Exercise – 16.1

1. Fill in the blanks:

(i) All points lying inside/outside a circle are called .... points/ .... points.

(ii) Circles having the same centre and different radii are called .... circles.

(iii) A point whose distance from the centre of a circle is greater than its radius lies in .... of the circle.

(iv) A continuous piece of a circle is .... of the circle.

(v) The longest chord of a circle is a . . . . of the circle.

(vi) An arc is a . . . . when its ends are the ends of a diameter.

(vii) Segment of a circle is the region between an arc and . . . . of the circle.

(viii) A circle divides the plane, on which it lies, in . .. parts.

Sol:

(i) Interior/exterior

(ii) Concentric

(iii) The exterior

(iv) Arc

(v) Diameter

(vi) Semi-circle

(vii) Centre

(viii) Three

2. Write the truth value (T/F) of the following with suitable reasons:

(i) A circle is a plane figure.

(ii) Line segment joining the centre to any point on the circle is a radius of the circle,

(iii) If a circle is divided into three equal arcs each is a major arc.

(iv) A circle has only finite number of equal chords.

(v) A chord of a circle, which is twice as long is its radius is a diameter of the circle.

(vi) Sector is the region between the chord and its corresponding arc.

(vii)The degree measure of an arc is the complement of the central angle containing the arc.

(viii) The degree measure of a semi-circle is 180°.

Sol:

(i) True

(ii) True

(iii) True

(iv) False

(v) True

(vi) True

(vii) False

(viii) True

Exercise – 16.2

1. The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Sol:

Given that

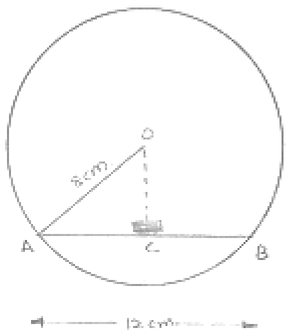
Radius of circles $(OA) = 8cm$

Chord $(AB) = 12cm$

Draw $OC \perp AB.$

WKT,

The perpendicular from center to chord bisects the chord



$\therefore AC = BC = \frac{12}{2} = 6cm$

Now in $\Delta OCA,$ by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow 6^2 + OC^2 = 8^2$

$\Rightarrow 36 + OC^2 = 64$

$\Rightarrow OC^2 = 64 - 36$

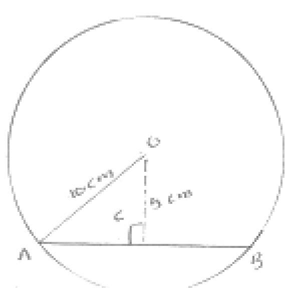
$\Rightarrow OC^2 = 28$

$\Rightarrow OC = \sqrt{28}$

$\Rightarrow OC = 5 \cdot 291\ cm$

2. Find the length of a chord which is at a distance of 5 cm from the centre of a circle ofradius 10 cm.

Sol:



Given that

Distance $(OC) = 5cm$

Radius of circle $(OA) = 10cm$

In ΔOCA by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow AC^2 + 5^2 = 10^2$

$\Rightarrow AC^2 = 100 - 25$

$\Rightarrow AC = \sqrt{75} = 8 \cdot 66cm$

WRK, the perpendicular from center to chord bisects the chord

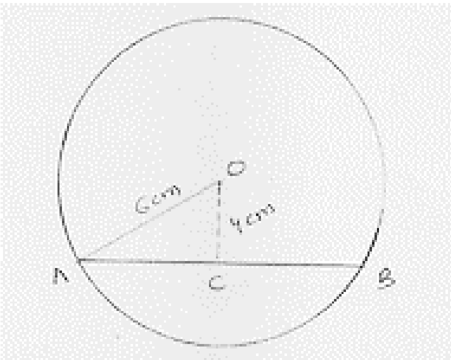
$\therefore AC = BC = 8 \cdot 66cm$

Then chord $AB = 8 \cdot 66 + 8 \cdot 66$

$= 17 \cdot 32cm$

3. Find the length of a chord which is at a distance of 4 cm from the centre of the circle of radius 6 cm.

Sol:



Radius of circle $(OA) = 6cm$

Distant $(OC) = 4cm$

In ΔOCA by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow AC^2 + 4^2 = 6^2$

$\Rightarrow AC^2 = 36 - 16$

$\Rightarrow AC = \sqrt{20} = 4 \cdot 47cm.$

WKT, the perpendicular distance from center to chord bisects the chord.

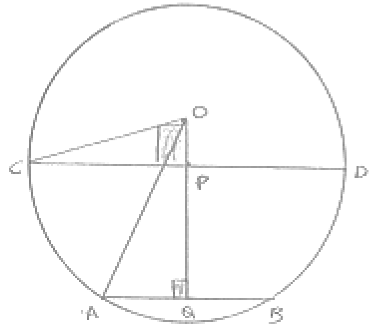
$AC = BC = 4 \cdot 47cm$

Then, $AB = 4 \cdot 47 + 4 \cdot 47$

$= 8 \cdot 94cm.$

4. Two chords AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

Sol:



Construction: Draw $OP \perp CD$

Chord $AB = 5cm$

Chord $CD = 11cm$

Distance $PQ = 3cm$

Let $OP = x\,cm$

And $OC = OA = rcm$

WKT perpendicular from center to chord bisects it

$\therefore CP = PD = \frac{11}{2}cm$

And $AQ = BQ = \frac{5}{2}cm$

In ΔOCP, by Pythagoras theorem

$OC^2 = OP^2 + CP^2$

$\Rightarrow r^2 = x^2 + \left(\frac{11}{2}\right)^2 \qquad ....(1)$

In ΔOQA, by Pythagoras theorem

$OA^2 = OQ^2 + AQ^2$

$\Rightarrow r^2 = (x+3)^2 + \left(\frac{5}{2}\right)^2 \qquad .....(2)$

Compare equation (1) and (2)

$(x+3)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2$

$\Rightarrow x^2 + 9 + 6x + \frac{25}{4} = x^2 + \left(\frac{121}{4}\right)$

$\Rightarrow x^2 + 6x - x^2 = \frac{121}{4} - \frac{25}{4} - 9$

$\Rightarrow 6x = 15$

$\Rightarrow x = \frac{15}{6} = \frac{5}{2}$

5. Give a method to find the centre of a given circle.

Sol:

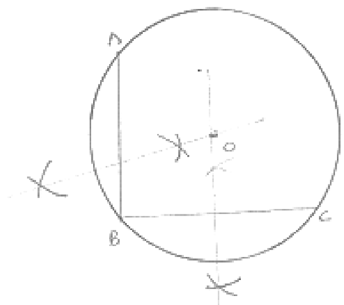
Steps of construction

(1) Take three point A,B and C on the given circle

(2) Join AB and BC

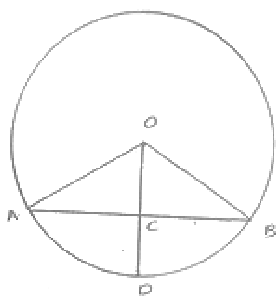
(3) Draw thee perpendicular bisectors of chord AB and BC which intersect each other at O

(4) Point will be required circle because WKT, perpendicular bisector of chord always passes through center



6. Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Sol:



Given: C is the midpoint of chord AB

To prove: D is the midpoint of arc AB

Proof:

In ΔOAC and ΔOBC

$OA = OB$ [Radius of circle]

$OC = OC$ [Common]

$AC = BC$ [C is the midpoint of AB]

Then, $\Delta OAC \cong \Delta OBC$ [By SSS condition]

$\therefore \angle AOC = \angle BOC$ [$c \cdot p \cdot c \cdot t$]

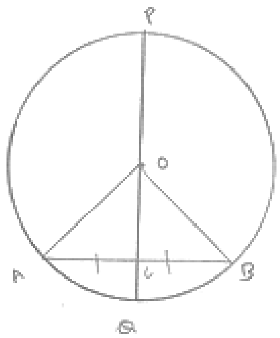
$\Rightarrow m\left(\overline{AD}\right) = m\left(\overline{BD}\right)$

$\Rightarrow \overline{AD} \cong \overline{BD}$

Here, D is the midpoint of arc AB

7. Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Sol:



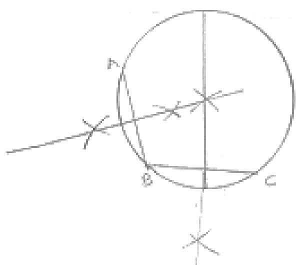
Given: PQ is a diameter of circle which Bisects
Chord AB at C
To prove: PQ bisects $\angle AOB$
Proof:
In ΔAOC and ΔBOC

$OA = OB$	[Radius of circle]
$OC = OC$	[Common]
$AC = BC$	[Given]
Then $\Delta AOC \cong BOC$	[by SSS condition]
$\therefore \angle AOC = \angle BOC$	[$c \cdot p \cdot c \cdot t$]

Hence PQ bisects $\angle AOB$

8. Given an arc of a circle, show how to complete the circle.

Sol:



Steps of construction:
(i) Take three point A, B and C on the given Arc
(ii) Join AB and BC
(iii) Draw the perpendicular bisectors of chords AB and BC which interest each other at point O, then O will required center of the required circle
(iv) Join OA
(v) With center O and radius OA, complete the circle

9. Prove that two different circles cannot intersect each other at more than two points.

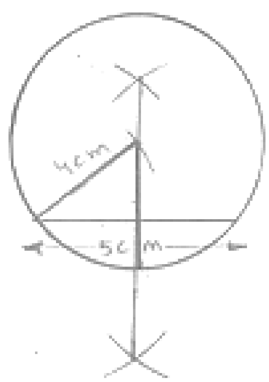
Sol:

Suppose two circles intersect in three points A,B,C,

Then A,B,C are non-collinear. So, a unique circle passes through these three points. This is contradiction to the face that two given circles are passing through A,B,C. Hence, two circles cannot intersect each other at more than two points.

10. A line segment AB is of length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B? Give reason in support of your answer.

Sol:



(i) Draw a line segment AB of 5cm

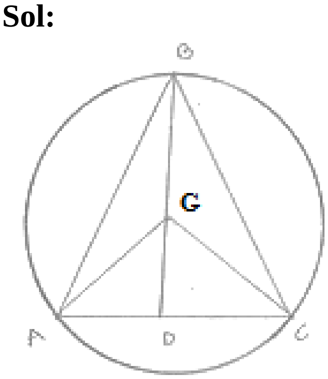
(ii) Draw the perpendicular bisectors of AB

(iii) Draw an arc which intersects the perpendicular bisectors at point O will be required center.

(iv) With center O and radius OA draw a circle. No, we cannot draw a circle of radius 2cm passing through A and B because when we draw an arc of radius 2cm with center A, the arc will not interest the perpendicular bisector and we will not find the center

11. An equilateral triangle of side 9cm is inscribed in a circle. Find the radius of the circle.

Sol:



Let ABC be an equilateral triangle of side 9cm and let AD one of its medians. Let G be the centroid of ΔABC. Then $AG : GD = 2:1$

WKT in an equilateral Δ^{le} centroid coincides with the circum center

Therefore, G is the center of the circumference with circum radius GA

Also G is the center and $GD \perp BC$. Therefore,

In right triangle ADB, we have

$AB^2 = AD^2 + DB^2$

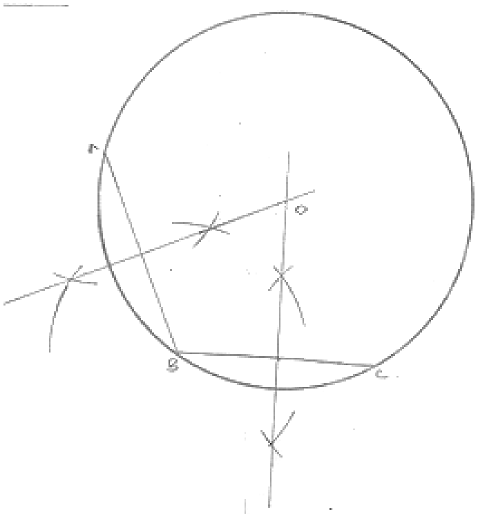
$\Rightarrow 9^2 = AB^2 + DB^2$

$\Rightarrow AD = \sqrt{81 - \frac{81}{4}} = \frac{9\sqrt{3}}{2} cm$

$\therefore$ Radius $= AG = \frac{2}{3} AD = 3\sqrt{3} cm.$

12. Given an arc of a circle, complete the circle.

Sol:



Steps of construction:

(i) Take three point A, B, C on the given Arc

(ii) Join AB and BC

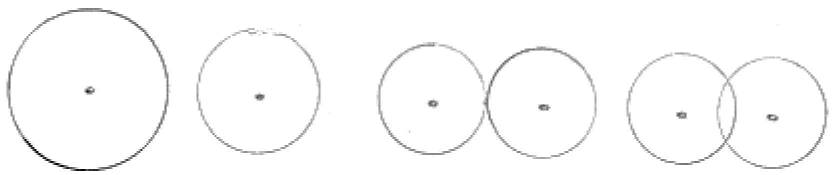
(iii) Draw the perpendicular bisectors of chords AB and BC which interest each other at point O, then O will required center of the required circle

(iv) Join OA

(v) With center O and radius OA, complete the circle

13. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Sol:

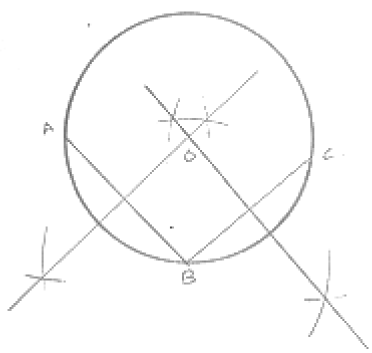


Each pair of circles have 0,1 or 2 points in common

The maximum number of points in common is '2'

14. Suppose you are given a circle. Give a construction to find its centre.

Sol:



Steps of constructions:

(1) Take three point A, B and C the given circle

(2) Join AB and BC

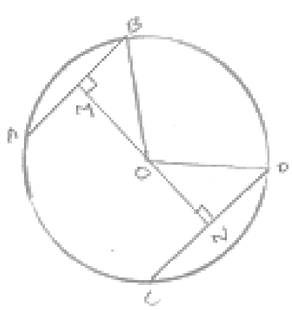
(3) Draw the perpendicular bisectors of chord AB and BC which intersect each other at O.

(4) Point O will be the required center of the circle because we know that the perpendicular bisector of the cord always passes through the center

15. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are opposite side of its center. If the distance between AB and CD is 6 cm. Find the radius of the circle.

Sol:

Draw $OM \perp AB$ *and* $ON \perp CD.$ Join OB and OD



$BM = \frac{AB}{2} = \frac{5}{2}$ (Perpendicular from center bisects the chord)

$ND = \frac{CD}{2} = \frac{11}{2}$

Let ON be x, So OM will be $6 - x$ in ΔMOB

$OM^2 + MB^2 = OB^2$

$(6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$36 + x^2 - 12x + \frac{25}{4} = OB^2$ .....(1)

In ΔNOD

$ON^2 + ND^2 = OD^2$

$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$

$x^2 + \frac{121}{4} = OD^2$ ......(2)

We have $OB = OD.$ (radii of same circle)

So, from equation (1) and (2).

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$

$\Rightarrow 12x = 36 + \frac{25}{4} - \frac{121}{4}$

$= \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$

$x = 1.$

From equation (2)

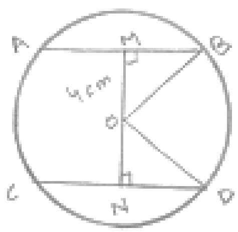
$(1)^2 + \left(\frac{121}{4}\right) = OD^2$

$OD^2 = 1 + \frac{121}{4} = \frac{121}{4}$

$OD = \frac{5\sqrt{5}}{2}$

So, radius of circle is found to be $\frac{5\sqrt{5}}{2}$ cm

16. The lengths of two parallel chords of a circle are 6 cm and 8 cm. if the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre?
Sol:



Distance of smaller chord AB from center of circle $=4cm$ $OM=4cm$

$MB=\frac{AB}{2}=\frac{6}{2}=3cm$

In ΔOMB

$OM^2+MB^2=OB^2$

$(4)^2+3^2=OB^2$

$16+9=OB^2$

$OB=\sqrt{25}$

$OB=5cm$

In ΔOND

$OD=OB=5cm$ [radii of same circle]

$ND=\frac{CD}{2}=\frac{8}{2}=4cm$

$ON^2+ND^2=OD^2$

$ON^2+(4)^2=(5)^2$

$ON^2=25-16=9$

$ON=3$

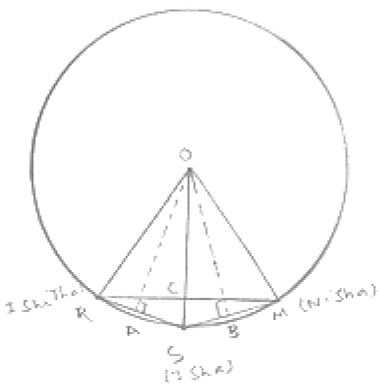
So, distance of bigger chord from circle is 3cm.

Exercise – 16.3

1. Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita throws a ball o Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24 m each, what is the distance between Ishita and Nisha.

Sol:

Let R, S and M be the position of Ishita, Isha and Nasha respectively



$AR = AS = \frac{24}{2} = 12cm$

$OR = OS = OM = 20m$ (radii of circle)

In OAR

$OA^2 + AR^2 = OR^2$

$OA^2 + (112m)^2 = (20m)^2$

$OD^2 = (400 - 144)m^2 = 256m^2$

$OA = 16m$

WKT, in an isosceles triangle altitude divides the base, So in $\Delta RSM \angle RCS$ will be $90°$ and $RC = CM$.

Area of $\Delta ORS = \frac{1}{2} \times OA \times RS$

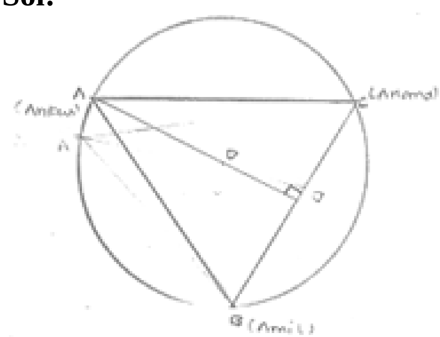
$\Rightarrow \frac{1}{2} \times RC \times OS = \frac{1}{2} \times 16 \times 24$

$\Rightarrow RC \times 20 = 16 \times 24 \Rightarrow RC = 192 \Rightarrow RM = 2(192) = 38 \cdot 4m$

So, distance between ishita and Nisha is $384m$.

2. A circular park of radius 40 m is situated in a colony. Three boys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Sol:



Given hat $AB = BC = CA$

So, ABC is an equilateral triangle

OA (radius) $= 40m.$

Medians of equilaterals triangles pass through the circum center (0) of the equilaterals triangles ABC

We also know that median intersect each other at the $2:1$ As AD is the median of equilaterals triangle ABC, we can write:

$$\frac{OA}{OD} = \frac{2}{7}$$

$$\Rightarrow \frac{40m}{OD} = \frac{2}{7}$$

$$\Rightarrow OD = 20m.$$

$$\therefore AD = OA + OD = (40 + 20)m$$

$$= 60m$$

In ΔADC

By using Pythagoras theorem

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = (60)^2 + \left(\frac{AC}{2}\right)^2$$

$$AC^2 = 3600 + \frac{AC^2}{4}$$

$$\Rightarrow \frac{3}{4}AC^2 = 3600$$

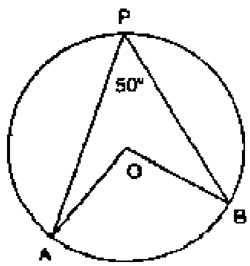
$$\Rightarrow AC^2 = 4800$$

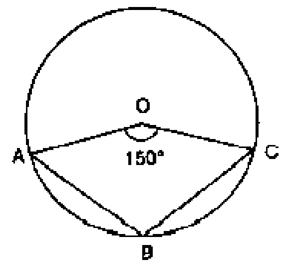
$$\Rightarrow AC = 40\sqrt{3}m$$

So, length of string of each phone will be $40\sqrt{3}m$

Exercise – 16.4

1. In the below fig. O is the centre of the circle. If ∠APB = 50°, find ∠AOB and ∠OAB.





Sol:

$\angle APB = 50°$

By degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow \angle APB = 2\times 50° = 100°$

Since $OA = OB$ [Radius of circle]

Then $\angle OAB = \angle OBA$ [Angle's opposite to equal sides]

Let $\angle OAB = x$

In $\Delta OAB \cdot$ by angle sum property

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow x + x + 100 = 180°$

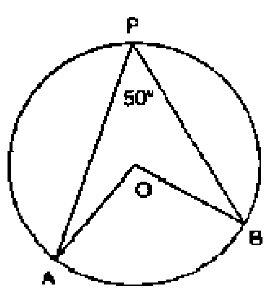
$\Rightarrow 2x + 100 = 180°$

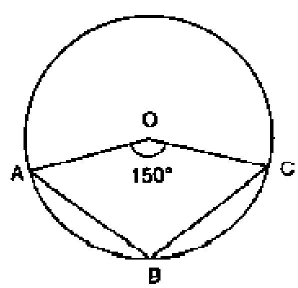
$\Rightarrow 2x = 80°$

$\Rightarrow x = 40°$

$\angle OAB = \angle OBA = 40°$

2. In the fig below, it is given that O is the centre of the circle and ∠AOC = 150°. Find ∠ABC.





Sol:

We have $\angle AOC = 150°$

$\therefore \angle AOC + reflex\ \angle AOC = 360°$ [complex angle]

$\Rightarrow 150° + reflex\ \angle AOC = 360°$

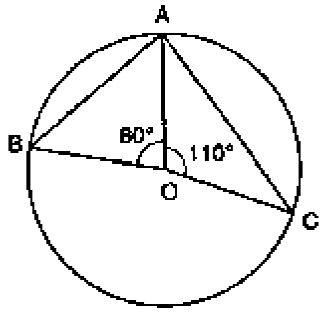
$\Rightarrow reflex\ \angle AOC = 360° - 150°$

$\Rightarrow reflex\ \angle AOC = 210°$

$\Rightarrow 2\angle ABC = 210°$ [By degree measure theorem]

$\Rightarrow \angle ABC = \frac{210}{2} = 105°$

3. In the below fig. O is the centre of the circle. Find ∠BAC.



Sol:

We have $\angle AOB = 80°$

And $\angle AOC = 110°$

$\therefore \angle AOB + \angle AOC + \angle BOC = 360°$ [Complete angle]

$\Rightarrow 80° + 110° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 360° - 80° - 110°$

$\Rightarrow \angle BOC = 170°$

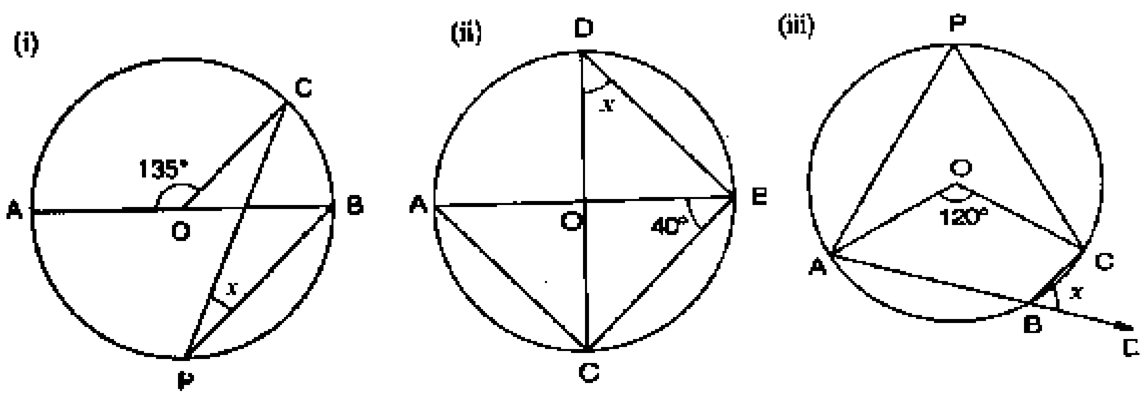
By degree measure theorem

$\angle BOC = 2\angle BAC$

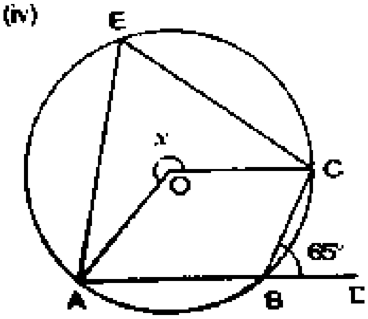
$\Rightarrow 170° = 2\angle BAC$

$\Rightarrow \angle BAC = \frac{170°}{2} = 85°$

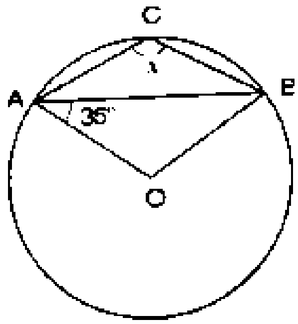
4. If O is the centre of the circle, find the value of x in each of the following figures:



(iv)



(v)



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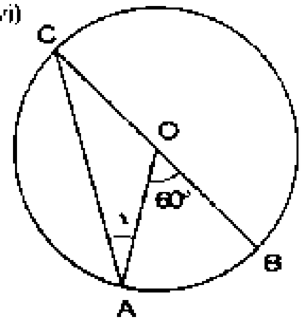

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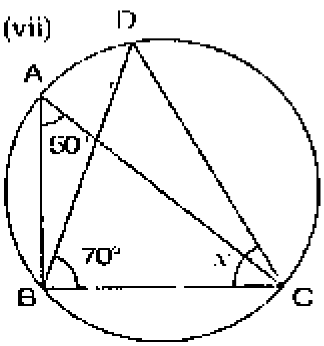
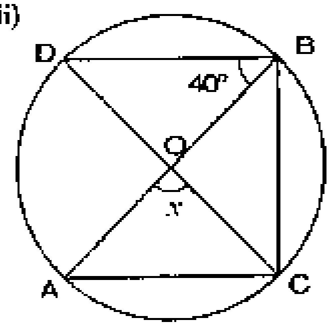
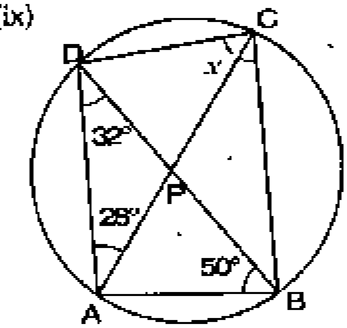


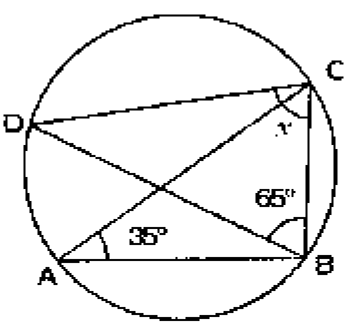
(viii)



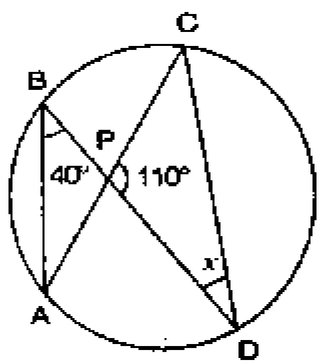
(ix)



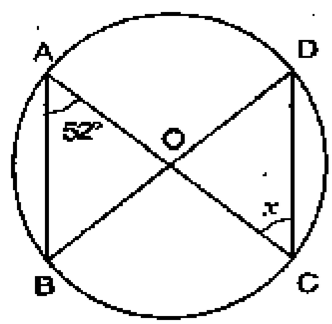
(x)



(xi)



(xii)



Sol:

(i) $\angle AOC = 135°$

$\therefore \angle AOC + \angle BOC = 180°$ [Linear pair of angles]

$\Rightarrow 135° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 135° = 45°$

By degree measures theorem

$\angle BOC = 2\angle CDB$

$\Rightarrow 45° = 2x$

$\Rightarrow x = \frac{45°}{2} = 22\frac{1°}{2}.$

(ii) We have

$\angle ABC = 40°$

$\angle ACB = 90°$ [Angle in semicircle]

In $\Delta ABC,$ by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180°$

$\Rightarrow \angle CAB + 90° + 40° = 180°$

$\Rightarrow \angle CAB = 180° - 90°$

$\Rightarrow \angle CAB = 50°$

Now,

$\angle COB = \angle CAB$ [Angle is same segment]

$\Rightarrow x = 50°$

(iii) We have

$\angle AOC = 120°$

By degree measure theorem

$\angle AOC = 2\angle APC$

$\Rightarrow 120° = 2\angle APC$

$\Rightarrow \angle APC = \frac{120°}{2} = 60°$

$\therefore \angle APC + \angle ABC = 180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow 60° + \angle ABC = 180°$

$\Rightarrow -60° + 180° = \angle ABC$

$\Rightarrow \angle ABC = 120°$

$\therefore \angle ABC + \angle DBC = 180°$ [Linear pair of angles]

$\Rightarrow 120 + x = 180°$

$\Rightarrow x = 180° - 120° = 60°$

(iv) We have

$\angle CBD = 65°$

$\therefore \angle ABC + \angle CBD = 180°$ [Linear pair of angles]

$\Rightarrow \angle ABC + 65° = 180°$

$\Rightarrow \angle ABC = 180° - 65° = 115°$

$\therefore$ Reflex $\angle AOC = 2\angle ABC$ [By degree measure theorem]

$\Rightarrow x = 2 \times 115°$

$\Rightarrow x = 230°$

(v) We have

$\angle OAB = 35°$

Then, $\angle OBA = \angle OAB = 35°$ [Angles opposite to equal radii]

In $\Delta AOB,$ by angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180°$

$\Rightarrow \angle AOB + 35° + 35° = 180°$

$\Rightarrow \angle AOB = 180° - 35° - 35° = 110°$

$\therefore \angle AOB + reflex\ \angle AOB = 360°$ [comple angle]

$\Rightarrow 110° + reflex\ \angle AOB = 360°$

$\Rightarrow reflex\ \angle AOB = 360° - 110° = 250°$

By degree measure theorem reflex $\angle AOB = 2\angle ACB$

$\Rightarrow 250° = 2x$

$\Rightarrow x = \frac{250°}{2} = 125°$

(vi) We have

$\angle AOB = 60°$

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow 60° = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{60°}{2} = 30°$ [Angles opposite to equal radii]

$\Rightarrow x = 30°$

(vii) We have

$\angle BAC = 50°$

And $\angle DBC = 70°$

$\therefore \angle BDC = \angle BAC = 50°$ [Angle in same segment]

In ΔBDC, by angles sum property

$\angle BDC + \angle BCD + \angle DBC = 180°$

$\Rightarrow 50° + x + 70 = 180°$

$\Rightarrow x = 180° - 70° - 50° = 60°$

(viii) We have

$\angle DBO = 40°$

$\angle DBC = 90°$ [Angle in semi circle]

$\Rightarrow \angle DBO + \angle OBC = 90°$

$\Rightarrow 40° + \angle OBC = 90°$

$\Rightarrow \angle OBC = 90° - 40° = 50°$

By degree measure theorem

$\angle AOC = 2\angle OBC$

$\Rightarrow x = 2 \times 50° = 100°$

(ix) In ΔDAB, by angle sum property

$\angle ADB + \angle DAB + \angle ABD = 180°$

$\Rightarrow 32° + \angle DAB + 50° = 180°$

$\Rightarrow \angle OAB = 180° - 32° - 50°$

$\Rightarrow \angle DAB = 95°$

Now,

$\angle OAB + \angle DCB = 180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow 98 + x = 180°$

$\Rightarrow x = 180 - 98° = 82°$

(x) We have

$\angle BAC = 35°$

$\angle BAC = \angle BAC = 35°$ [Angle in same segment]

In ΔBCD by angle sum property

$\angle BDC + \angle BCD + \angle DBC = 180°$

$\Rightarrow 35 + x + 65° = 180°$

$\Rightarrow x = 180° - 35° - 66° = 80°$

(xi) We have

$\angle ABD = 40°$

$\therefore \angle ACD = \angle ABD = 40°$ [Angle in same segment]

In $\Delta PCD,$ By angle sum property

$\angle PCD + \angle CPO + \angle PDC = 180°$

$\Rightarrow 40° + 110° + x° = 180°$

$\Rightarrow x° = 180° - 150°$

$\Rightarrow x = 30°$

(xii) Given that $\angle BAC = 52°$

Then, $\angle BDC = \angle BAC = 52°$ [Angle in same segment]

Since $OD = OC$

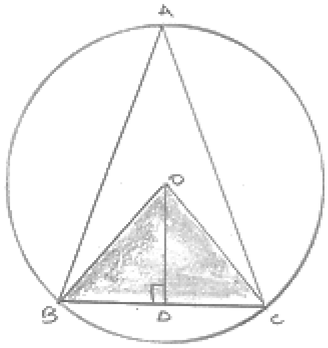
Then, $OD = OC$

Then, $\angle ODC = \angle OCD$ [Opposite angles to equal radii]

$\Rightarrow x = 52°$

5. O is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove that ∠BOD = ∠A.

Sol:



Given O is the circum center of ΔABC and $OD \perp BC$

To prove $\angle BOD = 2\angle A$

Proof:

In ΔOBA *and* ΔOCA

$\angle ODB = \angle ODC$ [Each $90°$]

$OB = OC$ [Radii of circle]

$OD = OD$ [Common]

Then, $\Delta OBD \cong \Delta OCD$ [By RHS condition]

$\therefore \angle BOD = \angle COD$ ....(1) $(P \cdot C \cdot T)$

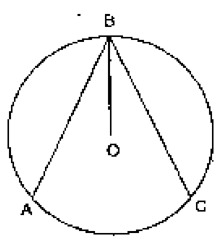
By degree measure theorem

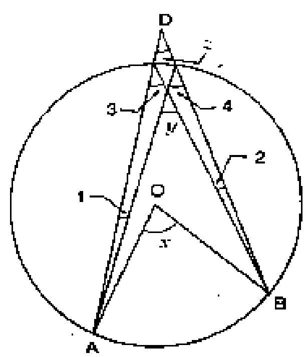
$\angle BOC = 2\angle BAC$

$\Rightarrow 2\angle BOD = 2\angle BAC$ [By using (1)]

$\Rightarrow \angle BOD = \angle BAC$

6. In the fig. below, O is the centre of the circle, BO is the bisector of ∠ABC. Show that AB = AC.





Sol:

Given, BO is the bisector of $\angle ABC$

To prove $AB = BC$

Proof:

Since, BO is the bisector of $\angle ABC$

Then, $\angle ABO = \angle DAB$ ....(2) [Opposite angles to equal sides]

Since $OB = OC$ [Radius of circle]

The $\angle CBO = \angle OCB$ ....(3) [Opposite angles to equal sides]

Compare equation (1), (2) and (3)

$\angle OAB = \angle OCB$ ....(4)

In $\angle OAB = \angle OCB$ [from (4)]

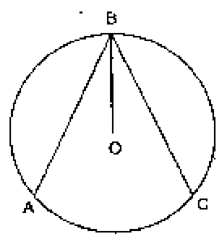
$\angle OBA = \angle OBC$ [Given]

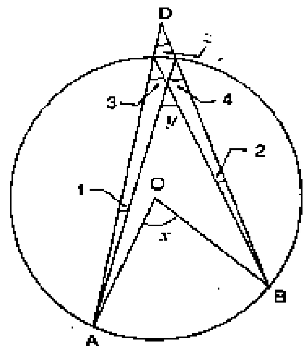
$OB = OB$ [Common]

Then, $\Delta OAB \cong OCB$ [By AAS condition]

$\therefore AB = BC$ $[c \cdot p \cdot c \cdot t]$

7. In the below fig. O is the centre of the circle, prove that ∠x = ∠y + ∠z.





Sol:

We have, $\angle 3 = \angle 4$ [Angles in same segment]

$\therefore \angle x = 2\angle 3$ [By degree measure theorem]

$\Rightarrow \angle x = \angle 3 + \angle 8$

$\Rightarrow \angle x = \angle 3 + \angle 4$ $.......(1)[\angle 3 = \angle 4]$

But $\angle y = \angle 3 + \angle 1$ [by exterior angle prop]

$\Rightarrow \angle 3 = \angle y - \angle 1$ $.......(2)$

From (1) and (2)

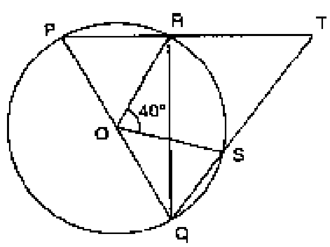
$\angle x = \angle y - \angle 1 + \angle 4$

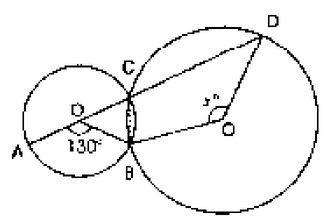
$\Rightarrow \angle x = \angle y + \angle y - \angle 1$

$\Rightarrow \angle x = \angle y + \angle z$ (By exterior angle prop)

$\Rightarrow \angle x = \angle y + \angle z$

8. In the below fig. O and O' are centres of two circles intersecting at B and C, ACD is a straight line, find x.





Sol:

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow 130° = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{130°}{2} = 65°$

$\therefore \angle ACB + \angle BCD = 180°$ [Linear pair of angle]

$\Rightarrow 65° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 65° = 115°$

By degree measure theorem

Reflex $\angle BOD = 2\angle BCA$

$\Rightarrow$ Reflex $\angle BOD = 2\times 115° = 230°$

Now, reflex $\angle BOD + \angle BOD = 360°$ [Complex angle]

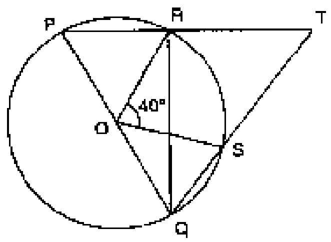
$\Rightarrow 230° + x = 360°$

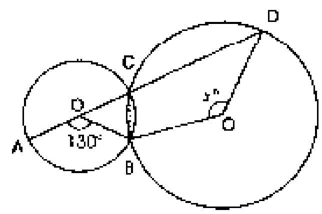
$\Rightarrow x = 360° - 230°$

$\Rightarrow 130°$

$x = 130°$

9. In the below fig. O is the centre and PQ is a diameter. If ∠ROS = 40°, find ∠RTS.





Sol:

Since PQ is diameter

Then, $\angle PRO = 90°$ [Angle in semi-circle]

$\therefore \angle PRQ + \angle TRQ = 180°$ [Linear pair of angle]

$\angle 90° + \angle TRQ = 180°$

$\angle TRQ = 180° - 90° = 90°$

By degree measure theorem

$\angle ROS = 2\angle RQS$

$\Rightarrow 40° = 2\angle RQS$

$\Rightarrow \angle RQS = \dfrac{40°}{2} = 20°$

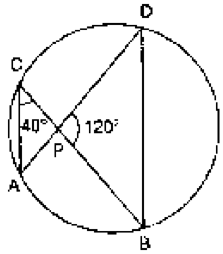
In ΔRQT, By angle sum property

$\angle RQT + QRT + \angle RTS = 180°$

$\Rightarrow 20° + 90° + \angle R + \angle S = 180°$

$\Rightarrow \angle RTS = 180° - 20° - 90° = 70°$

10. In the below fig. if ∠ACB = 40°, ∠DPB = 120°, find ∠CBD.



Sol:

We have

$\angle ACB = 40°, \angle DPB = 120°$

$\therefore \angle ADB = \angle ACB = 40°$ [Angle in same segment]

In $\Delta POB,$ by angle sum property

$\angle PDB + \angle PBD + \angle BPP = 180°$

$\Rightarrow 40° + \angle PBD + 120° = 180°$

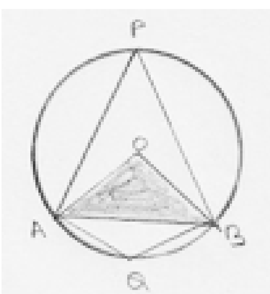
$\Rightarrow \angle PBD = 180° - 40° - 120°$

$\Rightarrow \angle PBD = 20°$

$\therefore \angle CBD = 20°$

11. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Sol:



We have

Radius $OA = chord\ \ AB$

$\Rightarrow OA = OB = AB$

Then ΔOAB is an equilateral triangle

$\therefore \angle AOB = 60°$ [one angle of equilateral]

By degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow 60° = 2\angle APB$

$\Rightarrow 60° = 2\angle APB$

$$\Rightarrow \angle APB = \frac{60°}{2} = 30°$$

Now, $\angle APB + \angle AQB = 180°$ [opposite angles of cyclic quadrilaterals]

$\Rightarrow 30° + \angle AQB = 180°$

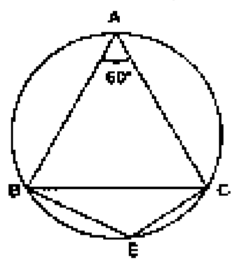
$\Rightarrow \angle AQB = 180° - 30° = 150°$

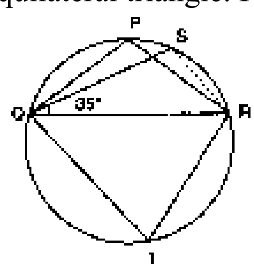
$\therefore$ Angle by chord AB at minor arc $= 150°$

Angle by chord AB at major arc $= 30°$

Exercise – 16.5

1. In the below fig. ΔABC is an equilateral triangle. Find m ∠BEC.





Sol:

Since, ΔABC is an equilateral triangles

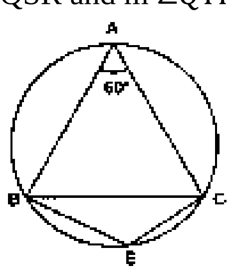
Then, $\angle BAC = 60°$

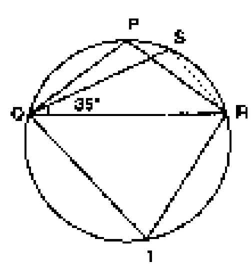
$\therefore \angle BAC + \angle BEC = 180°$ [Opposite angles of a quadrilaterals]

$\Rightarrow 60° + \angle BEC = 180° \Rightarrow \angle BEC = 180° - 60°$

$\Rightarrow \angle BEC = 180°$

2. In the below fig. ΔPQR is an isosceles triangle with PQ = PR and m ∠PQR = 35°. Find m ∠QSR and m ∠QTR.





Sol:

We have $\angle PQR = 35°$

Since, ΔPQR is an isosceles triangle with $PQ = RR$

Then $\angle PQR = \angle PRQ = 35°$

In ΔPQR by angle sum property

$\angle P + \angle PQR + \angle PRQ = 180°$

$\Rightarrow \angle P + 35° + 35° = 180°$

$\Rightarrow \angle P = 180° - 35° = 35°$

$\Rightarrow \angle P = 110°$ [Angles in same segment]

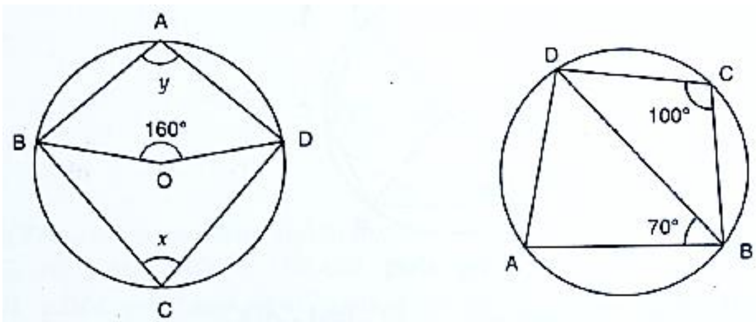
Now, $\angle QSR + \angle QTR = 180°$

$\Rightarrow 110° + \angle QTR = 180°$

$\Rightarrow \angle QTR = 180° - 110°$

$\Rightarrow \angle QTR = 70°$

3. In the below fig., O is the centre of the circle. If ∠BOD = 160°, find the values of x and y.



Sol:

Given that O is the center of the circle

We have, $\angle BOD = 160°$

By degree measure theorem

$\angle BOD = 2\angle BCD$

$\Rightarrow 160° = 2 \times x$

$\Rightarrow x = \frac{160°}{2} = 80°$

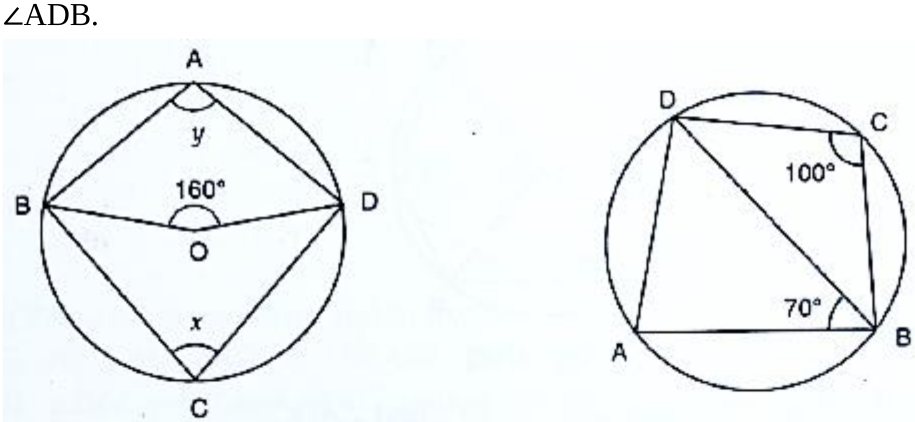
$\therefore \angle BAD + \angle BCD = 180°$ [Opposite angles of cyclic quadrilaterals]

$\Rightarrow y + x = 180°$

$\Rightarrow y + 80° = 180°$

$\Rightarrow y = 180° - 80° = 100°$

4. In the below fig. ABCD is a cyclic quadrilateral. If ∠BCD = 100° and ∠ABD = 70°, find ∠ADB.



Sol:

We have

$\angle BCD = 100°$ *and* $\angle ABD = 70°$

$\therefore \angle DAB + \angle BCD = 180°$ [Opposite angles of cyclic quadrilaterals]

$\Rightarrow \angle DAB + 100° = 180°$

$\Rightarrow \angle DAB = 180° - 100° = 80°$

$\Rightarrow \angle PAB = 80°$

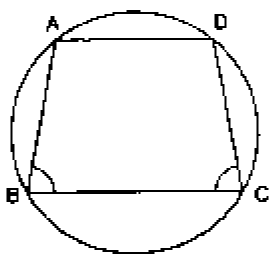
In $\Delta DAB,$ by angle sum property

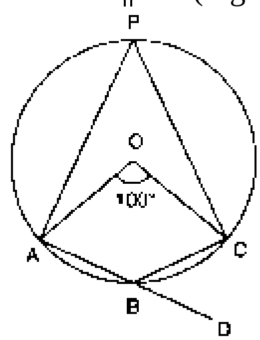
$\angle ADB + \angle DAB + \angle DBD = 180°$

$\Rightarrow \angle ABD + 80° + 70° = 180°$

$\Rightarrow \angle ABD = 180° - 150° = 30°$

5. If ABCD is a cyclic quadrilateral in which AD || BC (Fig below). Prove that ∠B = ∠C.





Sol:

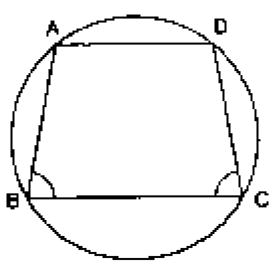
Since $ABCD$ is a cyclic quadrilateral with $AD \parallel BC$.

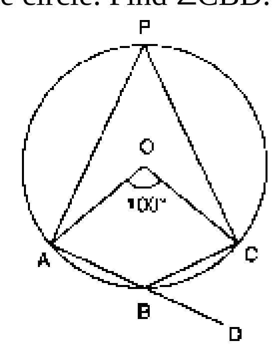
Then $\angle A + \angle C = 180°$ .....(1) [Opposite angles of cyclic quadrilaterals]

And, $\angle A + \angle B = 180°$ ......(2) [Co interior angles]

Compare (1) and (2) equations $\angle B = \angle C$

6. In the below fig. O is the centre of the circle. Find ∠CBD.





Sol:

Given that $\angle BOC = 100°$

By degree measure theorem

$\angle AOC = 2\angle APC$

$\Rightarrow 100° = 2\angle APC$

$\Rightarrow \angle APC = \frac{100°}{2} = 90°$

$\therefore \angle APC + \angle ABC = 180°$ [Opposite angles of cyclic quadrilaterals]

$\Rightarrow 50° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 50°$

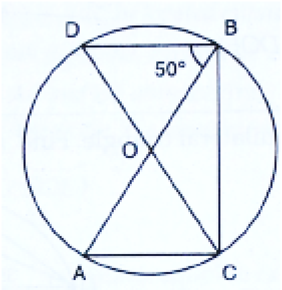
$= 130°$

$\therefore \angle ABC + \angle CBD = 180°$ [Linear pair of angles]

$\Rightarrow 130° + \angle CBD = 180°$

$\Rightarrow \angle CBD = 50°$

7. In the below fig. AB and CD are diameters of a circle with centre O. if ∠OBD = 50°, find ∠AOC.



Sol:

Given that,

$\angle OBD = 50°$

Since, $AB\, and\ CD$ are the diameter of circle then O is the center of the circle

$\therefore \angle PBC = 90°$ [Angle in semicircle]

$\Rightarrow \angle OBD + \angle DBC = 90°$

$\Rightarrow 50° + \angle DBC = 90°$

$\Rightarrow \angle DBC = 90° - 50° = 40°$

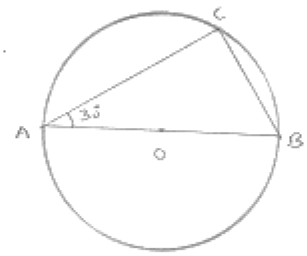
By degree measure theorem

$\angle AOC = 2\angle ABC$

$\Rightarrow \angle AOC = 2 \times 40° = 80°$

8. On a semi-circle with AB as diameter, a point C is a taken, so that m (∠CAB) = 30°. Find m (∠ACB) and m (∠ABC).

Sol:



We have, $\angle CAB = 30°$

$\angle ACB = 90°$ [Angle in semicircle]

In ΔABC, by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180°$

$\Rightarrow 30° + 90° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 120°$

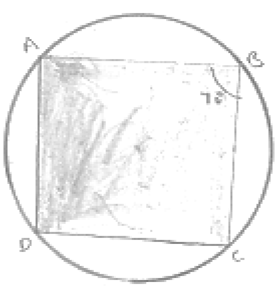
$= 60°$

9. In a cyclic quadrilateral ABCD if AB || CD and ∠B = 70°, find the remaining angles.

Sol:

Given that $\angle B = 70° = 70°$

Since $ABCD$ is a cyclic quadrilaterals



Then, $\angle B + \angle D = 180°$

$\Rightarrow 70° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 70° = 110°$

Since $AB \parallel DC$

Then $\angle B + \angle C = 180°$

$\Rightarrow 70° + \angle C = 180°$ [Cointerior angles]

$\Rightarrow \angle C = 180° - 70°$

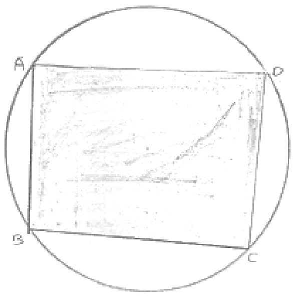
$= 110°$

Now, $\angle A + \angle C = 180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow \angle A + 110° = 180°$

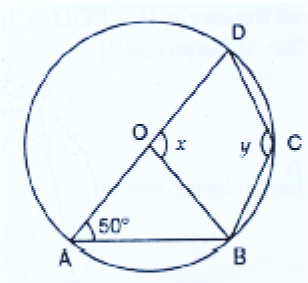
$\Rightarrow \angle A = 180° - 110°$
$\Rightarrow \angle A = 70°$

10. In a quadrilateral ABCD, if m ∠A = 3 (m ∠ C). Find m ∠A.
Sol:



We have, $\angle A = 3\angle C$
Let $\angle C = x$
Then $A = 3x$
$\therefore \angle A + \angle C = 180°$ [Opposite angles of cyclic quadrilaterals]
$\Rightarrow 3x + x = 180°$
$\Rightarrow 4x = 180° \Rightarrow x = \frac{180}{4} = 45°$
$\therefore \angle A = 3x$
$= 3 \times 45°$
$= 135°$
$\therefore \angle A = 135°$

11. In the below fig. O is the centre of the circle and ∠DAB = 50°. Calculate the values of x and y.



Sol:
We have $\angle DAB = 50°$
By degree measure theorem
$\angle BOD = 2\angle BAD$
$\Rightarrow x = 2 \times 50° = 100°$
Since, ABCD is a cyclic quadrilateral

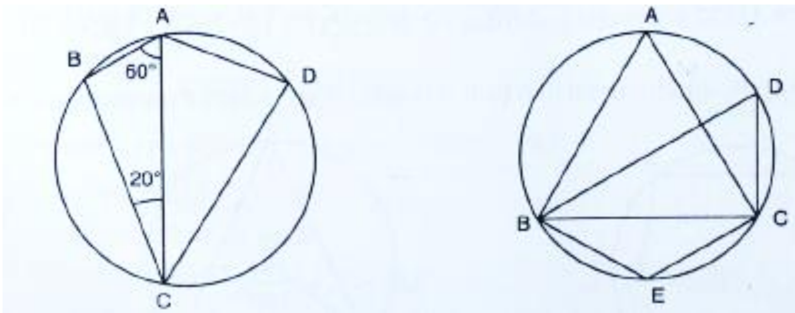
Then $\angle A + \angle C = 180°$

$\Rightarrow 50 + y = 180°$

$\Rightarrow y = 180° - 50°$

$= 130°$

12. In the below fig. if $\angle BAC = 60°$ and $\angle BCA = 20°$, find $\angle ADC$.



Sol:

By using angle sum property in ΔABC

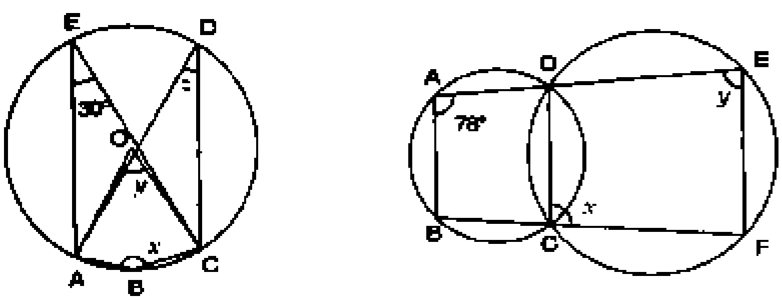
$\angle B = 180° - (60° + 120°) = 100°$

In cyclic quadrilaterals ABCD, we have:

$\angle B + \angle D = 180°$

$\angle D = 180° - 100° = 80°$

13. In the below fig. if ABC is an equilateral triangle. Find $\angle BDC$ and $\angle BEC$.



Sol:

Since ΔABC is an equilateral triangle

Then, $\angle BAC = 60°$

$\therefore \angle BDC = \angle BAC = 60°$ [Angles in same segment]

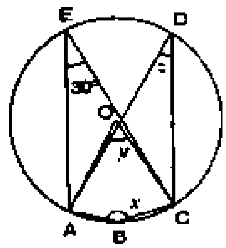
Since, quadrilaterals $ABEC$ is a cyclic quadrilaterals

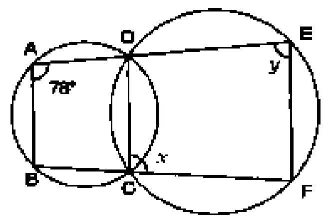
Then $\angle BAC + \angle BEC = 180°$

$\Rightarrow 60° + \angle BEC = 180°$

$\Rightarrow \angle BEC = 180° - 60° = 120°$

14. In the below fig. O is the centre of the circle, if ∠CEA = 30°, find the values of x, y and z.





Sol:

We have, $\angle AEC = 30°$

Since, quadrilateral $ABCE$ is a cyclic quadrilaterals

Then, $\angle ABC + \angle AEC = 180°$

$x + 30° = 180°$

$\Rightarrow x = 180° - 30° = 150°$

By degree measure theorem

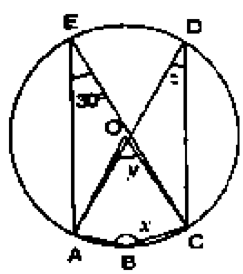
$\angle AOC = 2\angle AEC$

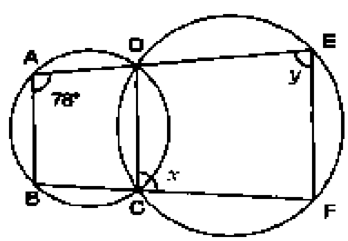
$\Rightarrow y = 2 \times 30° = 60°$

$\Rightarrow \therefore \angle ADC = \angle AEC$ [Angles in same segment]

$\Rightarrow z = 30°$

15. In the below fig. ∠BAD = 78°, ∠DCF = x° and ∠DEF y°. find the values of x, and y.





Sol:

We have, $\angle BAD = 78° \cdot \angle DCF = x°\ and\ \angle DEF = y°$

Since, ABCD is a cyclic quadrilateral

Then, $\angle BAD + \angle BCD = 180°$

$\Rightarrow 78° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 78° = 102°$

Now, $\angle BCD + \angle DCF = 180°$ [Linear pair of angles]

$\Rightarrow 102° = x - 180°$

$\Rightarrow x = 180° - 102° = 78°$

Since, DCEF is a cyclic quadrilateral

Then, $x + y = 180°$

$\Rightarrow 78° + y = 180°$

$\Rightarrow y = 180° - 78° = 102°$

$\therefore y = 102°$

16. In a cyclic quadrilateral ABCD, if ∠A − ∠C = 60°, prove that the smaller of two is 60°.

Sol:

We have

$\angle A - \angle C = 60°$ ....(1)

Since, ABCD is a cyclic quadrilaterals

Then $\angle A + \angle C = 180°$ ....(2)

Add equations (1) and (2)

$\angle A - \angle C + \angle A + \angle C = 60° + 180°$

$\Rightarrow 2\angle A = 240°$

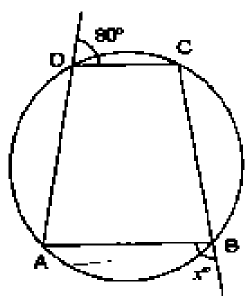
$\Rightarrow \angle A = \frac{240°}{2} = 120°$

Put value of $\angle A$ in equation (2)

$120° + \angle C = 180°$

$\Rightarrow \angle A = 180° - 120° = 60°$

17. In the below fig. ABCD is a cyclic quadrilateral. Find the value of x.



Sol:

$\angle EDC + \angle CDA = 180°$ [Linear pair of angles]

$\Rightarrow 80° + \angle CDA = 180°$

$\Rightarrow \angle CDA = 180° - 60° = 100°$

Since, ABCD is a cyclic quadrilateral

$\angle ADC + \angle ABC = 180°$

$\Rightarrow 100° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 180° - 100° = 80°$

Now, $\angle ABC + \angle ABF = 180°$ [Linear pair of angles]

$\Rightarrow 80 + x° = 180°$

$\Rightarrow x = 180° - 80° = 100°$

18. ABCD is a quadrilateral in which:

(i) BC || AD, ∠ADC = 110° and ∠BAC = 50°. Find ∠DAC.

(ii) ∠DBC = 80° and ∠BAC = 40°, find ∠BCD.

(iii) ∠BCD = 100° and ∠ABD = 70°, find ∠ADB.

Sol:

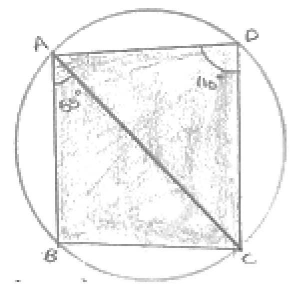
(i) Since, ABCD is a cyclic quadrilateral

Then, $\angle ABC + 110^\circ = 180^\circ$

$\Rightarrow \angle ABC + 110^\circ = 180^\circ$

$\Rightarrow \angle ABC = 180^\circ - 110^\circ$

$= 70^\circ$



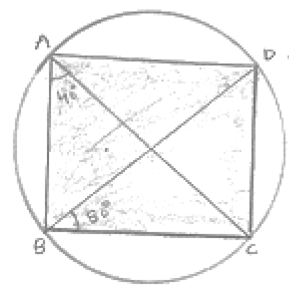
Since $AD \parallel BC$

Then, $\angle DAB + \angle ABC = 180^\circ$ [Co-interior angle]

$\Rightarrow \angle DAC + 50^\circ + 70^\circ = 180^\circ$

$\Rightarrow \angle DAC = 180^\circ - 120^\circ = 60^\circ$

(ii) $\angle BAC = \angle BDC = 40^\circ$ [Angle in same segment]



In ΔBDC, by angle sum property

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$\Rightarrow 80^\circ + \angle BCD + 40^\circ = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 40^\circ - 80^\circ$

$\Rightarrow \angle BCD = 60^\circ$

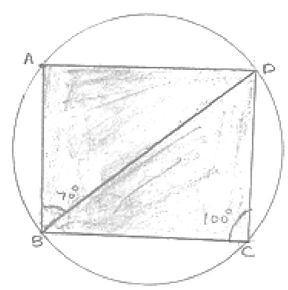
(iii) Given that $ABCD$ is a cyclic quadrilaterals

Ten $\angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \angle BAD + 100^\circ = 180^\circ$

$\Rightarrow \angle BAD = 180^\circ - 100^\circ$

$\Rightarrow \angle BAD = 80^\circ$



In ΔABD, by angle sum property

$\angle ABD + \angle ADB + \angle BAD = 180^\circ$

$\Rightarrow 70^\circ + \angle ADB + 80^\circ = 180^\circ$

$\Rightarrow \angle ADB = 180^\circ - 150^\circ$

$\Rightarrow \angle ADB = 30^\circ$

19. Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

Sol:

Let ABCD be a cyclic quadrilateral, and let O be the center of the corresponding circle
Then, each side of the equilateral ABCD is a chord of the circle and the perpendicular bisector of a chord always passes through the center of the circle
So, right bisectors of the sides of quadrilaterals ABCD, will pass through the circle O of the corresponding circle

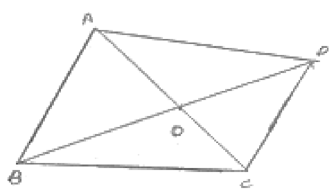
20. Prove that the centre of the circle circumscribing the cyclic rectangle ABCD is the point of intersection of its diagonals.

Sol:

Let O be the circle circumscribing the cycle rectangle ABCD. Since $\angle ABC = 90°$ and AC is a chord of the circle, so AC is a diameter of a circle. Similarly BD is a diameter
Hence, point of intersection of AC and BD is the center of the circle

21. Prove that the circles described on the four sides of a rhombus as diameters, pass through the point of intersection of its diagonals.

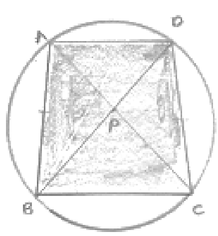
Sol:



Let ABCD be a rhombus such that its diagonals AC and BD interest at O
Since, the diagonals of a rhombus intersect at right angle
$\therefore \angle ACB = \angle BOC = \angle COD = \angle DOA = 90°$
Now, $\angle AOB = 90° \Rightarrow$ circle described on BC, AD and CD as diameter pass through O.

22. If the two sides of a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are equal.

Sol:



Given $ABCD$ is a cyclic quadrilateral in which $AB = DC$
To prove: $AC = BD$
Proof: In $\Delta PAB \ and \ \Delta PDC$

Given that $AB = DC$

$\angle BAD = \angle CDP$ [Angles in the same segment]

$\angle PBA = \angle PCD$ [Angles in same segment]

Then $\Delta PAB = \Delta PDC$ ......(1) $[c \cdot p \cdot c \cdot t]$

$PC = PB$ ......(2) $[c \cdot p \cdot c \cdot t]$

Add equation (1) and (2)

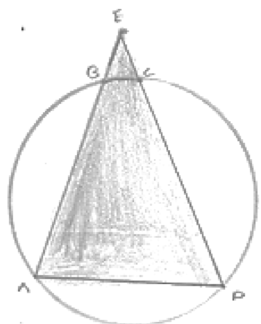
$PA + PC = PD + PB$

$\Rightarrow AC = BD$

23. ABCD is a cyclic quadrilateral in which BA and CD when produced meet in E and EA = ED. Prove that:

(i) AD || BC (ii) EB = EC

Sol:



Given ABCD is a cyclic quadrilateral in which $EA = ED$

To prove: (i) $AD \parallel BC$ (ii) $EB = EC$

Proof: (i) Since $EA = ED$

Then $\angle EAD = \angle EDA$ [Opposite angles to equal sides]

Since, ABCD is a cyclic quadrilaterals

Then, $\angle ABC + \angle ADC = 180°$

But $\angle ABC + \angle EBC = 180°$ [Linear pair of angles]

Then $\angle ADC = EBC$ ......(2)

Compare equations (1) and (2)

$\angle EAD = \angle CBA$ ......(3)

Since, corresponding angle are equal

Then $BC \parallel AD$

(ii) From equation (2)

$\angle EAD = \angle EBC$ .....(3)

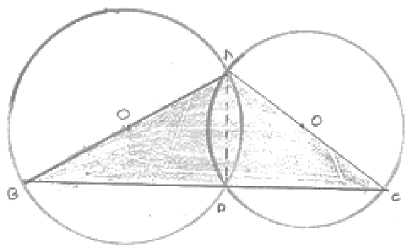
Similarly $\angle EDA = \angle ECB$ ....(4)

Compare equation (1), (3) and (4) $\angle EBC = \angle ECD$

$\Rightarrow EB = EC$ (Opposite angles to equal sides)

24. Circles are described on the sides of a triangle as diameters. Prove that the circles on any two sides intersect each other on the third side (or third side produced).

Sol:



Since AB is a diameter

Then $\angle ADB = 90°$ ......(1) [Angle in semicircle]

Since AC is a diameter

Then $\angle ADC = 90°$ ....(2) [Angle in semicircle]

Add equation (1) and (2)

$\angle ADB + \angle ADC = 90° + 90°$

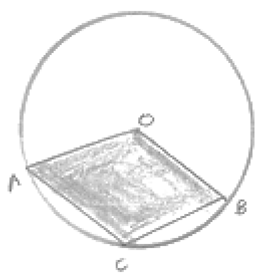
$\Rightarrow \angle BDC = 180°$

Then, BDC is a line

Hence, the circles on any two sides intersect each other on the third side

25. Prove that the angle in a segment shorter than a semicircle is greater than a right angle.

Sol:



Given: $\angle ACB$ is an angle in mirror segment

To prove: $\angle ACB > 90°$

Proof: By degree measure theorem

Reflex $\angle AOB > 180°$

And reflex $\angle AOB > 180°$

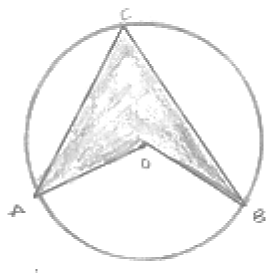
Then, $2\angle ACB > 180°$

$\angle ACB > \frac{180°}{2}$

$\Rightarrow \angle ACB > 90°$

26. Prove that the angle in a segment greater than a semi-circle is less than a right angle.

Sol:



Given:

$\angle ACB$ is an angle in major segment

To prove $\angle ACB < 90°$

Proof: by degree measure theorem

$\angle AOB = 2\angle ACB$

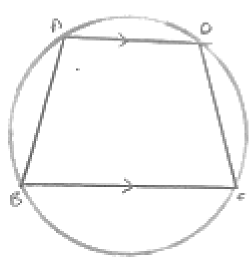
And $\angle AOB < 180°$

Then, $2\angle ACB < 180°$

$\angle ACB < 90°$

27. ABCD is a cyclic trapezium with AD || BC. If ∠B = 70°, determine other three angles of the trapezium.

Sol:



Given that

ABCD is a cyclic trapezium with $AD \parallel BC$ *and* $\angle B = 70°$

Since, $ABCD$ is a quadrilateral

Then $\angle B + \angle D = 180°$

$\Rightarrow 70° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 70° = 110°$

Since $AD \parallel BC$

Then $\angle A + \angle B = 180° \Rightarrow \angle A + 70° = 180°$ [Cointerior angles]

$\Rightarrow \angle A = 110°$

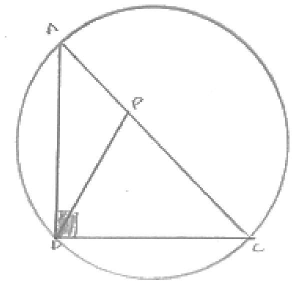
Since $ABCD$ is a cyclic quadrilateral then $\angle A + \angle c = 180°$

$\Rightarrow 110° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 110° = 70°$

28. Prove that the line segment joining the mid-point of the hypotenuse of a right triangle to its opposite vertex is half of the hypotenuse.

Sol:



Let ΔABC be a right angle triangle at angle B.

Let P be the midpoint of trypotenuse AC.

Draw a circle with center P and AC as a diameter

Since, $\angle ABC = 90°$, therefore the circle passes through B

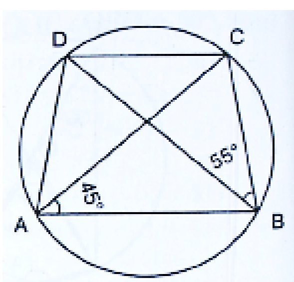
$\therefore BP = radius$

Also $AD = CP = Radius$

$\therefore AP = BP = CP$

Hence, $BP = \frac{1}{2} AC$.

29. In Fig. below, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If ∠DBC = 55° and ∠BAC = 45°, find ∠BCD.



Sol:

Since angles in the same segment of a circle are equal

$\therefore \angle CAD = \angle DBC = 65°$

$\therefore \angle DAB = \angle CAD + \angle BAC = 55° + 45° = 100°$

But, $\angle DAB + \angle BCD = 180°$ [Opposite angles of a cyclic]

$\therefore \angle BCD = 180° - 100°$

$= 80°$

$\therefore \angle BCD = 80°$