
Exercise – 5.1

Factorize:

1. $x^3 + x - 3x^2 - 3$

Sol:

$$x^3 + x - 3x^2 - 3$$

Taking x common in $(x^3 + x)$

$$= x(x^2 + 1) - 3x^2 - 3$$

Taking -3 common in $(-3x^2 - 3)$

$$= x(x^2 + 1) - 3(x^2 + 1)$$

Now, we take $(x^2 + 1)$ common

$$= (x^2 + 1)(x - 3)$$

$$\therefore x^3 + x - 3x^2 - 3 = (x^2 + 1)(x - 3)$$

2. $a(a+b)^3 - 3a^2b(a+b)$

Sol:Taking $(a+b)$ common in two terms

$$= (a+b)\{a(a+b)^2 - 3a^2b\}$$

Now, using $(a+b)^2 = a^2 + b^2 + 2ab$

$$= (a+b)\{a(a^2 + b^2 + 2ab) - 3a^2b\}$$

$$= (a+b)\{a^3 + ab^2 + 2a^2b - 3a^2b\}$$

$$= (a+b)\{a^3 + ab^2 - a^2b\}$$

$$= (a+b)a\{a^2 + b^2 - ab\}$$

$$= a(a+b)(a^2 + b^2 - ab)$$

$$\therefore a(a+b)^3 - 3a^2b(a+b) = a(a+b)(a^2 + b^2 - ab)$$

3. $x(x^3 - y^3) + 3xy(x - y)$

Sol:

Elaborating $x^3 - y^3$ using identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= x(x - y)(x^2 + xy + y^2) + 3xy(x - y)$$

Taking common $x(x - y)$ in both the terms

$$= x(x - y)\{x^2 + xy + y^2 + 3y\}$$

$$\therefore x(x^3 - y^3) + 3xy(x - y) = x(x - y)(x^2 + xy + y^2 + 3y)$$

4. $a^2x^2 + (ax^2 + 1)x + a$

Sol:

We multiply $x(ax^2 + 1) = ax^3 + x$

$$= a^2x^2 + ax^3 + x + a$$

Taking common ax^2 in $(a^2x^2 + ax^3)$ and 1 in $(x + a)$

$$= ax^2(a + x) + 1(x + a)$$

$$= ax^2(a + x) + 1(a + x)$$

Taking $(a + x)$ common in both the terms

$$= (a + x)(ax^2 + 1)$$

$$\therefore a^2x^2 + (ax^2 + 1)x + a = (a + x)(ax^2 + 1)$$

5. $x^2 + y - xy - x$

Sol:

On rearranging

$$x^2 - xy - x + y$$

Taking x common in the $(x^2 + y)$ and -1 in $(-x + y)$

$$= x(x - y) - 1(x - y)$$

Taking $(x - y)$ common in both the terms

$$= (x - y)(x - 1)$$

$$\therefore x^2 + y - xy - x = (x - y)(x - 1)$$

6. $x^3 - 2x^2y + 3xy^2 - 6y^3$

Sol:

Taking x^2 common in $(x^3 - 2x^2y)$ and $+3y^2$ common in $(3xy^2 - 6y^3)$

$$= x^2(x - 2y) + 3y^2(x - 2y)$$

$$= (x - 2y)(x^2 + 3y^2)$$

$$\therefore x^3 - 2x^2y + 3xy^2 - 6y^3 = (x - 2y)(x^2 + 3y^2)$$

7. $6ab - b^2 + 12ac - 2bc$

Sol:

Taking common b in $(6ab - b^2)$ and $2c$ in $(12ac - 2bc)$

$$= b(6a - b) + 2c(6a - b)$$

Taking $(6a - b)$ common in both terms

$$= (6a - b)(b + 2c)$$

$$\therefore 6ab - b^2 + 12ac - 2bc = (6a - b)(b + 2c)$$

8. $\left[x^2 + \frac{1}{x^2}\right] - 4\left[x + \frac{1}{x}\right] + 6$

Sol:

$$= x^2 + \frac{1}{x^2} - 4x - \frac{4}{x} + 4 + 2$$

$$= x^2 + \frac{1}{x^2} + 4 + 2 - \frac{4}{x} - 4x$$

$$= (x^2) + \left(\frac{1}{x}\right)^2 + (-2)^2 + 2 \times x \times x \frac{1}{x} + 2x \frac{1}{x} \times (-2) + 2(-2)x$$

Using identity

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$$

We get,

$$= \left[x + \frac{1}{x} + (-2)\right]^2$$

$$= \left[x + \frac{1}{x} - 2\right]^2$$

$$= \left[x + \frac{1}{x} - 2\right] \left[x + \frac{1}{x} - 2\right]$$

$$\therefore \left[x^2 + \frac{1}{x^2}\right] - 4\left[x + \frac{1}{x}\right] + 6 = \left[x + \frac{1}{x} - 2\right] \left[x + \frac{1}{x} - 2\right]$$

9. $x(x-2)(x-4)+4x-8$

Sol:

$$= x(x-2)(x-4)+4(x-2)$$

Taking $(x-2)$ common in both terms

$$= (x-2)\{x(x-4)+4\}$$

$$= (x-2)\{x^2-4x+4\}$$

Now splitting middle term of x^2-4x+4

$$= (x-2)\{x^2-2x-2x+4\}$$

$$= (x-2)\{x(x-2)-2(x-2)\}$$

$$= (x-2)\{(x-2)(x-2)\}$$

$$= (x-2)(x-2)(x-2)$$

$$= (x-2)^3$$

$$\therefore x(x-2)(x-4)+4x-8 = (x-2)^3$$

10. $(x+2)(x^2+25)-10x^2-20x$

Sol:

$$= (x+2)(x^2+25)-10x(x+2)$$

Taking $(x+2)$ common in both terms

$$= (x+2)(x^2+25-10x)$$

$$= (x+2)(x^2-10x+25)$$

Splitting middle term of $x^2-10x+25$

$$= (x+2)\{x^2-5x-5x+25\}$$

$$= (x+2)\{x(x-5)-5(x-5)\}$$

$$= (x+2)(x-5)(x-5)$$

$$\therefore (x+2)(x^2+25)-10x^2-20x = (x+2)(x-5)(x-5)$$

11. $2a^2+2\sqrt{6}ab+3b^2$

Sol:

$$= (2\sqrt{a})^2 + 2 \times \sqrt{2}a \times \sqrt{3}b + (\sqrt{3}b)^2$$

$$\begin{aligned}
 &\text{Using identity } a^2 + 2ab + b^2 = (a + b)^2 \\
 &= (\sqrt{2a} + \sqrt{3b})^2 \\
 &= (\sqrt{2a} + \sqrt{3b})(\sqrt{2a} + \sqrt{3b}) \\
 \therefore 2a^2 + 2\sqrt{6ab} + 3b^2 &= (\sqrt{2a} + \sqrt{3b})(\sqrt{2a} + \sqrt{3b})
 \end{aligned}$$

12. $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a)$

Sol:

$$\text{Let } (a - b + c) = x \text{ and } (b - c + a) = y$$

$$= x^2 + y^2 + 2xy$$

$$\text{Using identity } a^2 + b^2 + 2ab = (a + b)^2$$

$$= (x + y)^2$$

Now, substituting x and y

$$= (a - b + c + b - c + a)^2$$

Cancelling $-b, +b$ and $+c, -c$

$$= (2a)^2$$

$$= 4a^2$$

$$\therefore (a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a) = 4a^2$$

13. $a^2 + b^2 + 2(ab + bc + ca)$

Sol:

$$= a^2 + b^2 + 2ab + 2bc + 2ca$$

$$\text{Using identity } a^2 + b^2 + 2ab = (a + b)^2$$

We get,

$$= (a + b)^2 + 2bc + 2ca$$

$$= (a + b)^2 + 2c(b + a)$$

$$\text{or } (a + b)^2 + 2c(a + b)$$

Taking $(a + b)$ common

$$= (a + b)(a + b + 2c)$$

$$\therefore a^2 + b^2 + 2(ab + bc + ca) = (a + b)(a + b + 2c)$$

14. $4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2$

Sol:

$$\text{Let } (x-y) = a, (x+y) = b$$

$$= 4a^2 - 12ab + 9b^2$$

Splitting middle term $-12 = -6 - 6$ also $4 \times 9 = -6 \times -6$

$$= 4a^2 - 6ab - 6ab + 9b^2$$

$$= 2a(2a - 3b) - 3b(2a - 3b)$$

$$= (2a - 3b)(2a - 3b)$$

$$= (2a - 3b)^2$$

Substituting $a = x - y$ and $b = x + y$

$$= [2(x-y) - 3(x+y)]^2$$

$$= [2x - 2y - 3x - 3y]^2$$

$$= [2x - 3x - 2y - 3y]^2$$

$$= [-x - 5y]^2$$

$$= [(-1)(x+5y)]^2$$

$$= (x+5y)^2 \quad [\because (-1)^2 = 1]$$

$$\therefore 4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2 = (x+5y)^2$$

15. $a^2 - b^2 + 2bc - c^2$

Sol:

$$= a^2 - (b^2 - 2bc + c^2)$$

Using identity $a^2 - 2ab + b^2 = (a-b)^2$

$$= a^2 - (b-c)^2$$

Using identity $a^2 - b^2 = (a+b)(a-b)$

$$= (a+b-c)(a-(b-c))$$

$$= (a+b-c)(a-b+c)$$

$$\therefore a^2 - b^2 + 2bc - c^2 = (a+b-c)(a-b+c)$$

16. $a^2 + 2ab + b^2 - c^2$

Sol:

Using identity $a^2 + 2ab + b^2 = (a + b)^2$

$$= (a + b)^2 - c^2$$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$$= (a + b + c)(a + b - c)$$

$$\therefore a^2 + 2ab + b^2 - c^2 = (a + b + c)(a + b - c)$$

17. $a^2 + 4b^2 - 4ab - 4c^2$

Sol:

On rearranging

$$= a^2 - 4ab + 4b^2 - 4c^2$$

$$= (a)^2 - 2 \times a \times 2b + (2b)^2 - 4c^2$$

Using identity $a^2 - 2ab + b^2 = (a - b)^2$

$$= (a - 2b)^2 - 4c^2$$

$$= (a - 2b)^2 - (2c)^2$$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$$= (a - 2b + 2c)(a - 2b - 2c)$$

$$\therefore a^2 + 4b^2 - 4ab - 4c^2 = (a - 2b + 2c)(a - 2b - 2c)$$

18. $xy^9 - yx^9$

Sol:

$$xy^9 - yx^9$$

$$= xy(y^8 - x^8)$$

$$= xy\left((y^4)^2 - (x^4)^2\right)$$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$$= xy(y^4 + x^4)(y^4 - x^4)$$

$$= xy(y^4 + x^4)\left((y^2)^2 - (x^2)^2\right)$$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$$\begin{aligned}
&= xy(y^4 + x^4)(y^2 + x^2)(y^2 - x^2) \\
&= xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x) \\
&= xy(x^4 + y^4)(x^2 + y^2)(x + y)(-1)(x - y) \\
&\because (b - a) = -1(a - b) \\
&= -xy(x^4 + y^4)(x^2 + y^2)(x + y)(x - y) \\
\therefore xy^9 - yx^9 &= -xy(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)
\end{aligned}$$

19. $x^4 + x^2y^2 + y^4$

Sol:

Adding x^2y^2 and subtracting x^2y^2 to the given equation

$$\begin{aligned}
&= x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2 \\
&= x^4 + 2x^2y^2 + y^4 - x^2y^2 \\
&= (x^2)^2 + 2 \times x^2 \times y^2 + (y^2)^2 - (xy)^2
\end{aligned}$$

Using identity $a^2 + 2ab + b^2 = (a + b)^2$

$$= (x^2 + y^2)^2 - (xy)^2$$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$$\begin{aligned}
&= (x^2 + y^2 + xy)(x^2 + y^2 - xy) \\
\therefore x^4 + x^2y^2 + y^4 &= (x^2 + y^2 + xy)(x^2 + y^2 - xy)
\end{aligned}$$

20. $x^2 - y^2 - 4xz + 4z^2$

Sol:

On rearranging the terms

$$\begin{aligned}
&= x^2 - 4xz + 4z^2 - y^2 \\
&= (x)^2 - 2 \times x \times 2z + (2z)^2 - y^2
\end{aligned}$$

Using identity $a^2 - 2ab + b^2 = (a - b)^2$

$$= (x - 2z)^2 - y^2$$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$$\begin{aligned}
&= (x - 2z + y)(x - 2z - y) \\
\therefore x^2 - y^2 - 4xz + 4z^2 &= (x - 2z + y)(x - 2z - y)
\end{aligned}$$

21. $x^2 + 6\sqrt{2}x + 10$

Sol:

Splitting middle term,

$$= x^2 + 5\sqrt{2}x + \sqrt{2}x + 10 \quad \left[\because 6\sqrt{2} = 5\sqrt{2} + \sqrt{2} \text{ and } 5\sqrt{2} \times \sqrt{2} = 10 \right]$$

$$= x(x + 5\sqrt{2}) + \sqrt{2}(x + 5\sqrt{2})$$

$$= (x + 5\sqrt{2})(x + \sqrt{2})$$

$$\therefore x^2 + 6\sqrt{2}x + 10 = (x + 5\sqrt{2})(x + \sqrt{2})$$

22. $x^2 - 2\sqrt{2}x - 30$

Sol:

Splitting the middle term,

$$= x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30 \quad \left[\because -2\sqrt{2} = -5\sqrt{2} + 3\sqrt{2} \text{ also } -5\sqrt{2} \times 3\sqrt{2} = -30 \right]$$

$$x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$$

$$= (x - 5\sqrt{2})(x + 3\sqrt{2})$$

$$\therefore x^2 - 2\sqrt{2}x - 30 = (x - 5\sqrt{2})(x + 3\sqrt{2})$$

23. $x^2 - \sqrt{3}x - 6$

Sol:

Splitting the middle term,

$$= x^2 - 2\sqrt{3}x + \sqrt{3}x - 6 \quad \left[\because -\sqrt{3} = -2\sqrt{3} + \sqrt{3} \text{ also } -2\sqrt{3} \times \sqrt{3} = -6 \right]$$

$$= x(x - 2\sqrt{3}) + \sqrt{3}(x - 2\sqrt{3})$$

$$= (x - 2\sqrt{3})(x + \sqrt{3})$$

$$\therefore x^2 - \sqrt{3}x - 6 = (x - 2\sqrt{3})(x + \sqrt{3})$$

24. $x^2 + 5\sqrt{5}x + 30$

Sol:

Splitting the middle term,

$$= x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30 \quad \left[\because 5\sqrt{5} = 2\sqrt{5} + 3\sqrt{5} \text{ also } 2\sqrt{5} \times \sqrt{3} = 30 \right]$$

$$= x(x + 2\sqrt{5}) + 3\sqrt{5}(x + 2\sqrt{5})$$

$$= (x + 2\sqrt{5})(x + 3\sqrt{5})$$

$$\therefore x^2 + 5\sqrt{5}x + 30 = (x + 2\sqrt{5})(x + 3\sqrt{5})$$

25. $x^2 + 2\sqrt{3}x - 24$

Sol:

Splitting the middle term,

$$= x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24 \quad \left[\because 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3} \text{ also } 4\sqrt{3}(-2\sqrt{3}) = -24 \right]$$

$$= x(x + 4\sqrt{3}) - 2\sqrt{3}(x + 4\sqrt{3})$$

$$= (x + 4\sqrt{3})(x - 2\sqrt{3})$$

$$\therefore x^2 + 2\sqrt{3}x - 24 = (x + 4\sqrt{3})(x - 2\sqrt{3})$$

26. $2x^2 - \frac{5}{6}x + \frac{1}{12}$

Sol:

Splitting the middle term,

$$= 2x^2 - \frac{x}{2} - \frac{x}{3} + \frac{1}{12} \quad \left[\because -\frac{5}{6} = -\frac{1}{2} - \frac{1}{3} \text{ also } -\frac{1}{2} \times -\frac{1}{3} = 2 \times \frac{1}{12} \right]$$

$$= x\left(2x - \frac{1}{2}\right) - \frac{1}{6}\left(2x - \frac{1}{2}\right)$$

$$= \left(2x - \frac{1}{2}\right)\left(x - \frac{1}{6}\right)$$

$$\therefore 2x^2 - \frac{5}{6}x + \frac{1}{12} = \left(2x - \frac{1}{2}\right)\left(x - \frac{1}{6}\right)$$

27. $x^2 + \frac{12}{35}x + \frac{1}{35}$

Sol:

Splitting the middle term,

$$= x^2 + \frac{5}{35}x + \frac{7}{35}x + \frac{1}{35} \quad \left[\because \frac{12}{35} = \frac{5}{35} + \frac{7}{35} \text{ and } \frac{5}{35} \times \frac{7}{35} = \frac{1}{35} \right]$$

$$= x^2 + \frac{x}{7} + \frac{x}{5} + \frac{1}{35}$$

$$= x\left(x + \frac{1}{7}\right) + \frac{1}{5}\left(x + \frac{1}{7}\right)$$

$$= \left(x + \frac{1}{7}\right)\left(x + \frac{1}{5}\right)$$

$$\therefore x^2 + \frac{12}{35}x + \frac{1}{35} = \left(x + \frac{1}{7}\right)\left(x + \frac{1}{5}\right)$$

28. $21x^2 - 2x + \frac{1}{21}$

Sol:

$$= (\sqrt{21}x)^2 - 2 \times \sqrt{21}x \times \frac{1}{\sqrt{21}} + \left(\frac{1}{\sqrt{21}}\right)^2$$

Using identity $a^2 - 2ab + b^2 = (a - b)^2$

$$= \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$$

$$\therefore 21x^2 - 2x + \frac{1}{21} = \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$$

29. $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Sol:

Splitting the middle term,

$$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5} \quad \left[\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5 = 5\sqrt{5} \times 3\sqrt{5} \right]$$

$$= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$$

$$= (\sqrt{5}x + 3)(5x + \sqrt{5})$$

$$\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = (\sqrt{5}x + 3)(5x + \sqrt{5})$$

30. $2x^2 + 3\sqrt{5}x + 5$

Sol:

Splitting the middle term,

$$= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5 \quad \left[\because 3\sqrt{5} = 2\sqrt{5} + \sqrt{5} \text{ also } 2\sqrt{5} \times \sqrt{5} = 2 \times 5 \right]$$

$$= 2x(x + \sqrt{5}) + \sqrt{5}(x + \sqrt{5})$$

$$= (x + \sqrt{5})(2x + \sqrt{5})$$

$$\therefore 2x^2 + 3\sqrt{5}x + 5 = (x + \sqrt{5})(2x + \sqrt{5})$$

31. $9(2a - b)^2 - 4(2a - b) - 13$

Sol:

Let $2a - b = x$

$$= 9x^2 - 4x - 13$$

Splitting the middle term,

$$\begin{aligned} &= 9x^2 - 13x + 9x - 13 \\ &= x(9x - 13) + 1(9x - 13) \\ &= (9x - 13)(x + 1) \\ &\text{Substituting } x = 2a - b \\ &= [9(2a - b) - 13](2a - b + 1) \\ &= (18a - 9b - 13)(2a - b + 1) \\ \therefore 9(2a - b)^2 - 4(2a - b) - 13 &= (18a - 9b - 13)(2a - b + 1) \end{aligned}$$

32. $7(x - 2y)^2 - 25(x - 2y) + 12$

Sol:

Let $x - 2y = P$

$$= 7P^2 - 25P + 12$$

Splitting the middle term,

$$= 7P^2 - 21P - 4P + 12$$

$$= 7P(P - 3) - 4(P - 3)$$

$$= (P - 3)(7P - 4)$$

Substituting $P = x - 2y$

$$= (x - 2y - 3)(7(x - 2y) - 4)$$

$$= (x - 2y - 3)(7x - 14y - 4)$$

$$\therefore 7(x - 2y)^2 - 25(x - 2y) + 12 = (x - 2y - 3)(7x - 14y - 4)$$

33. $2(x + y)^2 - 9(x + y) - 5$

Sol:

Let $x + y = z$

$$= 2z^2 - 9z - 5$$

Splitting the middle term,

$$= 2z^2 - 10z + z - 5$$

$$= 2z(z - 5) + 1(z - 5)$$

$$= (z - 5)(2z + 1)$$

Substituting $z = x + y$

$$= (x + y - 5)(2(x + y) + 1)$$

$$= (x + y - 5)(2x + 2y + 1)$$

$$\therefore 2(x + y)^2 - 9(x + y) - 5 = (x + y - 5)(2x + 2y + 1)$$

34. Given possible expressions for the length and breadth of the rectangle having $35y^2 + 13y - 12$ as its area.

Sol:

$$\text{Area} = 35y^2 + 13y - 12$$

Splitting the middle term,

$$\text{Area} = 35y^2 + 28y - 15y - 12$$

$$= 7y(5y + 4) - 3(5y + 4)$$

$$\text{Area} = (5y + 4)(7y - 3)$$

Also area of rectangle = Length \times Breadth

$$\therefore \text{Possible length} = (5y + 4) \text{ and breadth} = (7y - 3)$$

$$\text{Or Possible length} = (7y - 3) \text{ and breadth} = (5y + 4)$$

35. What are the possible expressions for the dimensions of the cuboid whose volume is $3x^2 - 12x$.

Sol:

$$\text{Here volume} = 3x^2 - 12x$$

$$= 3x(x - 4)$$

$$= 3 \times x(x - 4)$$

Also volume = Length \times Breadth \times Height

$$\therefore \text{Possible expressions for dimensions of the cuboid are} = 3, x, (x - 4)$$

Exercise – 5.2**Factorize each of the following expressions :**

1. $p^3 + 27$

Sol:

$$p^3 + 27$$

$$= p^3 + 3^3$$

$$= (p+3)(p^2 - 3p + 3^2) \quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right]$$

$$= (p+3)(p^2 - 3p + 9)$$

$$\therefore p^3 + 27 = (p+3)(p^2 - 3p + 9)$$

2. $y^3 + 125$

Sol:

$$= y^3 + 5^3$$

$$= (y+5)(y^2 - 5y + 5^2) \quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right]$$

$$= (y+5)(y^2 - 5y + 25)$$

$$\therefore y^3 + 125 = (y+5)(y^2 - 5y + 25)$$

3. $1 - 27a^3$

Sol:

$$= (1)^3 - (3a)^3$$

$$= (1-3a)(1^2 + 1 \times 3a + (3a)^2) \quad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2) \right]$$

$$= (1-3a)(1 + 3a + 9a^2)$$

$$\therefore 1 - 27a^3 = (1-3a)(1 + 3a + 9a^2)$$

4. $8x^3y^3 + 27a^3$

Sol:

$$= (2xy)^3 + (3a)^3$$

$$= (2xy + 3a) \left((2xy)^2 - 2xy \times 3a + (3a)^2 \right) \quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right]$$

$$= (2xy + 3a)(4x^2y^2 - 6xya + 9a^2)$$

$$\therefore 8x^3y^3 + 27a^3 = (2xy + 3a)(4x^2y^2 - 6xya + 9a^2)$$

5. $64a^3 - b^3$

Sol:

$$= (4a)^3 - b^3$$

$$= (4a - b) \left((4a)^2 + 4a \times b + b^2 \right) \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$= (4a - b)(16a^2 + 4ab + b^2)$$

$$\therefore 64a^3 - b^3 = (4a - b)(16a^2 + 4ab + b^2)$$

6. $\frac{x^3}{216} - 8y^3$

Sol:

$$= \left(\frac{x}{6} \right)^3 - (2y)^3$$

$$= \left(\frac{x}{6} - 2y \right) \left(\left(\frac{x}{6} \right)^2 + \frac{x}{6} \times 2y + (2y)^2 \right) \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$= \left(\frac{x}{6} - 2y \right) \left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2 \right)$$

$$\therefore \frac{x^3}{216} - 8y^3 = \left(\frac{x}{6} - 2y \right) \left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2 \right)$$

7. $10x^4y - 10xy^4$

Sol:

$$10x^4y - 10xy^4$$

$$= 10xy(x^3 - y^3)$$

$$= 10xy(x - y)(x^2 + xy + y^2) \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$\therefore 10x^4y - 10xy^4 = 10xy(x - y)(x^2 + xy + y^2)$$

8. $54x^6y + 2x^3y^4$

Sol:

$$54x^6y + 2x^3y^4$$

$$= 2x^3y(27x^3 + y^3)$$

$$= 2x^3y((3x)^3 + y^3)$$

$$= 2x^3y(3x + y)((3x)^2 - 3 \times xy + y^2) \quad \left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2) \right]$$

$$= 2x^3y(3x + y)(9x^2 - 3xy + y^2)$$

$$\therefore 54x^6y + 2x^3y^4 = 2x^3y(3x + y)(9x^2 - 3xy + y^2)$$

9. $32a^3 + 108b^3$

Sol:

$$32a^3 + 108b^3$$

$$= 4(8a^3 + 27b^3)$$

$$= 4((2a)^3 + (3b)^3) \quad [\text{Using } a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= 4[(2a + 3b)((2a)^2 - 2a \times 3b + (3b)^2)]$$

$$= 4(2a + 3b)(4a^2 - 6ab + 9b^2)$$

$$\therefore 32a^3 + 108b^3 = 4(2a + 3b)(4a^2 - 6ab + 9b^2)$$

10. $(a - 2b)^3 - 512b^3$

Sol:

$$(a - 2b)^3 - 512b^3$$

$$= (a - 2b)^3 - (8b)^3$$

$$= (a - 2b - 8b)((a - 2b)^2 + (a - 2b)8b + (8b)^2) \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$= (a - 10b)(a^2 + 4b^2 - 4ab + 8b(a - 2b) + (8b)^2) \quad \left[\because (a - b)^2 = a^2 + b^2 - 2ab \right]$$

$$= (a - 10b)(a^2 + 4b^2 - 4ab + 8ab - 16b^2 + 64b^2)$$

$$= (a - 10b)(a^2 + 68b^2 - 16b^2 - 4ab + 8ab)$$

$$= (a - 10b)(a^2 + 52b^2 + 4ab)$$

$$\therefore (a - 2b)^3 - 512b^3 = (a - 10b)(a^2 + 4ab + 52b^2)$$

11. $(a + b)^3 - 8(a - b)^3$

Sol:

$$\begin{aligned}
 & (a + b)^3 - 8(a - b)^3 \\
 &= (a + b)^3 - [2(a - b)]^3 \\
 &= (a + b)^3 - (2a - 2b)^3 && \text{[Using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)\text{]} \\
 &= (a + b - (2a - 2b)) \left((a + b)^2 + (a + b)(2a - 2b) + (2a - 2b)^2 \right) \\
 &= (a + b - 2a + 2b) \left(a^2 + b^2 + 2ab + (a + b)(2a - 2b) + (2a - 2b)^2 \right) && \text{[}\because (a + b)^2 = a^2 + b^2 + 2ab\text{]} \\
 &= (3b - a) \left(a^2 + b^2 + 2ab + 2a^2 - 2ab + 2ab - 2b^2 + (2a - 2b)^2 \right) \\
 &= (3b - a) \left(3a^2 + 2ab - b^2 + (2a - 2b)^2 \right) \\
 &= (3b - a) \left(3a^2 + 2ab - b^2 + 4a^2 + 4b^2 - 8ab \right) && \text{[}\because (a - b)^2 = a^2 + b^2 - 2ab\text{]} \\
 &= (3b - a) \left(3a^2 + 4a^2 - b^2 + 4b^2 + 2ab - 8ab \right) \\
 &= (3b - a) \left(7a^2 + 3b^2 - 6ab \right) \\
 \therefore (a + b)^3 - 8(a - b)^3 &= (-a + 3b) \left(7a^2 - 6ab + 3b^2 \right)
 \end{aligned}$$

12. $(x + 2)^3 + (x - 2)^3$

Sol:

$$\begin{aligned}
 &= (x + 2 + x - 2) \left((x + 2)^2 - (x + 2)(x - 2) + (x - 2)^2 \right) && \text{[}\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)\text{]} \\
 &= 2x \left(x^2 + 4x + 4 - (x + 2)(x - 2) + x^2 - 4x + 4 \right) && \text{[}\because (a + b)^2 = a^2 + 2ab + b^2, (a - b)^2 = a^2 - 2ab + b^2\text{]} \\
 &= 2x \left(2x^2 + 8 - (x^2 - 2^2) \right) && \text{[}\because (a + b)(a - b) = a^2 - b^2\text{]} \\
 &= 2x \left(2x^2 + 8 - x^2 + 4 \right) \\
 &= 2x \left(x^2 + 12 \right) \\
 \therefore (x + 2)^3 + (x - 2)^3 &= 2x \left(x^2 + 12 \right)
 \end{aligned}$$

13. $8x^2y^3 - x^5$

Sol:

$$\begin{aligned}
 &= x^2 \left(8y^3 - x^3 \right) \\
 &= x^2 \left((2y)^3 - x^3 \right)
 \end{aligned}$$

$$\begin{aligned}
 &= x^2(2y-x)\left((2y)^2 + 2y(x) + x^2\right) && \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right] \\
 &= x^2(2y-x)(4y^2 + 2xy + x^2) \\
 \therefore 8x^2y^3 - x^5 &= x^2(2y-x)(4y^2 + 2xy + x^2)
 \end{aligned}$$

14. $1029 - 3x^3$

Sol:

$$\begin{aligned}
 &= 3(343 - x^3) \\
 &= 3(7^3 - x^3) \\
 &= 3(7-x)(7^2 + 7 \times x + x^2) && \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right] \\
 &= 3(7-x)(49 + 7x + x^2) \\
 \therefore 1029 - 3x^3 &= 3(7-x)(49 + 7x + x^2)
 \end{aligned}$$

15. $x^6 + y^6$

Sol:

$$\begin{aligned}
 &x^6 + y^6 \\
 &= (x^2)^3 + (y^2)^3 \\
 &= (x^2 + y^2)\left((x^2)^2 - x^2y^2 + (y^2)^2\right) && \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right] \\
 &= (x^2 + y^2)(x^4 - x^2y^2 + y^4) \\
 \therefore x^6 + y^6 &= (x^2 + y^2)(x^4 - x^2y^2 + y^4)
 \end{aligned}$$

16. $x^3y^3 + 1$

Sol:

$$\begin{aligned}
 &= (xy)^3 + 1^3 \\
 &= (xy+1)\left((xy)^2 - xy \times 1 + 1^2\right) && \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right] \\
 &= (xy+1)(x^2y^2 - xy + 1) \\
 \therefore x^3y^3 + 1 &= (xy+1)(x^2y^2 - xy + 1)
 \end{aligned}$$

17. $x^4y^4 - xy$

Sol:

$$= xy(x^3y^3 - 1)$$

$$= xy((xy)^3 - 1^3)$$

$$= xy(xy - 1)((xy)^2 + (xy)1 + 1^2) \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$= xy(xy - 1)(x^2y^2 + xy + 1)$$

$$\therefore x^4y^4 - xy = xy(xy - 1)(x^2y^2 + xy + 1)$$

18. $a^{12} + b^{12}$

Sol:

$$= (a^4)^3 + (b^4)^3$$

$$= (a^4)^3 + (b^4)^3$$

$$= (a^4 + b^4)((a^4)^2 - a^4 \times b^4 + (b^4)^2) \quad \left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2) \right]$$

$$= (a^4 + b^4)(a^8 - a^4b^4 + b^8)$$

$$\therefore a^{12} + b^{12} = (a^4 + b^4)(a^8 - a^4b^4 + b^8)$$

19. $x^3 + 6x^2 + 12x + 16$

Sol:

$$= x^3 + 6x^2 + 12x + 8 + 8$$

$$= x^3 + 3 \times x^2 \times 2 + 3 \times x \times 2^2 + 2^3 + 8$$

$$= (x + 2)^3 + 8 \quad \left[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3 \right]$$

$$= (x + 2)^3 + 2^3$$

$$= (x + 2 + 2)((x + 2)^2 - 2(x + 2) + 2^2) \quad \left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2) \right]$$

$$= (x + 4)(x^2 + 4x + 4 - 2x - 4 + 4) \quad \left[\because (a + b)^2 = a^2 + 2ab + b^2 \right]$$

$$= (x + 4)(x^2 + 2x + 4)$$

$$\therefore x^3 + 6x^2 + 12x + 16 = (x + 4)(x^2 + 2x + 4)$$

20. $a^3 + b^3 + a + b$

Sol:

$$\begin{aligned} &= (a^3 + b^3) + 1(a + b) \\ &= (a + b)(a^2 - ab + b^2) + 1(a + b) \\ &= (a + b)(a^2 - ab + b^2 + 1) \\ \therefore a^3 + b^3 + a + b &= (a + b)(a^2 - ab + b^2 + 1) \end{aligned}$$

21. $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$

Sol:

$$\begin{aligned} &= \left(a^3 - \frac{1}{a^3} \right) - 2 \left(a - \frac{1}{a} \right) \\ &= \left(a^3 - \left(\frac{1}{a} \right)^3 \right) - 2 \left(a - \frac{1}{a} \right) \\ &= \left(a - \frac{1}{a} \right) \left(a^2 + a \times \frac{1}{a} + \left(\frac{1}{a} \right)^2 \right) - 2 \left(a - \frac{1}{a} \right) \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right] \\ &= \left(a - \frac{1}{a} \right) \left(a^2 + 1 + \frac{1}{a^2} \right) - 2 \left(a - \frac{1}{a} \right) \\ &= \left(a - \frac{1}{a} \right) \left(a^2 + 1 + \frac{1}{a^2} - 2 \right) \\ &= \left(a - \frac{1}{a} \right) \left(a^2 + \frac{1}{a^2} - 1 \right) \\ \therefore a^3 - \frac{1}{a^3} - 2a + \frac{2}{a} &= \left(a - \frac{1}{a} \right) \left(a^2 + \frac{1}{a^2} - 1 \right) \end{aligned}$$

22. $a^3 + 3a^2b + 3ab^2 + b^3 - 8$

Sol:

$$\begin{aligned} &= (a + b)^3 - 8 \quad \left[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3 \right] \\ &= (a + b)^3 - 2^3 \\ &= (a + b - 2) \left((a + b)^2 + (a + b)2 + 2^2 \right) \\ &= (a + b - 2) (a^2 + b^2 + 2ab + 2a + 2b + 4) \\ \therefore a^3 + 3a^2b + 3ab^2 + b^3 - 8 &= (a + b - 2) (a^2 + b^2 + 2ab + 2a + 2b + 4) \end{aligned}$$

23. $8a^3 - b^3 - 4ax + 2bx$

Sol:

$$= 8a^3 - b^3 - 2x(2a - b)$$

$$= (2a)^3 - b^3 - 2x(2a - b)$$

$$= (2a - b)((2a)^2 + 2a \times b + b^2) - 2x(2a - b) \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$= (2a - b)(4a^2 + 2ab + b^2) - 2x(2a - b)$$

$$= (2a - b)(4a^2 + 2ab + b^2 - 2x)$$

$$\therefore 8a^3 - b^3 - 4ax + 2bx = (2a - b)(4a^2 + 2ab + b^2 - 2x)$$

24. Simplify:

Sol:

(i) $\frac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$

$$= \frac{173^3 + 127^3}{173^2 - 173 \times 127 + 127^2}$$

$$= \frac{(173 + 127)(173^2 - 173 \times 127 + 127^2)}{(173^2 - 173 \times 127 + 127^2)}$$

$$\left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2) \right]$$

$$= (173 + 127) = 300$$

(ii) $\frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

$$= \frac{155^3 - 55^3}{155^2 + 155 \times 55 + 55^2}$$

$$= \frac{(155 - 55)(155^2 + 155 \times 55 + 55^2)}{(155^2 + 155 \times 55 + 55^2)}$$

$$\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$= (155 - 55) = 100$$

Exercise – 5.3

Factorize:

1. $64a^3 + 125b^3 + 240a^2b + 300ab^2$

Sol:

$$\begin{aligned} & 64a^3 + 125b^3 + 240a^2b + 300ab^2 \\ &= (4a)^3 + (5b)^3 + 3 \times (4a)^2 \times 5b + 3(4a)(5b)^2 \\ &= (4a + 5b)^3 \quad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3 \right] \\ &= (4a + 5b)(4a + 5b)(4a + 5b) \\ \therefore 64a^3 + 125b^3 + 240a^2b + 300ab^2 &= (4a + 5b)(4a + 5b)(4a + 5b) \end{aligned}$$

2. $125x^3 - 27y^3 - 225x^2y + 135xy^2$

Sol:

$$\begin{aligned} & 125x^3 - 27y^3 - 225x^2y + 135xy^2 \\ &= (5x)^3 - (3y)^3 - 3 \times (5x)^2 \times 3y + 3 \times (5x)(3y)^2 \\ &= (5x - 3y)^3 \quad \left[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3 \right] \\ &= (5x - 3y)(5x - 3y)(5x - 3y) \\ \therefore 125x^3 - 27y^3 - 225x^2y + 135xy^2 &= (5x - 3y)(5x - 3y)(5x - 3y) \end{aligned}$$

3. $\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$

Sol:

$$\begin{aligned} &= \left(\frac{2}{3}x\right)^3 + (1)^3 + 3 \times \left(\frac{2}{3}x\right)^2 \times 1 + 3(1)^2 \times \left(\frac{2}{3}x\right) \\ &= \left(\frac{2}{3}x + 1\right)^3 \quad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3 \right] \\ &= \left(\frac{2}{3}x + 1\right) \left(\frac{2}{3}x + 1\right) \left(\frac{2}{3}x + 1\right) \\ \therefore \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x &= \left(\frac{2}{3}x + 1\right) \left(\frac{2}{3}x + 1\right) \left(\frac{2}{3}x + 1\right) \end{aligned}$$

4. $8x^3 + 27y^3 + 36x^2y + 54xy^2$

Sol:

$$8x^3 + 27y^3 + 36x^2y + 54xy^2$$

$$= (2x)^3 + (3y)^3 + 3 \times (2x)^2 \times 3y + 3 \times (2x)(3y)^2$$

$$= (2x + 3y)^3 \quad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3 \right]$$

$$= (2x + 3y)(2x + 3y)(2x + 3y)$$

$$\therefore 8x^3 + 27y^3 + 36x^2y + 54xy^2 = (2x + 3y)(2x + 3y)(2x + 3y)$$

5. $a^3 - 3a^2b + 3ab^2 - b^3 + 8$

Sol:

$$a^3 - 3a^2b + 3ab^2 - b^3 + 8$$

$$= (a - b)^3 + 8 \quad \left[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3 \right]$$

$$= (a - b)^3 + 2^3$$

$$= (a - b + 2) \left((a - b)^2 - (a - b)2 + 2^2 \right) \quad \left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2) \right]$$

$$= (a - b + 2)(a^2 + b^2 - 2ab - 2(a - b) + 4)$$

$$= (a - b + 2)(a^2 + b^2 - 2ab - 2a + 2b + 4)$$

$$\therefore a^3 - 3a^2b + 3ab^2 - b^3 + 8 = (a - b + 2)(a^2 + b^2 - 2ab - 2a + 2b + 4)$$

6. $x^3 + 8y^3 + 6x^2y + 12xy^2$

Sol:

$$= (x)^3 + (2y)^3 + 3 \times x^2 \times 2y + 3 \times x \times (2y)^2$$

$$= (x + 2y)^3 \quad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3 \right]$$

$$= (x + 2y)(x + 2y)(x + 2y)$$

$$\therefore x^3 + 8y^3 + 6x^2y + 12xy^2 = (x + 2y)(x + 2y)(x + 2y)$$

7. $8x^2 + y^3 + 12x^2y + 6xy^2$

Sol:

$$8x^2 + y^3 + 12x^2y + 6xy^2$$

$$= (2x)^3 + y^3 + 3 \times (2x)^2 \times y + 3(2x) \times y^2$$

$$= (2x + y)^3 \quad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3 \right]$$

$$= (2x + y)(2x + y)(2x + y)$$

$$\therefore 8x^3 + y^3 + 12x^2y + 6xy^2 = (2x + y)(2x + y)(2x + y)$$

8. $8a^3 + 27b^3 + 36a^2b + 54ab^2$

Sol:

$$\begin{aligned} &= (2a)^3 + (3b)^3 + 3 \times (2a)^2 \times 3b + 3 \times (2a)(3b)^2 \\ &= (2a + 3b)^3 \quad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3 \right] \\ &= (2a + 3b)(2a + 3b)(2a + 3b) \\ \therefore 8a^3 + 27b^3 + 36a^2b + 54ab^2 &= (2a + 3b)(2a + 3b)(2a + 3b) \end{aligned}$$

9. $8a^3 - 27b^3 - 36a^2b + 54ab^2$

Sol:

$$\begin{aligned} &8a^3 - 27b^3 - 36a^2b + 54ab^2 \\ &= (2a)^3 - (3b)^3 - 3 \times (2a)^2 \times 3b + 3(2a)(3b)^2 \\ &= (2a - 3b)^3 \quad \left[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a - b)^3 \right] \\ &= (2a - 3b)(2a - 3b)(2a - 3b) \\ \therefore 8a^3 - 27b^3 - 36a^2b + 54ab^2 &= (2a - 3b)(2a - 3b)(2a - 3b) \end{aligned}$$

10. $x^3 - 12x(x - 4) - 64$

Sol:

$$\begin{aligned} &x^3 - 12x(x - 4) - 64 \\ &= x^3 - 12x^2 + 48x - 64 \\ &= (x)^3 - 3 \times x^2 \times 4 + 3 \times 4^2 \times x - 4^3 \\ &= (x - 4)^3 \quad \left[\because a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3 \right] \\ &= (x - 4)(x - 4)(x - 4) \\ \therefore x^3 - 12x(x - 4) - 64 &= (x - 4)(x - 4)(x - 4) \end{aligned}$$

11. $a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$

Sol:

$$\begin{aligned} &= (ax)^3 - 3(ax)^2 \times b + 3(ax)b^2 - b^3 \\ &= (ax - b)^3 \quad \left[\because a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3 \right] \\ &= (ax - b)(ax - b)(ax - b) \\ \therefore a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3 &= (ax - b)(ax - b)(ax - b) \end{aligned}$$

Exercise – 5.4

1. $a^3 + 8b^3 + 64c^3 - 24abc$

Sol:

$$a^3 + 8b^3 + 64c^3 - 24abc$$

$$= (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$$

$$= (a + 2b + 4c) \left(a^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a \right)$$

$$\left[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \right]$$

$$= (a + 2b + 4c) \left(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac \right)$$

$$\therefore a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c) \left(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac \right)$$

2. $x^3 - 8y^3 + 27z^3 + 18xyz$

Sol:

$$= x^3 + (-2y)^3 + (3z)^3 - 3 \times x \times (-2y) \times (3z)$$

$$= (x + (-2y) + 3z) \left(x^2 + (-2y)^2 + (3z)^2 - x(-2y) - (-2y)(3z) - 3z(x) \right)$$

$$\left[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \right]$$

$$= (x - 2y + 3z) \left(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx \right)$$

$$\therefore x^3 - 8y^3 + 27z^3 + 18xyz = (x - 2y + 3z) \left(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx \right)$$

3. $\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

Sol:

$$\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$$

$$= \left(\frac{x}{3} \right)^3 + (-y)^3 + (5z)^3 - 3 \times \frac{x}{3} \times (-y) \times (5z)$$

$$= \left(\frac{x}{3} + (-y) + 5z \right) \left(\left(\frac{x}{3} \right)^2 + (-y)^2 + (5z)^2 - \frac{x}{3}(-y) - (-y)5z - 5z \left(\frac{x}{3} \right) \right)$$

$$= \left(\frac{x}{3} - y + 5z \right) \left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5xyz - \frac{5}{3}zx \right)$$

$$\therefore \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{x}{3} - y + 5z \right) \left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5}{3}zx \right)$$

4. $8x^3 + 27y^3 - 216z^3 + 108xyz$

Sol:

$$\begin{aligned} &= (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z) \\ &= (2x + 3y + (-6z)) \left((2x)^2 + (3y)^2 + (-6z)^2 - 2x \times 3y - 3y(-6z) - (-6z)2x \right) \\ &= (2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx) \\ \therefore 8x^3 + 27y^3 - 216z^3 + 108xyz &= (2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx) \end{aligned}$$

5. $125 + 8x^3 - 27y^3 + 90xy$

Sol:

$$\begin{aligned} &= 5^3 + (2x)^3 + (-3y)^3 - 3 \times 5 \times 2x \times (-3y) \\ &= (5 + 2x + (-3y)) \left(5^2 + (2x)^2 + (-3y)^2 - 5(2x) - 2x(-3y) - (-3y)5 \right) \\ &= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y) \\ \therefore 125 + 8x^3 - 27y^3 + 90xy &= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y) \end{aligned}$$

6. $(3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3$

Sol:

$$\begin{aligned} \text{Let } (3x - 2y) &= a, (2y - 4z) = b, (4z - 3x) = c \\ \therefore a + b + c &= 3x - 2y + 2y - 4z + 4z - 3x = 0 \\ \therefore a + b + c = 0 &\therefore a^3 + b^3 + c^3 = 3abc \\ \therefore (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 &= 3(3x - 2y)(2y - 4z)(4z - 3x) \end{aligned}$$

7. $(2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

Sol:

$$\begin{aligned} \text{Let } 2x - 3y &= a, 4z - 2x = b, 3y - 4z = c \\ \therefore a + b + c &= 2x - 3y + 4z - 2x + 3y - 4z = 0 \\ \therefore a + b + c = 0 &\therefore a^3 + b^3 + c^3 = 3abc \\ \therefore (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 &= 3(2x - 3y)(4z - 2x)(3y - 4z) \end{aligned}$$

$$8. \left[\frac{x}{2} + y + \frac{z}{3} \right]^3 + \left[\frac{x}{2} + \frac{2y}{3} + z \right]^3 + \left[-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3} \right]^3$$

Sol:

$$\text{Let } \left(\frac{x}{2} + y + \frac{z}{3} \right) = a, \left(\frac{x}{3} - \frac{2y}{3} + z \right) = b, \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3} \right) = c$$

$$a + b + c = \frac{x}{2} + y + \frac{z}{3} + \frac{x}{3} - \frac{2y}{3} + z - \frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}$$

$$a + b + c = \left(\frac{x}{2} + \frac{x}{3} - \frac{5x}{6} \right) + \left(y - \frac{2y}{3} - \frac{y}{3} \right) + \left(\frac{z}{3} + z - \frac{4z}{3} \right)$$

$$a + b + c = \frac{3x}{6} + \frac{2x}{6} - \frac{5x}{6} + \frac{3y}{3} - \frac{2y}{3} - \frac{y}{3} + \frac{z}{3} + \frac{3z}{3} - \frac{4z}{3}$$

$$a + b + c = \frac{5x - 5x}{6} + \frac{3y - 3y}{3} + \frac{4z - 4z}{3}$$

$$a + b + c = 0$$

$$\therefore a + b + c = 0 \quad \therefore a^3 + b^3 + c^3 = 3abc$$

$$\therefore \left[\frac{x}{2} + y + \frac{z}{3} \right]^3 + \left[\frac{x}{3} - \frac{2y}{3} + z \right]^3 + \left[-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3} \right]^3 = 3 \left(\frac{x}{2} + y + \frac{z}{3} \right) \left(\frac{x}{3} - \frac{2y}{3} + z \right) \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3} \right)$$

$$9. (a - 3b)^3 + (3b - c)^3 + (c - a)^3$$

Sol:

$$\text{Let } (a - 3b) = x, (3b - c) = y, (c - a) = z$$

$$x + y + z = a - 3b + 3b - c + c - a = 0$$

$$\therefore x + y + z = 0 \quad \therefore x^3 + y^3 + z^3 = 3xyz$$

$$\therefore (a - 3b)^3 + (3b - c)^3 + (c - a)^3 = 3(a - 3b)(3b - c)(c - a)$$

$$10. 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$$

Sol:

$$= (\sqrt{2}a)^3 + (\sqrt{3}b)^3 + c^3 - 3 \times \sqrt{2}a \times \sqrt{3}b \times c$$

$$= (\sqrt{2}a + \sqrt{3}b + c) \left((\sqrt{2}a)^2 + (\sqrt{3}b)^2 + c^2 - (\sqrt{2}a)(\sqrt{3}b) - (\sqrt{3}b)c - (\sqrt{2}a)c \right)$$

$$= (\sqrt{2}a + \sqrt{3}b + c) (2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ac)$$

$$\therefore 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = (\sqrt{2}a + \sqrt{3}b + c) (2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ac)$$

11. $3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$

Sol:

$$\begin{aligned} &= (\sqrt{3}a)^3 + (-b)^3 + (-\sqrt{5}c)^3 - 3 \times (\sqrt{3}a)(-b)(-\sqrt{5}c) \\ &= (\sqrt{3}a + (-b) + (-\sqrt{5}c)) \left((\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - \sqrt{3}a(-b) - (-b)(-\sqrt{5}c) - (-\sqrt{5}c)\sqrt{3}a \right) \\ &= (\sqrt{3}a - b - \sqrt{5}c) (3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac) \\ \therefore 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc &= (\sqrt{3}a - b - \sqrt{5}c) (3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac) \end{aligned}$$

12. $8x^3 - 125y^3 + 180xy + 216$

Sol:

$$\begin{aligned} &8x^3 - 125y^3 + 180xy + 216 \\ \text{or, } &8x^3 - 125y^3 + 216 + 180xy \\ &= (2x)^3 + (-5y)^3 + 6^3 - 3 \times (2x)(-5y)(6) \\ &= (2x + (-5y) + 6) \left((2x)^2 + (-5y)^2 + 6^2 - 2x(-5y) - (-5y)6 - 6(2x) \right) \\ &= (2x - 5y + 6) (4x^2 + 25y^2 + 36 + 10xy + 30y - 12x) \\ \therefore 8x^3 - 125y^3 + 180xy + 216 &= (2x - 5y + 6) (4x^2 + 25y^2 + 36 + 10xy + 30y - 12x) \end{aligned}$$

13. $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

Sol:

$$\begin{aligned} &= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3 \times \sqrt{2}a \times 2\sqrt{2}b \times c \\ &= (\sqrt{2}a + 2\sqrt{2}b + c) \left((\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b) - (2\sqrt{2}b)c - (\sqrt{2}a)c \right) \\ &= (\sqrt{2}a + 2\sqrt{2}b + c) (2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac) \\ 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc &= (\sqrt{2}a + 2\sqrt{2}b + c) (2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac) \end{aligned}$$