Exercise – 18.1

1. Find the lateral surface area and total surface area of a cuboid of length 80 cm, breadth 40 cm and height 20 cm.

Sol:

It is given that

Cuboid length $=80cm=L$

Breath $=40cm=$ b

Height $=20cm=h$

WKT,

Total surface area $=2[lb+bh+hl]$

$=2[(80)(40)+40(20)+20(80)]$

$=2[3200+800+1600]$

$=2[5600]$

$=11,200m^2$

Lateral surface area $=2[l+b]h=2[80+40]20$

$=40(120)$

$=4800cm^2$

2. Find the lateral surface area and total surface area of a cube of edge 10 cm.

Sol:

Cube of edge $a=10cm$

WKT,

Cube lateral surface area $=4a^2$

$=4\times 10\times 10 \quad [\because a=10]$

$=400cm^2$

Total surface area $a=6a^2$

$=6\times(10)^2$

$=600cm^2$

3. Find the ratio of the total surface area and lateral surface area of a cube.

Sol:

Cube total surface area $=6a^2$

Where, $a=$edge of cube

And, lateral surface area $=LSA=4a^2$

Where $a=$ edge of cube

$\therefore$ Ratio of TSA and LSA $=\frac{6a^2}{4a^2}$ is $\frac{3}{2}$ is $3:2$

4. Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden block covered with coloured paper with picture of Santa Claus on it. She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth and height as 80 cm, 40 cm and 20 cm respectively. How many square sheets of paper of side 40 cm would she require?

Sol:

Given that mary wants to paste the paper on the outer surface of the box; The quantity of the paper required would be equal to the surface area of the box which is of the shape of cuboid. The dimension of the box are

Length $(l) = 80cm$ Breath $(b) = 40cm$ and height (h) = 20cm

The surface area of thee box $= 2[lb + bh + hl]$

$= 2[80(40) + 40(20) + 20(80)]$

$= 2(5600) = 11,200cm^2$

The area of the each sheet of paper $= 40 \times 40cm^2$

$= 1600cm^2$

$\therefore$ Number of sheets required $= \dfrac{\text{Surface areaa of box}}{\text{area of one sheet of paper}}$

$= \dfrac{11,200}{1600} = 7$

5. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs. 7.50 m^2.

Sol:

Total area to be washed $= lb + 2(l + b)h$

Where length $(l) = 5m$

Breath $(b) = 4m$

Height $(h) = 3m$

$\therefore$ Total area to be white washed $= (5 \times 4) + 2(5 + 4) \times 3$

$= 20 + 54 = 74m^2$

Now,

Cost of white washing $1m^2$ is Rs. $7 \cdot 50$

$\therefore$ Cost of white washing $74m^2$ is $Rs.(74 \times 7 \cdot 50)$

$= Rs.\ 555$

6. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

Sol:

Length of new cuboid $=3a$

Breadth of cuboid $=a$

Height of new cuboid $=a$

The total surface area of new cuboid

$\Rightarrow (TSA)_1 = 2[lb+bh+hl]$

$\Rightarrow (TSA)_1 = 2[3a\times a + a\times a + 3a\times a]$

$\Rightarrow (TSA)_1 = 14a^2$

Total surface area of three cubes

$\Rightarrow (TSA)_2 = 3\times 6a^2 = 18a^2$

$\therefore \dfrac{(TSA)_1}{(TSA)_2} = \dfrac{14a^2}{18a^2} = \dfrac{7}{9}$

$\therefore$ Ratio is $7:9$

7. A 4 cm cube is cut into 1 cm cubes. Calculate the total surface area of all the small cubes.

Sol:

Edge of cube $=4cm$

Volume of $4cm$ cube $=(4cm)^3 = 64cm^2$

Edge of cube $=1cm$

Volume of $1cm$ cube $=(1cm)^3 = 1cm^3$

$\therefore$ Total number of small cubes $=\dfrac{64cm^3}{1cm^3} = 64$

$\therefore$ Total surface area of $64cm$ all cubes

$=64\times 6\times (1cm)^2$

$=384cm^2$

8. The length of a hall is 18 m and the width 12 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall.

Sol:

Length of the hall $=18m$

Width of hall $=12m$

Now given,

Area of the floor and the flat roof = sum of the areas of four walls.

$\Rightarrow 2lb = 2lh + 2bh$

$\Rightarrow lb = lh + bh$

$\Rightarrow h = \frac{lb}{l+b} = \frac{18\times 12}{18+12} = \frac{216}{30}$

$= 7\cdot 2m.$

9. Hameed has built a cubical water tank with lid for his house, with each other edge 1 .5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm. Find how much he would spend for the tiles, if the cost of tiles is Rs. 360 per dozen.

Sol:

Given that

Hameed is giving 5 outer faces of the tank covered with tiles he would need to know the surface area of the tank, to decide on the number of tiles required.

Edge of the cubic tank $= 1\cdot 5m = 150cm = a$

So, surface area of tank $= 5\times 150\times 150 cm^2$

Area of each square tile $= \frac{\text{surface area of tank}}{\text{area of each title}}$

$= \frac{5\times 150\times 150}{25\times 25} = 180$

Cost of 1 dozen tiles i.e., cost of 12 tiles = Rs. 360

Therefore, cost of 12 balls tiles $= Rs.\ 360$

$\therefore$ cost of one tile $= \frac{360}{12} = Rs.\ 30$

$\therefore$ The cost of 180 tiles $= 180\times Rs.\ 30$

$= Rs.\ 5,400$

10. Each edge of a cube is increased by 50%. Find the percentage increase in the surface area of the cube.

Sol:

Let d be the edge of the cube

$\therefore$ surface area of cube $= 6\times a^2$

i.e, $S_1 = 6a^2$

According to problem when edge increased by 50% then the new edge becomes

$= a + \frac{50}{100}\times a$

$= \frac{3}{2}a$

New surface area becomes $= 6\times\left(\frac{3}{2}a\right)^2$

i.e., $S_2 = 6\times\frac{9}{4}a^2$

$S_2 = \frac{27}{2}a^2$

$\therefore$ Increased surface Area $= \frac{27}{2}a^2 - 6a^2$

$= \frac{15}{2}a^2$

So, increase in surface area $= \frac{\frac{15}{2}a^2}{6a^2}\times 100$

$= \frac{15}{12}\times 100$

$= 125\%$

11. The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rates of Rs. 8 and Rs. 9.50 per m^2 is Rs. 1248. Find the dimensions of the box.

Sol:

Let the ratio be x

$\therefore$ length $= 2x$

Breath $= 3x$

Height $= 4x$

$\therefore$ Total surface area $= 2[lb + bh + hl]$

$= 2\left[6x^2 + 12x^2 + 8x^2\right]$

$= 52x^2m^2$

When cost is at Rs. 9.51 per m^2

$\therefore$ Total cost of $52x^2m^2 = Rs.8\times 52x^2$

$= Rs.\ 416x^2$

And when the cost is at $95\,per\ m^2$

$\therefore$ Total cost of $52x^2m^2 = Rs.\ 9\cdot 5\times 52x^2$

$= Rs.\ 499x^2$

$\therefore$ Different in cost $= Rs.\ 494x^2 - Rs.\ 416x^2$

$\Rightarrow 1248 = 494x^2 - 416x^2$

$\Rightarrow 78x^2 = 1248$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4$

12. A closed iron tank 12 m long, 9 m wide and 4 m deep is to be made. Determine the cost of iron sheet used at the rate of Rs. 5 per metre sheet, sheet being 2 m wide.

Sol:

Given length $= 12m$, Breadth $= 9m$ and Height $= 4m$.

Total surface area of tank $= 2(lb + bh + hl)$

$= 2[12 \times 9 + 9 \times 4 + 12 \times 4]$

$= 2[108 + 36 + 48]$

$= 384m^2$

Now length of iron sheet $= \dfrac{384}{\text{width of iron sheet}}$

$= \dfrac{384}{2} = 192m.$

Cost of iron sheet = length of sheet $\times$cost rate

$= 192 \times 5 = Rs. 960.$

13. Ravish wanted to make a temporary shelter for his car by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m with base dimensions 4 m × 3m?

Sol:

Given that

Shelter length = 4m

Breadth = 3m

Height = $2 \cdot 5m$

The tarpaulin will be required for four sides of the shelter

Area of tarpaulin in required $= 2(lb + bh + hl)$

$= \left[2(4) \times 2 \cdot 5 + (3 \times 2 \cdot 5)\right] + 4 \times 3\Big] m^2$

$= \left[2(10 + 7 \cdot 5) + 12\right] m^2$

$= 47m^2 = 47m^2.$

14. An open box is made of wood 3 cm thick. Its external length, breadth and height are 1.48 m, 1.16 m and 8.3 m. Find the cost of painting the inner surface of Rs 50 per sq. metre.

Sol:

Given

Length $=1\cdot 48m=148cm.$

Breath $=1\cdot 16m=116cm$

Height $=8\cdot 3m=83cm$

Thickness of wood = 3cm

$\therefore$ inner dimensions:

Length $(148-2\times 3)cm=142cm$

Breadth $(116-2\times 3)cm=110cm$

Height $=(83-3)cm=80cm.$

Inner surface area $=2(l+b)+lb$

$=2\left[(142)+100\right)80+142\times 110cm^2$

$=2(252)[80]+142\times 110cm^2=55,940cm^2$

$=559\ 40m^2$

Hence, cost of painting inner surface area

$=5,5940\times Rs.\ 50$

$=Rs.\ 279\cdot 70$

15. The cost of preparing the walls of a room 12 m long at the rate of Rs. 1.35 per square metre is Rs. 340.20 and the cost of matting the floor at 85 paise per square metre is Rs. 91.80. Find the height of the room.

Sol:

Given that

Length of room = 12m.

Let height of room be 'h' m.

Area of 4 walls $=2(l+b)\times h$

According to question

$\Rightarrow 2(l+b)\times h\times 1\cdot 35=340\cdot 20$

$\Rightarrow 2(12+b)\times h\times 1\cdot 35=340\cdot 20$

$\Rightarrow (12+b)\times h=\dfrac{170\cdot 10}{1\cdot 35}=126 \qquad ....(1)$

Also area of floor $=l\times b$

$\therefore l\times b\times 0\cdot 85=91\cdot 80$

$\Rightarrow 12\times b\times 0\cdot 85=91\cdot 80$

$\Rightarrow b = 9m$ ......(2)

Substituting $b = 9m$ in equation (1)

$\Rightarrow (12+9)\times h = 126$

$\Rightarrow h = 6m$

16. The dimensions of a room are 12.5 m by 9 m by 7 m. There are 2 doors and 4 windows in the room; each door measures 2.5 m by 1 .2 m and each window 1 .5 m by I m. Find the cost of painting the walls at Rs. 3.50 per square metre.

Sol:

Given length of room $= 12\cdot 5m$

Breadth of room $= 9m$

Height of room $= 7m$

$\therefore$ Total surface area of 4 walls

$= 2(l+b)\times h$

$= 2(12\cdot 5+9)\times 7$

$= 301m^2$

Area of 2 doors $= 2[2\cdot 5\times 1\cdot 2]$

$= 6m^2$

Area to be painted on 4 walls

$= 301-(6+6)$

$= 301-12 = 289m^2$

$\therefore$ cost of painting $= 289\times 3\cdot 50$

$Rs.\ 1011\cdot 5.$

17. The length and breadth of a hall are in the ratio 4: 3 and its height is 5.5 metres. The cost of decorating its walls (including doors and windows) at Rs. 6.60 per square metre is Rs. 5082. Find the length and breadth of the room.

Sol:

Let the length be 4x and breadth be 3x

Height $= 5\cdot 5m$ [given]

Now it is given that cost of decorating 4 walls at the rate of $Rs.\ 6\cdot 601m^2$ is $Rs.\ 5082$

$\Rightarrow$ Area of four walls $\times$rate = total cost of painting

$2(l+b)\times h\times 6.60 = 5082$

$2(4x+3x)\times 5.5\times 6.60 = 5082$

$\Rightarrow 7x = \dfrac{5082}{5\cdot 5\times 2\cdot 6\times 2}$

$\Rightarrow 7x = 10$

$\Rightarrow x = 10$

Length $= 4x = 4 \times 10 = 40m$

Breadth $= 3x = 3 \times 10 = 30m$

18. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (See Fig. 18.5). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2. Find the total expenses required for polishing and painting the surface of the bookshelf.

Sol:

External length of book shelf $= 85cm = l$

Breadth $= 25cm$

Height $= 110cm.$

External surface area of shelf while leaving front face of shelf

$= lh + 2(lb + bh)$

$= \left[85 \times 110 + 21\left(85 \times 25 + 25 \times 110\right)\right] cm^2$

$= 19100 cm^2$

Area of front face $= \left(85 \times 110 - 75 \times 100 + 2\left(75 \times 5\right)\right) cm^2$

$= 1850 + 750 cm^2$

$= 2600 cm^2$

Area to be polished $= 19100 + 2600 cm^2$

$= 21700 cm^2$

Cost of polishing $1cm^2$ area = $Rs.\ 0 \cdot 20$

Cost of polishing $21700 cm^2$ area $= Rs. \left[21700 \times 0 \cdot 20\right]$

$= Rs.\ 4340$

Now, length (l), breath (b), height (h) of each row of book shelf is 75cm, 20cm and 30cm

$= \left(\dfrac{110 - 20}{3}\right)$ respectively.

Area to be painted in row $= 2\left(l + h\right)b + lh$

$= \left[2\left(75 + 30\right) \times 20 + 75 \times 30\right] cm^2$

$= \left(4200 + 2250\right) cm^2$

$= 6450 cm^2$

Area to be painted in 3 rows $= \left(3 \times 6450\right) cm^2$

$= 19350 cm^2$

Cost of painting $1cm^2$ $area = Rs.\ 0 \cdot 10.$

Cost of painting 19350 $area = Rs.(19350 \times 0 \cdot 10) - Rs.1935$

Total expense required for polishing and painting the surface of the bookshelf
$= Rs.(4340 + 1935) = Rs.\ 6275.$

19. The paint in a certain container is sufficient to paint on area equal to 9.375 m^2. How many bricks of dimension 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Sol:

We know that

Total surface area of one brick $= 2(lb + bh + hl)$

$= 2[22 \cdot 5 \times 10 + 10 \times 7 \cdot 5 + 22 \cdot 5 \times 7.5]cm^2$

$= 2[468 \cdot 75)cm^2$

$= 937 \cdot 5cm^2$

Let n number of bricks be painted by the container

Area of brick $= 937 \cdot 50\ cm^2$

Area that can be painted in the container

$= 93755m^2 = 93750\,cm^2$

$93750 = 937 \cdot 5n$

$n = 100$

Thus, 100 bricks can be painted out by the container.

Exercise – 18.2

1. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold?

Sol:

Given length $= 6cm$

Breath $= 56m$

Height $= 4 \cdot 5m$

Volume of the tank $= l \times b \times h = 6 \times 5(4 \cdot 5) = 135m^3$ It is given that

$1m^3 = 1000$ liters

$\therefore 135m^3 = (135 \times 1000)$ liters

$= 1,35,000$ liters

$\therefore$ The tank can hold $1,35,000$ liters of water

2. A cubical vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Sol:

Given that

Length of vessel (l) = 10m

Width of vessel (b) = 8m

Let height of the cuboidal vessel be 'h'

Volume of vessel $= 380m^3$

$\therefore l \times b \times h = 380$

$10 \times 8 \times h = 380$

$h = 4 \cdot 75$

$\therefore$ height of the vessel should be $4 \cdot 75m$.

3. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs. 30 per m^3.

Sol:

Given length of the cuboidal Pit $(l) = 8m$

Width $(b) = 6m$

Depth $(h) = 3m$

Volume of cuboid pit $= l \times b \times h = (8 \times 6 \times 3)m^3$

$= 144m^3$

Cost of digging $1m^3 = Rs.\ 30$

Cost of digging $144m^3 = 144(Rs.\ 30) = Rs. 4320$.

4. If V is the volume of a cuboid of dimensions a, b, c and S is its surface area, then prove that $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

Sol:

Given that

Length $= a$

Breadth $= b$

Height $= c$

Volume $(v) = l \times b \times h$

$= a \times b \times c = abc$

Surface area $= 2(lb + bh + hl)$

$= 2(ab + bc + ac)$

Now, $\frac{2}{5}\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]=\frac{2}{2\left(ab+bc+ca\right)}\frac{\left[ab+bc+ca\right]}{abc}$

$=\frac{1}{abc}=\frac{1}{v}$

5. The areas of three adjacent faces of a cuboid are x, y and z. If the volume is V, prove that V^2 = xyz.

Sol:

Let a, b, d be the length, breath and height of cuboid then,

$x=ab$

$y=bd,$

$z=da,\ and$

$v=abd \qquad \left[v=l\times b\times h\right]$

$\Rightarrow xyz=ab\times bc\times ca=\left(abc\right)^2$

And $v=abc$

$v^2=\left(abc\right)^2$

$v^2=xyz$

6. If the area of three adjacent faces of a cuboid are 8 cm^2, 18 cm^3 and 25 cm^3. Find the volume of the cuboid.

Sol:

WKT, if x,y,z denote the areas of three adjacent faces of a cuboid

$\Rightarrow x=l\times b, y=b\times h, z=l\times h.$

Volume V is given by

$V=l\times b\times h.$

Now, $xyz=l\times b\times b\times h\times l\times h=V^2$

Here $x=8$

$y=18$

And $z=25$

$\therefore v^2=8\times 18\times 25=3600$

$\Rightarrow v=60cm^3.$

7. The breadth of a room is twice its height, one half of its length and the volume of the room is 512cu. m. Find its dimensions.

Sol:

We have,

$b = 2h \ and \ b = \frac{1}{2}.$

$\Rightarrow \frac{l}{2} = 2h$

$\Rightarrow l = 4h$

$\Rightarrow l = 4h, b = 2h$

Now,

Volume $= 512 \ m^3$

$\Rightarrow 4h \times 2h \times h = 512$

$\Rightarrow h^3 = 64$

$\Rightarrow h = 4$

So, $l = 4 \times h = 16m$

$b = 2 \times h = 8m$

And $h = 4m$

8. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Sol:

Radius of water flow $= 2km \ per \ hour = \left(\frac{2000}{60}\right) m/\min$

$= \left(\frac{100}{3}\right) m/\min$

Depth (h) of river = 3m

Width (b) of river = 40m

Volume of water followed in 1 min $= \frac{100}{3} \times 40 \times 3m^3 - 4000m^3$

Thus, 1 minute $4000 \ m^3 = 4000000$ liters of water will fall in sea.

9. Water in a canal 30 cm wide and 12 cm deep, is flowing with a velocity of l00 km per hour. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired?

Sol:

Given that,

Water in the canal forms a cuboid of

width $(h) = 300cm = 3m$

height $= 12 \ cm = 1 \cdot 2m$

length of cuboid is equal to the distance travelled in 30 min with the speed of 100 km per hour

$\therefore$ length of cuboid $= 100 \times \frac{30}{60} km = 50000$ meters

So, volume of water to be used for irrigation $= 50000 \times 3 \times 1 \cdot 2 m^3$

Water accumulated in the field forms a cuboid of base area equal to the area of the field and height equal to $\frac{8}{100}$ *meters*

$\therefore$ Area of field $\times \frac{8}{100} = 50,000 \times 3 \times 1 \cdot 2$

$\Rightarrow$ Area of field $= \frac{50000 \times 3 \times 1 \cdot 2 \times 100}{8}$

$= 2,250000$ meters

10. Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the volume, surface area and diagonal of the new cube.

Sol:

Let the length of each edge of the new cube be a cm

Then,

$a^3 = \left(6^3 + 8^3 + 10^3\right) cm^3$

$\Rightarrow a^3 = 1728$

$\Rightarrow a = 12$

$\therefore$ Volume of new cube $= a^3 = 1728 cm^3$

Surface area of the new cube $= 6a^2 = 6 \times 12^2 cm^2$

$= 864 cm^2$.

Diagonals of the new cube $= \sqrt{3a} = 12\sqrt{3} cm.$

11. Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Sol:

Given that

Volume of cube $= 512 cm^3$

$\Rightarrow side^3 = 512$

$\Rightarrow side^3 = 8^3$

$\Rightarrow side = 8 cm$

Dimensions of new cuboid formed

$l = 8 + 8 = 16 cm, b = 8 cm, h = 8 cm$

Surface area $= 2\left(lb + bh + hl\right)$

$= 2\left[16(8) + 8(8) + 16(8)\right] = 2\left[256 + 64\right]$

$= 640 cm^2$

$\therefore$ Surface area is $640 cm^2$.

12. Half cubic meter of gold-sheet is extended by hammering so as to cover an area of 1 hectare. Find the thickness of the gold-sheet.

Sol:

Given that

Volume of gold $= 0 \cdot 5 m^3$

Area of gold sheet = 1 hectare = 10000 m^2

$\therefore$ Thickness of gold sheet $= \dfrac{\text{Volume of gold}}{\text{Area of gold sheet}}$

$= \dfrac{0 \cdot 5 m^3}{1 Hectare}$

$= \dfrac{0 \cdot 5 m^3}{10000 m^2}$

$= \dfrac{5}{10000} \times 10 m$

$= \dfrac{100}{20000} m$

Thickness of gold sheet $= \dfrac{1}{200} cm.$

13. A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6 cm and 8 cm, find the edge of the third smaller cube.

Sol:

Volume of large cube = $V_1 + V_2 + V_3$

Let the edge of the third cube be x cm

$123^3 = 6^3 + 8^3 + x^3$ [Volume of cube $= side^3$]

$1728 = 216 + 512 + x^3$

$\Rightarrow x^3 = 1728 - 728 = 1000$

$\Rightarrow x = 10 cm$

$\therefore$ Side of third side $= 10 cm..$

14. The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many persons can sit in the hall, if each person requires 150 m^3 of air?

Sol:

Given that

Volume of cinema hall $= 100 \times 50 \times 18 m^3$

Volume air required by each person $= 150 m^3$

Number of person who can sit in the hall

$$= \frac{\text{volume of cinema hall}}{\text{volume of air req each person}}$$

$$= \frac{100 \times 50 \times 18 m^3}{150 m^3} = 600 \qquad [\because V = l \times b \times h]$$

$\therefore$ number of person who can sit in the hall

$= 600$ members

15. Given that 1 cubic cm of marble weighs 0.25 kg, the weight of marble block 28 cm in width and 5 cm thick is 112 kg. Find the length of the block.

Sol:

Let the length of the block be 1cm

Then, volume $= l \times 28 \times 5\ cm^3$

$\therefore$ weight $= 140 l \times 0 \cdot 25\ kg$

According to the question

$\Rightarrow 112 = 1401 \times 0 \cdot 25$

$$\Rightarrow l = \frac{112}{140 \times 0 \cdot 25} = 3 \cdot 2 cm$$

16. A box with lid is made of 2 cm thick wood. Its external length, breadth and height are 25 cm, 18 cm and 15 cm respectively. How much cubic cm of a liquid can be placed in it? Also, find the volume of the wood used in it.

Sol:

Given external dimensions of cuboid are

$l = 25 cm, b = 18 cm, h = 15 cm.$

$\therefore$ External volume $= l \times b \times h$

$= 25 \times 18 \times 15 cm^3$

$= 6750 cm^3.$

Internal dimension of cuboid.

l= $25 - 2 \times$ thickness=25 – 4 =21 cm.

h= 15 – 4 = 11 cm.

Internal volume $= l \times b \times h$

$= 21\times14\times11 cm^3$

$= 3234 cm^3$

$\therefore$ Volume of liquid that can be placed $= 3234 cm^3$

Now, volume of wood = external volume – Internal volume

$= 6750 - 3324$

$= 3516 cm^3$

17. The external dimensions of a closed wooden box are 48 cm, 36 cm, 30 cm. The box is made of 1.5 cm thick wood. How many bricks of size 6 cm x 3 cm x 0.75 cm can be put in this box?

Sol:

Given internal dimensions are

$l = 48 - 2\times \text{thickness} = 48 - 3 = 45 cm$

$b = 36 - 3 = 33 cm$

$h = 30 - 3 = 27 cm$

$\therefore$ Internal volume $= 45\times33\times27 cm^3$

Volume of brick $= 5\times3\times0\cdot75 cm^3$

Hence, number of bricks $= \dfrac{\text{Internal volume}}{\text{volume of 1 brick}}$

$= \dfrac{45\times33\times27}{6\times3\times0\cdot37}$

$= \dfrac{38880}{13\cdot5}$

$= 2970$

$\therefore 2970$ bricks can be kept inside the box

18. How many cubic centimeters of iron are there in an open box whose external dimensions are 36 cm, 25 cm and I 6.5 cm, the iron being 1.5 cm thick throughout? If I cubic cm of iron weighs 15g, find the weight of the empty box in kg.

Sol:

Outer dimensions

$l = 36\ cm$

$b = 25\ cm$

$h = 16\cdot5 cm$

Inner dimensions

$l = 36 - \left(2\times1\cdot5\right) = 33 cm$

$b = 25 - (3) = 22cm$

$h = 16 \cdot 5 - 1 \cdot 5 = 15cm$

Volume of iron = outer volume – inner volume

$= (36 \times 25 \times 16 \cdot 5 - 33 \times 12 \times 15)cm^3 = 3960cm^3$

Weight of iron $= 3960 \times 1 \cdot 5gm = 59400gm = 59 \cdot 4kg$

19. A cube of 9 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base are 15 cm and 12 cm, find the rise in water level in the vessel.

Sol:

Volume of cube $= S^3 = 9^3 = 729cm^3$

Area of base $l \times b = 15 \times 12 = 180cm^2$

Rise in water level $= \dfrac{\text{Volume of cube}}{\text{Area of base of rectangular vessel}}$

$= \dfrac{729}{180} = 4 \cdot 05cm$

20. A rectangular container, whose base is a square of side 5 cm, stands on a horizontal table, and holds water up to 1 cm from the top. When a cube is placed in the water it is completely submerged, the water rises to the top and 2 cubic cm of water overflows. Calculate the volume of the cube and also the length of its edge.

Sol:

Let the length of each edge of the cube be x cm

Then,

Volume of cube = volume of water inside the tank + volume of water that over flowed

$x^3 = (5 \times 5 \times 1) + 2 = 25 + 2$

$x^3 = 27$

$x = 3cm$

Hence, volume of cube = $27cm^3$

And edge of cube $= 3cm.$

21. A field is 200 m long and 150 m broad. There is a plot, 50 m long and 40 m broad, near the field. The plot is dug 7 m deep and the earth taken out is spread evenly on the field. By how many meters is the level of the field raised? Give the answer to the second place of decimal.

Sol:

Volume of earth dug out $= 50 \times 40 \times 7\, m^3$

$= 14000m^3$

Let the height of the field rises by h meters

$\therefore$ volume of filed (cuboidal) = Volume of earth dugout

$\Rightarrow 200\times150\times h=14000$

$\Rightarrow h=\dfrac{1400}{200\times150}=0\cdot47m.$

22. A field is in the form of a rectangle of length 18 m and width 15 m. A pit, 7.5 m long, 6 m broad and 0.8 m deep, is dug in a corner of the field and the earth taken out is spread over the remaining area of the field. Find out the extent to which the level of the field has been raised.

Sol:

Let the level of the field be risen by h meters volume of the earth taken out from the pit

$=7\cdot5\times6\times0\cdot8m^3$

Area of the field on which the earth taken out is to be spread (X)

$=18\times15-7\cdot5\times6=225m^2$

Now, area of the field $Xh=$ volume of the earth taken out from the pit

$\Rightarrow 225\times h=7\cdot5\times6\times0\cdot8$

$\Rightarrow h=\dfrac{36}{225}=0\cdot16m=16cm.$

23. A rectangular tank is 80 m long and 25 m broad. Water flows into it through a pipe whose cross-section is 25 cm^2, at the rate of 16 km per hour. How much the level of the water rises in the tank in 45 minutes.

Sol:

Let the level of water be risen by *h* cm.

Then,

Volume of water in the tank $=8000\times2500\times hcm^2$

Area of cross – section of the pipe $=25cm^2$.

Water coming out of the pipe forms a cuboid of base area $25cm^2$ and length equal to the distance travelled in 45 minutes with the speed 16km/hour.

i.e., length $=16000\times100\times\dfrac{45}{60}cm$

$\therefore$ Volume of water coming out of pipe in 45 minutes

$=25\times16000\times100\left(\dfrac{45}{60}\right)$

Now, volume of water in the tank $=$ volume of water coming out of the pipe in 45 minutes

$\Rightarrow 8000\times2500\times h=16000\times100\times\dfrac{45}{60}\times25$

$\Rightarrow h=\dfrac{16000\times100\times45\times25}{8000\times2500\times60}cm=1\cdot5cm.$

24. Water in a rectangular reservoir having base 80 m by 60 m i s 6.5 m deep. In what time can the water be emptied by a pipe ôf which the cross-section is a square of side 20 cm, if the water runs through the pipe at the rate of 15 km/hr.

Sol:

Given that,

Flow of water $=15km/hr$

$=15000m/hr.$

Volume of water coming out of the pipe in one hour

$=\frac{20}{100}\times\frac{20}{100}\times\pm5000=600m^3$

Volume of the tank $=80\times60\times6\cdot5$

$=31200m^3$

$\therefore$ Time taken to empty the tank

$=\frac{\text{Volume of tank}}{\text{volume of water coming out of the pipe in one hour}}$

$=\frac{31200}{600}$

$=52\ hours.$

25. A village having a population of 4000 requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Sol:

Given that

Length of the cuboidal tank $(l)=20m$

Breath of the cuboidal tank $(b)=15m.$

Height of cuboidal tank (h)=6 m

Height of the tank $=l\times b\times h=(20\times15\times6)m^3$

$=1800m^3$

$=1800000$ liters.

Water consumed by people of village in one day

$=4000\times150\ litres.$

$=600000\ litres.$

Let water of this tank lasts for n days

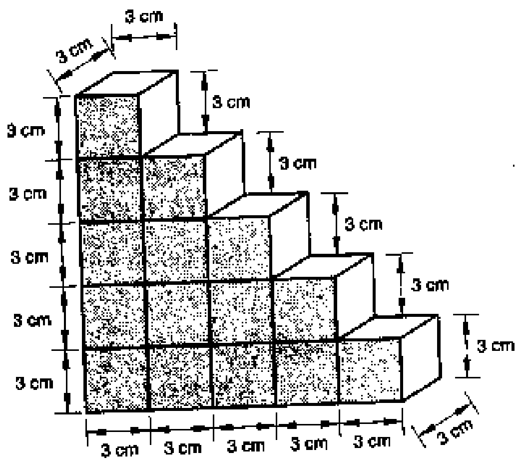
Water consumed by all people of village in n days = capacity of tank

$n\times600000=1800000$

$n=3$

Thus, the water of tank will last for 3 days.

26. A child playing with building blocks, which are of the shape of the cubes, has built a structure as shown in Fig. 18.12 If the edge of each cube is 3 cm, find the volume of the structure built by the child.



Sol:

Volume of each cube = edge $\times$ edge $\times$ edge

$=3\times3\times3cm^3=27cm^3$.

Number of cubes in the surface structure $=15$

$\therefore$ Volume of the structure $=27\times15cm^3$

$=405cm^3$.

27. A godown measures 40m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Sol:

Given go down length $(l_1)=40m$.

Breath $(b_1)=25m$.

Height $(h_1)=10m$.

Volume of wooden crate $=l_1\times b_1\times h_1=40\times25\times10m^3$

$=10000m^3$

Wood of wooden crate $=l_2\times b_2\times h_2$

$=1\cdot5\times1\cdot25\times0\cdot25m^3=0\cdot9375m^3$

Let m wooden creates be stored in the go down volume of m wood crates = volume of go down

$0\cdot9375\times n=10000$

$n=\dfrac{10000}{0\cdot9375}=10,666\cdot66,$

Thus, $10,666\cdot66$ wooden crates can be stored in go down.

28. A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, How many bricks would be required?

Sol:

Given that

The wall with all its bricks makes up the space occupied by it we need to find the volume of the wall, which is nothing but cuboid.

Here, length $=10m=1000cm$

Thickness $=24cm$

Height $=4m=400cm$

$\therefore$ The volume of the wall

= length × breadth × height

$=1000\times 24\times 400cm^3$

Now, each brick is a cuboid with length = 24cm,

Breadth $=12cm\ and\ height=8cm.$

So, volume of each brick = length $\times$breadth $\times$height

$=24\times 12\times 8cm^3.$

So, number of bricks required $=\dfrac{\text{Volume of the wall}}{\text{Volume of each brick}}$

$=\dfrac{1000\times 24\times 400}{24\times 12\times 8}$

$=4166\cdot 6.$

So, the wall requires 167 bricks.