THE SPECIAL THEORY OF RELATIVITY
CHAPTER - 47

1. $S = 1000\text{ km} = 10^6\text{ m}$
   The process requires minimum possible time if the velocity is maximum.
   We know that maximum velocity can be that of light i.e. $= 3 \times 10^8$ m/s.
   So, time $= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{10^6}{3\times10^8} = \dfrac{1}{300}$ s.
2. $\ell = 50$ cm, b = 25 cm, h = 10 cm, v = 0.6 c
   a) The observer in the train notices the same value of ℓ, b, h because relativity are in due to difference in frames.
   b) In 2 different frames, the component of length parallel to the velocity undergoes contraction but the perpendicular components remain the same. So length which is parallel to the x-axis changes and breadth and height remain the same.

   $$e' = e\sqrt{1-\frac{V^2}{C^2}} = 50\sqrt{1-\frac{(0.6)^2C^2}{C^2}}$$
   $$= 50\sqrt{1-0.36} = 50 \times 0.8 = 40\text{ cm.}$$

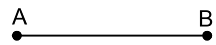
   The lengths observed are 40 cm × 25 cm × 10 cm.
3. L = 1 m
   a) v 3×10^5 m/s
   $$L' = 1\sqrt{1-\frac{9\times10^{10}}{9\times10^{16}}} = \sqrt{1-10^{-6}} = 0.9999995\text{ m}$$
   b) v = 3 x 10^6 m/s
   $$L' = 1\sqrt{1-\frac{9\times10^{12}}{9\times10^{16}}} = \sqrt{1-10^{-4}} = 0.99995\text{ m.}$$
   c) v = 3×10^7 m/s
   $$L' = 1\sqrt{1-\frac{9\times10^{14}}{9\times10^{16}}} = \sqrt{1-10^{-2}} = 0.9949 = 0.995\text{ m.}$$
4. v = 0.6 cm/sec ; t = 1 sec
   a) length observed by the observer = vt $\Rightarrow 0.6 \times 3 \times 10^6 \Rightarrow 1.8 \times 10^8$ m
   b) $\ell = \ell_0\sqrt{1-v^2/c^2} \Rightarrow 1.8 \times 10^8 = \ell_0\sqrt{1-\dfrac{(0.6)^2C^2}{C^2}}$

   $$\ell_0 = \frac{1.8\times10^8}{0.8} = 2.25 \times 10^8\text{ m/s.}$$
5. The rectangular field appears to be a square when the length becomes equal to the breadth i.e. 50 m.
   i.e. L′ = 50 ; L = 100 ; v = ?
   C = 3×10^8 m/s
   We know, $L' = L\sqrt{1-v^2/c^2}$
   $$\Rightarrow 50 = 100\sqrt{1-v^2/c^2} \Rightarrow v = \sqrt{3/2}C = 0.866\text{ C.}$$
6. $L_0 = 1000\text{ km} = 10^6$ m
   v = 360 km/h = (360 × 5) / 18 = 100 m/sec.
   a) $h' = h_0\sqrt{1-v^2/c^2} = 10^6\sqrt{1-\left(\dfrac{100}{3\times10^8}\right)^2} = 10^6\sqrt{1-\dfrac{10^4}{9\times10^6}} = 10^9.$

   Solving change in length = 56 nm.
   b) $\Delta t = \Delta L/v$ = 56 nm / 100 m = 0.56 ns.

7. v = 180 km/hr = 50 m/s

t = 10 hours



let the rest dist. be L.

$L' = L\sqrt{1-v^2/c^2} \Rightarrow L' = 10 \times 180 = 1800$ k.m.

$1800 = L\sqrt{1-\frac{180^2}{(3\times10^5)^2}}$

or, $1800 \times 1800 = L(1 - 36 \times 10^{-14})$

or, $L = \frac{3.24\times10^6}{1-36\times10^{-14}} = 1800 + 25 \times 10^{-12}$

or 25 nm more than 1800 km.

b) Time taken in road frame by Car to cover the dist = $\frac{1.8\times10^6+25\times10^{-9}}{50}$

$= 0.36 \times 10^5 + 5 \times 10^{-8}$ = 10 hours + 0.5 ns.

8. a) u = 5c/13

$\Delta t = \frac{t}{\sqrt{1-v^2/c^2}} = \frac{1y}{\sqrt{1-\frac{25c^2}{169c^2}}} = \frac{y\times13}{12} = \frac{13}{12}y$.

The time interval between the consecutive birthday celebration is 13/12 y.

b) The fried on the earth also calculates the same speed.

9. The birth timings recorded by the station clocks is proper time interval because it is the ground frame. That of the train is improper as it records the time at two different places. The proper time interval ΔT is less than improper.

i.e. $\Delta T' = v\,\Delta T$

Hence for – (a) up train → Delhi baby is elder (b) down train → Howrah baby is elder.

10. The clocks of a moving frame are out of synchronization. The clock at the rear end leads the one at from by $L_0\,V/C^2$ where L_0 is the rest separation between the clocks, and v is speed of the moving frame.

Thus, the baby adjacent to the guard cell is elder.

11. v = 0.9999 C ; Δt = One day in earth ; $\Delta t'$ = One day in heaven

$v = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\frac{(0.9999)^2C^2}{C^2}}} = \frac{1}{0.014141782} = 70.712$

$\Delta t' = v\,\Delta t$;

Hence, $\Delta t'$ = 70.7 days in heaven.

12. t = 100 years ; V = 60/100 K ; C = 0.6 C.

$\Delta t = \frac{t}{\sqrt{1-V^2/C^2}} = \frac{100y}{\sqrt{1-\frac{(0.6)^2C^2}{C^2}}} = \frac{100y}{0.8} = 125$ y.

13. We know

$f' = f\sqrt{1-V^2/C^2}$

f′ = apparent frequency ;

f = frequency in rest frame

v = 0.8 C

$f' = \sqrt{1-\frac{0.64C^2}{C^2}} = \sqrt{0.36} = 0.6\ s^{-1}$

14. V = 100 km/h, Δt = Proper time interval = 10 hours

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - V^2/C^2}} = \frac{10 \times 3600}{\sqrt{1 - \left(\frac{1000}{36 \times 3 \times 10^8}\right)^2}}$$

$$\Delta t' - \Delta t = 10 \times 3600 \left[\frac{1}{1 - \left(\frac{1000}{36 \times 3 \times 10^8}\right)^2} - 1\right]$$

By solving we get, $\Delta t' - \Delta t$ = 0.154 ns.

$\therefore$ Time will lag by 0.154 ns.

15. Let the volume (initial) be V.

$V' = V/2$

So, $V/2 = v\sqrt{1 - V^2/C^2}$

$\Rightarrow C/2 = \sqrt{C^2 - V^2} \Rightarrow C^2/4 = C^2 - V^2$

$\Rightarrow V^2 = C^2 - \frac{C^2}{4} = \frac{3}{4}C^2 \Rightarrow V = \frac{\sqrt{3}}{2}C$.

16. d = 1 cm, v = 0.995 C

a) time in Laboratory frame = $\frac{d}{v} = \frac{1 \times 10^{-2}}{0.995C}$

$= \frac{1 \times 10^{-2}}{0.995 \times 3 \times 10^8} = 33.5 \times 10^{-12} = 33.5$ PS

b) In the frame of the particle

$t' = \frac{t}{\sqrt{1 - V^2/C^2}} = \frac{33.5 \times 10^{-12}}{\sqrt{1 - (0.995)^2}} = 335.41$ PS.

17. x = 1 cm = 1×10^{-2} m ; K = 500 N/m, m = 200 g

Energy stored = ½ Kx^2 = ½ $\times 500 \times 10^{-4}$ = 0.025 J

Increase in mass = $\frac{0.025}{C^2} = \frac{0.025}{9 \times 10^{16}}$

Fractional Change of max = $\frac{0.025}{9 \times 10^{16}} \times \frac{1}{2 \times 10^{-1}} = 0.01388 \times 10^{-16} = 1.4 \times 10^{-8}$.

18. Q = MS $\Delta\theta \Rightarrow 1 \times 4200\,(100 - 0)$ = 420000 J.

E = $(\Delta m)C^2$

$\Rightarrow \Delta m = \frac{E}{C^2} = \frac{Q}{C^2} = \frac{420000}{(3 \times 10^8)^2}$

$= 4.66 \times 10^{-12} = 4.7 \times 10^{-12}$ kg.

19. Energy possessed by a monoatomic gas = 3/2 nRdt.

Now dT = 10, n = 1 mole, R = 8.3 J/mol-K.

E = $3/2 \times t \times 8.3 \times 10$

Loss in mass = $\frac{1.5 \times 8.3 \times 10}{C^2} = \frac{124.5}{9 \times 10^{15}}$

$= 1383 \times 10^{-16} = 1.38 \times 10^{-15}$ Kg.

20. Let initial mass be m

½ mv^2 = E

$\Rightarrow E = \frac{1}{2}m\left(\frac{12 \times 5}{18}\right)^2 = \frac{m \times 50}{9}$

$\Delta m = E/C^2$

$\Rightarrow \Delta m = \dfrac{m \times 50}{9 \times 9 \times 10^{16}} \Rightarrow \dfrac{\Delta m}{m} = \dfrac{50}{81 \times 10^{16}}$

$\Rightarrow 0.617 \times 10^{-16} = 6.17 \times 10^{-17}$.

21. Given : Bulb is 100 Watt = 100 J/s

So, 100 J in expended per 1 sec.

Hence total energy expended in 1 year = $100 \times 3600 \times 24 \times 365 = 3153600000$ J

Change in mass recorded = $\dfrac{\text{Total energy}}{C^2} = \dfrac{315360000}{9 \times 10^{16}}$

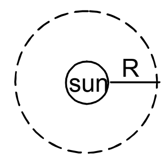
$= 3.504 \times 10^{8} \times 10^{-16}$ kg $= 3.5 \times 10^{-8}$ Kg.

22. $I = 1400$ w/m^2

Power = 1400 w/m$^2 \times A$

$= (1400 \times 4\pi R^2)w = 1400 \times 4\pi \times (1.5 \times 10^{11})^2$

$= 1400 \times 4\pi \times (1/5)^2 \times 10^{22}$



a) $\dfrac{E}{t} = \dfrac{\Delta m C^2}{t} = \dfrac{\Delta m}{t} = \dfrac{E/t}{C^2}$

$C^2 = \dfrac{1400 \times 4\pi \times 2.25 \times 10^{22}}{9 \times 10^{16}} = 1696 \times 10^{66} = 4.396 \times 10^{9} = 4.4 \times 10^{9}$.

b) 4.4×10^{9} Kg disintegrates in 1 sec.

2×10^{30} Kg disintegrates in $\dfrac{2 \times 10^{30}}{4.4 \times 10^{9}}$ sec.

$= \left(\dfrac{1 \times 10^{21}}{2.2 \times 365 \times 24 \times 3600}\right) = 1.44 \times 10^{-8} \times 10^{21}$ y $= 1.44 \times 10^{13}$ y.

23. Mass of Electron = Mass of positron = 9.1×10^{-31} Kg

Both are oppositely charged and they annihilate each other.

Hence, $\Delta m = m + m = 2 \times 9.1 \times 10^{-31}$ Kg

Energy of the resulting γ particle = $\Delta m\, C^2$

$= 2 \times 9.1 \times 10^{-31} \times 9 \times 10^{16}$ J $= \dfrac{2 \times 9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}}$ ev

$= 102.37 \times 10^{4}$ ev $= 1.02 \times 10^{6}$ ev $= 1.02$ Mev.

24. $m_e = 9.1 \times 10^{-31}$, $v_0 = 0.8$ C

a) $m' = \dfrac{Me}{\sqrt{1 - V^2/C^2}} = \dfrac{9.1 \times 10^{-31}}{\sqrt{1 - 0.64C^2/C^2}} = \dfrac{9.1 \times 10^{-31}}{0.6}$

$= 15.16 \times 10^{-31}$ Kg $= 15.2 \times 10^{-31}$ Kg.

b) K.E. of the electron : $m'C^2 - m_eC^2 = (m' - m_e)\, C^2$

$= (15.2 \times 10^{-31} - 9.1 \times 10^{-31})(3 \times 10^{8})^2 = (15.2 \times 9.1) \times 9 \times 10^{-31} \times 10^{18}$ J

$= 54.6 \times 10^{-15}$ J $= 5.46 \times 10^{-14}$ J $= 5.5 \times 10^{-14}$ J.

c) Momentum of the given electron = Apparent mass $\times$ given velocity

$= 15.2 \times 10^{-31} - 0.8 \times 3 \times 10^{8}$ m/s $= 36.48 \times 10^{-23}$ kg m/s

$= 3.65 \times 10^{-22}$ kg m/s

25. a) $ev - m_0C^2 = \dfrac{m_0C^2}{2\sqrt{1 - \dfrac{V^2}{C^2}}} \Rightarrow ev - 9.1 \times 10^{-31} \times 9 \times 10^{16}$

$= \dfrac{9.1 \times 9 \times 10^{-31} \times 10^{16}}{2\sqrt{1 - \dfrac{0.36C^2}{C^2}}} \Rightarrow eV - 9.1 \times 9 \times 10^{-15}$

$= \frac{9.1 \times 9 \times 10^{-15}}{2 \times 0.8} \Rightarrow eV - 9.1 \times 9 \times 10^{-15} = \frac{9.1 \times 9 \times 10^{-15}}{1.6}$

$\Rightarrow eV = \left(\frac{9.1 \times 9}{1.6} + 9.1 \times 9\right) \times 10^{-15} = eV\left(\frac{81.9}{1.6} + 81.9\right) \times 10^{-15}$

$eV = 133.0875 \times 10^{-15} \Rightarrow V = 83.179 \times 10^4 = 831$ KV.

b) $eV - m_0C^2 = \frac{m_0C^2}{2\sqrt{1 - \frac{V^2}{C^2}}} \Rightarrow eV - 9.1 \times 9 \times 10^{-19} \times 9 \times 10^{16} = \frac{9.1 \times 9 \times 10^{-15}}{2\sqrt{1 - \frac{0.81C^2}{C^2}}}$

$\Rightarrow eV - 81.9 \times 10^{-15} = \frac{9.1 \times 9 \times 10^{-15}}{2 \times 0.435}$

$\Rightarrow eV = 12.237 \times 10^{-15}$

$\Rightarrow V = \frac{12.237 \times 10^{-15}}{1.6 \times 10^{-19}} = 76.48$ kV.

$V = 0.99\ C = eV - m_0C^2 = \frac{m_0C^2}{2\sqrt{1 - \frac{V^2}{C^2}}}$

$\Rightarrow eV = \frac{m_0C^2}{2\sqrt{1 - \frac{V^2}{C^2}}} + m_0C^2 = \frac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{2\sqrt{1 - (0.99)^2}} + 9.1 \times 10^{-31} \times 9 \times 10^{16}$

$\Rightarrow eV = 372.18 \times 10^{-15} \Rightarrow V = \frac{372.18 \times 20^{-15}}{1.6 \times 10^{-19}} = 272.6 \times 10^4$

$\Rightarrow V = 2.726 \times 10^6 = 2.7$ MeV.

26. a) $\frac{m_0C^2}{\sqrt{1 - \frac{V^2}{C^2}}} - m_0C^2 = 1.6 \times 10^{-19}$

$\Rightarrow m_0C^2\left(\frac{1}{\sqrt{1 - V^2/C^2}} - 1\right) = 1.6 \times 10^{-19}$

$\Rightarrow \frac{1}{\sqrt{1 - V^2/C^2}} - 1 = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31} \times 9 \times 10^{16}}$

$\Rightarrow V = C \times 0.001937231 = 3 \times 0.001967231 \times 0^8 = 5.92 \times 10^5$ m/s.

b) $\frac{m_0C^2}{\sqrt{1 - \frac{V^2}{C^2}}} - m_0C^2 = 1.6 \times 10^{-19} \times 10 \times 10^3$

$\Rightarrow m_0C^2\left(\frac{1}{\sqrt{1 - V^2/C^2}} - 1\right) = 1.6 \times 10^{-15}$

$\Rightarrow \frac{1}{\sqrt{1 - V^2/C^2}} - 1 = \frac{1.6 \times 10^{-15}}{9.1 \times 9 \times 10^{15}}$

$\Rightarrow V = 0.584475285 \times 10^8 = 5.85 \times 10^7$ m/s.

c) K.E. = 10 Mev = 10×10^6 eV = $10^7 \times 1.6 \times 10^{-19}$ J = 1.6×10^{-12} J

$\Rightarrow \frac{m_0C^2}{\sqrt{1 - \frac{V^2}{C^2}}} - m_0C^2 = 1.6 \times 10^{-12}$ J

$\Rightarrow V^2 = 8..999991359 \times 10^{16} \Rightarrow V = 2.999987038 \times 10^8$.

27. $\Delta m = m - m_0 = 2m_0 - m_0 = m_0$

Energy $E = m_0c^2 = 9.1 \times 10^{-31} \times 9 \times 10^{16}$ J

E in e.v. = $\dfrac{9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}}$ = 51.18×10^4 ev = 511 Kev.

28. $$\frac{\left(\dfrac{m_0C^2}{\sqrt{1-\dfrac{V^2}{C^2}}} - m_0C^2\right) - \dfrac{1}{2}mv^2}{\dfrac{1}{2}m_0v^2} = 0.01$$

$$\Rightarrow \left[\frac{m_0C^2(1 + \dfrac{v^2}{2C^2} + \dfrac{1}{2} \times \dfrac{3}{4}\dfrac{V^2}{C^2} + \dfrac{1}{2} \times \dfrac{3}{4} \times \dfrac{5}{6}\dfrac{V^6}{C^6}) - m_0C^2}{\dfrac{1}{2}m_0v^2}\right] - \frac{1}{2}mv^2 = 0.1$$

$$\Rightarrow \frac{\dfrac{1}{2}m_0v^2 + \dfrac{3}{8}m_0\dfrac{V^4}{C^2} + \dfrac{15}{96}m_0\dfrac{V^4}{C^2} - \dfrac{1}{2}m_0v^2}{\dfrac{1}{2}m_0v^2} = 0.01$$

$$\Rightarrow \frac{3}{4}\frac{V^4}{C^2} + \frac{15}{96 \times 2}\frac{V^4}{C^4} = 0.01$$

Neglecting the v^4 term as it is very small

$$\Rightarrow \frac{3}{4}\frac{V^2}{C^2} = 0.01 \Rightarrow \frac{V^2}{C^2} = 0.04 / 3$$

$$\Rightarrow V/C = 0.2/\sqrt{3} = V = 0.2/\sqrt{3}\,C = \frac{0.2}{1.732} \times 3 \times 10^8$$

$= 0.346 \times 10^8$ m/s $= 3.46 \times 10^7$ m/s.

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