2. Powers

Exercise 2.1

Solution 1:

Express each of the following as a rational number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

(i) 2^{-3}

we know that, if a non-zero rational number and n is a positive integer, then

$$a^{-n} = \frac{1}{a^n}$$

Thus, we have

$$2^{-3} = \frac{1}{2^3}$$

$$= \frac{1}{8}$$

(ii) $(-4)^{-2}$

Thus, we have

$$(-4)^{-2} = \frac{1}{(-4)^2}$$

$$= \frac{1}{16}.$$

(iii) $\frac{1}{3^{-2}}$

Thus, we have

$$\frac{1}{3^{-2}} = 3^2 = 9.$$

(iv) $\left(\frac{1}{2}\right)^{-5}$.

$\left(\frac{1}{2}\right)^{-5} = \frac{1}{\left(\frac{1}{2}\right)^5} = \frac{2^5}{1^5}$ $\left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right.$ when n is a whole number$\left.\right]$.

$= 32$.

(v) $\left(\frac{2}{3}\right)^{-2}$

$\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{3^2}{2^2}$ $\left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right.$ when n is a whole number$\left.\right]$.

$= \frac{9}{4}$

Solution 2:

Find the values of each of the following

(i) $3^{-1}+4^{-1}$

Thus, we have

$3^{-1}+4^{-1} = \frac{1}{3}+\frac{1}{4}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

$= \frac{4+3}{12} = \frac{7}{12}$.

(ii) $(3^0+4^{-1}) \times 2^2$.

Thus, we have

$\left(1+\frac{1}{4}\right)\times 2^2 = \left(\frac{4+1}{4}\right)\times 4 = 5$.

$(3^0+4^{-1}) \times 2^2 = 5$.

(iii) $(3^{-1}+4^{-1}+5^{-1})^0$

Thus, we have

$(3^{-1}+4^{-1}+5^{-1})^0 = \left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)^0$

$= \left(\frac{20+15+12}{60}\right)^0$

$= \left(\frac{47}{60}\right)^0$

$= 1$ $[\because a^0 = 1]$

(iv) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

Thus, we have

$\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \left\{\left(\frac{1}{\frac{1}{3}}\right) - \left(\frac{1}{\frac{1}{4}}\right)\right\}^{-1}$

$= \{3-4\}^{-1}$

$= \{-1\}^{-1}$

$= \frac{1}{-1}$

$= -1$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

Solution 3:

(i) $\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\left(\frac{1}{4}\right)^{-1}$

Thus, we have

$$\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-1}+\left(\frac{1}{4}\right)^{-1}=\left[\frac{1}{\frac{1}{2}}+\frac{1}{\frac{1}{3}}+\frac{1}{\frac{1}{4}}\right]$$

$$= 2+3+4 \qquad \left[\because a^{-n}=\frac{1}{a^n}\right]$$

$$= 9.$$

(ii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

Thus, we have

$$\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}=\frac{1}{\frac{1}{2^2}}+\frac{1}{\frac{1}{3^2}}+\frac{1}{\frac{1}{4^2}}$$

$$= 4+9+16$$

$$\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}= 29.$$

(iii) $(2^{-1}\times 4^{-1})\div 2^{-2}$

Thus, we have

$$(2^{-1}\times 4^{-1})\div 2^{-2}=\left(\frac{1}{2}\times\frac{1}{4}\right)\times 2^2$$

$$=\frac{1}{2} \qquad \left[\because \frac{1}{a^{-n}}=a^n\right]$$

(iv) $(5^{-1}\times 2^{-1})\div 6^{-1}$

Thus, we have $(5^{-1}\times 2^{-1})\div 6^{-1}=\left(\frac{1}{5}\times\frac{1}{2}\right)\div\frac{1}{6}$

$$=\frac{1}{10}\times 6=\frac{3}{5} \quad \left[\because a^{-n}=\frac{1}{a^n}\right]$$

Solution 4:

Simplify

(i) $(4^{-1}\times 3^{-1})^2$

Thus, we have

$$(4^{-1}\times 3^{-1})^2=\left(\frac{1}{4}\times\frac{1}{3}\right)^2$$

$$=\left(\frac{1}{12}\right)^2$$

$$(4^{-1}\times 3^{-1})^2=\frac{1}{144}.$$

(ii) $(5^{-1}\div 6^{-1})^3$

Thus, we have

$$\left(\frac{1}{5}\div\frac{1}{6}\right)^3=\left(\frac{6}{5}\right)^3$$

$$=\frac{216}{125}.$$

(iii) $(2^{-1}+3^{-1})^{-1}$

Thus, we have

$$(2^{-1}+3^{-1})^{-1}=\left(\frac{1}{2}+\frac{1}{3}\right)^{-1}=\left(\frac{3+2}{6}\right)^{-1}$$

$$=\frac{6}{5}. \qquad \left[\because a^{-n}=\frac{1}{a^n}\right]$$

(iv) $(3^{-1}\times 4^{-1})^{-1}\times 5^{-1}$

Thus, we have

$$(3^{-1}\times 4^{-1})^{-1}\times 5^{-1}=\left(\frac{1}{3}\times\frac{1}{4}\right)^{-1}\times 5^{-1}=\left(\frac{1}{12}\right)^{-1}\times 5^{-1}$$

$$=\frac{12}{5} \qquad \left[\because a^{-n}=\frac{1}{a^n}\right]$$

Solution 5:

Simplify.

(i) $(3^2+2^2)\times\left(\frac{1}{2}\right)^3$

Thus, we have

$$(3^2+2^2)\times\left(\frac{1}{2}\right)^3 = (9+4)\times\frac{1}{8}$$

$$= \frac{13}{8}$$

(ii) $(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3}$

Thus, we have

$$(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3} = (9-4)\times\frac{1}{\left(\frac{2}{3}\right)^3}$$

$$= \frac{5\times 3^3}{2^3}$$

$$(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3} = \frac{135}{8}$$

(iii) $\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right]\div\left(\frac{1}{4}\right)^{-3}$

Thus, we have

$$\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right]\div\left(\frac{1}{4}\right)^{-3} = \left[3^3-2^3\right]\div 4^3$$

$$= \frac{27-8}{64} = \frac{27-8}{64} = \frac{19}{64}.$$

(iv) $(2^2+3^2-4^2)\div\left(\frac{3}{2}\right)^2$

Thus, we have

$$(2^2+3^2-4^2)\div\left(\frac{3}{2}\right)^2 = \frac{(4+9-16)}{\left(\frac{3}{2}\right)^2} = \frac{-3\times\cancel{8}\,4}{\cancel{27}\,9} = \frac{-4}{3}$$

$$(2^2+3^2-4^2)\div\left(\frac{3}{2}\right)^2 = \frac{-4}{3}.$$

Solution 6:

By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$

Let 5^{-1} be multiplied by x to get $(-7)^{-1}$. Then

$$x \times 5^{-1} = (-7)^{-1}$$

$$\frac{x}{5} = \frac{1}{-7}$$

By cross mutiplicating, we get

$$x = \frac{5}{-7}$$

Hence, the required number is $\frac{-5}{7}$.

By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product may be equal to $\left(\frac{4}{7}\right)^{-1}$

Let $\left(\frac{1}{2}\right)^{-1}$ be multiplied by x to get $\left(\frac{4}{7}\right)^{-1}$. Then.

$$x \times \left(\frac{1}{2}\right)^{-1} = \left(\frac{4}{7}\right)^{-1}$$

$$\frac{x}{\frac{1}{2}} = \frac{1}{\frac{4}{7}}$$

$$x = \frac{\frac{1}{2}}{\frac{4}{7}} = \frac{1}{2} \times \frac{7}{4} \quad \left[\because \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}\right]$$

$$x = \frac{7}{8}$$

$\therefore$ Hence, the required number is $\frac{7}{8}$.

Solution 7:

By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$

Let the number $(-15)^{-1}$ be divided by x to get $(-5)^{-1}$. Then,

$$(-15)^{-1} \div x = (-5)^{-1}$$

$$\Rightarrow \frac{1}{(-15)^{1}(x)} = \frac{1}{-5^{1}} \quad \left[\because a^{-n} = \frac{1}{a^{n}}\right]$$

$$\Rightarrow x = \frac{-5^{1}}{-15^{1}}$$

$$\Rightarrow x = \frac{-5}{-15}$$

$$\Rightarrow x = \frac{1}{3}$$

Hence, the required number is $\frac{1}{3}$.

Solution 8:

Solution 8:

Using the property $a^{-1} = \frac{1}{a}$ for every natural number 'a', we have $(-15)^{-1} = \frac{-1}{15}$ and $(-5)^{-1} = -\frac{1}{5}$.

We have to find a number x such that

$$\frac{\frac{-1}{15}}{\frac{x}{1}} = \frac{-1}{5}$$

$$\Rightarrow \frac{-1}{15} \times \frac{1}{x} = \frac{-1}{5}$$

$$\Rightarrow x = \frac{1}{3}$$

Hence, $(-15)^{-1}$ should be divided by $\frac{1}{3}$ to obtain $(-5)^{-1}$

Exercise 2.2

Solution 1:

Exercise - 2.2.

write each of the following in the exponential form

(i) $\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1}$

Thus, we have

$$\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} = \frac{1}{\frac{3}{2}} \times \frac{1}{\frac{3}{2}} \times \frac{1}{\frac{3}{2}} \times \frac{1}{\frac{3}{2}}$$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}.$$

$$= \left(\frac{2}{3}\right)^{1+1+1+1} \quad [\because a^m \times a^n = a^{m+n}]$$

$$= \underline{\underline{\left(\frac{2}{3}\right)^4}}$$

$$= \frac{2^4}{3^4} \quad \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$$

$$= \frac{16}{81}. = \left(\frac{3}{2}\right)^{-4}.$$

(ii) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2}$

Thus, we have

$$\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} = \left(\frac{2}{5}\right)^{-2+-2+-2}$$

$$= \left(\frac{2}{5}\right)^{-6}. \quad [\because a^m \times a^m \times a^m = a^{3m}]$$

Solution 2:

Evaluate :

(i) 5^{-2}

Thus, we have

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25} \qquad [\because a^{-n} = \frac{1}{a^n}]$$

(ii) $(-3)^{-2}$

Thus, we have.

$$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9} \qquad [\because a^{-n} = \frac{1}{a^n}]$$

(iii) $\left(\frac{1}{3}\right)^{-4}$

Thus, wehave

$$\left(\frac{1}{3}\right)^{-4} = \frac{1}{\left(\frac{1}{3}\right)^4} = 3^4 = 81. \qquad [\because a^{-n} = \frac{1}{a^n}]$$

(iv) $\left(\frac{-1}{2}\right)^{-1}$

Thus, we have

$$\left(\frac{-1}{2}\right)^{-1} = \frac{1}{\left(-\frac{1}{2}\right)} \qquad [\because a^{-n} = \frac{1}{a^n}]$$

$$= \frac{-2}{1}$$

$$\therefore \left(-\frac{1}{2}\right)^{-1} = -2.$$

Solution 3:

Express each of the following as a rational number in the form $\frac{p}{q}$:

(i) 6^{-1}

Thus, we have

$$6^{-1} = \frac{1}{6} \qquad [\because a^{-n} = \frac{1}{a^n}]$$

(ii) $(-7)^{-1}$

Thus, we have

$$(-7)^{-1} = \frac{1}{(-7)^1} = \frac{1}{-7} \qquad [\because a^{-n} = \frac{1}{a^n}]$$

(iii) $\left(\frac{1}{4}\right)^{-1}$

Thus, we have

$$\left(\frac{1}{4}\right)^{-1} = \frac{1}{\frac{1}{4}} = 4. \qquad [\because a^{-n} = \frac{1}{a^n}]$$

(iv) $(-4)^{-1} \times \left(\frac{-3}{2}\right)^{-1}$

Thus, we have

$$(-4)^{-1} \times \left(-\frac{3}{2}\right)^{-1} = \frac{1}{-4} \times \frac{1}{\left(-\frac{3}{2}\right)} = \frac{2}{12} = \frac{1}{6}.$$

(v) $\left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1}$

Thus, we have

$$\left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1} = \frac{1}{\frac{3}{5}} \times \frac{1}{\frac{5}{2}} = \frac{\cancel{5}}{3} \times \frac{2}{\cancel{5}} = \frac{2}{3}.$$

Solution 4:

Simplify.

(i) $[4^{-1} \times 3^{-1}]^2$

Thus, we have

$$[4^{-1} \times 3^{-1}]^2 = \left[\frac{1}{4} \times \frac{1}{3}\right]^2$$

$$= \left(\frac{1}{12}\right)^2$$

$$(4^{-1} \times 3^{-1})^2 = \frac{1}{144}.$$

(ii) $(5^{-1} \div 6^{-1})^3$

Thus, we have

$$[5^{-1} \div 6^{-1}]^3 = \left[\frac{1}{5} \div \frac{1}{6}\right]^3$$

$$= \left[\frac{6}{5}\right]^3$$

$$= \frac{216}{125}.$$

(iii) $(2^{-1} + 3^{-1})^{-1}$

Thus, we have

$$(2^{-1} + 3^{-1})^{-1} = \left(\frac{1}{2} + \frac{1}{3}\right)^{-1}$$

$$= \left(\frac{3+2}{6}\right)^{-1}$$

$$= \left(\frac{5}{6}\right)^{-1}$$

$$= \frac{6}{5}.$$

(iv) $\{3^{-1} \times 4^{-1}\}^{-1} \times 5^{-1}$

Thus, we have

$$\left\{\frac{1}{3} \times \frac{1}{4}\right\}^{-1} \times 5^{-1} = \left\{\frac{1}{12}\right\}^{-1} \times \frac{1}{5}$$

$$= \frac{12}{5}.$$

(v) $(4^{-1} - 5^{-1}) \div 3^{-1}$

Thus, we have

$$(4^{-1} - 5^{-1}) \div 3^{-1} = \left(\frac{1}{4} - \frac{1}{5}\right) \div \frac{1}{3}$$

$$= \left(\frac{5-4}{20}\right) \div \frac{1}{3}$$

$$= \frac{\frac{1}{20}}{\frac{1}{3}}$$

$$= \frac{3}{20} \qquad \left[\because \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}\right]$$

$$(4^{-1} - 5^{-1}) \div 3^{-1} = \frac{3}{20}.$$

Solution 5:

Express each of the following rational numbers with a negative exponent.

(i) $\left(\frac{1}{4}\right)^3$

Thus, we have

$$\left(\frac{1}{4}\right)^3 = \frac{1}{\left(\frac{1}{4}\right)^{-3}} = 4^{-3} \qquad \left[\because a^{-n} = \frac{1}{a^n} \text{ \& } \frac{1}{\left(\frac{a}{b}\right)^n} = \left(\frac{b}{a}\right)^n\right]$$

(ii) 3^5

Thus, we have

$$3^5 = \frac{1}{3^{-5}} \qquad \left[\because \frac{1}{a^{-n}} = a^n\right]$$

(iii) $\left(\frac{3}{5}\right)^4$

Thus, we have

$$\left(\frac{3}{5}\right)^4 = \left(\frac{5}{3}\right)^{-4} \qquad \left[\because \left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}\right]$$

(iv) $\left\{\left(\frac{3}{2}\right)^4\right\}^{-3}$

Thus, we have

$$\left\{\left(\frac{3}{2}\right)^4\right\}^{-3} = \left(\frac{3}{2}\right)^{4\times(-3)} = \left(\frac{3}{2}\right)^{-12} \qquad \left[\because (a^m)^n = a^{mn}\right]$$

(v) $\left\{\left(\frac{7}{3}\right)^4\right\}^{-3} \rightarrow \left\{\left(\frac{7}{3}\right)^4\right\}^{-3} = \left(\frac{7}{3}\right)^{4\times -3} = \left(\frac{7}{3}\right)^{-12}$.

Solution 6:

Express each of the following rational numbers with a positive exponent.

(i) $\left(\frac{3}{4}\right)^{-2}$

Thus, we have

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

(ii) $\left(\frac{5}{4}\right)^{-3}$

Thus, we have

$$\left(\frac{5}{4}\right)^{-3} = \left(\frac{4}{5}\right)^3$$

(iii) $4^3 \times 4^{-9}$.

Thus, we have.

$$4^3 \times 4^{-9} = 4^{3-9}$$

$$= 4^{-6} \qquad \left[\because a^m \times a^n = a^{m+n}\right]$$

$$= \left(\frac{1}{4}\right)^6$$

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4}$

Thus, we have

$$\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4} = \left\{\left(\frac{4}{3}\right)^{3\times 4}\right\} = \left(\frac{4}{3}\right)^{12}.$$

(v) $\left\{\left(\frac{3}{2}\right)^4\right\}^{-2}$

Thus, we have

$$\left\{\left(\frac{3}{2}\right)^4\right\}^{-2} = \left\{\left(\frac{3}{2}\right)^{4\times(-2)}\right\} = \left(\frac{3}{2}\right)^{-8} = \left(\frac{2}{3}\right)^8.$$

Solution 7:

Simplify.

(i) $\left\{\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$

Thus, we have.

$$\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3} = \frac{[3^3-2^3]}{4^3}$$

$$= \frac{27-8}{64}$$

$$= \frac{19}{64}.$$

(ii) $(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3}$.

Thus, we have

$$(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3} = (9-4)\times\left(\frac{3}{2}\right)^3$$

$$= \frac{5\times 27}{2^3}$$

$$= \frac{135}{2^3}$$

$$= \frac{135}{8}.$$

(iii) $\left\{\left(\frac{1}{2}\right)^{-1}\times 4^{-1}\right\}^{-1}$

Thus, we have

$$\left\{\left(\frac{1}{2}\right)^{-1}\times 4^{-1}\right\}^{-1} = \left\{2\times\left(\frac{1}{4}\right)\right\}^{-1} = \left\{\frac{-1}{2}\right\}^{-1}$$

$$= \frac{-1}{\frac{1}{2}} = -2.$$

(iv) $\left[\left\{\left(\frac{-1}{4}\right)^2\right\}^{-2}\right]^1$.

Thus, we have

$$\left[\left\{\left(\frac{-1}{4}\right)^2\right\}^{-2}\right]^1 = \left(\frac{-1}{4}\right)^{2(-2)(1)} \quad [\because (a^m)^n = a^{mn}]$$

$$= \left(\frac{-1}{4}\right)^{+4} \quad [\because ((a)^m)^n)^l = a^{mnl}]$$

$$= \frac{1}{\left(\frac{+1}{4}\right)^4}$$

$$= 4^4$$

$$= \frac{1}{256}.$$

(v) $\left\{\left(\frac{2}{3}\right)^2\right\}^3 \times \left(\frac{1}{3}\right)^{-4}\times 3^{-1}\times 6^{-1}$

Thus, we have

$$\left(\frac{2}{3}\right)^{2\times 3}\times\left(\frac{1}{3}\right)^{-4}\times 3^{-1}\times 6^{-1} = \frac{2^6}{3^6}\times 3^4\times\frac{1}{3}\times\frac{1}{6}$$

$$= \frac{2^6\times 3^4}{3^5\times 3\times 3\times 2}$$

$$= \frac{2\times 2^5\times 3^4}{3^4\times 3^4\times 2}$$

$$= \frac{2^5}{3^4} = \frac{32}{81}$$

Solution 8:

Simplify.

(i) $(3^2+2^2)\times\left(\frac{1}{2}\right)^3$

Thus, we have

$$(3^2+2^2)\times\left(\frac{1}{2}\right)^3 = (9+4)\times\frac{1}{8}$$

$$= \frac{13}{8}.$$

(ii) $(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3}$

Thus, we have

$$(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3} = (9-4)\times\frac{1}{\left(\frac{2}{3}\right)^3}$$

$$= \frac{5\times3^3}{2^3}$$

$$(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3} = \frac{135}{8}.$$

(iii) $\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right]\div\left(\frac{1}{4}\right)^{-3}$

Thus, we have

$$\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right]\div\left(\frac{1}{4}\right)^{-3} = [\,3^3-2^3]\div4^3$$

$$= \frac{\cancel{91}-\cancel{9}}{64} = \frac{27-8}{64} = \frac{19}{64}.$$

(iv) $(2^2+3^2-4^2)\div\left(\frac{3}{2}\right)^{\cancel{2}}$

Thus, we have

$$(2^2+3^2-4^2)\div\left(\frac{3}{2}\right)^{\cancel{2}} = \frac{(4+9-16)}{\left(\frac{3}{2}\right)^3} = \frac{-3\times\cancel{8}4}{\cancel{2}9} = \frac{-4}{3}$$

$$(2^2+3^2-4^2)\div\left(\frac{3}{2}\right)^2 = \frac{-4}{3}.$$

Solution 9:

By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$

Let 5^{-1} be multiplied by x to get $(-7)^{-1}$. Then

$$x\times5^{-1} = (-7)^{-1}$$

$$\frac{x}{5} = \frac{1}{-7}$$

By cross mutiplicating, we get.

$$x = \frac{5}{-7}$$

Hence, the required number is $\frac{-5}{7}$.

By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product may be equal to $\left(\frac{4}{7}\right)^{-1}$

Let $\left(\frac{1}{2}\right)^{-1}$ be multiplied by x to get $\left(\frac{4}{7}\right)^{-1}$. Then.

$$x\times\left(\frac{1}{2}\right)^{-1} = \left(\frac{4}{7}\right)^{-1}$$

$$\frac{x}{\frac{1}{2}} = \frac{1}{\frac{4}{7}}$$

$$x = \frac{\frac{1}{2}}{\frac{4}{7}} = \frac{1}{2}\times\frac{7}{4} \quad \left[\because \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\times\frac{d}{c}\right]$$

$$x = \frac{7}{8}$$

$\therefore$ Hence, the required number is $\frac{7}{8}$.

Solution 10:

By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$

Let the number $(-15)^{-1}$ be divided by x to get $(-5)^{-1}$. Then,

$(-15)^{-1} \div x = (-5)^{-1}$

$\Rightarrow \frac{1}{(-15)^{1}(x)} = \frac{1}{-5^{1}}$ $\quad [\because a^{-n} = \frac{1}{a^{n}}]$

$\Rightarrow x = \frac{-5^{1}}{-15^{1}}$

$\Rightarrow x = \frac{-5}{-15}$

$\Rightarrow x = \frac{1}{3}$

Hence, the required number is $\frac{1}{3}$.

Solution 11:

By what number should $\left(\frac{5}{3}\right)^{-2}$ be multiplied so that the product may be $\left(\frac{7}{3}\right)^{-1}$?

Let the required number be x. Then

$\left(\frac{5}{3}\right)^{-2} \times x = \left(\frac{7}{3}\right)^{-1}$

$\left(\frac{3}{5}\right)^{2} \times x = \left(\frac{3}{7}\right)$

$\frac{9}{25} x = \frac{3}{7}$

$x = \frac{3 \times 25}{7 \times 9}$

$x = \frac{25}{21}$.

Hence, required number is $\frac{25}{21}$

Solution 12:

12. Find x, if.

(i) $\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$

$$\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$$

$$\left(\frac{1}{4}\right)^{-4-8} = \left(\frac{1}{4}\right)^{-4x}$$

$$+12 = +4x \qquad [\because a^m = a^n \; \therefore m = n]$$

$$x = \frac{12}{4}$$

$$x = 3.$$

(ii) $\left(\frac{-1}{2}\right)^{-19} \div \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{2x+1}$

$$\left(\frac{-1}{2}\right)^{-19} \div \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{2x+1}$$

$$\left(\frac{-1}{2}\right)^{-19-8} = \left(\frac{-1}{2}\right)^{2x+1}$$

$$\left(\frac{-1}{2}\right)^{-27} = \left(\frac{-1}{2}\right)^{2x+1}$$

$$2x+1 = -27$$

$$2x = -27-1$$

$$x = \frac{-27-1}{2}$$

$$x = -14.$$

(iii) $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$

$$\left(\frac{3}{2}\right)^{-3+5} = \left(\frac{3}{2}\right)^{2x+1}$$

$$2x+1 = 5-3$$

$$2x = 2-1$$

$$2x = 1$$

$$x = \frac{1}{2}.$$

(iv) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$

$$\left(\frac{2}{5}\right)^{15-3} = \left(\frac{2}{5}\right)^{2+3x}$$

$$3x+2 = 12$$

$$3x = 12-2$$

$$x = \frac{10}{3}$$

(v) $\left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^{5}$

$$\frac{\left(\frac{5}{4}\right)^{-x}}{\left(\frac{5}{4}\right)^{-4}} = \left(\frac{5}{4}\right)^{5} \Rightarrow \left(\frac{5}{4}\right)^{-x} = \left(\frac{5}{4}\right)^{5} \times \left(\frac{5}{4}\right)^{-4}$$

$$\Rightarrow \left(\frac{5}{4}\right)^{-x} = \left(\frac{5}{4}\right)^{5-4}$$

$$\Rightarrow -x = 1$$

$$\Rightarrow x = -1.$$

(vi) $\left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$

$$\left(\frac{8}{3}\right)^{2x+1+5} = \left(\frac{8}{3}\right)^{x+2}$$

$$2x+6 = x+2$$

$$2x-x = 2-6$$

$$x = -4.$$

Solution 13:

13. (i) If $x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$, find the value of x^{-2}.

We have, $x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$

$$\Rightarrow x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{3}{2}\right)^{4} \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$$

$$\Rightarrow x = \left(\frac{3}{2}\right)^{2+4}$$

$$\Rightarrow x = \left(\frac{3}{2}\right)^{6}$$

$$\therefore x^{-2} = \left(\left(\frac{3}{2}\right)^{6}\right)^{-2} = \left(\frac{3}{2}\right)^{6\times(-2)} = \left(\frac{3}{2}\right)^{-12} = \left(\frac{2}{3}\right)^{12}.$$

(ii) If $x = \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$, find the value of x^{-1}

we have,

$$x = \left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$$

$$\Rightarrow x = \left(\frac{5}{4}\right)^{2} \div \left(\frac{1}{4}\right)^{2} \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$$

$$\Rightarrow x = \frac{\frac{5^2}{4^2}}{\frac{1^2}{4^2}} \qquad \left[\because \left(\frac{a}{b}\right)^{m} = \frac{a^m}{b^m}\right]$$

$$\Rightarrow x = 25 \qquad \therefore x^{-1} = \frac{1}{25}.$$

Solution 14:

Find the value of x for which $5^{2x} \div 5^{-3} = 5^{5}$

Thus, we have

$$\Rightarrow 5^{2x} \div 5^{-3} = 5^{5}$$

$$\Rightarrow \frac{5^{2x}}{5^{-3}} = 5^{5}$$

$$\Rightarrow 5^{2x} = 5^{5} \times 5^{-3}$$

$$\Rightarrow 5^{2x} = 5^{5-3} \qquad [\because a^m \times a^n = a^{m+n}]$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1.$$

Hence, the required value is '1'

Exercise 2.3

Solution 1:

Express the following numbers in the standard form.

(i) 602000000000000000.

$602000000000000000 = 6.02 \times 10^{15}$

[∵ The decimal point is moved 15 Places to the Left]

(ii) 0.00000000000942.

To express 0.00000000000000492 in standard form the decimal point is moved through one place only to the right so that there is just one digit on the Left of the decimal point.

$0.00000000000942 = 9.42 \times 10^{-12}$.

(iii) 0.00000000085.

$0.00000000085 = 8.5 \times 10^{-10}$

[∵ The decimal point is moved 10 Places to the right]

(iv) 846×10^{7}

$846 \times 10^{7} = 8.46 \times 10^{9}$

[∵ The decimal point is moved two Places to the right]

(v) 3759×10^{-4}.

$3759 \times 10^{-4} = 3.759 \times 10^{-1}$

[The decimal point is moved Three places to the right].

(vi) 0.00072984.

$0.00072984 = 7.2984 \times 10^{-4}$

[The decimal point moved four Places to the Right]

(vii) 0.000437×10^{4}.

$0.000437 \times 10^{4} = 4.37$.

[The decimal point moved 4 places to the Right].

(viii) $4 \div 100000$

$4 \div 100000 = 4 \times 10^{-5}$. [∵ $\frac{4}{10^5} = 4 \times 10^{5}$ & $10^5 = 1,00,000$ & $\frac{1}{a^n} = a^{-n}$].

Solution 2:

write the following numbers in the usual form

(i) 4.83×10^7

$4.83 \times 10^7 = 48300000$

(ii) 3.02×10^{-6}

$\frac{3.02}{1000000} = 0.00000302.$

(iii) 4.5×10^4

$4.5 \times 10000 = 45000.$

(iv) 3×10^{-8}

$\frac{3}{100000000} = 0.00000003.$

(v) 1.0001×10^9

$1.0001 \times 1000000000 = 1000100000$

[∵ The decimal point ~~Point~~ is moved 9 places to the right]

(vi) 5.8×10^2

$5.8 \times 100 = 580$

[The decimal point is moved 2 places to the right]

(vii) 3.61492×10^6

$3.61492 \times 10^6 = 3614920.$

(viii) 3.25×10^{-7}

$3.25 \times 10^{-7} = 0.000000325.$