

SOLUTIONS TO CONCEPTS CHAPTER 11

1. Gravitational force of attraction,

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 10 \times 10}{(0.1)^2} = 6.67 \times 10^{-7} \text{ N}$$

2. To calculate the gravitational force on 'm' at online due to other mouse.

$$\vec{F}_{OD} = \frac{G \times m \times 4m}{(a/r^2)^2} = \frac{8Gm^2}{a^2}$$

$$\vec{F}_{OI} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{6Gm^2}{a^2}$$

$$\vec{F}_{OB} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{4Gm^2}{a^2}$$

$$\vec{F}_{OA} = \frac{G \times m \times m}{(a/r^2)^2} = \frac{2Gm^2}{a^2}$$

$$\text{Resultant } F_{OF} = 64 \left(\frac{Gm^2}{a^2} \right)^2 + 36 \left(\frac{Gm^2}{a^2} \right)^2 = 10 \frac{Gm^2}{a^2}$$

$$\text{Resultant } F_{OE} = 64 \left(\frac{Gm^2}{a^2} \right)^2 + 4 \left(\frac{Gm^2}{a^2} \right)^2 = 25 \frac{Gm^2}{a^2}$$

The net resultant force will be,

$$F = 100 \left(\frac{Gm^2}{a^2} \right)^2 + 20 \left(\frac{Gm^2}{a^2} \right)^2 - 2 \left(\frac{Gm^2}{a^2} \right)^2 \times 20.5$$

$$= \left(\frac{Gm^2}{a^2} \right)^2 (120 - 40.5) = \left(\frac{Gm^2}{a^2} \right)^2 (79.5)$$

$$= \frac{Gm^2}{a^2} \sqrt{40.4} = 4.2 \sqrt{\frac{Gm^2}{a^2}}$$

3. a) if 'm' is placed at mid point of a side

$$\text{then } \vec{F}_{OA} = \frac{4Gm^2}{a^2} \text{ in OA direction}$$

$$\vec{F}_{OB} = \frac{4Gm^2}{a^2} \text{ in OB direction}$$

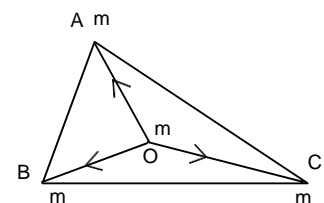
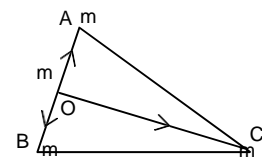
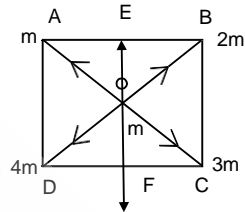
Since equal & opposite cancel each other

$$\vec{F}_{OC} = \frac{Gm^2}{\left[\left(\frac{3}{2}a \right)^2 \right]} = \frac{4Gm^2}{3a^2} \text{ in OC direction}$$

$$\text{Net gravitational force on } m = \frac{4Gm^2}{a^2}$$

- b) If placed at O (centroid)

$$\text{the } \vec{F}_{OA} = \frac{Gm^2}{(a/r_3)^2} = \frac{3Gm^2}{a^2}$$



$$\vec{F}_{OB} = \frac{3Gm^2}{a^2}$$

$$\text{Resultant } F = \sqrt{2 \left(\frac{3Gm^2}{a^2} \right)^2 - 2 \left(\frac{3Gm^2}{a^2} \right)^2 \frac{1}{2}} = \frac{3Gm^2}{a^2}$$

Since $\vec{F}_{OC} = \frac{3Gm^2}{a^2}$, equal & opposite to F , cancel

Net gravitational force = 0

$$4. \quad \vec{F}_{CB} = \frac{Gm^2}{4a^2} \cos 60^\circ \hat{i} - \frac{Gm^2}{4a^2} \sin 60^\circ \hat{j}$$

$$\vec{F}_{CA} = \frac{Gm^2}{4a^2} \cos 60^\circ \hat{i} - \frac{Gm^2}{4a^2} \sin 60^\circ \hat{j}$$

$$\begin{aligned} \vec{F} &= \vec{F}_{CB} + \vec{F}_{CA} \\ &= \frac{-2Gm^2}{4a^2} \sin 60^\circ \hat{j} = \frac{-2Gm^2}{4a^2} \frac{r_3}{2} = \frac{r_3 Gm^2}{4a^2} \end{aligned}$$

5. Force on M at C due to gravitational attraction.

$$\vec{F}_{CB} = \frac{Gm^2}{2R^2} \hat{j}$$

$$\vec{F}_{CD} = \frac{-GM^2}{4R^2} \hat{i}$$

$$\vec{F}_{CA} = \frac{-GM^2}{4R^2} \cos 45^\circ \hat{j} + \frac{GM^2}{4R^2} \sin 45^\circ \hat{j}$$

So, resultant force on C,

$$\begin{aligned} \therefore F_C &= F_{CA} + F_{CB} + F_{CD} \\ &= \frac{GM^2}{4R^2} \left(\frac{1}{2} + 1 \right) \hat{j} - \frac{GM^2}{4R^2} \left(\frac{1}{2} \right) \hat{i} \\ &= \frac{GM^2}{4R^2} \left(\frac{3}{2} \hat{j} - \frac{1}{2} \hat{i} \right) \end{aligned}$$

$$\text{For moving along the circle, } F = \frac{mv^2}{R}$$

$$\text{or } \frac{GM^2}{4R^2} \left(\frac{3}{2} + 1 \right) = \frac{MV^2}{R} \quad \text{or } V = \sqrt{\frac{GM}{R} \left(\frac{2\sqrt{2} + 1}{4} \right)}$$

$$\begin{aligned} 6. \quad \frac{GM}{(R+h)^2} &= \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(1740 + 1000)^2 \times 10^6} = \frac{49.358 \times 10^{11}}{2740 \times 2740 \times 10^6} \\ &= \frac{49.358 \times 10^{11}}{0.75 \times 10^{13}} = 65.8 \times 10^{-2} = 0.65 \text{ m/s}^2 \end{aligned}$$

7. The linear momentum of 2 bodies is 0 initially. Since gravitational force is internal, final momentum is also zero.

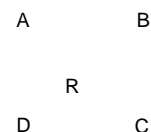
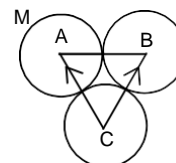
$$\text{So } (10 \text{ kg})v_1 = (20 \text{ kg})v_2$$

$$\text{Or } v_1 = v_2 \quad \dots(1)$$

Since P.E. is conserved

$$\text{Initial P.E.} = \frac{-6.67 \times 10^{-11} \times 10 \times 20}{1} = -13.34 \times 10^{-9} \text{ J}$$

When separation is 0.5 m,



$$-13.34 \times 10^{-9} + 0 = \frac{-13.34 \times 10^{-9}}{(1/2)} + (1/2) \times 10 v_1^2 + (1/2) \times 20 v_2^2 \quad \dots(2)$$

$$\Rightarrow -13.34 \times 10^{-9} = -26.68 \times 10^{-9} + 5 v_1^2 + 10 v_2^2$$

$$\Rightarrow -13.34 \times 10^{-9} = -26.68 \times 10^{-9} + 30 v_2^2$$

$$\Rightarrow v_2^2 = \frac{13.34 \times 10^{-9}}{30} = 4.44 \times 10^{-10}$$

$$\Rightarrow v_2 = 2.1 \times 10^{-5} \text{ m/s.}$$

$$\text{So, } v_1 = 4.2 \times 10^{-5} \text{ m/s.}$$

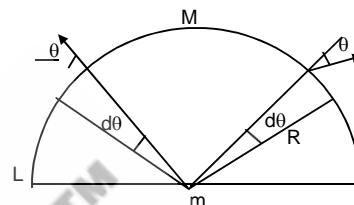
8. In the semicircle, we can consider, a small element of d , then $R d\theta = (M/L) R d\theta = dm$.

$$F = \frac{GMRd\theta m}{LR^2}$$

$$dF_3 = 2 dF \text{ since } = \frac{2GMm}{LR} \sin \theta d\theta.$$

$$\therefore F = \int_0^{\pi/2} \frac{2GMm}{LR} \sin \theta d\theta = \frac{2GMm}{LR} [-\cos \theta]_0^{\pi/2}$$

$$\therefore = -2 \frac{GMm}{LR} (-1) = \frac{2GMm}{LR} = \frac{2GMm}{L \times L/A} = \frac{2\pi GMm}{L^2}$$



9. A small section of rod is considered at 'x' distance mass of the element = $(M/L) \cdot dx = dm$

$$dE_1 = \frac{G(dm) \times 1}{(d^2 + x^2)} = dE_2$$

$$\text{Resultant } dE = 2 dE_1 \sin \theta = 2 \times \frac{G(dm)}{(d^2 + x^2)} \times \frac{x}{(d^2 + x^2)} = \frac{2 \times GM \times d}{L(d^2 + x^2)^{3/2}}$$

Total gravitational field

$$E = \int_0^{L/2} \frac{2Gm dx}{L(d^2 + x^2)^{3/2}}$$

Integrating the above equation it can be found that,

$$E = \frac{2GM}{d \sqrt{L^2 + 4d^2}}$$

10. The gravitational force on 'm' due to the shell of M_2 is 0.

$$M \text{ is at a distance } \frac{R_1 + R_2}{2}$$

Then the gravitational force due to M is given by

$$= \frac{GM_1 m}{(R_1 + R_2/2)} = \frac{4GM_1 m}{(R_1 + R_2)^2}$$

11. Mass of earth $M = (4/3) \pi R^3 \rho$

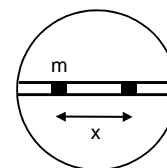
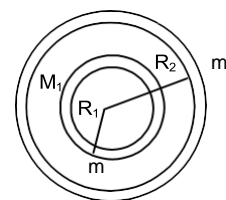
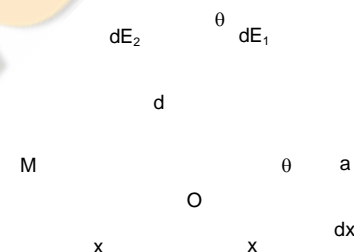
Mass of the imaginary sphere, having

Radius = x, $M' = (4/3) \pi x^3 \rho$

$$\text{or } \frac{M'}{M} = \frac{x^3}{R^3}$$

$$\therefore \text{Gravitational force on } F = \frac{GM'm}{m^2}$$

$$\text{or } F = \frac{GMx^3 m}{R^3 x^2} = \frac{GMmx}{R^3}$$



12. Let d be the distance from centre of earth to man 'm' then

$$D = \sqrt{x^2 + \left(\frac{R}{4}\right)^2} = (1/2) \sqrt{4x^2 + R^2}$$

M be the mass of the earth, M' the mass of the sphere of radius $d/2$.

Then $M = (4/3) \pi R^3 \rho$

$M' = (4/3) \pi d^3 \rho$

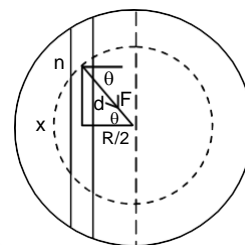
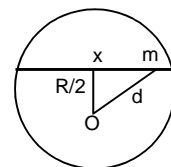
or $\frac{M'}{M} = \frac{d^3}{R^3}$

\therefore Gravitational force is m ,

$$F = \frac{Gm'm}{d^2} = \frac{Gd^3Mm}{R^3d^2} = \frac{GMmd}{R^3}$$

So, Normal force exerted by the wall = $F \cos \theta$.

$$= \frac{GMmd}{R^3} \times \frac{R}{2d} = \frac{GMm}{2R^2} \quad (\text{therefore I think normal force does not depend on } x)$$



R M

m r
 O

13. a) m' is placed at a distance x from 'O'.

If $r < x < 2r$, Let's consider a thin shell of man

$$dm = \frac{m}{\frac{4}{3}\pi r^3} \times \pi x^2 dx = \frac{3m}{4r^3} x^2 dx$$

Thus $\int dm = \frac{m}{r^3} x^3$

Then gravitational force $F = \frac{Gmdm}{x^2} = \frac{Gmx^3/r^3}{x^2} = \frac{Gmx}{r^3}$

- b) $2r < x < 2R$, then F is due to only the sphere.

$$\therefore F = \frac{Gmm'}{(x-r)^2}$$

- c) if $x > 2R$, then Gravitational force is due to both sphere & shell, then due to shell,

$$F = \frac{Gmm'}{(x-R)^2}$$

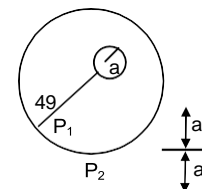
due to the sphere = $\frac{Gmm'}{(x-r)^2}$

So, Resultant force = $\frac{Gmm'}{(x-r)^2} + \frac{Gmm'}{(x-R)^2}$

14. At P_1 , Gravitational field due to sphere $M = \frac{GM}{(3a+a)^2} = \frac{GM}{16a^2}$

At P_2 , Gravitational field is due to sphere & shell,

$$= \frac{GM}{(a+4a+a)^2} + \frac{GM}{(4a+a)^2} = \frac{GM}{a^2} \left(\frac{1}{36} + \frac{1}{25} \right) = \frac{61}{900} \frac{GM}{a^2}$$

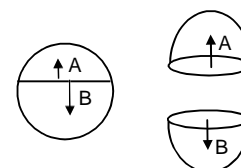


15. We know in the thin spherical shell of uniform density has gravitational field at its internal point is zero.

At A and B point, field is equal and opposite and cancel each other so Net field is zero.

zero.

Hence, $E_A = E_B$



16. Let 0.1 kg man is x m from 2kg mass and $(2 - x)$ m from 4 kg mass.

$$\therefore \frac{2 \times 0.1}{x^2} = \frac{4 \times 0.1}{(2-x)^2}$$

$$\text{or } \frac{0.2}{x^2} = - \frac{0.4}{(2-x)^2}$$

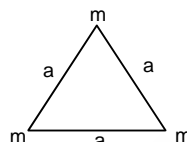
$$\text{or } \frac{1}{x^2} = \frac{2}{(2-x)^2} \text{ or } (2-x)^2 = 2x^2$$

$$\text{or } 2-x = \sqrt{2}x \text{ or } x(r_2 + 1) = 2$$

$$\text{or } x = \frac{2}{2.414} = 0.83 \text{ m from } 2\text{kg mass.}$$

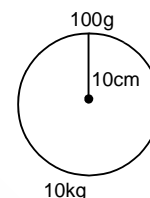
17. Initially, the side of Δ is a
To increase it to $2a$,

$$\text{work done} = \frac{Gm^2}{2a} + \frac{Gm^2}{a} = \frac{3Gm^2}{2a}$$



18. Work done against gravitational force to take away the particle from sphere,

$$= \frac{G \times 10 \times 0.1}{0.1 \times 0.1} = \frac{6.67 \times 10^{-11} \times 1}{1 \times 10^{-1}} = 6.67 \times 10^{-10} \text{ J}$$



19. $E = (5 \text{ N/kg}) \hat{i} + (12 \text{ N/kg}) \hat{j}$

a) $F = E m$

$$= 2\text{kg} [(5 \text{ N/kg}) \hat{i} + (12 \text{ N/kg}) \hat{j}] = (10 \text{ N}) \hat{i} + (24 \text{ N}) \hat{j}$$

$$|\vec{F}| = \sqrt{100 + 576} = 26 \text{ N}$$

b) $V = E r$

At $(12 \text{ m}, 0)$, $V = - (60 \text{ J/kg}) \hat{i}$ $\vec{V} = 60 \text{ J}$

At $(0, 5 \text{ m})$, $V = - (60 \text{ J/kg}) \hat{j}$ $\vec{V} = - 60 \text{ J}$

$$\text{c) } \Delta V = \int_{(0,0)}^{(12,5)} \vec{E} \cdot d\vec{r} = \int_{(0,0)}^{(12,5)} [(10\text{N})\hat{i} + (24\text{N})\hat{j}] \cdot d\vec{r}$$

$$= - (120 \text{ J}) \hat{i} + 120 \text{ J} \hat{j} = 240 \text{ J}$$

$$\text{d) } \Delta v = - [r(10\text{N}\hat{i} + 24\text{N}\hat{j})]_{(12\text{m},0)}^{(0,5\text{m})}$$

$$= -120 \hat{j} + 120 \hat{i} = 0$$

20. a) $V = (20 \text{ N/kg}) (x + y)$

$$\frac{GM}{R} = \frac{MLT^{-2}}{M} \text{ or } \frac{M^{-1}L^3T^{-2}M^1}{L} = \frac{ML^2T^{-2}}{M}$$

$$\text{Or } M^0 L^2 T^{-2} = M^0 L^2 T^{-2}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

b) $E_{(x,y)} = - 20(\text{N/kg}) \hat{i} - 20(\text{N/kg}) \hat{j}$

c) $F = E m$

$$= 0.5\text{kg} [- (20 \text{ N/kg}) \hat{i} - (20 \text{ N/kg}) \hat{j}] = - 10\text{N} \hat{i} - 10 \text{N} \hat{j}$$

$$\therefore |\vec{F}| = \sqrt{100 + 100} = 10\sqrt{2} \text{ N}$$

21. $E = 2 \hat{i} + 3 \hat{j}$

The field is represented as

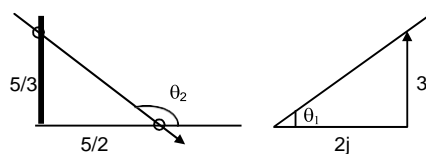
$$\tan \theta_1 = 3/2$$

Again the line $3y + 2x = 5$ can be represented as

$$\tan \theta_2 = - 2/3$$

$$m_1 m_2 = -1$$

Since, the direction of field and the displacement are perpendicular, is done by the particle on the line.



22. Let the height be h

$$\therefore (1/2) \frac{GM}{R^2} = \frac{GM}{(R+h)^2}$$

$$\text{Or } 2R^2 = (R+h)^2$$

$$\text{Or } \sqrt{2} R = R+h$$

$$\text{Or } h = (\sqrt{2} - 1)R$$

23. Let
- g'
- be the acceleration due to gravity on mount everest.

$$g' = g \left(1 - \frac{2h}{R} \right)$$

$$= 9.8 \left(1 - \frac{17696}{6400000} \right) = 9.8 (1 - 0.00276) = 9.773 \text{ m/s}^2$$

24. Let
- g'
- be the acceleration due to gravity in mine.

$$\text{Then } g' = g \left(1 - \frac{2h}{R} \right)$$

$$= 9.8 \left(1 - \frac{640}{6400 \times 10^3} \right) = 9.8 \times 0.9999 = 9.799 \text{ m/s}^2$$

25. Let
- g'
- be the acceleration due to gravity at equator & that of pole = g

$$g' = g - \omega^2 R$$

$$= 9.81 - (7.3 \times 10^{-5})^2 \times 6400 \times 10^3$$

$$= 9.81 - 0.034$$

$$= 9.776 \text{ m/s}^2$$

$$mg' = 1 \text{ kg} \times 9.776 \text{ m/s}^2$$

$$= 9.776 \text{ N or } 0.997 \text{ kg}$$

The body will weigh 0.997 kg at equator.

26. At equator,
- $g' = g - \omega^2 R$
- ... (1)

Let at 'h' height above the south pole, the acceleration due to gravity is same.

$$\text{Then, here } g' = g \left(1 - \frac{2h}{R} \right) \dots (2)$$

$$\therefore g - \omega^2 R = g \left(1 - \frac{2h}{R} \right)$$

$$\text{or } 1 - \frac{\omega^2 R}{g} = 1 - \frac{2h}{R}$$

$$\text{or } h = \frac{\omega^2 R^2}{2g} = \frac{(7.3 \times 10^{-5})^2 \times (6400 \times 10^3)^2}{2 \times 9.81}$$

$$= 11125 \text{ N} = 10 \text{ Km (approximately)}$$

27. The apparent 'g' at equator becomes zero.

$$\text{i.e. } g' = g - \omega^2 R = 0$$

$$\text{or } g = \omega^2 R$$

$$\text{or } \omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6400 \times 10^3}} = \sqrt{1.5 \times 10^{-6}} = 1.2 \times 10^{-3} \text{ rad/s.}$$

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.2 \times 10^{-3}} = 1.5 \times 10^{-6} \text{ sec.} = 1.41 \text{ hour}$$

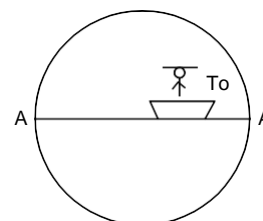
28. a) Speed of the ship due to rotation of earth
- $v = \omega R$

$$\text{b) } T_0 = mgr = mg - m\omega^2 R$$

$$\therefore T_0 - mg = m\omega^2 R$$

- c) If the ship shifts at speed 'v'

$$T = mg - m\omega_1^2 R$$



$$= T_0 - \left(\frac{(v - \omega R)^2}{R^2} \right) R$$

$$= T_0 - \left(\frac{v^2 + \omega^2 R^2 - 2\omega Rv}{R} \right) m$$

$$\therefore T = T_0 + 2\omega v m$$

29. According to Kepler's laws of planetary motion,

$$T^2 \propto R^3$$

$$\frac{T_m^2}{T_e^2} = \frac{R_{ms}^3}{R_{es}^3}$$

$$\left(\frac{R_{ms}}{R_{es}} \right)^3 = \left(\frac{1.88}{1} \right)^2$$

$$\therefore \frac{R_{ms}}{R_{es}} = (1.88)^{2/3} = 1.52$$

30. $T = 2\pi \sqrt{\frac{r^3}{GM}}$

$$27.3 = 2 \times 3.14 \sqrt{\frac{(3.84 \times 10^5)^3}{6.67 \times 10^{-11} \times M}}$$

$$\text{or } 2.73 \times 2.73 = \frac{2 \times 3.14 \times (3.84 \times 10^5)^3}{6.67 \times 10^{-11} \times M}$$

$$\text{or } M = \frac{2 \times (3.14)^2 \times (3.84)^3 \times 10^{15}}{3.335 \times 10^{-11} (27.3)^2} = 6.02 \times 10^{24} \text{ kg}$$

\(\therefore\) mass of earth is found to be 6.02×10^{24} kg.

31. $T = 2\pi \sqrt{\frac{r^3}{GM}}$

$$\Rightarrow 27540 = 2 \times 3.14 \sqrt{\frac{(9.4 \times 10^3 \times 10^3)^3}{6.67 \times 10^{-11} \times M}}$$

$$\text{or } (27540)^2 = (6.28)^2 \frac{(9.4 \times 10^6)^3}{6.67 \times 10^{-11} \times M}$$

$$\text{or } M = \frac{(6.28)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (27540)^2} = 6.5 \times 10^{23} \text{ kg.}$$

32. a) $V = \sqrt{\frac{GM}{r+h}} = \sqrt{\frac{gr^2}{r+h}}$

$$= \sqrt{\frac{9.8 \times (6400 \times 10^3)^2}{10^6 \times (6.4 + 2)}} = 6.9 \times 10^3 \text{ m/s} = 6.9 \text{ km/s}$$

b) K.E. = $(1/2) mv^2$

$$= (1/2) 1000 \times (47.6 \times 10^6) = 2.38 \times 10^{10} \text{ J}$$

c) P.E. = $\frac{GMm}{-(R+h)}$

$$= - \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10^3}{(6400 + 2000) \times 10^3} = - \frac{40 \times 10^{13}}{8400} = - 4.76 \times 10^{10} \text{ J}$$

d) $T = \frac{2\pi(r+h)}{V} = \frac{2 \times 3.14 \times 8400 \times 10^3}{6.9 \times 10^3} = 76.6 \times 10^2 \text{ sec} = 2.1 \text{ hour}$

33. Angular speed of earth & the satellite will be same

$$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$$

$$\text{or } \frac{1}{24 \times 3600} = \frac{1}{2\pi \sqrt{\frac{(R+h)^3}{gR^2}}} \quad \text{or } 12 \times 3600 = 3.14 \sqrt{\frac{(R+h)^3}{gR^2}}$$

$$\text{or } \frac{(R+h)^2}{gR^2} = \frac{(12 \times 3600)^2}{(3.14)^2} \quad \text{or } \frac{(6400+h)^3 \times 10^9}{9.8 \times (6400)^2 \times 10^6} = \frac{(12 \times 3600)^2}{(3.14)^2}$$

$$\text{or } \frac{(6400+h)^3 \times 10^9}{6272 \times 10^9} = 432 \times 10^4$$

$$\text{or } (6400+h)^3 = 6272 \times 432 \times 10^4$$

$$\text{or } 6400+h = (6272 \times 432 \times 10^4)^{1/3}$$

$$\text{or } h = (6272 \times 432 \times 10^4)^{1/3} - 6400$$

$$= 42300 \text{ cm.}$$

- b) Time taken from north pole to equator =
- $(1/2)t$

$$= (1/2) \times 6.28 \frac{(42300 + 6400)^3}{10 \times (6400)^2 \times 10^6} = 3.14 \frac{(497)^3 \times 10^6}{(64)^2 \times 10^{11}}$$

$$= 3.14 \frac{497 \times 497 \times 497}{64 \times 64 \times 10^5} = 6 \text{ hour.}$$

34. For geo stationary satellite,

$$r = 4.2 \times 10^4 \text{ km}$$

$$h = 3.6 \times 10^4 \text{ km}$$

$$\text{Given } mg = 10 \text{ N}$$

$$mgh = mg \left(\frac{R^2}{(R+h)^2} \right)$$

$$= 10 \left[\frac{(6400 \times 10^3)^2}{(6400 \times 10^3 + 3600 \times 10^3)^2} \right] = \frac{4096}{17980} = 0.23 \text{ N}$$

- 35.
- $T = 2\pi$

$$\frac{R_2^3}{gR_1^2}$$

$$\text{Or } T^2 = 4\pi^2 \frac{R_2^3}{gR_1^2}$$

$$\text{Or } g = \frac{4\pi^2 R_2^3}{T^2 R_1^2}$$

$$\therefore \text{Acceleration due to gravity of the planet is } = \frac{4\pi^2 R_2^3}{T^2 R_1^2}$$

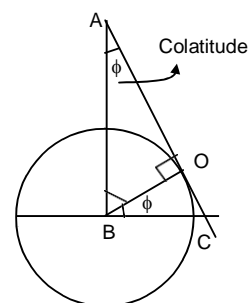
36. The colatitude is given by
- ϕ
- .

$$\angle OAB = 90^\circ - \angle ABO$$

$$\text{Again } \angle OBC = \phi = \angle OAB$$

$$\therefore \sin \phi = \frac{6400}{42000} = \frac{8}{53}$$

$$\therefore \phi = \sin^{-1} \left(\frac{8}{53} \right) = \sin^{-1} 0.15.$$



37. The particle attain maximum height = 6400 km.

On earth's surface its P.E. & K.E.

$$E = (1/2) mv^2 + \left(-\frac{GMm}{R} \right) \quad \dots(1)$$

In space, its P.E. & K.E.

$$E_s = \left(-\frac{GMm}{R+h} \right) + 0$$

$$E_s = \left(-\frac{GMm}{2R} \right) \quad \dots(2) \quad (\because h=R)$$

Equating (1) & (2)

$$-\frac{GMm}{R} + \frac{1}{2} mv^2 = -\frac{GMm}{2R}$$

$$\text{Or } (1/2) mv^2 = GMm \left(\frac{1}{2R} + \frac{1}{R} \right)$$

$$\text{Or } v^2 = \frac{GM}{R}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}$$

$$= 40.02 \times 10^{13}$$

$$= 6.4 \times 10^6$$

$$= 6.2 \times 10^7 = 0.62 \times 10^8$$

$$\text{Or } v = 0.62 \times 10^8 = 0.79 \times 10^4 \text{ m/s} = 7.9 \text{ km/s.}$$

38. Initial velocity of the particle = 15 km/s

Let its speed be 'v' at interstellar space.

$$\therefore (1/2) m[(15 \times 10^3)^2 - v^2] = \int_R^\infty \frac{GMm}{x^2} dx$$

$$\Rightarrow (1/2) m[(15 \times 10^3)^2 - v^2] = GMm \left[-\frac{1}{x} \right]_R^\infty$$

$$\Rightarrow (1/2) m[(225 \times 10^6) - v^2] = \frac{GMm}{R}$$

$$\Rightarrow 225 \times 10^6 - v^2 = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}$$

$$\Rightarrow v^2 = 225 \times 10^6 - \frac{40.02}{32} \times 10^8$$

$$\Rightarrow v^2 = 225 \times 10^6 - 1.2 \times 10^8 = 10^8 (1.05)$$

$$\text{Or } v = 1.01 \times 10^4 \text{ m/s or}$$

$$= 10 \text{ km/s}$$

39. The mass of the sphere = 6×10^{24} kg.

Escape velocity = 3×10^8 m/s

$$V_c = \sqrt{\frac{2GM}{R}}$$

$$\text{Or } R = \frac{2GM}{V_c^2}$$

$$= \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2} = \frac{80.02}{9} \times 10^{-3} = 8.89 \times 10^{-3} \text{ m} \approx 9 \text{ mm.}$$

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