4. Cubes And Cube Roots

CUBES AND CUBE ROOTS (Exercise - 14.1)

(1) We should find cubes

(i) 7

Cube of 7 $= 7 \times 7 \times 7$

$= 343$

(ii) 12

cube of 12 $= 12 \times 12 \times 12$

or

Here $a = 1$ $b = 2$

Using Column method.

Column I	Column II	Column III	Column IV
a^3	$3 \times a^2 \times b$	$3 \times a \times b^2$	b^3
$1^3 = 1$	$3 \times 1^2 \times 2 = 6$ $+ 1$	$3 \times 1 \times 4 = 1\underline{2}$	$2^3 = \underline{8}$
1	7	2	8

$12^3 = 1728$

(iii) 16

Here $a = 1$ $b = 6$

using column method

Column I	Column II	Column III	Column IV
a^3	$3 \times a^2 \times b$	$3 \times a \times b^2$	b^3
$1^3 = 1$ $+ 3$	$3 \times 1^2 \times 6 = 18$ $+ 12$	$3 \times 1 \times 6^2 = 108$ $+ 21$	$6^3 = 21\underline{6}$
$\underline{4}$	$3\underline{0}$	$12\underline{9}$	6

$16^3 = 4096$

iv) 21

Here $a = 2$ $b = 1$

Using column method

column I	column II	column III	column IV
a^3	$3 \times a^2 \times b$	$3 \times a \times b^2$	b^3
$2^3 = 8$ $+1$ $\underline{9}$	$3 \times 2^2 \times 1 = 1\underline{2}$	$3 \times 2 \times 1 = \underline{6}$	$1^3 = \underline{1}$
9	2	6	1

$21^3 = 9261$

v) 40

Here $a = 4$, $b = 0$

using column method

Column I	column II	column III	column IV
a^3	$3 \times a^2 \times b$	$3 \times a \times b^2$	b^3
$4^3 = \underline{64}$	$3 \times 4^2 \times 0 = \underline{0}$	$3 \times 4 \times a^2 = \underline{0}$	$\underline{0}$

$40^3 = 64000$

vi) 55

Here $a = 5$, $b = 5$

by column method

Column I	column II	column III	column IV
a^3	$3a^2b$	$2ab^2$	b^3
$5^3 = 125$ 41 166	$3 \times 5^2 \times 5 = 375$ $+38$ $41\underline{3}$	$3 \times 5 \times 5^2 = 375$ $+12$ $38\underline{7}$	$5^3 = 12\underline{5}$ 5

$55^3 = 166375$

(vii) $(100)^3 = 100 \times 100 \times 100$

$= 10,00,000$

(viii) $(302)^3 = (300+2)^3$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$= 300^3 + 2^3 + 2.300.2\,[302]$

$= 27000000 + 8 + 362400$

$= 27362408$

(ix) $(301)^3 = (300+1)^3$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$= 300^3 + 1^3 + 2.300.1\,[300+1]$

$= 27000000 + 1 + 180600$

$= 27180601$

(2) Cubes of natural numbers upto 10

$1^3 = 1 \times 1 \times 1 = 1$; $4^3 = 4 \times 4 \times 4 = 64$

$2^3 = 2 \times 2 \times 2 = 8$; $5^3 = 5 \times 5 \times 5 = 125$

$3^3 = 3 \times 3 \times 3 = 27$; $6^3 = 6 \times 6 \times 6 = 216$

$7^3 = 7 \times 7 \times 7 = 343$.

$8^3 = 8 \times 8 \times 8 = 512$

$9^3 = 9 \times 9 \times 9 = 729$

$10^3 = 10 \times 10 \times 10 = 1000$

(i) cubes of all even numbers are even

and

(ii) Cubes of all odd numbers are odd.

(3) Given pattern.

$$1^3 = 1$$
$$1^3 + 2^3 = (1+2)^2$$
$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$

Now, similarly

$$1^3 + 2^3 + 3^3 + \dots n^3 = (1+2+3 \dots n)^2$$

$$\Rightarrow \left(1^3+2^3+3^3 \dots + 9^3 + 10^3\right) = (1+2+3+4 \dots + 9 + 10)^2$$
$$= (55)^2 = 55 \times 55$$
$$= 3025$$

(4) First 5 natural numbers which are multiples of 3 are 3, 6, 9, 12, 15.

Cube of 3 = $3^3 = 3\times3\times3 = 27$

$6^3 = 6\times6\times6 = 216$

$9^3 = 9\times9\times9 = 729$

$12^3 = 12\times12\times12 = 1728$

$15^3 = 15\times15\times15 = 3375$

all the cubes got are divisible by 27.

There, fore

'the cube of a natural number which is a multiple of 3 is a multiple of 27'.

(5) First 5 natural numbers in the form of (3n+1) are :-

4, 7, 10, 13, 16

Cubes of these numbers :

$4^3 = 4\times4\times4 = 64$

$7^3 = 7\times7\times7 = 343$

$10^3 = 10 \times 10 \times 10 = 1000$ (5) +(1)

$13^3 = 13 \times 13 \times 13 = 2197$

$16^3 = 16 \times 16 \times 16 = 4096$

64, 343, 1000, 2197, 4096 gives remainder '1' when divided by '7'.

$\Rightarrow$ Give statement is true

(6) first five natural numbers in $3n+2$ form are

5, 8, 11, 14, 17

Cubes of given numbers are :-

$5^3 = 5 \times 5 \times 5 = 125$

$8^3 = 8 \times 8 \times 8 = 512$

$11^3 = 11 \times 11 \times 11 = 1331$

$14^3 = 14 \times 14 \times 14 = 2744$

$17^3 = 17 \times 17 \times 17 = 4913$

125, 512, 1331, 2744, 4913 gives remainder '2'. when divided by '3'.

$\Rightarrow$ Given statement is true.

(7) First 5 natural numbers which are multiples of 7 are :- 7, 14, 21, 28, 35

Cubes of these numbers

$7^3 = 7 \times 7 \times 7 = 343$

$14^3 = 14 \times 14 \times 14 = 2744$

$21^3 = 21 \times 21 \times 21 = 9261$

$28^3 = 28 \times 28 \times 28 = 21952$

$35^3 = 35 \times 35 \times 35 = 42875$

343, 2744, 9261, 21952, 42875 are the multiples of 7^3.

(8) (i) $64 = 2^6$

$= 4^3$ (perfect cube)

$$\begin{array}{r|l} 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

(ii) $216 = 2^3 \times 3^3$

$= 6^3$

(perfect cube)

$$\begin{array}{r|l} 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

(iii)

$243 = 3^5$

$= 3^3 \cdot 3^2$

(Not a perfect cube)

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(iv) $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$= 2^3 \times 5^3$

$= 10^3$ (perfect cube).

$$\begin{array}{r|l} 2 & 1000 \\ \hline 2 & 500 \\ \hline 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

(v) $1728 = 2^6 \cdot 3^3$

$= 2^6 \cdot 3^3$

$= (4 \times 3)^3$

$= (12)^3$

(perfect cube)

$$\begin{array}{r|l} 2 & 1728 \\ \hline 2 & 864 \\ \hline 2 & 432 \\ \hline 2 & 216 \\ \hline 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(vi) $3087 = 3^2 \cdot 7^3$

(Not perfect-cube)

$$\begin{array}{r|l} 3 & 3087 \\ \hline 3 & 1029 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

(vii) $4068 = 2^2 \times 3 \times 113$

(Not perfect cube)

$$\begin{array}{r|l} 2 & 4068 \\ \hline 2 & 2034 \\ \hline 3 & 1017 \\ \hline 3 & 339 \\ \hline 8 & 113 \\ \hline & 8 \end{array}$$

(viii) 106480

$= 2^4 \times 5 \times 11^3$

= (Not perfect cube)

$$\begin{array}{r|l} 2 & 106480 \\ 2 & 53240 \\ 2 & 26620 \\ 2 & 13310 \\ 11 & 6655 \\ 5 & 605 \\ 11 & 121 \\ & 11 \end{array}$$

(ix) 166375

$= 5^3 \times 11^3$

$= (55)^3$

[Perfect cube]

$$\begin{array}{r|l} 5 & 166375 \\ 5 & 33275 \\ 5 & 6655 \\ 11 & 1331 \\ 11 & 121 \\ & 11 \end{array}$$

(x) 456533

$= 11^3 \times 7^3$

$= (11 \times 7)^3$

(perfect cube)

$$\begin{array}{r|l} 11 & 456533 \\ 7 & 41503 \\ 7 & 5929 \\ 7 & 847 \\ 11 & 121 \\ & 11 \end{array}$$

9 (i) $216 = 2^3 \times 3^3$

$= (6)^3$

cube of even natural

$$\begin{array}{r|l} 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

(ii) $512 = 2^9$

$= (2^3)^3$

$= (8)^3$

$\boxed{\text{Cube of even natural number}}$

$$\begin{array}{r|l} 2 & 512 \\ 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

(iii) $729 = 3^3 \times 3^3$

$= (9)^3$

Not a cube of even natural number

$$\begin{array}{r|l} 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

(iv) $1000 = 2^3 \times 5^3$

$= (10)^3$

Cube of even natural number.

$$\begin{array}{r|l} 2 & 1000 \\ 2 & 500 \\ 2 & 250 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

(v) $1728 = 2^6 \times 3^3$

$= (12)^3$

Cube of even natural number.

$$\begin{array}{r|l} 2 & 1728 \\ 2 & 864 \\ 2 & 432 \\ 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

(11)

$13284 = 2^2 \times 3^4 \times 41$

[Not even a cube Here]

2 | 13284
2 | 6642
3 | 3321
3 | 1107
3 | 369
3 | 123
41

(10)

$\underline{125} = 5 \times 5 \times 5 = 5^3$

[Cube of odd natural]

5 | 125
5 | 25
5

$\underline{343} = 7^3$

[Cube of odd natural]

7 | 343
7 | 49
7

$\underline{1728} = 2^6 \times 3^3$

$= (4 \times 3)^3$

$= (12)^3$

[Not a cube of odd natural]

2 | 1728
2 | 864
2 | 432
2 | 216
2 | 108
2 | 54
3 | 27
3 | 9
3

$\underline{4096} = 2^{12}$

$= (2^6)^2$

$= (64)^2$

[Not a cube of odd natural]

2 | 4096
2 | 2048
2 | 1024
2 | 512
2 | 256
2 | 128
2 | 64
2 | 32
4 | 16
4

$\underline{32768} = 2^{15}$

$= (2^5)^3$

$= (32)^3$

Not a cube of odd number

2 | 32768
2 | 16384
2 | 8192
2 | 4096
2 | 2048
2 | 1024
2 | 512
2 | 256
2 | 128
2 | 64
2 | 32
4 | 16
4