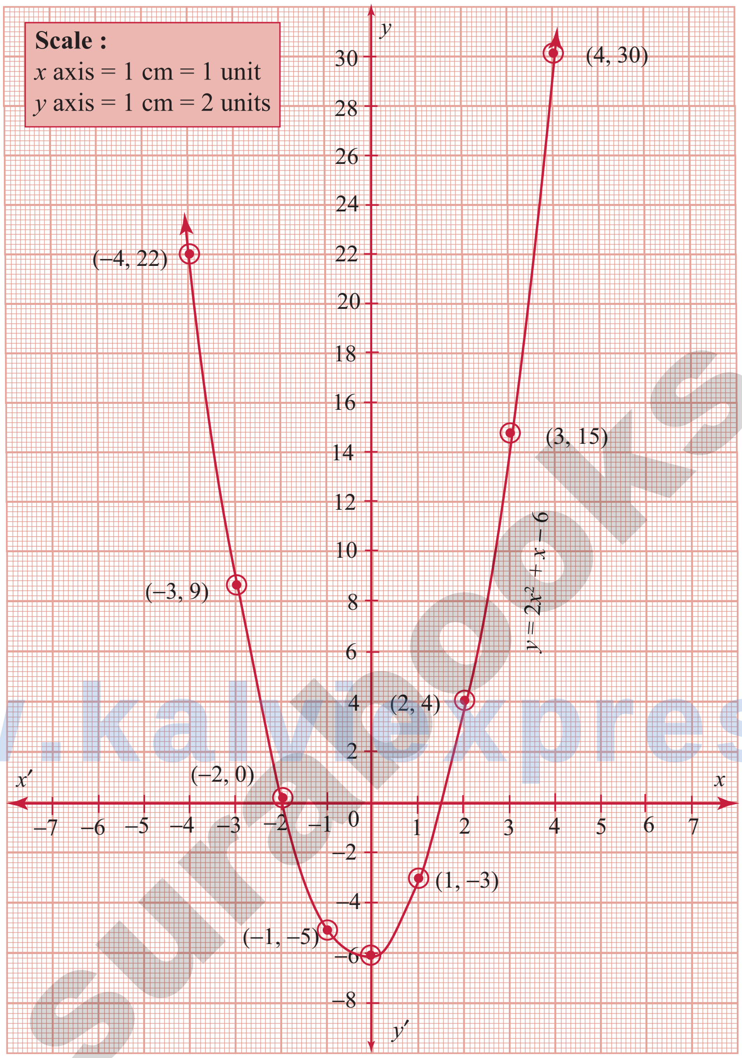
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
x	–4	–3	–2	–1	0	1	2	3	4
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
$y = 2x^2 + x - 6$	22	9	0	–5	–6	–3	4	15	30

Step 2:

The points to be plotted: (–4, 22), (–3, 9), (–2, 0), (–1, –5), (0, –6), (1, –3), (2, 4), (3, 15), (4, 30)

Step 3:

Draw the parabola and mark the co-ordinates of the intersecting point of the parabola with the *x*-axis.



Step 4:

The points of intersection of the parabola with the *x*-axis are (–2, 0) and (1.5, 0).

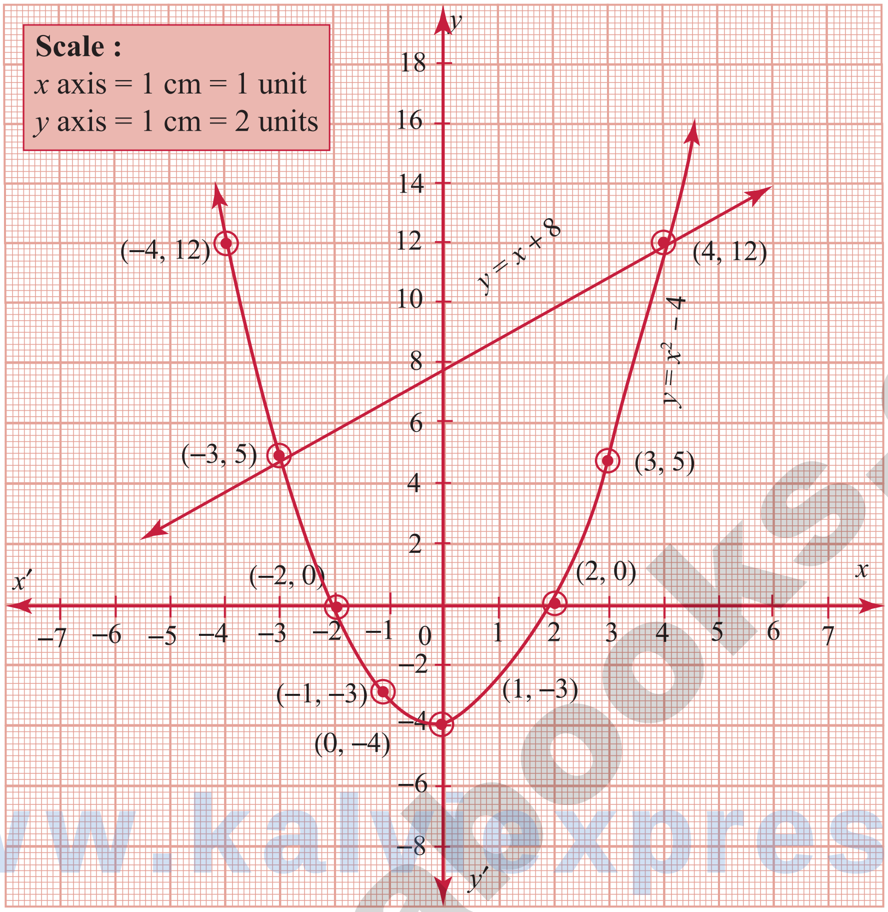
Since the parabola intersects the *x*-axis at two points, the equation has real and unequal roots.

∴ Solution {–2, 1.5}

2. **Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$**

Sol.

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
– 4	–4	–4	–4	–4	–4	–4	–4	–4	–4
$x^2 - 4$	12	5	0	–3	–4	–3	0	5	12



$$\begin{array}{rcl} \textbf{To solve } x^2 - x - 12 &=& 0 \\ \cancel{x^2} + 0x - 4 &=& y \\ \cancel{x^2} - x - 12 &=& 0 \\ (-)\quad (+)\quad (+) && (-) \\ \hline x + 8 &=& y \\ y &=& x + 8 \end{array}$$

x	–4	–3	–2	–1	0	1	2	3	4
8	8	8	8	8	8	8	8	8	8
$x – 8$	4	5	6	7	8	9	10	11	12

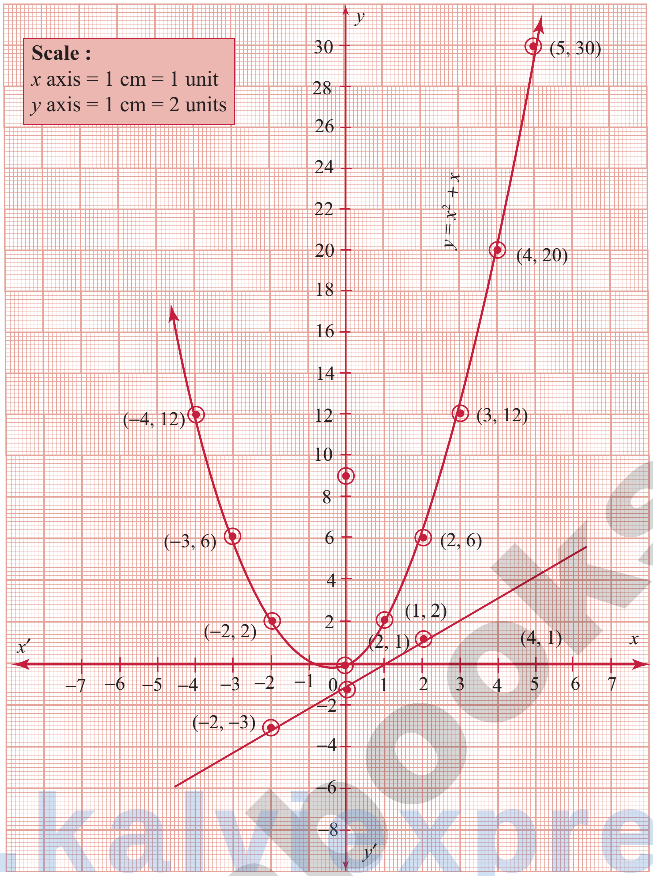
Point of intersection (–3, 5), (4, 12) solution of $x^2 – x – 12 = 0$ is –3, 4

3. **Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.**

Sol.

x	–4	–3	–2	–1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$+x$	–4	–3	–2	–1	0	1	2	3	4	5
$y = x^2 + x$	12	6	2	0	0	2	6	12	20	30

Draw the parabola by the plotting the points (–4, 12), (–3, 6), (–2, 2), (–1, 0), (0, 0), (1, 2), (2, 6), (3, 12), (4, 20), (5, 30)



To solve: $x^2 + 1 = 0$, subtract $x^2 + 1 = 0$ from $y = x^2 + x$.

$x^2 + 1 = 0$ from $y = x^2 + x$

i.e.

$$\begin{array}{rcl} y & = & \cancel{x^2} + x \\ 0 & = & \cancel{x^2} + 1 \\ (-) & (-) & (-) \\ \hline y & = & x - 1 \\ \hline \end{array}$$

This is a straight line.

Draw the line $y = x - 1$.

x	–2	0	2
–1	–1	–1	–1
y	–3	–1	1

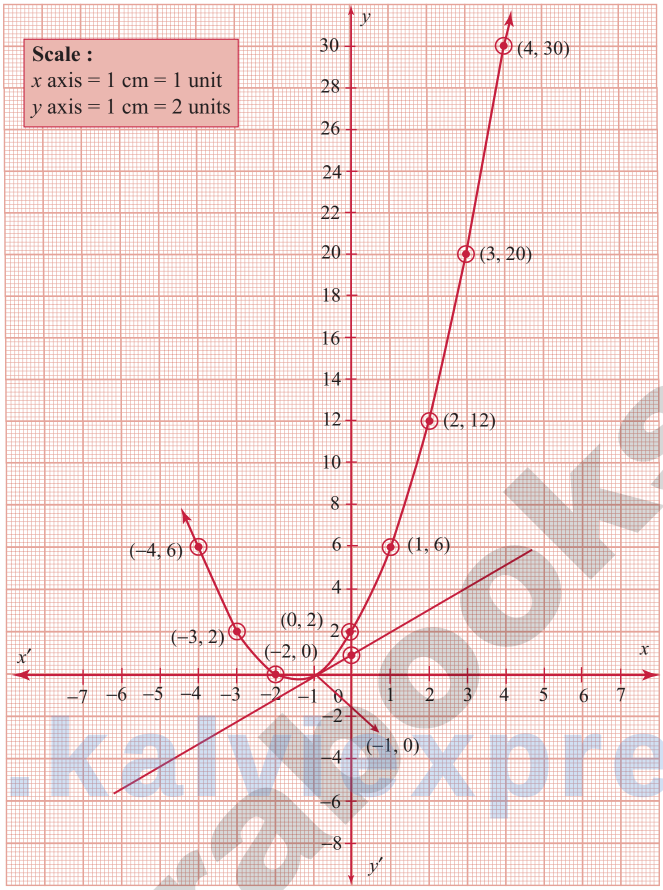
Plotting the points (–2, –3), (0, –1), (2, 1) we get a straight line. This line does not intersect the parabola. Therefore there is no real roots for the equation $x^2 + 1 = 0$.

4. Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$. **[PTA - 5; FRT - 2022]**

Sol.

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
3x	–12	–9	–6	–3	0	3	6	9	12
2	2	2	2	2	2	2	2	2	2
$y = x^2 + 3x + 2$	6	2	0	0	2	6	12	20	30

Draw the parabola by plotting the point (–4, 6), (–3, 2), (–2, 0), (–1, 0), (0, 2), (1, 6), (2, 12), (3, 20), (4, 30).



To solve $x^2 + 2x + 1 = 0$, subtract $x^2 + 2x + 1 = 0$ from $y = x^2 + 3x + 2$

$$\begin{array}{l} y = x^2 + 3x + 2 \\ 0 = x^2 + 2x + 1 \\ (-) \quad (-) \quad (-) \qquad (-) \\ \hline y = x + 1 \end{array}$$

x	–2	0	2
1	1	1	1
$y = x + 1$	–1	1	3

Draw the straight line by plotting the points (–2, –1), (0, 1), (2, 3)

The straight line touches the parabola at the point (–1, 0)

Therefore the x coordinate –1 is the only solution of the given equation

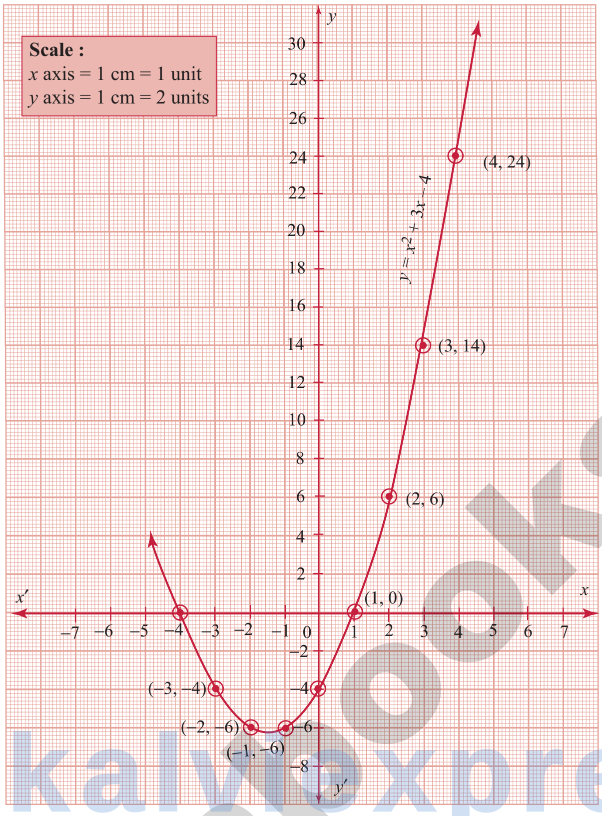
5. Draw the graph of $y = x^2 + 3x – 4$ and hence use it to solve $x^2 + 3x – 4 = 0$.

[Govt. MQP - 2019 ; Qy. - 2019; Sep. - 2021]

Sol.

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$3x$	–12	–9	–6	–3	0	3	6	9	12
–4	–4	–4	–4	–4	–4	–4	–4	–4	–4
$y = x^2 + 3x – 4$	0	–4	–6	–6	–4	0	6	14	24

Draw the parabola using the points (–4, 0), (–3, –4), (–2, –6), (–1, –6), (0, –4), (1, 0), (2, 6), (3, 14), (4, 24).



To solve: $x^2 + 3x - 4 = 0$ subtract $x^2 + 3x - 4 = 0$ from $y = x^2 + 3x - 4$

$$\begin{array}{rcl} y & = & x^2 + 3x - 4 \\ 0 & = & x^2 + 3x - 4 \\ & & (-)\ (-)\ \ (+) \\ \hline y & = & 0 \text{ is the equation of the } x \text{ axis.} \end{array}$$

The points of intersection of the parabola with the x axis are the points (–4, 0) and (1, 0), whose x - co-ordinates (–4, 1) is the solution, set for the equation $x^2 + 3x - 4 = 0$.

6. Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$. **[PTA - 2 & 6]**

Sol.

x	–5	–4	–3	–2	–1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
$-5x$	25	20	15	10	5	0	–5	–10	–15	–20
-6	–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
$y = x^2 + 5x - 6$	44	30	18	8	0	–6	–10	–12	–12	–10

Draw the parabola using the points (–5, 44), (–4, 30), (–3, 18), (–2, 8), (–1, 0), (0, –6), (1, –10), (2, –12), (3, –12), (4, –10)

EXERCISE 7.1

1. The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Sol. [Aug. - 2022]

$$\text{Given } r : h = 5 : 7$$
$$\Rightarrow \quad r = 5x \text{ and}$$
$$h = 7x$$
$$\text{CSA of a cylinder} = 2\pi rh$$
$$\therefore 5500 = 2\times\frac{22}{7} \times 5x \times 7x$$
$$22\cancel{0}x^2 = 550\cancel{0}$$
$$x^2 = \frac{550}{22} = 25$$
$$\therefore x = 5$$
$$\therefore \text{Radius} = 5 \times 5 = 25 \text{ cm}$$
$$[\because r = 5x \text{ and } x = 5]$$
$$\text{Height} = 7 \times 5 = 35 \text{ cm.}$$

2. A solid iron cylinder has total surface area of 1848 sq.m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.

Sol.

$$\text{Given C.S.A.} = \frac{5}{6} \text{ T.S.A}$$
$$\text{TSA} = 1848 \text{ m}^2$$
$$2\pi r\,(h + r) = 1848 \text{ m}^2$$
$$2\pi rh + 2\pi r^2 = 1848 \text{ m}^2$$
$$\frac{5}{6} \times 1848 + 2\pi r^2 = 1848 \quad [\because 2rh = \frac{5}{6} \times \text{TSA}]$$
$$1540 + 2\pi r^2 = 1848$$
$$2\pi r^2 = 1848 - 1540 = 308$$
$$2 \times 22 \times r^2 = 308$$
$$\Rightarrow \quad r^2 = 308 \times \frac{1}{2}\times\frac{7}{22}$$
$$\Rightarrow \quad r^2 = 49$$
$$\Rightarrow \quad r = 7 \ m.$$
$$2\pi rh = \frac{5}{6} \times 1848$$
$$2\times\frac{22}{\cancel{7}}\times\cancel{7}\times h = \frac{5}{6} \times 1848$$
$$\Rightarrow \quad h = 35 \text{ m}$$

$\therefore$ Radius $r = 7$ m, Height = 35 m.

3. The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.

Sol. Given R = 16 cm $\quad r$ = R – thickness

h = 13 cm $\quad$ = 16 – 4 = 12 cm

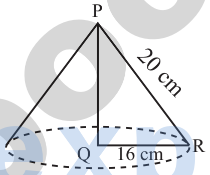
Thickness = 4 cm $\quad \Rightarrow r$ = 12 cm.

Total surface area of hollow cylinder = $2\pi\,(\text{R} + r)\,(\text{R} - r + h)$ sq. units.

$$\text{T.S.A} = 2\times\frac{22}{7}\,(16 + 12)\,(16 - 12 + 13)$$
$$= \frac{44}{\cancel{7}}\left(\overset{4}{\cancel{28}}\right)(17)$$
$$\text{T.S.A} = 2992 \text{ sq.cm}$$

4. A right angled triangle PQR where ∠Q = 90° is rotated about QR and PQ. If QR = 16 cm and PR = 20 cm, compare the curved surface areas of the right circular cones so formed by the triangle.

Sol. When it is rotated about PQ the C.S.A of the cone formed = πrl. [Here $r = 16$, $l = 20$]



$$= \frac{22}{7} \times 16 \times 20$$
$$= \frac{7040}{7} = 1005.71 \text{ cm}^2$$

When it is rotated about QR CSA of the cone formed.

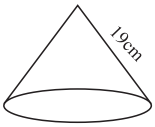
$$= \pi rl. \text{ Here } h = 16,\ l = 20$$
$$\text{here } r = \sqrt{l^2 - h^2} = \sqrt{20^2 - 16^2}$$
$$= \sqrt{400 - 256} = \sqrt{144} = 12 \text{ cm}$$
$$\text{CSA} = \pi rl = \frac{22}{7} \times 12 \times 20 = \frac{5280}{7}$$
$$= 754.28 \text{ cm}^2.$$

1005.71 > 754.28.

$\therefore$ CSA of the cone when rotated about its PQ is larger.

5. 4 persons live in a conical tent whose slant height is 19 m. If each person require 22 m² of the floor area, then find the height of the tent.

Sol. Given l = 19 m



Base area of the cone

$$= \pi r^2 = \text{sq units.}$$
$$\pi r^2 = 4 \times 22 \text{ m}^2$$
$$\frac{22}{7}\times r^2 = 88$$
$$[\because \text{Area for 4 persons} = 4 \times 22]$$
$$r^2 = \overset{4}{\cancel{88}}\times\frac{7}{\cancel{22}}$$

10th STD

INSTANT SUPPLEMENTARY EXAM AUGUST - 2022

Reg. No.

PART - III

Mathematics (With Answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 100

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

Note : This question paper contains **four** parts.

Part - I

Note : (1) Answer **all** the questions. **14 × 1 = 14**

(2) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to set B, then the number of elements in B is:

(a) 3 (b) 2 (c) 4 (d) 8

2. The range of the relation R ={(x, x^2)/ x is a Prime number less than 13} is :

(a) {2, 3, 5, 7} (b) {2, 3, 5, 7, 11}
(c) {4, 9, 25, 49, 121} (d) {1, 4, 9, 25, 49, 121}

3. The sum of the exponents of the Prime factors in the Prime factorization of 1729 is :

(a) 1 (b) 2 (c) 3 (d) 4

4. A system of three linear equations in the three variable is inconsistent if their planes :

(a) Intersect only at a point
(b) Intersect in a line
(c) Coincide with each other
(d) Do not intersect

5. The solution of the system $x + y - 3z = -6$, $-7y + 7z = 7$, $3z = 9$ is

(a) $x = 1, y = 2, z = 3$
(b) $x = -1, y = 2, z = 3$
(c) $x = -1, y = -2, z = 3$
(d) $x = 1, y = -2, z = 3$

6. $y^2 + \frac{1}{y^2}$ is not equal to :

(a) $\frac{y^4+1}{y^2}$ (b) $\left[y+\frac{1}{y}\right]^2$
(c) $\left[y-\frac{1}{y}\right]^2+2$ (d) $\left(y+\frac{1}{y}\right)^2-2$

7. If ΔABC , DE || BC, AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then, the length of AE is :

(a) 1.4 cm (b) 1.8 cm
(c) 1.2 cm (d) 1.05 cm

8. How many tangents can be drawn to the circle from an exterior point ?

(a) one (b) two
(c) infinite (d) zero

9. The point of intersection of $3x - y = 4$ and $x + y = 8$ is

(a) (5, 3) (b) (2, 4)
(c) (3, 5) (d) (4, 4)

10. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then, slope of the perpendicular bisector of PQ is :

(a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 0

11. The angle of elevation of a cloud from a point h metre above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake (in meters) is

(a) $\frac{h(1+\tan\beta)}{1-\tan\beta}$ (b) $\frac{h(1-\tan\beta)}{1+\tan\beta}$
(c) $h \tan (45° - \beta)$ (d) None of these

12. If the radius of the base of a right circular cylinder is halved keeping the same height then, the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is :

(a) 1 : 2 (b) 1 : 4 (c) 1 : 6 (d) 1 : 8

13. The total surface area of hemi-sphere is how much times the square of its radius :

(a) π (b) 4π (c) 3π (d) 2π

14. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is :

(a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) $\frac{3}{9}$ (d) $\frac{7}{9}$

Part - II

Note : Answer **any 10** questions. Question No.**28** is **compulsory**. **10 × 2 = 20**

15. If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.

16. If A = {5,6}, B = {4, 5, 6}, C = {5, 6, 7}. Show that A × A = (B × B) ∩ (C × C).

17. Find the least number that is divisible by the first ten natural numbers.

18. Find the 19th term of an A.P. – 11, –15, – 19, ...

19. Find the square root of $\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$.

20. ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively, such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm find AD.

21. If area of triangle formed by the vertices A (–1, 2), B(K, –2) and C (7, 4) is 22 sq. units, find the value of K.

22. The line *p* passes through the points (3, –2), (12, 4) and the line *q* passes through the points (6, – 2) and (12, 2). Is *p* parallel to *q* ?

23. Find the slope of a line joining the points $(5, \sqrt{5})$with the origin.

24. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of tower of height $10\sqrt{3}$ m.

25. If the total surface area of a cone of radius 7 cm is 704 cm^2, then, find its slant height.

26. The radius and height of a cylinder are in the ratio 5 : 7 and its curved surface area is 5500 sq.cm. Find its radius and height.

27. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is :

(i) White (ii) Black or Red

28. Find the value of *x*, in $x^2 - 4x - 12$.

Part - III

Note : Answer **any 10** questions. Question No.**42** is **compulsory**. **10 × 5 = 50**

29. Represent the given relation by :

(i) an arrow diagram
(ii) a graph and
(iii) a set in roster form, wherever possible

$\{(x, y) / y = x + 3, x, y$ are natural number $< 10\}$

30. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

31. If nine times ninth term is equal to fifteen times fifteenth term show that six times twenty fourth term is zero.

32. Simplify : $\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14}$

33. Find the square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$.

34. Solve $x^2 + 2x - 2 = 0$ by Formula method.

35. State and prove Angle Bisector Theorem.

36. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point ?

37. Find the area of quadrilateral formed by the points (8, 6), (5, 11), (–5, 12) and (–4, 3).

38. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall. What is the height of the window? ($\sqrt{3}$ = 1.732).

39. A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.

40. If the circumference of conical wooden piece is 484 cm then, find its volume when its height is 105 cm.

41. Two unbiased dice are rolled once. Find the probability of getting :

(i) A doublet (equal numbers on both dice).
(ii) The product as a Prime number.
(iii) The sum as a Prime number.
(iv) The sum as 1.

42. A Cat is located at the point (6, 4) in *xy* plane. A bottle of milk is kept at (–5, –11). The Cat wishes to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

Part - IV

Note : Answer **all** the questions. **2 × 8 = 16**

43. (a) Construct a triangle similar to a triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR. (Scale factor $\frac{7}{3} > 1$).

(OR)

(b) Draw a circle of diameter 6 cm. From a point P, which is 8 cm away from its centre, draw the two tangents PA and PB to the circle and measure their lengths.

44. (a) Draw the graph of $x^2 - 9x + 20 = 0$ and state the nature of their solution.

(OR)

(b) Draw the graph of $y = x^2 - 4x + 3 = 0$ and use it to solve $x^2 - 6x + 9 = 0$.

* * *