Exercise – 24.1

1. If the heights of 5 persons are 140 cm, 150 cm, 152 cm, 158 cm and 161 cm respectively, find the mean height.

Sol:

It is given that,

The heights of 5 persons are $-140cm,150cm,152cm,158cm\ and\ 161cm.$

$$\therefore \text{Mean height} = \frac{Sum\ of\ heights}{Total\ No.of\ persons}$$

$$= \frac{140+150+152+158+161}{5}$$

$$= \frac{761}{5}$$

$$= 152 \cdot 2.$$

2. Find the mean of 994, 996, 998, 1002 and 1000.

Sol:

Given numbers are $-994,996,998,1000\ and\ 1002.$

$$\therefore Mean = \frac{Sum\ of\ Numbers}{Total\ Numbers}$$

$$= \frac{994+996+998+1000+1002}{5}$$

$$= \frac{4990}{5}$$

$$= 998.$$

3. Find the mean of first five natural numbers.

Sol:

Given that,

The first five natural numbers are $1,2,3,4,5$

$$\therefore Mean = \frac{Sum\ of\ Numbers}{Total\ Numbers}$$

$$= \frac{1+2+3+4+5}{5}$$

$$= \frac{15}{5}$$

$$\boxed{Mean = 3}$$

4. Find the mean of all factors of 10.

Sol:

All factors of 10 are $-1, 2, 5, 10$

$$\therefore Mean = \frac{Sum\ of\ \ factors}{Total\ \ factors}$$

$$= \frac{1+2+5+10}{4}$$

$$= \frac{18}{4}$$

$$= \frac{9}{2}$$

$$= 4 \cdot 5$$

$$\boxed{\therefore Mean = 4 \cdot 5}$$

5. Find the mean of first 10 even natural numbers.

Sol:

Given that,

The first 10 natural numbers be $-2, 4, 6, 8, 10, 12, 14, 16, 18, 20$

$$\therefore Mean = \frac{Sum\ of\ all\ Numbers}{Total\ \ Numbers}$$

$$= \frac{2+4+6+8+10+12+14+16+18+20}{10} = \frac{110}{10}$$

$$= \frac{110}{10} = 11$$

$$\boxed{Mean = 11}$$

6. Find the mean of x, x + 2, x + 4, x +6, x + 8.

Sol:

Numbers be $x, x+2, x+4, x+6$ *and* $x+8$

$$\therefore Mean = \frac{Sum\ of\ \ Numbers}{Total\ \ Numbers}$$

$$= \frac{x+x+2+x+4+x+6+x+8}{5}$$

$$= \frac{5x+20}{5}$$

$$= \frac{5(x+4)}{5}$$

$$= x+4$$

7. Find the mean of first five multiples of 3.

Sol:

First five multiple of 3: $3,6,9,12,15$

$$\therefore Mean = \frac{Sum\ of\ Numbers}{Total\ \ Numbers}$$

$$= \frac{3+6+9+12+15}{5}$$

$$= \frac{45}{5} = 9.$$

8. Following are the weights (in kg) of 10 new born babies in a hospital on a particular day:

3.4, 3.6, 4.2, 4.5, 3.9, 4.1, 3.8, 4.5, 4.4, 3.6. Find the mean $\bar{X}$.

Sol:

The weight (in kg) of 10 new born babies

$$= 3\cdot4, 3\cdot6, 4\cdot2, 4\cdot5, 3\cdot9, 4\cdot1, 3\cdot8, 4\cdot5, 4\cdot4, 3\cdot6$$

$$\therefore Mean(\bar{x}) = \frac{Sum\ of\ weights}{Total\ \ babies}$$

$$= \frac{3\cdot4+3\cdot6+4\cdot2+4\cdot5+3\cdot9+4\cdot1+3\cdot8+4\cdot5+4\cdot4+3\cdot6}{10}$$

$$= \frac{40}{10} 4kg.$$

9. The percentage of marks obtained by students of a class in mathematics are : 64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1. Find their mean.

Sol:

The percentage marks obtained by students are

$$= 64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1.$$

$$\therefore \text{Mean marks } = \frac{64+36+47+23+0+19+81+93+72+35+3+1}{12}$$

$$= \frac{474}{12} = 39\cdot5$$

$$\therefore \text{Mean marks } = 39\cdot5$$

10. The numbers of children in 10 families of a locality are:

2, 4, 3, 4, 2, 0, 3, 5, 1, 1, 5. Find the mean number of children per family.

Sol:

The number of children in 10 families is

$$\Rightarrow 2, 4, 3, 4, 2, 3, 5, 1, 1, 5.$$

$\therefore$ Mean number of children per family

$= \dfrac{Total\ \ no.\ of\ children}{Total\ \ families}$

$= \dfrac{2+4+3+4+2+3+5+1+1+5}{10}$

$= \dfrac{30}{10}$

$= 3.$

11. If M is the mean of x_1 , x_2 , x_3 , x_4 , x_5 and x_6, prove that

$(x_1 - M) + (x_2 - M) + (x_3 - M) + (x_4 - M) + (x_5 - M) + (x_6 - M) = 0.$

Sol:

Let m be the mean of x_1, x_2, x_3, x_4, x_5 *and* x_6

Then $M = \dfrac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6}$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 6M$

To prove: $(x_1 - M) + (x_2 - M) + (x_3 - M) + (x_4 - M) + (x_5 - M) + (x_6 - M)$

$= (x_1 + x_2 + x_3 + x_4 + x_5 + x_6) - (M + M + M + M + M + M)$

$= 6M - 6M$

$= 0$

$= RHS$

12. Durations of sunshine (in hours) in Amritsar for first 10 days of August 1997 as reported by the Meteorological Department are given below:

9.6, 5.2, 3.5, 1.5, 1.6, 2.4, 2.6, 8.4, 10.3, 10.9

(i) Find the mean $\bar{X}$ (ii) Verify that $= \sum_{i=1}^{10} (x_i - \bar{x}) = 0$

Sol:

Duration of sunshine (in hours) for 10 days are

$= 9 \cdot 6, 5 \cdot 2, 3 \cdot 5, 1 \cdot 5, 1 \cdot 6, 2 \cdot 4, 2 \cdot 6, 8 \cdot 4, 10 \cdot 3, 10 \cdot 9$

(i) $Mean\ \bar{x} = \dfrac{Sum\ of\ all\ numbers}{Total\ \ numbers}$

$= \dfrac{9 \cdot 6 + 5 \cdot 2 + 3 \cdot 5 + 1 \cdot 5 + 1 \cdot 6 + 2 \cdot 4 + 2 \cdot 6 + 8 \cdot 4 + 10 \cdot 3 + 10 \cdot 9}{10}$

$= \dfrac{56}{10} = 5 \cdot 6$

(ii) $\text{LHS} = \sum_{i=1}^{10}(x_i - \bar{x})$

$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + \ldots\ldots\ldots\ldots + (x_{10} - \bar{x})$

$= (9 \cdot 6 - 5 \cdot 6) + (5 \cdot 2 - 5 \cdot 6) + (3 \cdot 5 - 5 \cdot 6) + (1 \cdot 5 - 5 \cdot 6) + (1 \cdot 6 - 5 \cdot 6) + (2 \cdot 4 - 5 \cdot 6)$

$= (4) + (-0 \cdot 4) + (-2 \cdot 1) - 4 \cdot 1 - 4 - 3 \cdot 2 - 3 + 2 \cdot 8 + 4 \cdot 7 + 5 \cdot 3$

$= 16 \cdot 8 - 16 \cdot 8$

$= 0.$

13. Explain, by taking a suitable example, how the arithmetic mean alters by (i) adding a constant k to each term, (ii) subtracting a constant k from each them, (iii) multiplying each term by a constant k and (iv) dividing each term by a non-zero constant k.

Sol:

Let us say numbers are be $3, 4, 5$

$\therefore Mean = \frac{Sum\ of\ number}{Total\ number}$

$= \frac{3+4+5}{3}$

$= \frac{12}{3}$

$= 4$

(i) Adding constant term $k = 2$ in each term

New numbers are $= 5, 6, 7.$

$\therefore$ New mean $= \frac{5+6+7}{3}$

$= \frac{18}{3} = 6 = 4 + 2$

$\therefore$ New mean will be 2 more than the original mean.

(ii) Subtracting constant term $k = 2$ in each term New number are $= 1, 2, 3.$

$\therefore$ New mean $= \frac{1+2+3}{3} = \frac{6}{3} = 2 = 4 - 2.$

$\therefore$ New mean will be 2 less than the original mean

(iii) Multiplying by constant term $k = 2$ in each term

New numbers are = 6, 8, 10

New mean = $\frac{6+8+10}{3}$

$= \frac{24}{3}$

$= 8$

$= 4 \times 2$

$\therefore$ New mean will be 2 times of the original mean.

(iv) Divide by constant term $k = 2$ in each term

New number are $= 1 \cdot 5, 2, 2 \cdot 5$

$\therefore$ New mean $= \frac{1 \cdot 5 + 2 + 2 \cdot 5}{3}$

$= \frac{6}{3} = 2 = \frac{4}{2}$

$\therefore$ New mean will be half of the original mean.

14. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

Sol:

Mean marks of 100 students = 40

$\Rightarrow$ Sum of marks of 100 students $= 100 \times 40 = 4000$

Correct value $= 53.$

Incorrect value $= 83.$

Correct sum $= 4000 - 83 + 53$

$= 3970$

$\therefore$ Correct mean $= \frac{3970}{100}$

$= 39 \cdot 7.$

15. The traffic police recorded the speed (in kmlhr) of 10 motorists as 47, 53, 49, 60, 39, 42, 55, 57, 52, 48. Later on an error in recording instrument was found. Find the correct overage speed of the motorists if the instrument recorded 5 km/hr less in each case.

Sol:

The speed of 10 motorists are $47, 53, 49, 60, 39, 42, 55, 57, 52, 48$

Later on it was discovered that the instrument recorded 5km/hr less than in each case

Corrected values are 52, 58, 54, 65, 44, 47, 60, 62, 57, 53

$\therefore$ Correct mean $= \frac{52 + 58 + 54 + 65 + 44 + 47 + 60 + 62 + 57 + 53}{10}$

$= \frac{552}{10}$

$= 55 \cdot 2 \ \ km / hr$

16. The mean of five numbers is 27. If one number is excluded, their mean is 25. Find the excluded number.

Sol:

The mean of the numbers 27

The, sum of five numbers $= 5 \times 27$

$= 135.$

If one number is excluded, then the new mean is 25

$\therefore$ Sum of numbers $= 4 \times 25 = 100$

$\therefore$ Excluded number $= 135 - 100$

$= 35$

17. The mean weight per student in a group of 7 students is 55 kg. The individual weights of 6 of them (in kg) are 52, 54, 55, 53, 56 and 54. Find the weight of the seventh student.

Sol:

The mean weight per student in a group of 7 students is 55kg.

Weight of 6 students (in kg) $= 52, 54, 55, 53, 56$ and 54.

Let weight of 7^{th} student = x kg

$\therefore \text{Mean} = \dfrac{Sum\ of\ all\ weights}{Total\ students}$

$\Rightarrow 55 = \dfrac{52 + 54 + 55 + 53 + 56 + 54 + x}{7}$

$\Rightarrow 385 = 324 + x$

$\Rightarrow x = 385 - 324$

$\Rightarrow x = 61\ kg$

$\therefore$ Weight of 7^{th} student = 61kg

18. The mean weight of 8 numbers is 15. If each number is multiplied by 2, what will be the new mean?

Sol:

We have,

The mean weight of 8 numbers is 15

Then, The sum of 8 numbers $= 8 \times 15 = 120.$

If each number is multiplied by 2

Then, new mean $= 120 \times 2$

$= 240$

$\therefore \text{New mean} = \dfrac{240}{8} = 30.$

19. The mean of 5 numbers is 18. If one number is excluded, their mean is 16. Find the excluded number.

Sol:

The mean of 5 numbers is 18

Then, the sum of 5 numbers $= 5 \times 18$

$= 90$

If the one number is excluded

Then, the mean of 4 numbers = 16.

$\therefore$ Sum of 4 numbers $= 4 \times 16$

$= 64$

Excluded number $= 90 - 64$

$= 26.$

20. The mean of 200 items was 50. Later on, it was discovered that the two items were misread as 92 and 8 instead of 192 and 88. Find the correct mean.

Sol:

The mean of 200 items = 50

Then the sum of 200 items $= 200 \times 50$

$= 10,000$

Correct values $= 192$ *and* 88

Incorrect values $= 92 = 8$

$\therefore$ Correct sum $= 10000 - 92 - 8 + 192 + 88$

$= 10180$

$\therefore$ Correct mean $= \dfrac{10180}{200} = 50 \cdot 9$

$= \dfrac{101 \cdot 8}{2} = 50 \cdot 9.$

21. Find the values of n and $\bar{X}$ in each of the following cases:

(i) $\sum_{i=1}^{n}(x_i - 12) = -10 \quad \sum_{i=1}^{n}(x_i - 3) = 62$

(ii) $\sum_{i=1}^{n}(x_i - 10) = 30 \quad \sum_{i=1}^{n}(x_i - 6) = 150.$

Sol:

(i) Given $\sum_{i=1}^{n}(x_n - 12) = -10$

$\Rightarrow (x_1 - 12) + (x_2 - 12) + ... + (x_n - 12) = -10$

$\Rightarrow (x_1 + x_2 + x_3 + x_4 + x_5 + ... + x_n) - (12 + 12 + 12 + ... + 12) = -10$

$\Rightarrow \sum x - 12n = -10 \qquad .....(1)$

And $\sum_{i=1}^{n}(x_i - 3) = 62$

$\Rightarrow (x_1 - 3) + (x_2 - x_3) + (x_3 - 3) + ... + (x_n - 3) = 62.$

$\Rightarrow (x_1 + x_2 + ... + x_n) - (3 + 3 + 3 + 3 + ... + 37) = 62$

$\Rightarrow \Sigma x - 3n = 62 \qquad ....(2)$

By subtracting equation (1) from equation (2)

We get

$\Sigma x - 3n - \Sigma x + 12n = 62 + 10$

$\Rightarrow 9n = 72$

$\Rightarrow n = \frac{72}{9} = 8.$

Put value of n in equation (1)

$\Sigma x - 12 \times 8 = -10$

$\Rightarrow \Sigma x - 96 = -10$

$\Rightarrow \Sigma x = -10 + 96 = 86$

$\therefore \bar{x} = \frac{\Sigma x}{x} = \frac{86}{8} = 10 \cdot 75$

(ii) Given $\sum_{i=1}^{n}(x_i - 10) = 30$

$\Rightarrow (x_1 - 10) + (x_2 - 10) + ... + (x_n - 10) = 30$

$\Rightarrow (x_1 + x_2 + x_3 + ... + x_n) - (10 + 10 + 10 + ... + 10) = 30$

$\Rightarrow \sum x - 10n = 30 \qquad .....(1)$

And $\sum_{i=1}^{n}(x_i - 6) = 150.$

$\Rightarrow (x_1 - 6) + (x_2 - 6) + ... + (x_n - 6) = 150.$

$\Rightarrow (x_1 + x_2 + x_3 + ... + x_n) - (6 + 6 + 6 + ... + 6) = 150$

$\Rightarrow \Sigma x - 6n = 150 \qquad ....(2)$

By subtracting equation (1) from equation (2)

$\Sigma x - 6n - \Sigma x + 10n = 150 - 30$

$\Rightarrow \Sigma x - \Sigma x + 4n = 120$

$\Rightarrow n = \frac{120}{4}$

$\Rightarrow n = 30$

Put value of n in equation (1)

$\Sigma x - 10 \times 30 = 30$

$\Rightarrow \Sigma x - 300 = 30$

$\Rightarrow \Sigma x = 30 + 300 = 330$

$\therefore \bar{x} = \frac{\Sigma x}{n} = \frac{330}{30} = 11.$

22. The sums of the deviations of a set of n values $x_1, x_2, \ldots . x_{11}$ measured from 15 and -3 are $-$ 90 and 54 respectively. Find the valùe of n and mean.

Sol:

(i) Given $\sum_{i=1}^{n}(x_i + 5) = -90$

$\Rightarrow (x_1 - 15) + (x_2 - 15) + ... + (x_n - 15) = -90$

$\Rightarrow (x_1 + x_2 + ... + x_n) - (15 + 15 + ... + 15) = -90$

$\Rightarrow \sum x - 15n = -90 \qquad .....(1)$

And $\sum_{i=1}^{n}(x_i + 3) = 54$

$\Rightarrow (x_1 - 3) + (x_2 - 3) + ... + (x_n + 3) = 54.$

$\Rightarrow (x_1 + x_2 + x_3 + ... + x_n) + (3 + 3 + 3 + ... + 37) = 54$

$\Rightarrow \Sigma x + 3n = 54 \qquad ....(2)$

By subtracting equation (1) from equation (2)

$\Sigma x - 30 - \Sigma x + 15n = 54 + 90$

$\Rightarrow 18n = 144$

$\Rightarrow n = \frac{144}{18} = 8.$

Put value of n in equation (1)

$\Sigma x - 15 \times 8 = -90$

$\Rightarrow \Sigma x - 120 = -90$

$\Rightarrow \Sigma x = -90 + 120 = 30$

$\therefore Mean = \frac{\Sigma x}{n} = \frac{30}{8} = \frac{15}{4}.$

23. Find the sum of the deviations of the variate values 3, 4, 6, 7, 8, 14 from their mean.

Sol:

Values are $3, 4, 6, 7, 8, 14.$

$\therefore Mean = \frac{Sum\ of\ \ numbers}{Total\ \ number}$

$= \frac{3+4+6+7+8+14}{6}$

$= \frac{42}{6}$

$= 7.$

$\therefore$ Sum of deviation of values from their mean

$\Rightarrow (3-7)+(4-7)+(6-7)+(7-7)+(8-7)+(14-7)$

$\Rightarrow (-4)+(-3)+(-1)+(0)+(1)+(7)$

$\Rightarrow -8+8$

$= 0.$

24. If $\bar{X}$ is the mean of the ten natural numbers x_1, x_2, x_3 , ..., x_{10} , show that, $(x_1-\bar{X})+(x_2-\bar{X})+.....+(x_{10}-\bar{X})=0$

Sol:

We have, $\bar{x} = \frac{x_1+x_2+....+x_{10}}{10}$

$\Rightarrow x_1+x_2+.....+x_{10}=10\bar{x}$ $.....(i)$

Now, $(x_1-\bar{x})+(x_2-\bar{x})+.....+(x_{10}-\bar{x})$

$=(x_1+x_2+.....+x_{10})-(\bar{x}+\bar{x}+.....$up to 10 terms)

$\Rightarrow 10\bar{x}-10\bar{x}$ [By equation (i)]

$= 0$

$\therefore (x_1-\bar{x})+(x_2-\bar{x})+.....+(x_{10}-\bar{x})=0$ Hence proved.

Exercise – 24.2

1. Calculate the mean for the following distribution:

x:	5	6	7	8	9
f:	4	8	14	11	3

Sol:

x	f	Fx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	N = 40	$\Sigma fx = 281.$

$\therefore$ Mean $\bar{x} = \dfrac{\Sigma fx}{N}$

$= \dfrac{281}{40}$

$= 7 \cdot 025$

2. Find the mean of the following data:

x:	19	21	23	25	27	29	31
f:	13	15	16	18	16	15	13

Sol:

x	f	fx
19	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	N = 106	$\Sigma fx = 2650$

$\therefore$ Mean $\bar{x} = \dfrac{\Sigma fx}{N} = \dfrac{2650}{106} = 25.$

3. The mean of the following data is 20.6. Find the value of p.

x:	10	15	p	25	35
f:	3	10	25	7	5

Sol:

x	f	Fx
10	3	30
15	10	150
P	25	25p
25	7	175
35	5	175
	N = 50	$\Sigma fx = 25P + 530$

It is given that

$\text{Mean} = 20 \cdot 6$

$\Rightarrow \frac{\Sigma fx}{N} = 20 \cdot 6$

$\Rightarrow \frac{25p + 530}{50} = 20 \cdot 6$

$\Rightarrow 25p + 530 = 20 \cdot 6(50) = 1030$

$\Rightarrow 25p = 1030 - 530$

$\Rightarrow 25p = 500$

$\Rightarrow p = \frac{500}{25} = 20$

$\Rightarrow p = 20$

$\boxed{\therefore P = 20}$

4. If the mean of the following data is 15, find p.

x:	5	10	15	20	25
f:	6	p	6	10	5

Sol:

x	f	Fx
5	6	30
10	P	10p
15	6	90
20	10	200
25	5	125
	N = P + 27	$\Sigma fx = 10P + 445$

Given mean = 15

$\Rightarrow \frac{\Sigma xf}{N} = 15$

$$\Rightarrow \frac{10p+445}{p+27}=15$$

$$\Rightarrow 10p+445=15p+405$$

$$\Rightarrow 15p-10p=445-405$$

$$\Rightarrow 5p=40$$

$$\Rightarrow p=\frac{40}{5}$$

$$\therefore p=8.$$

5. Find the value of p for the following distribution whose mean is 16.6

x:	8	12	15	p	20	25	30
f:	12	16	20	24	16	8	4

Sol:

x	f	fx
8	12	96
12	16	192
15	20	300
P	24	24p
20	16	320
25	8	200
30	4	120
	N = 100	$\Sigma fx=24P+1228$

Given mean $=16\cdot6$

$$\Rightarrow \frac{\Sigma fx}{N}=16\cdot6$$

$$\Rightarrow \frac{24p+1228}{100}=16\cdot6$$

$$\Rightarrow 24p=1660-1228$$

$$\Rightarrow 24p=432$$

$$\Rightarrow p=\frac{432}{24}=18$$

6. Find the missing value of p for the following distribution whose mean is 12.58.

x:	5	8	10	12	p	20	25
f:	2	5	8	22	7	4	2

Sol:

x	f	fx
5	2	10
8	5	40
10	8	80
12	22	264
P	7	7p
20	4	80
25	2	50
	N = 50	$\Sigma fx = 7P + 524.$

Given mean $= 12 \cdot 58$

$\Rightarrow \frac{\Sigma fx}{N} = 12 \cdot 58$

$\Rightarrow \frac{7p + 524}{50} = 12 \cdot 58$

$\Rightarrow 7p + 524 = 629$

$\Rightarrow 7p = 629 - 524$

$\Rightarrow 7p = 105$

$\Rightarrow p = \frac{105}{7} = 15$

7. Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

Sol:

x	f	Fx
3	6	18
5	8	40
7	15	105
9	P	9p
11	8	88
13	4	52
	N = P + 41	$\Sigma fx = 9P + 303.$

Given mean $= 7 \cdot 68$

$\Rightarrow \frac{\Sigma fx}{N} = 7 \cdot 68$

$\Rightarrow \frac{9p + 303}{p + 41} = 7 \cdot 68$

$\Rightarrow 9p + 303 = 7 \cdot 68p + 314 \cdot 88$

$\Rightarrow 9p - 7 \cdot 68p = 314 \cdot 88 - 303$

$\Rightarrow 1 \cdot 32p = 11 \cdot 88$

$\Rightarrow p = \frac{11 \cdot 88}{1 \cdot 32}$

$\Rightarrow p = 9.$

8. Find the mean of the following distribution:

x:	10	12	20	25	35
f:	3	10	15	7	5

Sol:

x	f	Fx
10	3	30
12	10	120
20	15	300
25	7	175
35	5	175
	N = 40	$\Sigma fx = 800$

$\therefore Mean(\bar{x}) = \frac{\Sigma fx}{N}$

$= \frac{800}{40} = 20$

$\bar{x} = 20.$

9. Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of candidates	Average score
I	60	75
II	48	80
III	N A	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

Sol:

Let no. of candidates appeared from school $III = x.$

School	No. of candidates	Average score
I	60	75
II	48	80
III	x	55
IV	40	50

Given, average score of all school = 66.

$$\Rightarrow \frac{N_1\bar{x}_1 + N_2\bar{x}_2 + N_3\bar{x}_3 + N_4\bar{x}_4}{N_1 + N_2 + N_3 + N_4} = 66$$

$$\Rightarrow \frac{60 + 75 + 48 + 80 + x \times 55 + 40 \times 50}{60 + 48 + x + 40} = 66$$

$$\Rightarrow \frac{4500 + 3840 + 55x + 2000}{148 + x} = 66$$

$$\Rightarrow \frac{10340 + 55x}{148 + x} = 66$$

$$\Rightarrow 10340 + 55x = 66x + 9768$$

$$\Rightarrow 10340 - 9768 = 66x - 55x$$

$$\Rightarrow 11x = 572$$

$$\Rightarrow x = \frac{572}{11} = 52.$$

$\therefore$ No. of candidates appeared from school (3) – 52.

10. Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

Sol:

No. of heads per toss (x)	No. of tosses (f)	fx
0	38	0
1	144	144
2	342	684
3	287	861
4	164	656

5	25	125
	N = 100	$\Sigma fx = 2470$

$\therefore$ Mean number of heads per toss $= \dfrac{\Sigma fx}{N}$

$= \dfrac{2470}{1000}$

$= 2 \cdot 47.$

11. Find the missing frequencies in the following frequency distribution if its known that the mean of the distribution is 50.

x: 10 30 50 70 90

f: 17 f_1 32 f_2 19 Total 120

Sol:

x	f	fx
10	17	170
30	f_1	$30\ f_1$
50	32	1600
70	f_2	$70\ f_2$
90	19	1710
	N = 120	$\Sigma fx = 3480 + 30 f_1 + 70 f_2$

It is give that

Mean = 50

$\Rightarrow \dfrac{\Sigma fx}{N} = 50$

$\Rightarrow \dfrac{3480 + 30 f_1 + 70 f_2}{N} = 50$

$\Rightarrow 3480 + 30 f_1 + 70 f_2 = 50(120)$

$\Rightarrow 30 f_1 + 70 f_2 = 6000 - 3480$

$\Rightarrow 10(3 f_1 + 7 f_2) = 10(252)$

$\Rightarrow 3 f_1 + 7 f_2 = 252 \quad .....(1)$ [$\because$ Divide by 10]

And $N = 120$

$\Rightarrow 17 + f_1 + 32 + f_2 + 19 = 120$

$\Rightarrow 68 + f_1 + f_2 = 120$

$\Rightarrow f_1 + f_2 = 120 - 68$

$\Rightarrow f_1 + f_2 = 52$

Multiply with '3' on both sides

$\Rightarrow 3 f_1 + 3 f_2 = 156 \quad .....(2)$

Subtracting equation (2) from equation (1)

$3f_1 + 7f_2 - 3f_1 - 3f_2 = 252 - 156$

$\Rightarrow 4f_2 = 96$

$\Rightarrow f_2 = \frac{96}{4}$

$\Rightarrow f_2 = 24$

Put value of f_2 in equation (1)

$\Rightarrow 3f_1 + 7 \times 24 = 250$

$\Rightarrow 3f_1 = 252 - 168 - 84$

$\Rightarrow f_1 = \frac{84}{3} = 28.$

Exercise – 24.3

Find the median of the following data (1-8)

1. 83, 37, 70, 29, 45, 63, 41, 70, 34, 54

Sol:

Given numbers are

83, 37, 70, 29, 45, 63, 41, 70, 34, 54

Arrange the numbers is ascending order

29, 34, 37, 41, 45, 54, 63, 70, 70, 83

n = 10 (even)

$$\therefore \text{Median} = \frac{\frac{n}{2}^{th} \ value + \left(\frac{n}{2}+1\right)^{th} value}{2}$$

$$= \frac{\frac{10}{2}^{th} \ value + \left(\frac{10}{2}+1\right)^{th} value}{2}$$

$$= \frac{5^{th} \ value + 6^{th} value}{2}$$

$$= \frac{45+54}{2} = \frac{99}{2} = 49 \cdot 5$$

2. 133, 73, 89, 108, 94, 1O4, 94, 85, 100, 120

Sol:

Given numbers are 133, 73, 89, 108, 94, 104, 94, 85, 100, 120

Arrange in ascending order

73, 85, 89, 94, 94, 100, 104, 105, 120, 133

$n = 10$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th}\ value + \left(\frac{n}{2}+1\right)^{th}\ value}{2}$$

$$= \frac{\frac{10}{2}^{th}\ value + \left(\frac{10}{2}+1\right)^{th}\ value}{2}$$

$$= \frac{5^{th}\,value + 6^{th}\,value}{2}$$

$$= \frac{90+104}{2} = 97$$

3. 31 , 38, 27, 28, 36, 25, 35, 40

Sol:

Given numbers are 31, 38, 27, 28, 36, 35, 40

Arranging in increasing order

25, 27, 28, 31, 35, 36, 38, 40

$n = 8$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th}\ value + \left(\frac{n}{2}+1\right)^{th}\ value}{2}$$

$$= \frac{\frac{8}{2}^{th}\ value + \left(\frac{8}{2}+1\right)^{th}\ value}{2}$$

$$= \frac{4^{th}\,value + 5^{th}\,value}{2} = \frac{31+35}{2}$$

$$= \frac{66}{2} = 33$$

4. 15, 6, 16, 8, 22, 21, 9, 18, 25

Sol:

Given numbers are 15, 6, 16, 8, 22, 21, 9, 18, 25

Arrange in increasing order

6, 8, 9, 15, 16, 18, 21, 22, 25

$n = 9$ (odd)

$$\therefore Median = \left(\frac{n+1}{2}\right)^{th}\ value = \left(\frac{9+1}{2}\right)^{th}\ value$$

$= 5^{th}\ value$

$= 16$

5. 41, 43, 127, 99, 71, 92, 71, 58, 57

Sol:

Given numbers are 41, 43, 127, 99, 71, 92, 71, 58, 57

Arrange in increasing order

41, 43, 57, 58, 71, 71, 92, 99, 127

$n = 9\,(\text{odd})$

$\therefore Median = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{9+1}{2}\right)^{th} value$

$= 5^{th}\ value$

$= 71$

6. 25, 34, 31, 23, 22, 26, 35, 29, 20, 32

Sol:

Given number are 25, 34, 31, 23, 22, 26, 35, 29, 20, 32

Arranging in increasing order

20, 22, 23, 25, 26, 29, 31, 32, 34, 35

$n = 10\,(\text{even})$

$\therefore Median = \frac{\frac{n}{2}^{th}\ value + \left(\frac{n}{2}+1\right)^{th} value}{2}$

$= \frac{\frac{10}{2}^{th}\ value + \left(\frac{10}{2}+1\right)^{th} value}{2}$

$= \frac{5^{th}\ value + 6^{th}\ value}{2}$

$= \frac{26+29}{2} = \frac{55}{2}.$

7. 12, 17, 3, 14, 5, 8, 7, 15

Sol:

Given numbers are 12, 17, 3, 14, 5, 8, 7, 15

Arranging in increasing order 3, 5, 7, 8, 12, 14, 15, 17

$n = 8\,(\text{even})$

$$\therefore Median = \frac{\frac{n}{2}^{th}\ value + \left(\frac{n}{2}+1\right)^{th}\ value}{2}$$

$$= \frac{\frac{8}{2}^{th}\ value + \left(\frac{8}{2}+1\right)^{th}\ value}{2}$$

$$= \frac{4^{th}\,value + 5^{th}\,value}{2}$$

$$= \frac{8+12}{2} = \frac{20}{2}$$

$\therefore$ Median = 10

8. 92, 35, 67, 85, 72, 81, 56, 51, 42, 69

Sol:

Given number are

92, 35, 67, 85, 72, 81, 56, 51, 42, 69

Arranging in increasing order

35, 42, 51, 56, 67, 69, 72, 81, 85, 92

$n = 10$ (even)

$$\therefore Median = \frac{\frac{n}{2}^{th}\ value + \left(\frac{n}{2}+1\right)^{th}\ value}{2}$$

$$= \frac{5^{th}\,value + 6^{th}\,value}{2}$$

$$= \frac{67+69}{2} = 68.$$

9. Numbers 50, 42, 35, 2x + 10, 2x − 8, 12, 11, 8 are written in descending order and their median is 25, find x.

Sol:

Given number of observation, $n = 8$

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\ observation + \left(\frac{n}{2}+1\right)^{th}\ observation}{2}$$

$$= \frac{2x+10+2x-8}{2}$$

$$= 2x+1$$

Given median $= 25$

$\therefore 2x+1=25$

$\Rightarrow 2x=24$

$\Rightarrow x=12$

10. Find the median of the following observations : 46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33. If 92 is replaced by 99 and 41 by 43 in the above data, find the new median?

Sol:

Given numbers are

46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33

Arrange in increasing order

33, 35, 41, 46, 55, 58, 64, 77, 87, 90, 92

$n=1\,(\text{odd})$

$\therefore Median=\left(\frac{n+1}{2}\right)^{th} value$

$=\left(\frac{11+1}{2}\right)^{th} value$

$=6^{th} value=58$

If 92 is replaced by 99 and 41 by 43

Then, the new values are

33, 35, 43, 46, 55, 58, 64, 77, 87, 90, 99

$\therefore n=11(\text{odd})$

New median $=\left(\frac{n+1}{2}\right)^{th} value$

$=\left(\frac{11+1}{2}\right)^{th} value$

$=6^{th} value$

$=58.$

11. Find the median of the following data : 41, 43, 127, 99, 61, 92, 71, 58, 57 If 58 is replaced by 85, what will be the new median.

Sol:

Given numbers are

41, 43, 127, 99, 61, 92, 71, 58 and 57

Arrange in ascending order

41, 43, 57, 58, 61, 71, 92, 99, 127

$n=9\,(\text{odd})$

$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{9+1}{2}\right)^{th} value$

$= 5^{th} value$

$= 61$

If 58 is replaced by 85

Then new values be in order are

41, 43, 57, 61, 71, 85, 92, 99, 27

$n = 9 \text{ (odd)}$

$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{9+1}{2}\right)^{th} value$

$= 5^{th} value$

$= 71$

12. The weights (in kg) of 15 students are: 31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30. Find the median. If the weight 44 kg is replaced by 46 kg and 27 kg by 25 kg, find the new median.

Sol:

Given numbers are

31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30

Arranging increasing order

27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 44, 45

$n = 15 \text{ (odd)}$

$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{15+1}{2}\right)^{th} value$

$= 8^{th} value$

$= 35kg$

If the weight 44kg is replaced by 46 kg and 27 kg is replaced by 25 kg

Then, new values in order be

25, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 45, 46

$n = 15 \text{ (odd)}$

$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} value$

$= \left(\frac{15+1}{2}\right)^{th} value$

$= 8^{th} value$

$= 35kg$

13. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x: 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95

Sol:

Total number of observation in the given data is 10 (even number). So median of this data will be mean of $\frac{10}{2}$ i.e., 5^{th} *and* $\frac{10}{2}+1$ i.e., 6^{th} observations.

So, median of data $= \frac{5^{th} observation + 6^{th} observation}{2}$

$\Rightarrow 63 = \frac{x + x + 2}{2}$

$\Rightarrow 63 = \frac{2x + 2}{2}$

$\Rightarrow 63 = x + 1$

$\Rightarrow x = 62$

Exercise – 24.4

1. Find out the mode of the following marks obtained by 15 students in a class:
Marks : 4, 6, 5, 7, 9, 8, 10, 4, 7, 6, 5, 9, 8, 7, 7.

Sol:

Marks	4	5	6	7	8	9	10
No. of students	2	2	2	4	2	2	1

Since, the maximum frequency corresponds to the value 7 then mode = 7 marks.

2. Find the mode from the following data:
125, 175, 225, 125, 225, 175, 325, 125, 375, 225, 125

Sol:

Values	125	175	225	325	375
Frequency	4	2	3	1	1

Since, maximum frequency 4 corresponds value 125 then mode = 125

3. Find the mode for the following series :

7.5, 7.3, 7.2, 7.2, 7.4, 7.7, 7.7,7.5, 7.3, 7.2, 7.6, 7.2

Sol:

Values	7·2	7·3	7·4	7·5	7·6	7·7
Frequency	4	2	1	2	1	2

Since, maximum frequency 4 corresponds to value $7 \cdot 2$ then mode $= 7 \cdot 2$

4. Find the mode of the following data in each case:

(i) 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18

(ii) 7, 9, 12, 13, 7, 12, 15, 7, 12, 7, 25, 18, 7

Sol:

(i) Arranging the data in an ascending order

14, 14, 14, 14, 17, 18, 18, 18, 22, 23, 25, 28

Here observation 14 is having the highest frequency

i.e., 4 in given data, so mode of given data is 14.

(ii)

Values	7	9	12	13	15	18	25
Frequency	5	1	3	1	1	1	1

Since, maximum frequency 5 corresponds to value 7 then the mode = 7

5. The demand of different shirt sizes, as obtained by a survey, is given below:

Size:	38	39	40	41	42	43	44	Total
No of persons (wearing it)	26	39	20	15	13	7	5	125

Find the modal shirt sizes, as observed from the survey.

Sol:

Size	38	39	40	41	42	43	44	Total
No. of persons	26	39	20	15	13	7	5	125

Since, maximum frequency 39 corresponds to value – 39 then mode size = 39.