Real Numbers

Exercise 1A

Question 1:

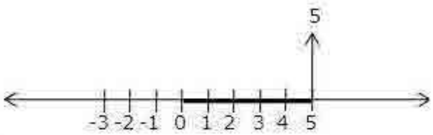
The numbers of the form $\frac{p}{q}$ where p and q are integers and q $\neq$ 0 are known as rational numbers.

Ten examples of rational numbers are:

$\frac{2}{3}, \frac{4}{5}, \frac{7}{9}, \frac{8}{11}, \frac{15}{23}, \frac{23}{27}, \frac{25}{31}, \frac{26}{32}, 1, \frac{12}{5}$

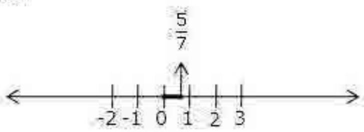
Question 2:

(i) 5

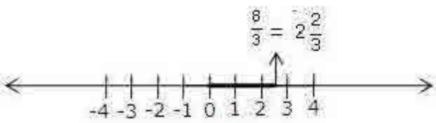


(ii) -3

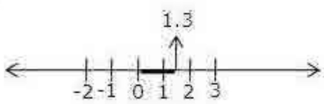
(iii) $\frac{5}{7}$



(iv) $\frac{8}{3} = 2\frac{2}{3}$

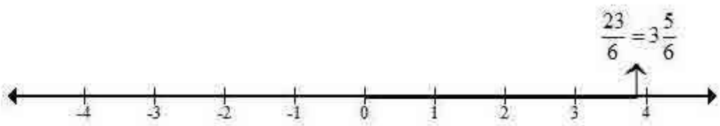


(v) 1.3



(vi) -2.4

(vii) $\frac{23}{6} = 3\frac{5}{6}$



Question 3:

(i) $\frac{1}{4}$ and $\frac{1}{3}$

Let $x = \frac{1}{4}$ and $y = \frac{1}{3}$

Then, $x < y$ because $\frac{1}{4} < \frac{1}{3}$

∴ Rational number lying between x and y

$$= \frac{1}{2}(x + y)$$
$$= \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}\right)$$
$$= \frac{1}{2}\left(\frac{3 + 4}{12}\right)$$
$$= \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

Hence, $\frac{7}{24}$ is a rational number lying between $\frac{1}{4}$ and $\frac{1}{3}$.

(ii) $\frac{3}{8}$ and $\frac{2}{5}$

Let $x = \frac{3}{8}$ and $y = \frac{2}{5}$

Then, $x < y$ because $\frac{3}{8} < \frac{2}{5}$

∴ Rational number lying between x and y

$$= \frac{1}{2}(x + y)$$
$$= \frac{1}{2}\left(\frac{3}{8} + \frac{2}{5}\right)$$
$$= \frac{1}{2}\left(\frac{15 + 16}{40}\right)$$
$$= \frac{1}{2} \times \frac{31}{40} = \frac{31}{80}$$

Hence, $\frac{31}{80}$ is a rational number lying between $\frac{3}{8}$ and $\frac{2}{5}$.

$$\frac{\frac{1}{5}+\frac{9}{40}}{2}=\frac{\frac{17}{40}}{2}=\frac{17}{80}$$

A rational number lying between $\frac{9}{40}$ and $\frac{1}{4}$ is

$$\frac{\frac{9}{40}+\frac{1}{4}}{2}=\frac{\frac{19}{40}}{2}=\frac{19}{80}$$

$$\frac{\frac{1}{5}+\frac{1}{4}}{2}=\frac{\frac{9}{20}}{2}=\frac{9}{40}$$

Therefore, we have $\frac{1}{5} < \frac{17}{80} < \frac{9}{40} < \frac{19}{80} < \frac{1}{4}$

Or we can say that, $\frac{1}{5} < \frac{17}{80} < \frac{9\times2}{40\times2} < \frac{19}{80} < \frac{1}{5}$

That is, $\frac{1}{5} < \frac{17}{80} < \frac{18}{80} < \frac{19}{80} < \frac{1}{5}$

Therefore, three rational numbers between $\frac{1}{5}$ and $\frac{1}{4}$

$\frac{17}{80}$, $\frac{18}{80}$ and $\frac{19}{80}$

Question 5:

Let $x = \frac{2}{5}$ and $y = \frac{3}{4}$

Then, x < y because $\frac{2}{5} < \frac{3}{4}$

Or we can say that, $\frac{2\times4}{5\times4} = \frac{3\times5}{4\times5}$

That is, $\frac{8}{20} < \frac{15}{20}$.

We know that, 8 < 9 < 10 < 11 < 12 < 13 < 14 < 15.

Therefore, we have, $\frac{8}{20} < \frac{9}{20} < \frac{10}{20} < \frac{11}{20} < \frac{12}{20} < \frac{13}{20} < \frac{14}{20} < \frac{15}{20}$

Thus, 5 rational numbers between, $\frac{8}{20} < \frac{15}{20}$ are:

$\frac{9}{20}, \frac{10}{20}, \frac{11}{20}, \frac{12}{20}$ and $\frac{13}{20}$

Question 6:

Let x = 3 and y = 4

Then, x < y, because 3 < 4

We can say that, $\frac{21}{7} < \frac{28}{7}$.

We know that, 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28.

Therefore, we have, $\frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$

Therefore, 6 rational numbers between 3 and 4 are:

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}$ and $\frac{26}{7}$

Question 7:

Let x = 2.1 and y = 2.2

Then, x < y because 2.1 < 2.2

Or we can say that, $\frac{21}{10} < \frac{22}{10}$

Or, $\frac{21\times100}{10\times100} = \frac{22\times100}{10\times100}$

That is, we have, $\frac{2100}{1000} < \frac{2200}{1000}$

We know that, 2100 < 2105 < 2110 < 2115 < 2120 < 2125 < 2130 < 2135 < 2140 < 2145 < 2150 < 2155 < 2160 < 2165 < 2170 < 2175 < 2180 < 2185 < 2190 < 2195 < 2200

Therefore, we can have,

$$\frac{2100}{1000}<\frac{2105}{1000}<\frac{2110}{1000}<\frac{2115}{1000}<\frac{2120}{1000}<\frac{2125}{1000}<\frac{2130}{1000}<\frac{2135}{1000}<\frac{2140}{1000}<\frac{2145}{1000}<\frac{2150}{1000}$$
$$<\frac{2155}{1000}<\frac{2160}{1000}<\frac{2165}{1000}<\frac{2170}{1000}<\frac{2175}{1000}<\frac{2180}{1000}<\frac{2185}{1000}<\frac{2190}{1000}<\frac{2195}{1000}<\frac{2200}{1000}$$

Therefore, 16 rational numbers between, 2.1 and 2.2 are:

$$\frac{2105}{1000},\frac{2110}{1000},\frac{2115}{1000},\frac{2120}{1000},\frac{2125}{1000},\frac{2130}{1000},\frac{2135}{1000},\frac{2140}{1000},\frac{2145}{1000},$$
$$\frac{2150}{1000},\frac{2155}{1000},\frac{2160}{1000},\frac{2165}{1000},\frac{2170}{1000},\frac{2175}{1000},\frac{2180}{1000}$$

So, 16 rational numbers between 2.1 and 2.2 are:
2.105, 2.11, 2.115, 2.12, 2.125, 2.13, 2.135, 2.14, 2.145, 2.15, 2.155, 2.16, 2.165, 2.17, 2.175, 2.18

Exercise 1B

Question 1:

(i)

$\frac{13}{80}$

$$\frac{13}{80} = \frac{13}{2 \times 2 \times 2 \times 2 \times 5} = \frac{13}{2^4 \times 5}$$

If the prime factors of the denominator are 2 and/or 5 then the rational number is a terminating decimal.

Since, 80 has prime factors 2 and 5, $\frac{13}{80}$ is a terminating decimal.

(ii) $\frac{7}{24}$

$$\frac{7}{24} = \frac{7}{2 \times 2 \times 2 \times 3} = \frac{7}{2^3 \times 3}$$

If the prime factors of the denominators of the fraction are other than 2 and 5, then the rational number is not a terminating decimal.

Since, 24 has prime factors 2 and 3 and 3 is different from 2 and 5,

$\frac{7}{24}$is not a terminating decimal.

(iii) $\frac{5}{12}$

$$\frac{5}{12} = \frac{5}{2 \times 2 \times 3} = \frac{5}{2^2 \times 3}$$

If the prime factors of the denominators of the fraction are other than 2 and 5, then the rational number is not a terminating decimal.

Since 12 has prime factors 2 and 3 and 3 is different from 2 and 5,

$\frac{5}{12}$is not a terminating decimal.

(iv) $\frac{8}{35}$

$$\frac{8}{35} = \frac{8}{5 \times 7}$$

If the prime factors of the denominators of the fraction are other than 2 and 5, then the rational number is not a terminating decimal.

Since 35 has prime factors 5 and 7, and 7 is different from 2 and 5,

$\frac{8}{35}$is not a terminating decimal.

(v) $\frac{16}{125}$

$$\frac{16}{125} = \frac{16}{5 \times 5 \times 5} = \frac{16}{5^3}$$

If the prime factors of the denominator are 2 and/or 5 then the rational number is a terminating decimal.

Since 125 has prime factor 5 only

$\frac{16}{125}$is a terminating decimal.

Question 2:

(i)$\frac{5}{8}$

```
    0.625
8)  5.000
    48
   ----
     20
     16
    ----
      40
      40
    ----
       0
```

$\frac{5}{8}$= 0.625

(ii) $\frac{9}{16}$

```
      0.5625
16) 9.0000
    80
    100
     96
      40
      32
       80
       80
        0
```

$\frac{9}{16}$= 0.5625

(iii) $\frac{7}{25}$

```
     0.28
25) 7.00
    50
    200
    200
      0
```

$\frac{7}{25}$= 0.28

(iv)$\frac{11}{24}$

```
      0.45833
24) 11.00000
     96
     140
     120
      200
      192
        80
        72
         80
         72
          8
```

$\frac{11}{24}$= $0.458\bar{3}$

(v) $2\frac{5}{12} = \frac{29}{12}$

```
      2.4166
12) 29.0000
    24
     50
     48
      20
      12
       80
       72
        80
        72
         8
```

$2\frac{5}{12}$= $2.41\bar{6}$

Question 3:

(i) Let x = $0.\bar{3}$

i.e x = 0.333 .... (i)

⇒10x = 3.333 .... (ii)

Subtracting (i) from (ii), we get

9x = 3

⇒ x = $\frac{3}{9}$= $\frac{1}{3}$

Hence, $0.\bar{3}$ = $\frac{1}{3}$

(ii) Let x = $1\bar{3}$

i.e x = 1.333 .... (i)

⇒10x = 13.333 .... (ii)

Subtracting (i) from (ii) we get;

9x = 12

⇒ x = $\frac{12}{9}$ $\frac{4}{3}$

Hence, 1.$\bar{3}$ = $\frac{4}{3}$

(iii) Let x = 0$\overline{34}$

i.e x = 0.3434 .... (i)

⇒100x = 34.3434 .... (ii)

Subtracting (i) from (ii), we get

99x = 34

⇒ x = $\frac{34}{99}$

Hence, 0.$\overline{34}$ = $\frac{33}{99}$

(iv) Let x = 3.$\overline{14}$

i.e x = 3.1414 .... (i)

⇒100x = 314.1414 .... (ii)

Subtracting (i) from (ii), we get

99x = 311

⇒ x = $\frac{311}{99}$

Hence, 3.$\overline{14}$ = $\frac{311}{99}$

(v) Let x = 0.$\overline{324}$

i.e. x = 0.324324 ....(i)

⇒1000x = 324.324324....(ii)

Subtracting (i) from (ii), we get

999x = 324

⇒ x = $\frac{324}{999}$ $\frac{12}{37}$

Hence, 0.$\overline{324}$= $\frac{12}{37}$

(vi) Let x = 0.$1\bar{7}$

i.e. x = 0.177 .... (i)

⇒10x = 1.777 .... (ii)

and 100x = 17.777.... (iii)

Subtracting (ii) from (iii), we get

90x = 16

⇒ x = $\frac{16}{90}$ $\frac{8}{45}$

Hence, 0.$1\bar{7}$ = $\frac{8}{45}$

(vii) Let x = 0.$5\bar{4}$

i.e. x = 0.544 .... (i)

⇒10 x = 5.44 .... (ii)

and 100x = 54.44 ....(iii)

Subtracting (ii) from (iii), we get

90x = 49

⇒ x = $\frac{49}{90}$

Hence, 0.$5\bar{4}$ = $\frac{49}{90}$

(vii) Let x = Let x = 0.$1\overline{63}$

i.e. x = 0.16363 .... (i)

⇒10x = 1.6363 .... (ii)

and 1000 x = 163.6363 .... (iii)

Subtracting (ii) from (iii), we get

$990x = 162$

$\Rightarrow x = \frac{162}{990} = \frac{9}{55}$

Hence, $0.1\overline{63} = \frac{9}{55}$

Question 4:

(i) True. Since the collection of natural number is a sub collection of whole numbers, and every element of natural numbers is an element of whole numbers

(ii) False. Since 0 is whole number but it is not a natural number.

(iii) True. Every integer can be represented in a fraction form with denominator 1.

(iv) False. Since division of whole numbers is not closed under division, the value of $\frac{p}{q}$, p and q are integers and $q \neq 0$, may not be a whole number.

(v) True. The prime factors of the denominator of the fraction form of terminating ***decimal contains 2 and/or 5, which are integers and are not equal to zero.***

(vi) True. The prime factors of the denominator of the fraction form of repeating decimal contains integers, which are not equal to zero.

(vii) True. 0 can considered as a fraction , which is a rational number.