Exercise -10.1

1. In Fig. 10.22, the sides BA and CA have been produced such that: BA = AD and CA = AE. Prove that segment DE || BC.

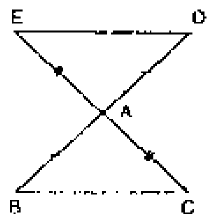


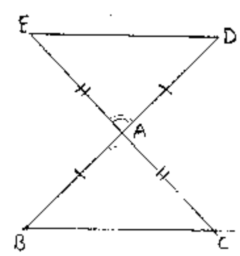
Fig. 10.22

Sol:

Given that, the sides BA and CA have been produced such that $BA = AD$ and $CA = AE$ and given to prove $DE \parallel BC$

Consider triangle BAC and DAE,

We have



$BA = AD$ and $CA = AE$ [$\because$ given in the data]

And also $\angle BAC = \angle DAE$ [$\because$ vertically opposite angles]

So, by SAS congruence criterion, we have $\Delta BAC \cong \Delta DAE$

$\Rightarrow BC = DE$ and $\angle DEA = \angle BCA, \angle EDA = \angle CBA$

[Corresponding parts of congruent triangles are equal]

Now, DE and BC are two lines intersected by a transversal DB such that $\angle DEA = \angle BCA,$ i.e., alternate angles are equal

Therefore, $DE \parallel BC$

2. In a ΔPQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that: LN = MN.

Sol:

Given that, in ΔPQR, $PQ = QR$ and L,M,N are midpoints of the sides PQ, QP and RP respectively and given to prove that $LN = MN$

Here we can observe that PQR is and isosceles triangle

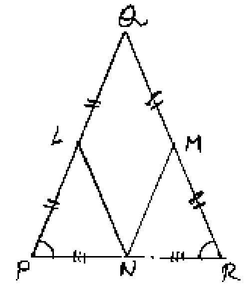
$\Rightarrow PQ = QR$ and $\angle QPR = \angle QRP$ ……..(1)

And also, L and M are midpoints of PQ and QR respectively

$\Rightarrow PL = LQ = \frac{PQ}{2}, QM = MR = \frac{QR}{2}$

And also, $PQ = QR$

$\Rightarrow PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2}$ ....(2)



Now, consider ΔLPN and ΔMRN,

$LP = MR$ [From – (2)]

$\angle LPN = \angle MRN$ [From – (1)]

$\because \angle QPR$ and $\angle LPN$ and $\angle QRP$ and $\angle MRN$ are same

$PN = NR$ [$\because N$ is midpoint of PR]

So, by SAS congruence criterion, we have $\Delta LPN \cong \Delta MRN$

$\Rightarrow LN = MN$

[$\because$ Corresponding parts of congruent triangles are equal]

3. In Fig. 10.23, PQRS is a square and SRT is an equilateral triangle. Prove that
(i) PT = QT (ii) ∠TQR = 15°

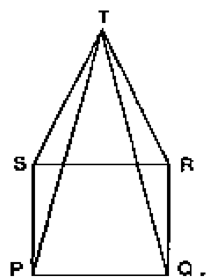


Fig. 10.23

Sol:

Given that PQRS is a square and SRT is an equilateral triangle. And given to prove that (i) $PT = QT$ and (ii) $\angle TQR = 15°$

Now, PQRS is a square

$\Rightarrow PQ = QR = RS = SP$ ....(1)

And $\angle SPQ = \angle PQR = \angle QRS = \angle RSP = 90° =$ right angle

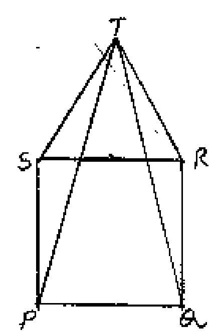
And also, SRT is an equilateral triangle.

$\Rightarrow SR = RT = TS$ ....(2)

And $\angle TSR = \angle SRT = \angle RTS = 60°$

From (1) and (2)

$PQ = QR = SP = SR = RT = TS$ ....(3)



And also,

$\angle TSR = \angle TSR + \angle RSP = 60° + 90° + 150°$

$\angle TRQ = \angle TRS + \angle SRQ = 60° + 90° + 150°$

$\Rightarrow \angle TSR = \angle TRQ = 150°$ ....(4)

Now, in ΔTSR and ΔTRQ

$TS = TR$ [From (3)]

$\angle TSP = \angle TRQ$ [From (4)]

$SP = RQ$ [From (3)]

So, by SAS congruence criterion we have

$\Delta TSP \cong \Delta TRQ$

$\Rightarrow \boxed{PT = QT}$ [Corresponding parts of congruent triangles are equal]

Consider ΔTQR,

$QR = TR$ [From (3)]

$\Rightarrow \Delta TQR$ is a isosceles triangle

$\Rightarrow \angle QTR = \angle TQR$ [angles opposite to equal sides]

Now,

Sum of angles in a triangle is equal to $180°$

$\Rightarrow \angle QTR + \angle TQR + \angle TRQ = 180°$

$\Rightarrow 2\angle TQR + 150° = 180°$ [From (4)]

$\Rightarrow 2\angle TQR = 180° - 150°$

$\Rightarrow 2\angle TQR = 30° \Rightarrow \boxed{\angle TQR = 15°}$

$\therefore$ Hence proved

4. Prove that the medians of an equilateral triangle are equal.

Sol:

Given to prove that the medians of an equilateral triangle are equal

Median: The line joining the vertex and midpoint of opposite side.

Now, consider an equilateral triangle ABC

Let D,E,F are midpoints of BC, CA and AB.

Then, AD, BE and CF are medians of ΔABC.

Now,

D is midpoint of BC $\Rightarrow BD = DC = \dfrac{BC}{2}$

Similarly, $CE = EA = \dfrac{AC}{2}$

$AF = FB = \dfrac{AB}{2}$

Since ΔABC is an equilateral triangle $\Rightarrow AB = BC = CA$ …….(1)

$\Rightarrow BD = DC = CE = EA = AF = FB = \dfrac{BC}{2} = \dfrac{AC}{2} = \dfrac{AB}{2}$ …….(2)

And also, $\angle ABC = \angle BCA = \angle CAB = 60°$ …….(3)

Now, consider ΔABD and ΔBCE

$AB = BC$ [From (1)]

$BD = CE$ [From (2)]

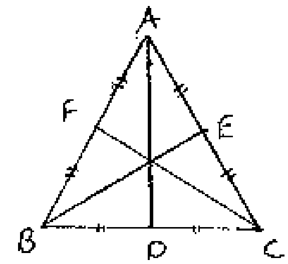
$\angle ABD = \angle BCE$ [From (3)] [$\angle ABD$ and $\angle ABC$ and $\angle BCE$ and $\angle BCA$ are same]

So, from SAS congruence criterion, we have

$\boxed{\Delta ABD \cong \Delta BCE}$

$\Rightarrow \boxed{AD = BE}$ ………(4)

[Corresponding parts of congruent triangles are equal]



Now, consider ΔBCE and ΔCAF,

BC = CA [From (1)]

$\angle BCE = \angle CAF$ [From (3)]

[$\angle BCE$ and $\angle BCA$ and $\angle CAF$ and $\angle CAB$ are same]

CE = AF [From (2)]

So, from SAS congruence criterion, we have $\boxed{\Delta BCE \cong \Delta CAF}$

$\Rightarrow \boxed{BE = CF}$ ……….(5)

[Corresponding parts of congruent triangles are equal]

From (4) and (5), we have

$AD = BE = CF$

$\Rightarrow$ Median $AD =$ Median $BE =$ Median CF

$\therefore$ The medians of an equilateral triangle are equal

$\therefore$ Hence proved

5. In a ΔABC, if ∠A=l20° and AB = AC. Find ∠B and ∠C.

Sol:

Consider a ΔABC

Given that $\angle A = 120°$ and $AB = AC$ and given to find $\angle B$ and $\angle C$

We can observe that ΔABC is an isosceles triangle since $AB = AC$

$\Rightarrow \boxed{\angle B = \angle C}$ ………(1) [Angles opposite to equal sides are equal]

We know that sum of angles in a triangle is equal to $180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

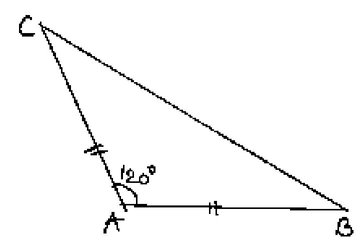
$\Rightarrow \angle A + \angle B + \angle B = 180°$ [From (1)]

$\Rightarrow 120° + 2\angle B = 180°$

$\Rightarrow 2\angle B = 180° - 120°$

$\Rightarrow 2\angle B = 60° \Rightarrow \boxed{\angle B = 30°}$

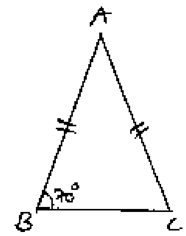
$\Rightarrow \angle C = \angle B = 30°$



6. In a ΔABC, if AB = AC and ∠B = 70°, find ∠A.

Sol:

Consider ΔABC, we have $\angle B = 70°$ and $AB = AC$



Since, $AB = AC$ ΔABC is an isosceles triangle

$\Rightarrow \angle B = \angle C$ [Angles opposite to equal sides are equal]

$\Rightarrow \angle B = \angle C = 70°$

And also,

Sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 70° + 70° = 180°$

$\Rightarrow \angle A + 140° = 180°$

$\Rightarrow \angle A = 180° - 140° \Rightarrow \boxed{\angle A = 40°}$

7. The vertical angle of an isosceles triangle is 100°. Find its base angles.

Sol:

Consider an isosceles ΔABC such that $AB = AC$

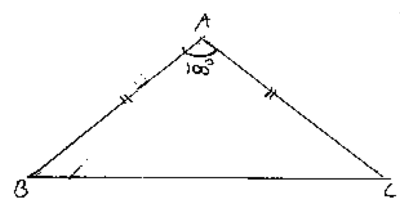
Given that vertical angle A is $100°$. Given to find the base angles

Since ΔABC is isosceles

$\angle B = \angle C$ [Angles opposite to equal angles are equal]

And also,

Sum of the interior angles of a triangle $= 180°$



$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 100° + \angle B + \angle B = 180°$

$\Rightarrow 2\angle B = 180° - 100° \Rightarrow 2\angle B = 80°$

$\Rightarrow \boxed{\angle B = 40°}$

$\therefore \angle B = \angle C = 40°$

8. In Fig. 10.24, AB = AC and ∠ACD =105°, find ∠BAC.

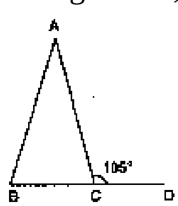


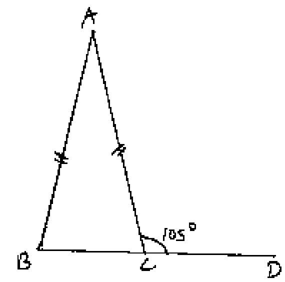
Fig. 10.24

Sol:

Consider the given figure

We have,

$AB = AC$ and $\angle ACD = 105°$



Since,

$\angle BCD = 180° =$ Straight angle

$\Rightarrow \angle BCA + \angle ACD = 180°$

$\Rightarrow \angle BCA + 105° = 180°$

$\Rightarrow \angle BCA = 180° - 105° \Rightarrow \boxed{\angle BCA = 75°}$ ......(1)

And also,

ΔABC is an isosceles triangle $[\because AB = AC]$

$\Rightarrow \angle ABC = \angle ACB$ [Angles opposite to equal sides are equal]

From (1), we have

$\angle ACB = 75° \Rightarrow \angle ABC = \angle ACB = 75°$

And also,

Sum of interior angles of a triangle $= 180°$

$\Rightarrow \angle ABC = \angle BCA + \angle CAB = 180°$

$\Rightarrow 75° + 75° + \angle CAB = 180°$

$\Rightarrow 150° + \angle BAC = 180° \Rightarrow \angle BAC = 180° - 150° = 30°$

$\therefore \boxed{\angle BAC = 30°}$

9. Find the measure of each exterior angle of an equilateral triangle.

Sol:

Given to find the measure of each exterior angle of an equilateral triangle consider an equilateral triangle ABC.

We know that for an equilateral triangle

$AB = BC = CA$ and $\angle ABC = \angle BCA = \angle CAB = \frac{180°}{3} = 60°$ ……(1)

Now,

Extend side BC to D, CA to E and AB to F.

Here

BCD is a straight line segment

$\Rightarrow \angle BCD =$ Straight angle $= 180°$

$\angle BCA + \angle ACD = 180°$

$\Rightarrow 60° + \angle ACD = 180°$ [*From* (1)]

$\Rightarrow \boxed{\angle ACD = 120°}$

Similarly, we can find $\angle FAB$ and $\angle FBC$ also as $120°$ because ABC is an equilateral triangle

$\therefore \angle ACD = \angle EAB = \angle FBC = 120°$

Hence, the median of each exterior angle of an equilateral triangle is $120°$

10. If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Sol:

ED is a straight line segment and B and C are points on it.

$\Rightarrow \angle EBC = \angle BCD =$ straight angle $=180°$

$\Rightarrow \angle EBA + \angle ABC = \angle ACB + \angle ACD$

$\Rightarrow \angle EBA = \angle ACD + \angle ACB - \angle ABC$

$\Rightarrow \angle EBA = \angle ACD$ $\quad\left[From\ (1)\ \angle ABC = \angle ACD\right]$

$\Rightarrow \boxed{\angle ABE = \angle ACD}$

$\therefore$ Hence proved

11. In Fig. 10.25, AB = AC and DB = DC, find the ratio ∠ABD : ∠ACD.

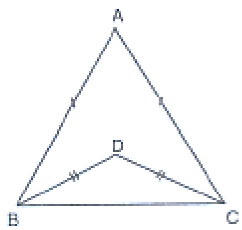
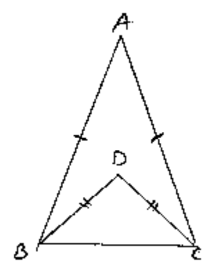


Fig. 10.25

Sol:

Consider the figure



Given

$AB = AC, DB = DC$ and given to find the ratio

$\angle ABD = \angle ACD$

Now, ΔABC and ΔDBC are isosceles triangles since $AB = AC$ and $DB = DC$ respectively

$\Rightarrow \angle ABC = \angle ACB$ and $\angle DBC = \angle DCB$ $\quad$ [$\because$ angles opposite to equal sides are equal]

Now consider,

$\angle ABD : \angle ACD$

$\Rightarrow (\angle ABC - \angle DBC):(\angle ACB - \angle DCB)$

$\Rightarrow (\angle ABC - \angle DBC):(\angle ABC - \angle DBC)$ $\quad\left[\because \angle ABC - \angle ACB\ and\ \angle DBC = \angle DCB\right]$

$\Rightarrow 1:1$

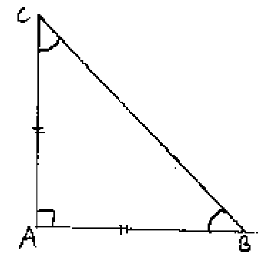
$\therefore \angle ABD : \angle ACD = 1:1$

12. Determine the measure of each of the equal angles of a right-angled isosceles triangle.

OR

ABC is a right-angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Sol:



Given to determine the measure of each of the equal angles of right – angled isosceles triangle

Consider on a right – angled isosceles triangle ABC such that

$\angle A = 90°$ and $AB = AC$

Since, $AB = AC \Rightarrow \angle C = \angle B$ ……….(1)

[Angles opposite to equal sides are equal]

Now,

Sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + \angle B + \angle B = 180°$ $[\because \angle A = 90° \text{ and } \angle B = \angle C]$

$\Rightarrow 2\angle B = 90°$

$\Rightarrow \boxed{\angle B = 45°} \Rightarrow \angle C = 45°$

$\therefore \angle B = \angle C = 45°$

Hence, the measure of each of the equal angles of a right-angled isosceles triangle is $45°$.

13. AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (See Fig. 10.26). Show that the line PQ is perpendicular bisector of AB.

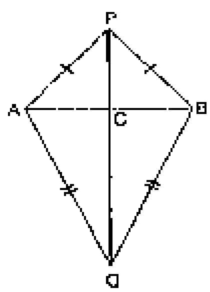


Fig. 10.26

Sol:

Consider the figure,

We have

AB is a line segment and P,Q are points on opposite sides of AB such that

$AP = BP$ ....(1)

$AQ = BQ$ ....(2)

We have to prove that PQ is perpendicular bisector of AB.

Now consider ΔPAQ and ΔPBQ,

We have $AP = BP$ [$\because$ From (1)]

$AQ = BQ$ [$\because$ From (2)]

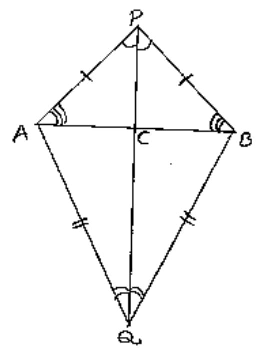
And $PQ = PQ$ [Common site]

$\Rightarrow \boxed{\Delta PAQ \cong \Delta PBQ}$ …….(3) [From SSS congruence]

Now, we can observe that ΔAPB and ΔABQ are isosceles triangles.(From 1 and 2)

$\Rightarrow \angle PAB = \angle PBA$ and $\angle QAB = \angle QBA$

Now consider ΔPAC and ΔPBC,



C is the point of intersection of AB and PQ.

$PA = PB$ [From (1)]

$\angle APC = \angle BPC$ [From (3)]

$PC = PC$ [Common side]

So, from SAS congruency of triangle $\Delta PAC \cong \Delta PBC$

$\Rightarrow AC = CB$ and $\angle PCA = \angle PCB$ …….(4)

[∵ Corresponding parts of congruent triangles are equal]

And also, ACB is line segment

$\Rightarrow \angle ACP + \angle BCP = 180°$

But $\angle ACP = \angle PCB$

$\Rightarrow \boxed{\angle ACP = \angle PCB = 90°}$ ………(5)

We have $AC = CB \Rightarrow C$ is the midpoint of AB

From (4) and (5)

We can conclude that PC is the perpendicular bisector of AB

Since C is a point on the line PQ, we can say that PQ is the perpendicular bisector of AB.

Exercise -10.2

1. In Fig. 10.40, it is given that RT = TS, ∠1 = 2∠2 and ∠4 = 2∠3. Prove that ΔRBT ≅ ΔSAT.

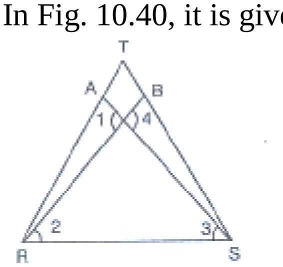
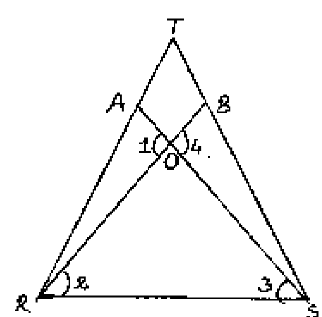


Fig. 10.40

Sol:

In the figure given that



$RT = TS$ ……..(1)

$\angle 1 = 2\angle 2$ ……..(2)

And $\angle 4 = 2\sqrt{3}$ ……..(3)

And given to prove $\Delta RBT \cong \Delta SAT$

Let the point of intersection of RB and SA be denoted by O

Since RB and SA intersect at O.

$\therefore \angle AOR = \angle BOS$ [Vertically opposite angles]

$\Rightarrow \angle 1 = \angle 4$

$\Rightarrow 2\angle 2 = 2\angle 3$ [From (2) and (3)]

$\Rightarrow \boxed{\angle 2 = \angle 3}$ ……..(4)

Now we have $RT = TS$ in ΔTRS

$\Rightarrow \Delta TRS$ is an isosceles triangle

$\therefore \boxed{\angle TRS = \angle TSR}$ ……..(5) [Angles opposite to equal sides are equal]

But we have

$\angle TRS = \angle TRB + \angle 2$ ………(6)

And $\angle TSR = \angle TSA + \angle 3$ ……….(7)

Putting (6) and (7) in (5) we get

$\angle TRB + \angle 2 = \angle TSA + \angle B$

$\Rightarrow \boxed{\angle TRB = \angle TSA}$ $[\because From\ (4)]$

Now consider ΔRBT and ΔSAT

$RT = ST$ [From (1)]

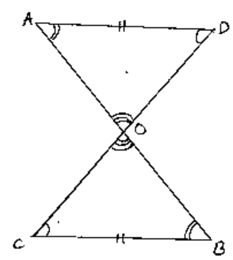
$\angle TRB = \angle TSA$ [From (4)]

$\angle RTB = \angle STA$ [Common angle]

From ASA criterion of congruence, we have $\boxed{\Delta RBT \cong \Delta SAT}$

2. Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Sol:



Given that lines AB and CD intersect at O

Such that $BC \| AD$ and $BC = AD$ ……..(1)

We have to prove that AB and CD bisect at O.

To prove this first we have to prove that $\Delta AOD \cong \Delta BOC$

3. BD and CE are bisectors of ∠B and ∠C of an isosceles ΔABC with AB = AC. Prove that BD = CE.

Sol:

Given that ΔABC is isosceles with $AB = AC$ and BD and CE are bisectors of $\angle B$ and $\angle C$

We have to prove $BD = CE$

Since $AB = AC \Rightarrow \angle ABC = \angle ACB$ …….(1)

[$\because$ Angles opposite to equal sides are equal]

Since BD and CE are bisectors of $\angle B$ and $\angle C$

$$\Rightarrow \angle ABD = \angle DBC = \angle BCE = ECA = \frac{\angle B}{2} = \frac{\angle C}{2} \quad …….(2)$$

Now,

Consider ΔEBC and ΔDCB

$\angle EBC = \angle DCB$ [$\because \angle B = \angle C$] from (1)

$BC = BC$ [Common side]

$\angle BCE = \angle CBD$ [$\because$ From (2)]

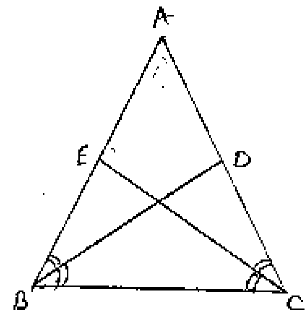
So, by ASA congruence criterion, we have $\boxed{\Delta EBC \cong \Delta DCB}$

Now,

$CE = BD$ [$\because$ Corresponding parts of congruent triangles are equal]

or $\boxed{BD = CE}$

$\therefore$ Hence proved



Since $AD \parallel BC$ and transversal AB cuts at A and B respectively

$\therefore \boxed{\angle DAO = \angle OBC}$ ....(2) [alternate angle]

And similarly $AD \parallel BC$ and transversal DC cuts at D ad C respectively

$\therefore \boxed{\angle ADO = \angle OCB}$ ....(3) [alternate angle]

Since AB and CD intersect at O.

$\therefore \angle AOD = \angle BOC$ [Vertically opposite angles]

Now consider ΔAOD and ΔBOD

$\angle DAO = \angle OBC$ [$\because$ From (2)]

$AD = BC$ [$\because$ From (1)]

And $\angle ADO = \angle OCB$ [From (3)]

So, by ASA congruence criterion, we have

$\boxed{\Delta AOD \cong \Delta BOC}$

Now,

$AO = OB$ and $DO = OC$ [$\because$ Corresponding parts of congruent triangles are equal]

$\Rightarrow$ Lines AB and CD bisect at O.

$\therefore$ Hence proved

Exercise -10.3

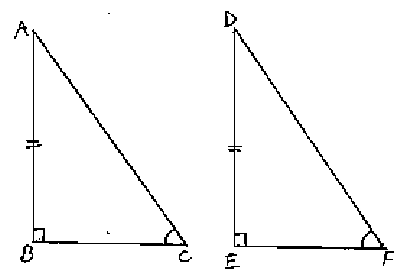
1. In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other.

We have to prove that the triangles are congruent.

Let us consider two right triangles such that



$\angle B = \angle C = 90°$ ....(1)

$AB = DE$ ....(2)

$\angle C = \angle F$ ....(3)

Now observe the two triangles ABC and DEF

$\angle C = \angle F$ [From (3)]

$\angle B = \angle E$ [From (1)]

and $AB = DE$ [From (2)]

So, by AAS congruence criterion, we have $\boxed{\Delta ABC \cong \Delta DEF}$

$\therefore$ The two triangles are congruent

Hence proved

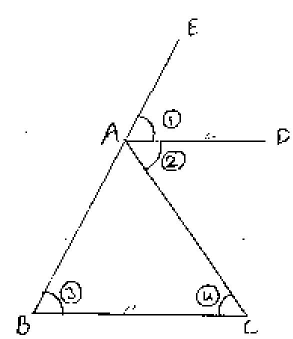
2. If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Sol:

Given that the bisector of the exterior vertical angle of a triangle is parallel to the base and we have to prove that the triangle is isosceles

Let ABC be a triangle such that AD is the angular bisector of exterior vertical angle EAC and $AD \parallel BC$

Let $\angle EAD = (1), \angle DAC = (2), \angle ABC = (3)$ and $\angle ACB = (4)$



We have,

(1) = (2) [$\because AD$ is bisector of $\angle EAC$]

(1) = (3) [Corresponding angles]

and (2) = (4) [alternative angle]

$\Rightarrow (3) = (4) \Rightarrow \boxed{AB = AC}$

Since, in $\Delta ABC,$ two sides AB and AC are equal we can say that ΔABC is isosceles

3. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Sol:

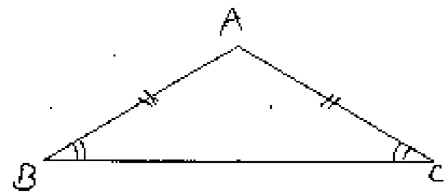
Let ΔABC be isosceles such that $AB = AC$.

$\Rightarrow \angle B = \angle C$

Given that vertex angle A is twice the sum of the base angles B and C.

i.e., $\angle A = 2(\angle B + \angle C)$

$\Rightarrow \angle A = 2(\angle B + \angle B)$ $[\because \angle B = \angle C]$



$\Rightarrow \angle A = 2(2\angle B)$

$\Rightarrow \boxed{\angle A = 4\angle B}$

Now,

We know that sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 4\angle B + \angle B + \angle B = 180°$ $[\because \angle A = 4\angle B \text{ and } \angle B = \angle C]$

$\Rightarrow 6\angle B = 180°$

$\Rightarrow \angle B = \frac{180°}{6} = 30°$ $\therefore \boxed{\angle B = 30°}$

Since, $\angle B = \angle C \Rightarrow \angle B = \angle C = 30°$

And $\angle A = 4\angle B \Rightarrow \angle A = 4 \times 30° = 120°$

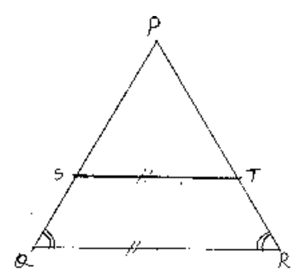
$\therefore$ Angles of the given triangle are $120°, 30°, 30°$.

4. PQR is a triangle in which PQ = PR and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

Sol:

Given that PQR is a triangle such that $PQ = PR$ and S is any point on the side PQ and $ST \parallel QR$.

We have to prove $PS = PT$



Since, $PQ = PR \Rightarrow \Delta PQR$ is isosceles

$\Rightarrow \angle Q = \angle R$ (or) $\angle PQR = \angle PRQ$

Now,

$\angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$ [Corresponding angles as $ST \parallel QR$]

Since, $\angle PQR = \angle PRQ \Rightarrow \boxed{\angle PST = \angle PTS}$

Now, In ΔPST, $\angle PST = \angle PTS$

$\Rightarrow \Delta PST$ is an isosceles triangle

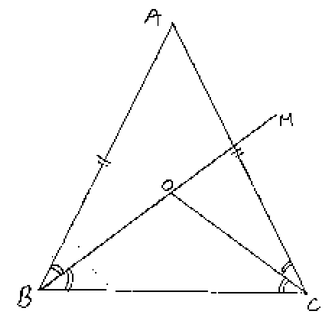
$\Rightarrow \boxed{PS = PT}$

5. In a ΔABC, it is given that AB = AC and the bisectors of ∠B and ∠C intersect at O. If M is a point on BO produced, prove that ∠MOC = ∠ABC.

Sol:

Given that in ΔABC,

$AB = AC$ and the bisector of $\angle B$ and $\angle C$ intersect at O and M is a point on BO produced



We have to prove $\angle MOC = \angle ABC$

Since,

$AB = AC \Rightarrow \Delta ABC$ is isosceles $\Rightarrow \angle B = \angle C$ (or) $\angle ABC = \angle ACB$

Now,

BO and CO are bisectors of $\angle ABC$ and $\angle ACB$ respectively

$\Rightarrow ABO = \angle OBC = \angle ACO = \angle OCB = \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$ ……….(1)

We have, in ΔOBC

$\angle OBC + \angle OCB + \angle BOC = 180°$ ……….(2)

And also

$\angle BOC + \angle COM = 180°$ ……….(3) [Straight angle]

Equating (2) and (3)

$\Rightarrow \angle OBC + \angle OCB + \angle \cancel{BOC} = \angle \cancel{BOC} + \angle MOC$

$\Rightarrow \angle OBC + \angle OBC = \angle MOC$ [∵ From (1)]

$\Rightarrow 2\angle OBC = \angle MOC$

$\Rightarrow 2\left(\frac{1}{2}\angle ABC\right) = \angle MOC$ [∵ *From* (1)]

$\Rightarrow \angle ABC = \angle MOC$

$\therefore \boxed{\angle MOC = \angle ABC}$

6. P is a point on the bisector of an angle ∠ABC. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.

Sol:

Given that P is a point on the bisector of an angle $\angle ABC$, and $PQ \parallel AB$.

We have to prove that ΔBPQ is isosceles

Since,

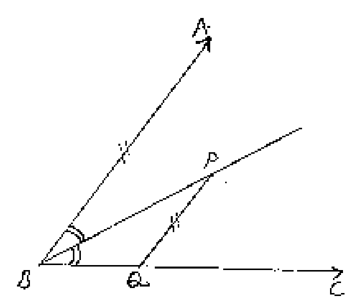
BP is bisector of $\angle ABC \Rightarrow \angle ABP = \angle PBC$ ………(1)

Now,

$PQ \parallel AB$

$\Rightarrow \angle BPQ = \angle ABP$ ………(2)

[alternative angles]



From (1) and (2), we get

$\boxed{\angle BPQ = \angle PBC \ (or)\ \angle BPQ = \angle PBQ}$

Now,

In ΔBPQ,

$\angle BPQ = \angle PBQ$

$\Rightarrow \Delta BPQ$ is an isosceles triangle.

$\therefore$ Hence proved

7. Prove that each angle of an equilateral triangle is 60°.

Sol:

Given to prove that each angle of an equilateral triangle is $60°$

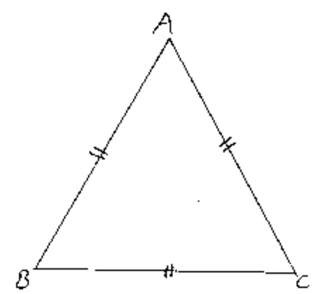
Let us consider an equilateral triangle ABC

Such that $AB = BC = CA$

Now,

$AB = BC \Rightarrow \angle A = \angle C$ ……..(1) [Opposite angles to equal sides are equal]

and $BC = AC \Rightarrow \angle B = \angle A$ ……..(2)



From (1) and (2), we get

$\boxed{\angle A = \angle B = \angle C}$ ……..(3)

We know that

Sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle A + \angle A = 180°$ $\left[\because From(3)\right]$

$\Rightarrow 3\angle A = 180°$

$\Rightarrow \angle A = \dfrac{180°}{3} = 60°$

$\therefore \boxed{\angle A = \angle B = \angle C = 60°}$

Hence, each angle of an equilateral triangle is $60°$.

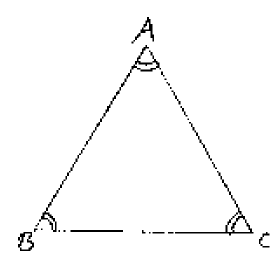
8. Angles A, B, C of a triangle ABC are equal to each other. Prove that ΔABC is equilateral.

Sol:

Given that angles A, B, C of a triangle ABC equal to each other.

We have to prove that ΔABC is equilateral

We have, $\angle A = \angle B = \angle C$



Now,

$\angle A = \angle B \Rightarrow BC = AC$

[Opposite sides to equal angles are equal]

and $\angle B = \angle C \Rightarrow AC = AB$

From the above we get

$\boxed{AB = BC = AC}$

$\Rightarrow \Delta ABC$ is equilateral

9. ABC is a triangle in which ∠B = 2 ∠C . D is a point on BC such that AD bisects ∠BAC and AB = CD. Prove that ∠BAC = 72°.

Sol:

Given that in $\Delta ABC, \angle B = 2\angle C$ and D is a point on BC such that AD bisectors $\angle BAC$ and $AB = CD$.

We have to prove that $\angle BAC = 72°$

Now, draw the angular bisector of $\angle ABC$, which meets AC in P. join PD

Let $C = \angle ACB = y \Rightarrow \angle B = \angle ABC = 2\angle C = 2y$ and also

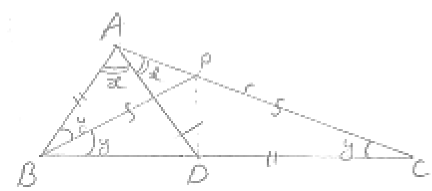
Let $\angle BAD = \angle DAC \Rightarrow \angle BAC = 2x$ [$\because AD$ is the bisector of $\angle BAC$]

Now, in ΔBPC,

$\angle CBP = y$ [$\because BP$ is the bisector of $\angle ABC$]

$\angle PCB = y$

$\Rightarrow \angle CBP = \angle PCB = y$ $\therefore \boxed{PC = BP}$



Consider, ΔABP and ΔDCP, we have

$\angle ABP = \angle DCP = y$

$AB = DC$ [Given]

And $PC = BP$ [From above]

So, by SAS congruence criterion, we have $\boxed{\Delta ABP \cong \Delta DCP}$

Now,

$\angle BAP = \angle CDF$ and $AP = DP$ [Corresponding parts of congruent triangles are equal]

$\Rightarrow \angle BAP = \angle CDP = 2x$

Consider, ΔAPD,

We have $AP = DP \Rightarrow \angle ADP = \angle DAP$

But $\angle DAP = x \Rightarrow \angle ADP = \angle DAP = x$

Now

In ΔABD,

$\angle ABD + \angle BAD + \angle ADB = 180°$

And also $\angle ADB + \angle ADC = 180°$ [Straight angle]

From the above two equations, we get

$\angle ABD + \angle BAD + \angle ADB = \angle ADB + \angle ADC$

$\Rightarrow 2y + x = \angle ADP + \angle PDC$

$\Rightarrow 2y + x = x + 2x$

$\Rightarrow 2y = 2x$

$\Rightarrow y = x \ (or) \ \boxed{x = y}$

We know,

Sum of angles in a triangle $= 180°$

So, in ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 2x + 2y + y = 180°$ $\quad [\because \angle A = 2x, \angle B = 2y, \angle C = y]$

$\Rightarrow 2(y) + 3y = 180°$ $\quad [\because x = y]$

$\Rightarrow 5y = 180°$

$\Rightarrow y = \frac{180°}{5} = 36°$ $\quad \therefore \boxed{x = y = 36°}$

Now,

$\angle A = \angle BAC = 2x = 2 \times 36° = 72°$

$\therefore \boxed{\angle BAC = 72°}$

10. ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Sol:

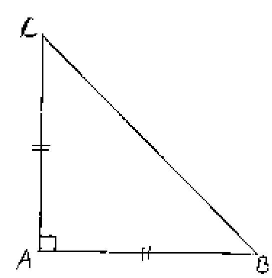
Given that ABC is a right angled triangle such that $\angle A = 90°$ and $AB = AC$

Since,

$AB = AC \Rightarrow \Delta ABC$ is also isosceles

$\therefore$ We can say that ΔABC is right angled isosceles triangle

$\Rightarrow \angle C = \angle B$ and $\angle A = 90°$ ……….(1)



Now, we have

Sum of angled in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + \angle B + \angle B = 180°$ $\quad [\because From (1)]$

$\Rightarrow 2\angle B = 180° - 90°$

$\Rightarrow \angle B = \frac{90°}{2} = 45°$

$\therefore \boxed{\angle B = \angle C = 45°}$

Exercise -10.4

1. In Fig. 10.92, it is given that AB = CD and AD = BC. Prove that ΔADC ≅ ΔCBA.

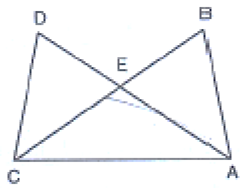


Fig. 10.92

Sol:

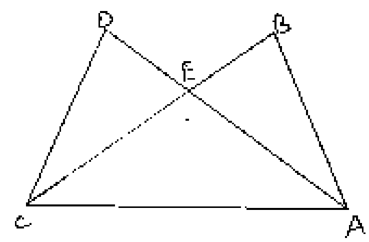
Given that in the figure $AB = CD$ and $AD = BC$.

We have to prove

$\Delta ADC \cong \Delta CBA$

Now,

Consider ΔADC and ΔCBA,



We have

$AB = CD$ [Given]

$BC = AD$ [Given]

And $AC = AC$ [Common side]

So, by SSS congruence criterion, we have

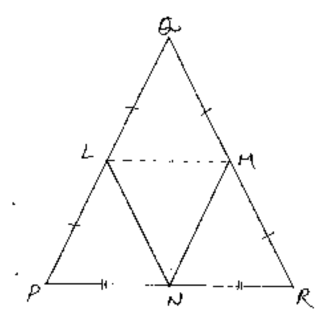
$\boxed{\Delta ADC \cong \Delta CBA}$

$\therefore$ Hence proved

2. In a ΔPQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Sol:

Given that in $\Delta PQR, PQ = QR$ and L, M and N are mid-points of PQ, QR and RP respectively



We have to prove $LN = MN$.

Join L and M, M and N, N and L

We have

$PL = LQ, QM = MR$ and $RN = NP$

[$\because L, M$ and N are mid-points of PQ, QR and RP respectively]

And also

$$PQ = QR \Rightarrow PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2} \qquad ....(1)$$

Using mid-point theorem, we have

$$MN \parallel PQ \text{ and } MN = \frac{1}{2}PQ \Rightarrow \boxed{MN = PL = LQ} \qquad ....(2)$$

Similarly, we have

$LN \parallel QR$ and $LN = \frac{1}{2}QR \Rightarrow \boxed{LN = QM = MR}$ ....(3)

From equation (1), (2) and (3), we have

$PL = LQ = QM = MR = MN = LN$

$\therefore \boxed{LN = MN}$

Exercise -10.5

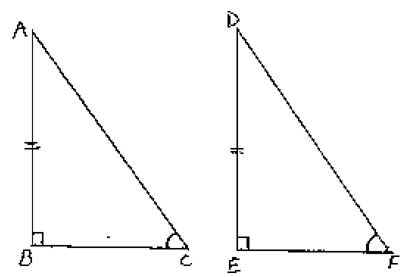
1. ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angle of the other

We have to prove that the triangles are congruent

Let us consider two right triangles such that



$\angle B = \angle E = 90°$ ....(1)

$AB = DE$ ....(2)

$\angle C = \angle F$ ....(3)

Now observe the two triangles ABC and DEF

$\angle C = \angle F$ $\left[From\ (3)\right]$

$\angle B = \angle E$ $\left[From\ (1)\right]$

$and\ AB = DE$ $\left[From\ (2)\right]$

So, by AAS congruence criterion, we have

$\boxed{\Delta ABC \cong \Delta DEF}$

$\therefore$ The two triangles are congruent

Hence proved

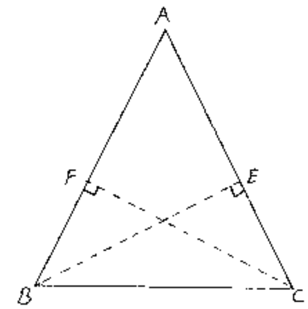
2. ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that ΔABC is isosceles.

Sol:

Given that ABC is a triangle in which BE and CF are perpendicular to the sides AC and AB respectively such that $BE = CF$.

We have to prove that ΔABC is isosceles

Now, consider ΔBCF and ΔCBE,



We have

$\angle BFC = CEB = 90°$ [Given]

$BC = CB$ [Common side]

And $CF = BE$ [Given]

So, by RHS congruence criterion, we have $\boxed{\Delta BFC \cong CEB}$

Now,

$\angle FBC = \angle EBC$ [$\because$ Incongruent triangles corresponding parts are equal]

$\Rightarrow \angle ABC = \angle ACB$

$\Rightarrow AC = AB$ [$\because$ Opposite sides to equal angles are equal in a triangle]

$\therefore \Delta ABC$ is isosceles

3. If perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

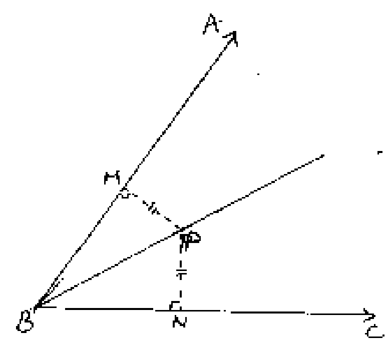
Sol:

Given that, if perpendicular from any point within, an angle on its arms is congruent, prove that it lies on the bisector of that angle

Now,

Let us consider an angle ABC and let BP be one of the arm within the angle

Draw perpendicular PN and PM on the arms BC and BA such that they meet BC and BA in N and M respectively.



Now, in ΔBPM and ΔBPN

We have $\angle BMP = BNP = 90°$ [given]

$BP = BP$ [Common side]

And $MP = NP$ [given]

So, by RHS congruence criterion, we have

$\boxed{\Delta BPM \cong \Delta BPN}$

Now,

$\angle MBP = \angle NBP$ [$\because$ Corresponding parts of congruent triangles are equal]

$\Rightarrow BP$ is the angular bisector of $\angle ABC$.

$\therefore$ Hence proved

4. In Fig. 10.99, AD ⊥ CD and CB ⊥. CD. If AQ = BP and DP = CQ, prove that ∠DAQ = ∠CBP.

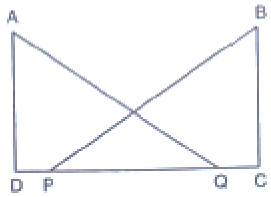


Fig. 10.99

Sol:

Given that, in the figure $AD \perp CD$ and $CB \perp CD$ and $AQ = BP, DP = CQ$

We have to prove that $\angle DAQ = \angle CBP$

Given that $DP = QC$

Add PQ on both sides

Given that $DP = QC$

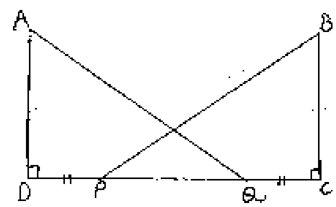
Add PQ on both sides

$\Rightarrow DP + PQ = PQ + QC$

$\Rightarrow \boxed{DQ = PC}$ ....(1)

Now, consider triangle DAQ and CBP,

We have



$\angle ADQ = \angle BCP = 90°$ [given]

$AQ = BP$ [given]

And $DQ = PC$ [From (1)]

So, by RHS congruence criterion, we have $\boxed{\Delta DAQ \cong \Delta CBP}$

Now,

$\boxed{\angle DAQ = \angle CBP}$ [∵ Corresponding parts of congruent triangles are equal]

∴ Hence proved

5. ABCD is a square, X and Yare points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and ∠BAY = ∠4BX.

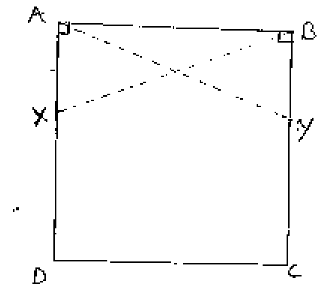
Sol:

Given that ABCD is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$.

We have to prove $BY = AX$ and $\angle BAY = \angle ABX$

Join B and X, A and Y.

Since, ABCD is a square $\Rightarrow \angle DAB = \angle CBA = 90°$



$\Rightarrow \boxed{\angle XAB = \angle YBA = 90°}$ ....(1)

Now, consider triangle XAB and YBA

We have

$\angle XAB = \angle YBA = 90°$ [From (1)]

$BX = AY$ [given]

And $AB = BA$ [Common side]

So, by RHS congruence criterion, we have $\boxed{\Delta XAB \cong \Delta YBA}$

Now, we know that corresponding parts of congruent triangles are equal.

$\therefore \boxed{BY = AX \text{ and } \angle BAY = \angle ABX}$

$\therefore$ Hence proved

6. Which of the following statements are true (T) and which are false (F):

(i) Sides opposite to equal angles of a triangle may be unequal.

(ii) Angles opposite to equal sides of a triangle are equal

(iii) The measure of each angle of an equilateral triangle is 60°

(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.

(v) The bisectors of two equal angles of a triangle are equal.

(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.

(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.

(viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.

(ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Sol:

(i) False (F)

Reason: Sides opposite to equal angles of a triangle are equal

(ii) True (F)

Reason: Since the sides are equal, the corresponding opposite angles must be equal

(iii) True (T)

Reason: Since all the three angles of an equilateral triangles are equal and sum of the three angles is 180°, each angle will be equal to $\frac{180°}{3} \Rightarrow 60°$

(iv) False (F)

Reason: Here the altitude from thee vertex is also the perpendicular bisector of the opposite side.

$\Rightarrow$ The triangle must be isosceles and may be an equilateral triangle.

(v) True (T)

Reason: Since it an isosceles triangle, the lengths of bisectors of the two equal angles are equal

(vi) False (F)
Reason: The angular bisector of the vertex angle is also a median
$\Rightarrow$ The triangle must be an isosceles and also may be an equilateral triangle.

(vii) False (F)
Reason: Since two sides are equal, the triangle is an isosceles triangle.
$\Rightarrow$ The two altitudes corresponding to two equal sides must be equal.

(viii) False (F)
Reason: The two right triangles may or may not be congruent

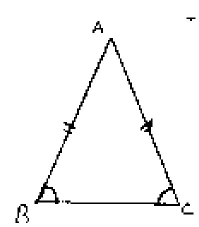
(ix) True (T)
Reason: According to RHS congruence criterion the given statement is true.

7. Fill the blanks in the following so that each of the following statements is true.

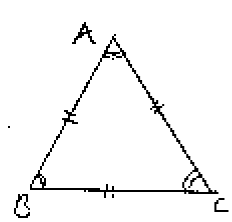
(i) Sides opposite to equal angles of a triangle are ......
(ii) Angle opposite to equal sides of a triangle are .....
(iii) In an equilateral triangle all angles are .....
(iv) In a ΔABC if $\angle A = \angle C$, then AB = ...
(v) If altitudes CE and BF of a triangle ABC are equal, then AB = ...
(vi) In an isosceles triangle ABC with AB = AC, if BD and CE are its altitudes, then BD is …… CE.
(vii) In right triangles ABC and DEF, if hypotenuse AB = EF and side AC = DE, then $\Delta ABC \cong \Delta$ ……

Sol:

(i) Sides opposite to equal angles of a triangle are equal
(ii) Angles opposite to equal sides of a triangle are equal

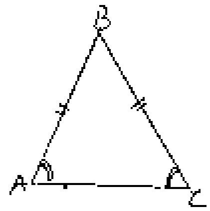


(iii) In an equilateral triangle all angles are equal
Reason: Since all sides are equal in a equilateral triangle, the angles opposite to equal sides will be equal

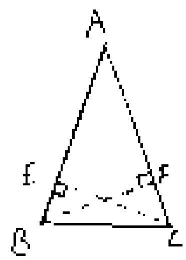


(iv) In a ΔABC if $\angle A = \angle C$, then $AB = BC$

Reason: Since, the sides opposite to equal angles are equal, the side opposite to $\angle A$ i.e., BC and $\angle C$ i.e., AB are equal



(v) If altitudes CE and BF of a triangle ABC are equal, then $AB = AC$

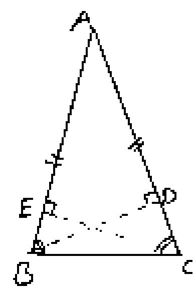


Reason: From RHS congruence criterion $\Delta BEC \cong \Delta CFB$

$\Rightarrow \angle EBC = \angle FCB \Rightarrow \angle ABC = \angle ACB \Rightarrow AC = AB$

[$\because$ Sides opposite to equal angels are equal]

(vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is equal to CE



Reason: Since angles opposite to equal sides are equal, so

$\angle ABC = \angle ACB$

$\Rightarrow \angle EBC = \angle DCB$

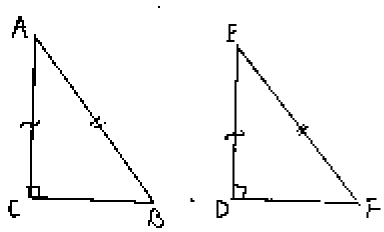
So, by ASA congruence criterion

$\Delta EBC \cong \Delta DCB$

$\Rightarrow CE = BD$ [Corresponding parts of congruent triangles are equal]

(vii) In right triangles ABC and DEF, if hypotenuse $AB = EF$ and side $AC = DE$, then. $\Delta ABC \cong \Delta EFD$

Reason: From RHS congruence criterion we have $\Delta ABC \cong \Delta EFD$



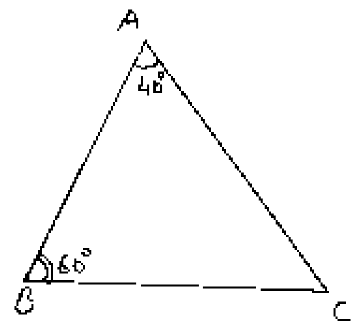
Exercise -10.6

1. In ΔABC, if ∠A = 40° and ∠B = 60°. Determine the longest and shortest sides of the triangle.

Sol:

Given that in ΔABC, $\angle A = 40°$ and $\angle B = 60°$

We have to find longest and shortest side



We know that,

Sum of angles in a triangle $180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow 40° + 60° + \angle C = 180°$

$\Rightarrow \angle C = 180° - (40° + 60°)$

$= 180° - 100° = 80°$

$\therefore \boxed{\angle C = 80°}$

Now,

$\Rightarrow 40° < 60° < 80° \Rightarrow \angle A < \angle B < \angle C$

$\Rightarrow \angle C$ is greater angle and $\angle A$ is smaller angle

Now, $\angle A < \angle B < \angle C$

$\Rightarrow BC < AAC < AB$

[$\because$ Side opposite to greater angle is larger and side opposite to smaller angle is smaller]

$\therefore AB$ is longest and BC is smallest or shortest side.

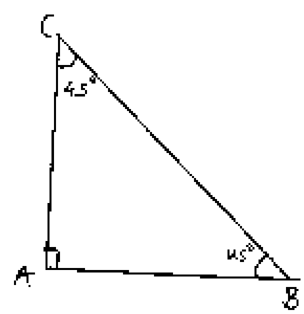
2. In a ΔABC, if ∠B = ∠C = 45°, which is the longest side?

Sol:

Given that in ΔABC,

$\angle B = \angle C = 45°$

We have to find longest side



We know that,

Sum of angles in a triangle $= 180°$

$\Rightarrow \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 45° + 45° = 180°$

$\Rightarrow \angle A = 180° - (45° + 45°) = 180° - 90° = 90°$

$\therefore \boxed{\angle A = 90°}$

3. In ΔABC, side AB is produced to D so that BD = BC. If ∠B = 60° and ∠A = 70°, prove that: (i) AD > CD (ii) AD > AC

Sol:

Given that in ΔABC, side AB is produced to D So that $BD = BC$ and $\angle B = 60°, \angle A = 70°$

We have to prove that

(i) $AD > CD$ (ii) $AD > AC$

First join C and D

Now, in ΔABC

$\angle A + \angle B + \angle C = 180^\circ$ [$\because$ Sum of angles in a triangle $= 180^\circ$]

$\Rightarrow \angle C = 180^\circ - 70^\circ - 60^\circ$

$= 180^\circ - 130^\circ = 50^\circ$

$\therefore \boxed{\angle C = 50^\circ} \Rightarrow \angle ACB = 50^\circ$ ....(1)

And also in ΔBDC,

$\angle DBC = 180^\circ - \angle ABC$ [$\because ABD$ is a straight angle]

$= 180^\circ - 60^\circ = 120^\circ$

and also $BD = BC$ [given]

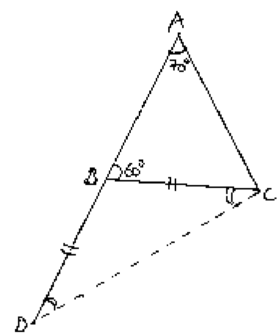
$\Rightarrow \angle BCD = \angle BDC$ [$\because$ Angles opposite to equal sides are equal]

Now,

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$ [$\because$ Sum of angles in a triangle $= 180^\circ$]

$\Rightarrow 120^\circ + \angle BCD + \angle BCD = 180^\circ$

$\Rightarrow 2\angle BCD = 180^\circ - 120^\circ \Rightarrow \angle BCD = \dfrac{60^\circ}{2} = 30^\circ$



$\therefore \boxed{\angle BCD = \angle BDC = 30^\circ}$ ....(2)

Now, consider ΔADC,

$\angle BAC \Rightarrow \angle DAC = 70^\circ$ [given]

$\angle BDC \Rightarrow \angle ADC = 30^\circ$ [$\because$ From (2)]

$\angle ACD = \angle ACB + \angle BCD$

$= 50^\circ + 30^\circ$ [$\because$ From (1) and (2)]

$= 80^\circ$

Now, $\angle ADC < \angle DAC < \angle ACD$

$\Rightarrow \boxed{AC < DC < AD}$

[$\because$ Side opposite to greater angle is longer and smaller angle is smaller]

$\Rightarrow AD > CD$ and $AD > AC$

$\therefore$ Hence proved

Or

We have, $\angle ACD > \angle DAC$ and $\angle ACD > \angle ADC$

$\Rightarrow \boxed{AD > DC}$ and $\boxed{AD > AC}$

[$\because$ Side opposite to greater angle is longer and smaller angle is smaller]

4. Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

Sol:

Given lengths of sides are 2cm, 3cm and 7cm we have to check whether it is possible to draw a triangle with ten the given lengths of sides

We know that,

A triangle can be drawn only when the sum of any two sides is greater than the third side.

So, let's check the rule.

$2 + 3 \ngtr 7 \; or \; 2 + 3 < 7$

$2 + 7 > 3$

and $3 + 7 > 2$

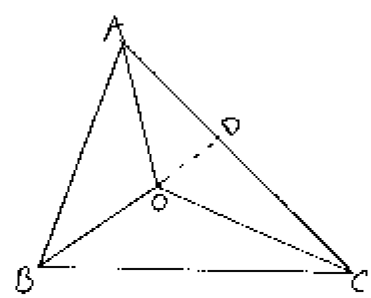
Here, $2 + 3 \ngtr 7$ So, the triangle does not exit.

5. O is any point in the interior of ΔABC. Prove that

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > $\frac{1}{2}$ (AB + BC + CA)



Sol:

Given that O is any point in the interior of ΔABC

We have to prove

(i) AB + AC > OB + OC

(ii) AB + BC + CA > OA + QB + OC

(iii) OA + OB + OC > $\frac{1}{2}$ (AB + BC + CA)

We know that, in a triangle the sum of any two sides is greater than the third side

So, we have

In ΔABC

$AB + BC > AC$

$BC + AC > AB$

$AC + AB > BC$

In ΔOBC

$OB + OC > BC$ ....(1)

In ΔOAC

$OA + OC > AC$ ....(2)

In ΔOAB

$OA + OB > AB$ ....(3)

Now, extend (or) produce BO to meet AC in D.

Now, in ΔABD, we have

$AB + AD > BD$

$\Rightarrow AB + AD > BO + OD$ ....(4) $[\because BD = BO + OD]$

Similarly in ΔODC, we have

$OD+DC>OC$ ....(5)

(i) Adding (4) and (5), we get

$AB+AD+OD+DC>BO+OD+OC$

$\Rightarrow AB+(AD+DC)>OB+OC$

$\Rightarrow \boxed{AB+AC>OB+OC}$ ....(6)

Similarly, we have

$BC+BA>OA+OC$ ....(7)

$and\ CA+CB>OA+OB$ ....(8)

(ii) Adding equation (6), (7) and (8), we get

$AB+AC+BC+BA+CA+CB>OB+OC+OA+OC+OA+OB$

$\Rightarrow 2AB+2BC+2CA>2OA+2OB+2OC$

$\Rightarrow 2(AB+BC+CA)>2(OA+OB+OC)$

$\Rightarrow \boxed{AB+BC+CA>OA+OB+OC}$

(iii) Adding equations (1), (2) and (3)

$OB+OC+OA+OC+OA+OB>BC+AC+AB$

$\Rightarrow 2OA+2OB+2OC>AB+BC+CA$

We get $\Rightarrow 2(OA+OB+OC)>AB+BC+CA$

$\therefore \boxed{(OA+OB+OC)>\frac{1}{2}(AB+BC+CA)}$

6. Prove that the perimeter of a triangle is greater than the sum of its altitudes.

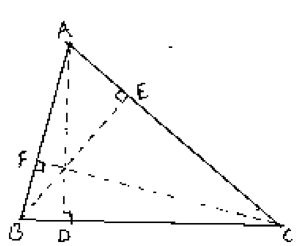
Sol:

Given: A ΔABC in which $AD\perp BC, BE\perp AC$ and $CF\perp AB.$

To prove:

$AD+BE+CF<AB+BC+AC$

Figure:



Proof:

We know that of all the segments that can be drawn to a given line, from a point not lying on it, the perpendicular distance i.e., the perpendicular line segment is the shortest.

Therefore,

$AD \perp BC$

$\Rightarrow AB > AD \;\; and \;\; AC > AD$

$\Rightarrow \boxed{AB + AC > 2AD}$ ....(1)

Similarly $BE \perp AC$

$\Rightarrow BA > BE$ and $BC > BE$

$\Rightarrow \boxed{BA + BC > 2BE}$ ......(2)

And also $CF \perp AB$

$\Rightarrow CA > CF$ and $CB > CF$

$\Rightarrow \boxed{CA + CB > 2CF}$ ......(3)

Adding (1), (2) and (3), we get

$AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF$

$\Rightarrow 2AB + 2BC + 2CA > 2(AD + BE + CF)$

$\Rightarrow 2(AB + BC + CA) > 2(AD + BE + CF)$

$\Rightarrow \boxed{AB + BC + CA > AD + BE + CF}$

$\Rightarrow$ The perimeter of the triangle is greater than the sum of its altitudes

$\therefore$ Hence proved

7. In Fig. 10.131, prove that: (i) CD + DA + AB + BC > 2AC (ii) CD + DA + AB > BC

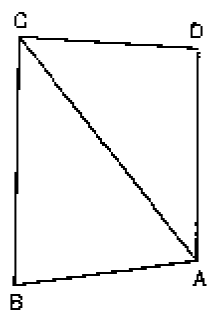


Fig. 10.131

Sol:

Given to prove

(i) $CD + DA + AB + BC > 2AC$

(ii) $CD + DA + AB > BC$

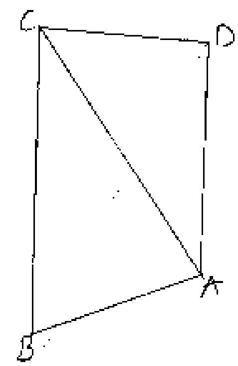
From the given figure,

We know that, in a triangle sum of any two sides is greater than the third side

(i) So,

In ΔABC, we have

$AB + BC > AC$ ....(1)



In ΔADC, we have

$CD + DA > AC$ ....(2)

Adding (1) and (2) we get

$AB + BC + CD + DA > AC + AC$

$\Rightarrow \boxed{CD + DA + AB + BC > 2AC}$

(ii) Now, in ΔABC, we have

$\boxed{AB + AC > BC}$ ....(3)

and in ΔADC, we have

$CD + DA > AC$

Add AB on both sides

$\Rightarrow \boxed{CD + DA + AB > AC + AB}$ ....(4)

From equation (3) and (4), we get

$CD + DA + AB > AC + AB > BC$

$\Rightarrow \boxed{CD + DA + AB > BC}$

$\therefore$ Hence proved

8. Which of the following statements are true (T) and which are false (F)?

(i) Sum of the three sides of a triangle is less than the sum of its three altitudes.

(ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.

(iii) Sum of any two sides of a triangle is greater than the third side.

(iv) Difference of any two sides of a triangle is equal to the third side.

(v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.

(vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.

Sol:

(i) False (F)

Reason: Sum of these sides of a triangle is greater than sum of its three altitudes

(ii) True (F)

(iii) True (T)

(iv) False (F)

Reason: The difference of any two sides of a triangle is less than third side.

(v) True (T)

Reason: The side opposite to greater angle is longer and smaller angle is shorter in a triangle

(vi) True (T)

Reason: The perpendicular distance is the shortest distance from a point to a line not containing it.

9. Fill in the blanks to make the following statements true.

(i) In a right triangle the hypotenuse is the .... side.

(ii) The sum of three altitudes of a triangle is ..... than its perimeter.

(iii) The sum of any two sides of a triangle is .... than the third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the side opposite to it.

(v) Difference of any two sides of a triangle is than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has .... angle opposite to it.

Sol:

(i) In a right triangle the hypotenuse is the largest side

Reason: Since a triangle can have only one right angle, other two angles must be less than $90°$

$\Rightarrow$ The right angle is the largest angle

$\Rightarrow$ Hypotenuse is the largest side.

(ii) The sum of three altitudes of a triangle is less than its perimeter

(iii) The sum of any two sides of a triangle is greater than the third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the smaller side opposite to it.

(v) Difference of any two sides of a triangle is less than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has greater angle opposite to it.