Exercise – 21.1

1. Find the surface area of a sphere of radius:
(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm
Sol:
(i) Given radius $= 10 \cdot 5 cm$

$\boxed{\text{Surface area} = 4\pi r^2}$

$= 4 \times \frac{22}{7} \times (10 \cdot 5)^2$

$= 1386 cm^2$

(ii) Given radius $= 5 \cdot 6 cm$

Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (5 \cdot 6)^2 = 394 \cdot 24 cm^2$

(iii) Given radius = 14cm

Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (14)^2 = 2464 cm^2$

2. Find the surface area of a sphere of diameter:
(i) 14 cm (ii) 21 cm (iii) 3.5 cm
Sol:
(i) Diameter = 14cm

Radius $= \frac{Diameter}{2} = \frac{14}{2} = 7 cm$

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2 = 616 cm^2$

(ii) Diameter = 21cm

Radius $= \frac{Diameter}{2} = \frac{21}{2} = 10 \cdot 5 cm$

$\therefore$ Surface area $= 4\pi r^2 = 4\pi \times (10 \cdot 5)^2 = 4 \times \frac{22}{7} \times 10 \cdot 5^2 = 1386 cm^2$

(iii) Diameter = $3 \cdot 5 cm$
Radius $= 3 \cdot 5 cm / 2 = 1 \cdot 75 cm$

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{3 \cdot 5}{2^2} = 38 \cdot 5 cm^2$

3. Find the total surface area of a hemisphere and a solid hemisphere each of radius 10 cm. (Use π = 3.14)

Sol:

The surface area of the hemisphere $= 2\pi r^2$

$= 2 \times 3\cdot 14 \times (10)^2$

$= 628 cm^2$

The surface area of solid hemisphere $= 3\pi r^2$

$= 3 \times 3\cdot 14 \times (10)^2$

$= 942 cm^2$

4. The surface area of a sphere is 5544 cm^2, find its diameter.

Sol:

Surface area of a sphere is $5544 cm^2$

$\Rightarrow 4\pi r^2 = 5544$

$\Rightarrow \dfrac{4 \times 22}{7} \times r^2 = 5544$

$\Rightarrow r^2 = \dfrac{5544 \times 7}{88}$

$\Rightarrow r = \sqrt{21cm \times 21cm} = \sqrt{(21)^2} cm$

$\Rightarrow r = 21cm.$

Diameter $= 2$ (radius)

$= 2(21cm)$

$= 42cm.$

5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin- plating it on the inside at the rate of Rs. 4 per 100 cm^2.

Sol:

Given

Inner diameter of hemisphere bowl $= 10\cdot 5cm$

Radius $= \dfrac{10\cdot 5}{2} cm = 5\cdot 25cm.$

Surface area of hemispherical bowl $= 2\pi r$

$= 2\left[\dfrac{22}{7}\right] \times (5\cdot 25)^2 \; cm^2$

$= 173\cdot 25cm^2.$

Cost of tin planning $100cm^2$ area = Rs. 4

Cost of tin planning $173\cdot 25cm^2$ area = $Rs.\left(\frac{4\times 173\cdot 25}{100}\right)$

$= Rs.\ 6\cdot 93$

Thus, The cost of tin plating the inner side of hemisphere bowl is $Rs. 6\cdot 93$

6. The dome of a building is in the form of a hemisphere. Its radius is 63 dm. Find the cost of painting it at the rate of Rs. 2 per sq. m.

Sol:

Dome Radius $= 63d\ m = 6\cdot 3m$

Inner S · A of dome $= 2\pi r^2 = 2\times\frac{22}{7}\times(6\cdot 3)^2 = 249\cdot 48m^2$

Now, cost of $1m^2 = Rs.\ 2.$

$\therefore$ Cost of $249\cdot 48m^2 = Rs.[2\times 249\cdot 48]$

$= Rs.\ 498\cdot 96.$

7. Assuming the earth to be a sphere of radius 6370 km, how many square kilo metres is area of the land, if three-fourth of the earth's surface is covered by water?

Sol:

$\frac{3}{4}^{th}$ of earth surface is covered by water

$\therefore \frac{1}{4}^{th}$ earth surface is covered by land

$\therefore$ Surface area covered by land $= \frac{1}{4}\times 4\pi r^2$

$= \frac{1}{4}\times 4\times\frac{22}{7}\times 6370^2$

$= 127527.4\,km^2$

8. A cylinder of same height and radius is placed on the top of a hemisphere. Find the curved surface area of the shape if the length of the shape be 7 cm.

Sol:

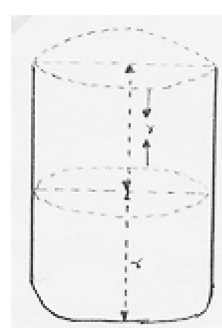
Given length of the shape $= 7cm$

But length $= r + r$

$\Rightarrow 2r = 7cm$

$\Rightarrow r = \frac{7}{2}cm$

$\Rightarrow r = 3\cdot 5cm$



Also; $h=r$

Total S.A of shape $=2\pi rh+2\pi r^2=2\pi r\times r+2\pi r^2$

$=2\pi r^2+2\pi r^2$

$=4\pi r^2$

$=4\times\frac{22}{7}\times(3\cdot5)^2$

$=154cm^2$

9. A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 16 cm and its height is 15 cm. Find the cost of painting the toy at Rs. 7 per 100 cm^2.

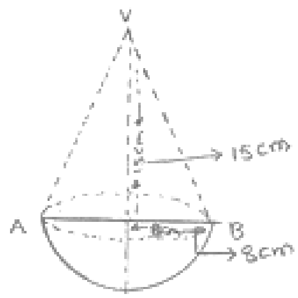
Sol:

Diameter of cone = 16cm.

$\therefore$ Radius of cone = 8cm.

Height of cone = 15cm

Slant height of cone $=\sqrt{8^2+15^2}$



$=\sqrt{64+225}$

$=\sqrt{289}$

$=17cm$

$\therefore$ Total curved surface area of toy

$=\pi rl+2\pi r^2$

$=\frac{22}{7}\times8\times17+2\times\frac{22}{7}\times8^2$

$=\frac{5808}{7}cm^2$

Now, cost of $100cm^2=Rs.7$

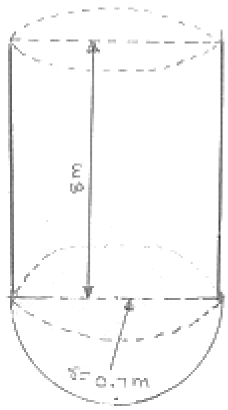
$1cm^2=Rs.\frac{7}{100}$

Hence, cost of $\frac{5808}{7}cm^2=Rs.\left(\frac{5808}{7}\times\frac{7}{100}\right)$

$=Rs.58\cdot08$

10. A storage tank consists of a circular cylinder with a hemisphere adjoined on either end. If the external diameter of the cylinder be 1.4 m and its length be 8 m, find the cost of painting it on the outside at the rate of Rs. 10 per m^2.

Sol:



Diameter of cylinder $= 1 \cdot 4\, m$

$\therefore$ Radius of cylinder $= \frac{1 \cdot 4}{2} = 0 \cdot 7m$

Height of cylinder = 8m.

$\therefore S \cdot A$ of tank $= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 0 \cdot 7 \times 8 + 2 \times \frac{22}{7} \times (0 \cdot 7)^2$

$= \frac{176}{5} + \frac{77}{25}$

$= \frac{957}{25} = 38 \cdot 28 cm^2$

Now, cost of $1m^2 = Rs.\ 10.$

$\therefore$ Cost of $38 \cdot 28m^2 = Rs.[10 \times 38 \cdot 28]$

$= Rs. 382 \cdot 80$

11. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol:

Let the diameter of the earth is d then, diameter of moon will be $\frac{d}{4}$

Radius of earth $= \frac{d}{2}$

Radius of moon $= \frac{\frac{d}{2}}{4} = \frac{d}{8}$

$S \cdot A$ of moon $= 4\pi\left(\frac{d}{8}\right)^2$

Surface area of earth $= 4\pi\left(\frac{d}{2}\right)^2$

Required ratio $= \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} = \frac{4}{64} = \frac{1}{16}$

Thus, the required ratio of the surface areas is $\frac{1}{16}$.

12. A hemi-spherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 cm, find the cost of painting it, given the cost of painting is Rs. 5 per l00 cm^2

Sol:

Given that only the rounded surface of the dome to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done.

Now, circumference of the dome $= 17 \cdot 6m.$

Therefore, $17 \cdot 6 = 2\pi r.$

$2 \times \frac{22}{7} r = 17 \cdot 6m.$

So, the radius of the dome $= 17 \cdot 6 \times \frac{7}{2 \times 22} m = 2 \cdot 8m$

The curved surface area of the dome $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 2 \cdot 8 \times 2 \cdot 8cm^2$

$= 49 \cdot 28m^2$

Now, cost of painting $100cm^2$ is Rs. 5.

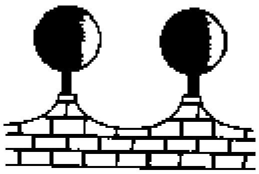
So, cost of painting $1m^2 = Rs.\ 500$

Therefore, cost of painting the whole dome

$= Rs.\ 500 \times 49 \cdot 28$

$= Rs. 24640$

13. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed in small supports as shown in Fig. below. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1 .5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Sol:

Wooden sphere radius $=\left(\frac{21}{2}\right)cm = 10\cdot 5cm.$

Surface area of a wooden sphere

$= 4\pi r^2 = 4\left[\frac{22}{7}\right][10\cdot 5]^2\ cm^2 = 1386cm^2$

Radius $\left(r^1\right)$ of cylindrical support $= 1\cdot 5cm$

Height $\left(h^1\right)$ of cylindrical support $= 7cm$

CSA of cylindrical support $= 2\pi r^1 h\left[2\times\frac{22}{7}\times 1\cdot 5\times 7\right]$

$= 66cm^2$

Area of circular end of cylindrical support $= \pi r^2 = \left[\frac{22}{7}(1\cdot 5)^2\right] = 7\cdot 07cm^2$

Area to be painted silver $= \left[8\times\left(1386 - 7\cdot 07\right)\right]cm^2$

$= 8\left(1378\cdot 93\right)cm^2$

$= 11031\cdot 44cm^2$

Cost occurred in painting silver color

$= Rs.\ \left(11031\cdot 44\times 0\cdot 25\right) = Rs.\ 2757\cdot 86$

Area to painted black $= \left(8\times 66\right)cm^2 = 528\ cm^2$

Cost occurred in painting black color $= Rs.\left(528\times 0\cdot 05\right) = Rs.\ 26\cdot 40$

$\therefore$ Total cost occurred in painting $= Rs.\left(2757\cdot 86 + 26\cdot 40\right) = Rs. 2784\cdot 26$

Exercise – 21.2

1. Find the volume of a sphere whose radius is:
(i) 2 cm (ii) 3.5 cm (iii) 10.5 cm
Sol:

(i) Radius $(r) = 2cm$

$\therefore$ Volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2)^3 = 33 \cdot 52cm^3$

(ii) Radius $(r) = 3 \cdot 5cm$

$\therefore$ Volume $= (3 \cdot 5)^3 \times \pi \times \frac{4}{3} = \frac{4}{3} \times \frac{22}{7} \times (3 \cdot 5)^3 = 179 \cdot 666cm^3$

(iii) Radius $(r) = 10 \cdot 5cm$

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (10 \cdot 5)^3 = 4851cm^3$

2. Find the volume of a sphere whose diameter is:
(i) 14 cm (ii) 3.5 dm (iii) 2.1 m
Sol:

(i) Diameter $= 14cm$, radius $= \frac{14}{2} = 7cm$

$\Rightarrow$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (7)^3 = 1437 \cdot 33cm^3$

(ii) Diameter $= 3 \cdot 5d\ m$, radius $= \frac{3 \cdot 5}{2} dm = 1 \cdot 75dm$

$\therefore$ Volume $= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{3 \cdot 5}{2}\right)^3 = 22 \cdot 46dm^3$

(iii) Diameter $= 2 \cdot 1m \Rightarrow r = \frac{2 \cdot 1}{2} m$

$\therefore$ Volume $= \frac{4}{3} \times \left(\frac{22}{7}\right) \times \left(\frac{2 \cdot 1}{2}\right)^3$

$= 4 \cdot 851m^3.$

3. A hemispherical tank has inner radius of 2.8 m. Find its capacity in litres.

Sol:

Radius of tank $= 2\cdot 8m$

$\therefore$ Capacity $= \frac{2}{3} \times \frac{22}{7} \times (2\cdot 8)^3$

$= 45\cdot 994m^3$

$\therefore$ Capacity in liters $= 45994$ liters $\left[1m^3 = 1000\right]$

4. A hemispherical bowl is made of steel 0.25 cm thick. The inside radius of the bowl is 5 cm. Find the volume of steel used in making the bowl.

Sol:

Inner radius $= 5cm$

Outer radius $= 5 + 0\cdot 25$

$= 5\cdot 25$

Volume of steel used = outer volume – inner volume

$= \frac{2}{3} \times \pi \times \left(R^3 - r^3\right)$

$= \frac{2}{3} \times \frac{22}{7}\left[5\cdot 25^3 - 5^3\right]$

$= 41\cdot 282cm^3$

5. How mañy bullets can be made out of a cube of lead, whose edge measures 22 cm, each bullet being 2 cm in diameter?

Sol:

Cube edge $= 22cm$

$\therefore$ Volume of cube $= (22)^3$

$= 10648cm^3$

And,

Volume of each bullet $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{2}{2}\right)^3$

$= \frac{4}{3} \times \frac{22}{7}$

$= \frac{88}{21}cm^3$

$\therefore \text{No. of bullets } = \dfrac{\text{Volume of cube}}{\text{Volume of bullet}}$

$= \dfrac{10648}{\frac{88}{21}} = 2541$

6. A shopkeeper has one laddoo of radius 5 cm. With the same material, how many laddoos of radius 2.5 cm can be made.

Sol:

Volume of laddoo having radius $= 5cm$

i.e volume $(V_1) = \dfrac{4}{3}\pi r^3$

$V_1 = \dfrac{4}{3} \times \dfrac{22}{7} \times (5)^3$

$V_1 = \dfrac{11000}{21} cm^3$

Also volume of laddoo having radius $= 2 \cdot 5 cm$

i.e., $V_2 = \dfrac{4}{3}\pi r^3$

$V_2 = \dfrac{4}{3} \times \dfrac{22}{7} \times (2 \cdot 5)^3$

$V_2 = \dfrac{1375}{21} cm^3$

$\therefore \text{No. of laddoos} = \dfrac{V_1}{V_2} = \dfrac{11000}{1375} = 8.$

7. A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. If the diameters of two balls be $\frac{3}{2}$ cm and 2 cm, find the diameter of the third ball.

Sol:

Volume of lead ball $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times \left(\dfrac{3}{2}\right)^3$

$\therefore$ According to question,

$\text{Volume of lead ball} = \dfrac{4}{3} \times \pi \left(\dfrac{3}{4}\right)^3 + \dfrac{4}{3}\pi \left(\dfrac{2}{2}\right)^3 + \dfrac{4}{3}\pi \left(\dfrac{d}{2}\right)^3$

$$\Rightarrow \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 + \frac{4}{3}\left[\pi\left(\frac{2}{2}\right)^3 + \left(\frac{d}{2}\right)^3\right]$$

$$\Rightarrow \frac{4}{3}\pi\left[\left(\frac{3}{2}\right)^3\right] = \frac{4}{3}\pi\left[\left(\frac{3}{4}\right)^3 + \left(\frac{2}{2}\right)^3 + \left(\frac{d}{2}\right)^3\right]$$

$$\Rightarrow \frac{27}{8} = \frac{27}{64} + \frac{8}{8} + \frac{d^3}{8}$$

$$\Rightarrow \left[\frac{27}{8} - \frac{27}{64} - 1\right]8 = d^3$$

$$\Rightarrow \frac{d^3}{8} = \frac{125}{64}$$

$$\Rightarrow \frac{d}{2} = \frac{5}{4}$$

$$\Rightarrow d = \frac{10}{4}$$

$$\Rightarrow d = 2\cdot 5cm$$

8. A sphere of radius 5 cm is immersed in water filled in a cylinder, the level of water rises $\frac{5}{3}$ cm. Find the radius of the cylinder.

Sol:

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi(5)^3$$

$\therefore$ Volume of water rise in cylinder = Volume of sphere

Let r be the radius of the cylinder

$$\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\Rightarrow r^2 \times \frac{5}{3} = \frac{4}{3}(5)^3$$

$$\Rightarrow r^2 = 20 \times 5$$

$$\Rightarrow r^2 = 100$$

$$\Rightarrow r^1 = 10cm$$

9. If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

Sol:

Let $V_1 \text{ and } V_2$ be the volumes of first sphere and second sphere respectively

Radius of 1st sphere = r

2nd sphere radius $= 2r$

$$\therefore \frac{Volume\ 1^{st}}{Volume\ 2^{nd}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(2r)^3} = \frac{1}{8}.$$

10. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Sol:

Given that

Volume of thee cone = Volume of the hemisphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$\Rightarrow r^2 h = 2r^3$$

$$\Rightarrow h = 2r$$

$$\Rightarrow \frac{h}{r} = \frac{1}{1} \times 2 = \frac{2}{1}$$

$\therefore$ Ratio of the their height is $2:1$

11. A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.

Sol:

Given that

Volume of water in the hemisphere bowl = Volume of water in the cylinder

Let n be the height to which water rises in the cylinder.

Inner radii of bowl $= 3\cdot 5cm = r_1$

Inner radii of bowl $= 7cm = r_2$

$$\Rightarrow \frac{2}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\Rightarrow h = \frac{2r_1^3}{3r_2^2} = \frac{2(3\cdot 5)^3}{3(7)^2}$$

$$\Rightarrow h = \frac{7}{12} cm.$$

12. A cylinder whose height is two thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

Sol:

Given that,

Height of cylinder $= \frac{2}{3}$(diameter)

We know that,

Diameter $= 2$(radius)

$h = \frac{2}{3} \times 2r = \frac{4}{3} r$

Volume of the cylinder = volume of the sphere

$\Rightarrow \cancel{\pi} r^2 \times \frac{\cancel{4}}{\cancel{3}} r = \frac{\cancel{4}}{\cancel{3}} \cancel{\pi} (4)^3$

$\Rightarrow r^3 = 4^3$

$\Rightarrow r = 4cm$

13. A vessel in the form of a hemispherical bowl is full of water. The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively 6 cm and 4 cm. Find the height of water in the cylinder.

Sol:

It is given that,

Volume of water is hemisphere bowl = volume of cylinder

$\Rightarrow \frac{2}{3} \pi (6)^3 = \pi (4)^2 h$

$\Rightarrow h = \frac{2}{3} \times \frac{6 \times 6 \times 6}{4 \times 4}$

$\Rightarrow h = 9cm$

$\therefore$ Height of cylinder $= 9cm.$

14. A cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the tub and thus level of water is raised by 9 cm. What is the radius of the ball?

Sol:

Let r be the radius of the iron ball

Then, Volume of iron ball = Volume of water raised in the hub

$\Rightarrow \frac{4}{3} \pi r^3 = \pi r^2 h$

$\Rightarrow \frac{4}{3} r^3 = (16)^2 \times 9$

$\Rightarrow r^3 = \frac{27 \times 16 \times 16}{4}$

$\Rightarrow r^3 = 1728$

$\Rightarrow r = 12\,cm$

Therefore, radius of the ball = 12cm.

15. A cylinder of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 6.75 cm. Find the radius of the ball. (Use π = 22/7).

Sol:

Given that,

Radius of cylinder $=12cm=r_1$

Raised in height $=6\cdot 75cm=h$

$\Rightarrow$ Volume of water raised = Volume of the sphere

$$\Rightarrow \pi r_1^2 h=\frac{4}{3}\pi r_2^3$$

$$\Rightarrow 12\times 12\times 6\cdot 75=\frac{4}{3}r_2^3$$

$$\Rightarrow \frac{12\times 12\times 6\cdot 75\times 3}{4}=r_2^3$$

$$\Rightarrow r_2^3=729$$

$$\Rightarrow \boxed{r_2=9cm}$$

Radius of sphere is 9cm.

16. The diameter of a coper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 108 m, find its diameter.

Sol:

Given that diameter of a coper sphere = 18cm.

Radius of the sphere = 9cm

Length of the wire = 108m

= 10,800cm

Volume of cylinder = volume of sphere

$$\Rightarrow \pi r_1^2 h=\frac{4}{3}\pi r_2^3$$

$$\Rightarrow r_1^2\times 10800=\frac{4}{3}\times 9\times 9\times 9\Rightarrow r_1^2=0\cdot 09$$

$\therefore$ Diameter = $2\times 0\cdot 3=0\cdot 6cm$

17. A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1 .5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimetres?

Sol:

Given that,

Radius of cylinder jar $=6cm=r_1$

Level to be rised = 2cm=h

Radius of each iron sphere $=1\cdot 5cm = r_2$

Number of sphere $= \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$

$$= \frac{\pi r_1^2 h}{4\pi r_2^3}$$

$$= \frac{r_1^2 h}{r_2^3 \times \frac{4}{3}} = \frac{6\times 6\times 2}{\frac{4}{3}\times 1\cdot 5\times 1\cdot 5\times 1\cdot 5}$$

Number of sphere = 16.

18. A measuring jar of internal diameter 10 cm is partially filled with water. Four equal spherical balls of diameter 2 cm each are dropped in it and they sink down in water completely. What will be the change in the level of water in the jar?

Sol:

Given that,

Diameter of jar = 10cm

Radius of jar = 5cm

Let the level of water raised by 'h'

Diameter of spherical ball = 2cm

Radius of the ball =1cm

Volume of jar = 4(Volume of spherical)

$$\Rightarrow \pi r_1^2 h = 4\left(\frac{4}{3}\pi r_2^3\right)$$

$$\Rightarrow r_1^2 h = 4\times\frac{4}{3} r_2^3$$

$$\Rightarrow r_1^2 h = 4\times\frac{4}{3}\times 1\times 1\times 1$$

$$\Rightarrow h = \frac{4\times 4\times 1}{3\times 5\times 5}$$

$$\Rightarrow h = \frac{16}{75} cm.$$

$\therefore$ Height of water in jar $= \frac{16}{75} cm.$

19. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.

Sol:

Given that,

Diameter of sphere = 6cm

Radius of sphere $= \frac{d}{2} = \frac{6}{2} cm = 3cm = r_1$

Diameter of the wire = $0 \cdot 2cm$

Radius of the wire $= 0 \cdot 1cm = r_2$

Volume of sphere = Volume of wire

$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

$\Rightarrow \frac{4}{3} \times 3 \times 3 \times 3 = 0 \cdot 1 \times 0 \cdot 1 \times h$

$\Rightarrow \frac{4 \times 3 \times 3}{0 \cdot 1 \times 0 \cdot 1} = h$

$\Rightarrow h = 3600$

$\Rightarrow h = 36m.$

$\therefore$ Length of wire = 36m.

20. The radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted and recast into a solid cylinder of height $2\frac{2}{3}$ cm. Find the diameter of the cylinder.

Sol:

Given that,

Internal radius of the sphere $= 3cm = r_1$

External radius of the sphere $= 5cm = r_2$

Height of cylinder $= 2\frac{2}{3} cm = \frac{8}{3} cm = h$

Volume of spherical shell = Volume of the cylinder

$\Rightarrow \frac{4}{3}\pi\left(r_2^3 - r_1^3\right) = \pi r_3^2 h$

$\Rightarrow \frac{4}{3}\left(5^3 - 3^3\right) = \frac{8}{3} r_3^2$

$\Rightarrow \frac{4 \times 98 \times 3}{3 \times 8} = r_3^2$

$\Rightarrow r_3^2 = \sqrt{49}$

$\Rightarrow r_3 = 7cm$

$\therefore$ Diameter of the cylinder = 2 (radius) = 14cm

21. A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

Sol:

Given radius of hemisphere = 7cm = r_1

Height of cone $h = 49cm$

Volume of hemisphere = Volume of cone

$\Rightarrow \frac{2}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$

$\Rightarrow \frac{2}{3}\times 7^3 = \frac{1}{3} r_2^2 \times 49$

$\Rightarrow \frac{2\times 7\times 7\times 7\times 3}{3\times 49} = r_2^2$

$\Rightarrow r_2^2 = 3\cdot 74cm$

$\therefore$ Radius of the base $= 3\cdot 74cm.$

22. A hollow sphere of internal and external radii 2 cm and 4 cm respectively is melted into a cone of base radius 4 cm. Find the height and slant height of the cone.

Sol:

Given that

Hollow sphere external radii $= 4cm = r_2$

Internal radii $(r_1) = 2cm$

Cone base radius $(R) = 4cm$

Height = ?

Volume of cone = Volume of sphere

$\Rightarrow \frac{1}{3}\pi r^2 H = \frac{4}{3}\pi\left(R_2^3 - R_1^3\right)$

$\Rightarrow 4^2 H = 4\left(4^3 - 2^3\right)$

$\Rightarrow H = H = \frac{4\times 56}{16} = 14cm$

Slant height $= \sqrt{R^2 + H^2} = \sqrt{4^2 + 14^2}$

$\Rightarrow l = \sqrt{16+196} = \sqrt{212}$

$= 14\cdot 56cm.$

23. A metallic sphere of radius 10.5 cm is melted and thus recast into small cones, each of radius 3.5 cm and height 3 cm. Find how many cones are obtained.

Sol:

Given that

Metallic sphere of radius $= 10 \cdot 5cm$

Cone radius $= 3 \cdot 5cm$

Height of radius $= 3cm$

Let the number of cones obtained be x

$V_s = x \times V$ cone

$\Rightarrow \frac{4}{3}\pi r^3 = x \times \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{4 \times 10 \cdot 5 \times 10 \cdot 5 \times 10 \cdot 5}{3 \cdot 5 \times 3 \cdot 5 \times 3} = x$

$\Rightarrow x = 126$

$\therefore$ Number of cones $= 126$

24. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Sol:

Given that

A cone and a hemisphere have equal bases and volumes

$V_{cone} = V_{hemisphere}$

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$\Rightarrow r^2 h = 2r^3$

$\Rightarrow h = 2r$

$\Rightarrow h : r - 2r : r - 2 : 1$

25. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1 : 2 : 3.

Sol:

Given that,

A cone, hemisphere and a cylinder stand one equal bases and have the same weight

We know that

$V_{cone} : V_{hemisphere} : V_{cylinder}$

$\Rightarrow \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

Multiplying by 3

$\Rightarrow \pi r^2 h : 2\pi r^3 : 3\pi r^2 h$ or

$\pi r^3 : 2\pi r^3 : 3\pi r^3 \left[\therefore r = h \because r^2 h = r^3\right]$

Or $1 : 2 : 3$

26. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

Sol:

A cylindrical tub of radius $=12cm$

Depth $=20cm.$

Let $r\,cm$ be the radius of the ball

Then, volume of ball = volume of water raised

$$=\frac{4}{3}\pi r^3=\pi r^2 h$$

$$=\frac{4}{3}\pi r^3=\pi\times(12)^2\times 6\cdot 75$$

$$\Rightarrow r^3=\frac{144\times 6\cdot 75\times 3}{4}$$

$$\Rightarrow r^3=729$$

$$\Rightarrow r=9cm$$

Thus, radius of the ball $=9cm.$

27. The largest sphere is carved out of a cube of side 10.5 cm. Find the volume of the sphere.

Sol:

Given that,

The largest sphere is carved out of a cube of side $=10\cdot 5cm$

Volume of the sphere = ?

We have,

Diameter of the largest sphere $=10\cdot 5cm$

$2r=10\cdot 5$

$$\Rightarrow r=5\cdot 25cm$$

Volume of sphere $=\frac{4}{3}\times\frac{22}{7}\times(5\cdot 25)^3=\frac{4}{3}\times\frac{22}{7}\times 5\cdot 25\times 5\cdot 25\times 5\cdot 25$

$$\Rightarrow \text{Volume }=\frac{11\times 441}{8}cm^3=606\cdot 37cm^3.$$

28. A sphere, a cylinder and a cone have the same diameter. The height of the cylinder and also the cone are equal to the diameter of the sphere. Find the ratio of their volumes.

Sol:

Let r be the common radius thus,

$h=$height of the cone = height of the cylinder = 2r

Let

$V_1 = \text{Volume of sphere} = \frac{4}{3}\pi r^3$

$V_2 = \text{Volume of cylinder } = \pi r^2 \times 2r = 2\pi r^3$

$V_3 = \text{Volume of the cone} = \frac{1}{3}\pi r^2 \times 2r = \frac{2}{3}\pi r^3$

Now,

$V_1 : V_2 : V_3 = \frac{4}{3}\pi r^3 : 2\pi r^3 : \frac{2}{3}\pi r^3$

$= 4 : 6 : 2$

$= 2 : 3 : 1$

29. A cube of side 4 cm contains a sphere touching its side. Find the volume of the gap in between.

Sol:

It is given that

Cube side = 4cm

Volume of cube $= (4cm)^3 = 64cm^3$

Diameter of the sphere = Length of the side of the cube = 4cm

$\therefore$ Radius of sphere = 2cm

Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2)^3 = 33\cdot 52cm^3$

$\therefore$ Volume of gap $=$ Volume of gap – Volume of sphere

$= 64cm^2 - 33\cdot 52cm^3 = 30\cdot 48cm^3$.

30. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol:

Given that,

Inner radius (r_1) of hemispherical tank $= 1m = r_1$

Thickness of hemispherical tank $= 1cm = 0\cdot 01m$

Outer radius (r_2) of the hemispherical $= (1 + 0\cdot 01m) = 1\cdot 01m = r_2$

Volume of iron used to make the tank $= \frac{2}{3}\pi\left(r_2^3 - r_1^3\right)$

$= \frac{2}{3} \times \frac{22}{7}\left[(1\cdot 01)^3 - 1^3\right]$

$= \frac{44}{21}[1\cdot 030301 - 1]m^3$

$= 0\cdot 06348m^3$ (Approximately)

31. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Sol:

Given that,

Diameter of capsule $=3\cdot5\,mm$

Radius $=\frac{3\cdot5}{2}=1\cdot75mm$

Volume of spherical capsule $=\frac{4}{3}\pi r^3$

$=\frac{4}{3}\times\frac{22}{7}\times(1\cdot75)^3\,mm^3$

$=22\cdot458mm^3$

$\therefore 22\cdot46\,mm^3$ of medicine is required.

32. The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol:

Given that,

The diameter of the moon is approximately one fourth of the diameter of the earth.

Let diameter of earth bed. So radius $=\frac{d}{2}$

Then, diameter of moon $=\frac{d}{4}$, radius $=\frac{\frac{d}{2}}{4}=\frac{d}{8}$

Volume of moon $=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{d}{8}\right)^3=\frac{4}{3}\times\frac{1}{512}\pi d^3$

Volume of earth $=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{d}{2}\right)^3=\frac{1}{8}\times\frac{4}{3}\pi d^3$

$$\frac{\text{Volume of moon}}{\text{Volume of earth}}=\frac{\frac{1}{512}\times\frac{4}{3}\pi d^3}{\frac{1}{8}\times\frac{4}{3}\pi d^3}=\frac{1}{64}.$$

Thus, the volume of moon is $\frac{1}{64}$ of volume of earth.