Areas

Exercise 7A

Question 1:

Here, b = 24 cm and h = 14.5 cm

$$\text{Area of triangle} = \left(\frac{1}{2} \times \text{base} \times \text{height}\right) \text{sq units}$$

$$= \left(\frac{1}{2} \times 24 \times 14.5\right) cm^2$$

$$= 174\ cm^2$$

Question 2:

Let height = x and base = 3x

$$\text{Area of triangle} = \left(\frac{1}{2} \times \text{base} \times \text{height}\right) \text{sq units}$$

$$\therefore \text{Area of triangle} = \frac{1}{2} \times x \times 3x$$

$$= \frac{3}{2} x^2$$

We know that, 1 hectare = 10000 sq metre

Rate of sowing the field per hectare = Rs.58

Total cost of sowing the triangular field = Rs.783

$\Rightarrow$ Total cost = Area of the triangular field $\times$ Rs. 58

$$\Rightarrow \frac{3}{2} x^2 \times \frac{58}{10000} = 783$$

$$\Rightarrow x^2 = \frac{783}{58} \times \frac{2}{3} \times 10000 \text{ sq metre}$$

$$\Rightarrow x^2 = 90000 \text{ sq metre}$$

$$\Rightarrow x = 300\text{m}$$

Hence, height = 300 m and base = 900 m.

Question 3:

Here, a = 42 cm, b = 34 cm and c = 20 cm

Therefore, $s = \frac{42+34+20}{2} = 48$

Area = $\sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{48(48-42)(48-34)(48-20)}$$
$$= \sqrt{48 \times 6 \times 14 \times 28}$$
$$= \sqrt{4 \times 4 \times 3 \times 3 \times 2 \times 14 \times 14 \times 2}$$
$$= 4 \times 3 \times 2 \times 14$$
$$= 336 \text{ cm}^2$$

Longest side = 42 cm

$\Rightarrow b = 42$ cm

Let h be the height corresponding to the longest side.

Area of the triangle $= \frac{1}{2} \times b \times h$

$\Rightarrow \frac{1}{2} \times b \times h = 336$

$\Rightarrow 42 \times h = 336 \times 2$

$\Rightarrow h = \frac{336 \times 2}{42} = 16$ cm

Question 4:

Here, $a = 18$ cm, $b = 24$ cm and $c = 30$ cm

Therefore, $s = \frac{18+24+30}{2} = 36$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{36(36-18)(36-24)(36-30)}$$
$$= \sqrt{36 \times 18 \times 12 \times 6}$$
$$= \sqrt{6 \times 6 \times 6 \times 3 \times 3 \times 4 \times 6}$$
$$= 6 \times 6 \times 3 \times 2$$
$$= 216 \text{ cm}^2$$

Smallest side = 18 cm

Let h be the height corresponding to the smallest side.

Area of the triangle $= \frac{1}{2} \times b \times h$

$\Rightarrow \frac{1}{2} \times b \times h = 216$

$\Rightarrow 18 \times h = 216 \times 2$

$\Rightarrow h = \frac{216 \times 2}{18} = 24$ cm

Question 5:

Here, a = 91 m, b = 98 m and c = 105 m

Therefore, $s = \frac{91+98+105}{2} = \frac{294}{2} = 147$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{147(147-91)(147-98)(147-105)}$$
$$= \sqrt{147 \times 56 \times 49 \times 42}$$
$$= \sqrt{49 \times 3 \times 7 \times 2 \times 2 \times 2 \times 49 \times 7 \times 3 \times 2}$$
$$= 49 \times 3 \times 2 \times 2 \times 7$$
$$= 4116 \text{ m}^2$$

Longest side = 105m $\Rightarrow$ b=105

Let h be the height corresponding to the longest side.

Area of the triangle $= \frac{1}{2} \times b \times h$

$\Rightarrow \frac{1}{2} \times b \times h = 4116$

$\Rightarrow 105 \times h = 2 \times 4116$

$\Rightarrow \quad h = \frac{2 \times 4116}{105} = 78.4$ m

Question 6:

Let the sides of the triangle be 5x, 12x and 13x.

Its perimeter $= (5x + 12x + 13x) = 30x$

$\therefore 30x = 150$ m [given]

$\Rightarrow x = \frac{150}{30} = 5$ m

Thus, sides of the triangle are;

$5x = 5 \times 5 = 25$m

$12x = 12 \times 5 = 60$ m

$13x = 13 \times 5 = 65$ m

Let a = 25 m, b = 60 m and c = 65 m.

Now $\quad s = \frac{1}{2}(a+b+c)$

$$= \left(\frac{25+60+65}{2}\right) \text{ m} = \frac{150}{2} = 75 \text{ m.}$$

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{75(75-25)(75-60)(75-65)}$$
$$= \sqrt{75 \times 50 \times 15 \times 10}$$
$$= \sqrt{25 \times 3 \times 25 \times 2 \times 5 \times 3 \times 5 \times 2}$$
$$= \sqrt{25 \times 25 \times 5 \times 5 \times 3 \times 3 \times 2 \times 2}$$
$$= 25 \times 5 \times 3 \times 2 = 750 \text{ sq m.}$$

$\therefore$ area of the triangle = 750 sq m.

Question 7:

Let the sides of the triangle be 25x, 17x and 12x.
Then, its perimeter = (25x + 17x + 12z) = 54x

$\Rightarrow \quad 54x = 540$

$\Rightarrow \quad x = \frac{540}{54} = 10\text{m}.$

Thus, sides of the triangle are:
$25x = 25 \times 10 = 250$ m
$17x = 17 \times 10 = 170$ m
$12x = 12 \times 10 = 120$ m
Let, a = 250 m, b = 170 m and c = 120 m

Now, $s = \frac{1}{2}(a + b + c)$

$= \left(\frac{250 + 170 + 120}{2}\right)\text{m}$

$= \left(\frac{540}{2}\right)\text{m} = 270\text{m}$

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-250)(270-170)(270-120)}$

$= \sqrt{3 \times 3 \times 3 \times 10 \times 10 \times 2 \times 10 \times 10 \times 10 \times 5 \times 3}$

$= 3 \times 3 \times 10 \times 10 \times 10 = 9000\text{m}^2$

$\therefore$ Cost of ploughing the field at the rate of Rs. 18.80 per 10m^2

$= \frac{18.80}{10} \times 9000 = \text{Rs. } 16920$

$\therefore$ Cost of ploughing the field = Rs. 16920.

Question 8:

One side of a triangular field = 85 m
Second side of a triangular field = 154 m
Let the third side of a triangular field be x m
Perimeter (given) = 324 m

$\therefore \quad 85\text{m} + 154\text{m} + x\text{m} = 324$ m

$\Rightarrow \quad x = 324 - 239$

$\Rightarrow \quad x = 85$ m

$\therefore$ the third side = 85 m

Let a = 85 m, b = 154 m and c = 85 m

Now $S = \frac{1}{2}(a + b + c)$

$= \left(\frac{85 + 154 + 85}{2}\right) = \frac{324}{2} = 162$

$\therefore$ area of the triangle $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{162(162-85)(162-154)(162-85)}$

$= \sqrt{162 \times 77 \times 8 \times 77}$

$= \sqrt{2 \times 9 \times 9 \times 7 \times 11 \times 2 \times 2 \times 2 \times 7 \times 11}$

$= \sqrt{11 \times 11 \times 9 \times 9 \times 7 \times 7 \times 2 \times 2 \times 2 \times 2}$

$= 11 \times 9 \times 7 \times 2 \times 2 = 2772\text{ m}^2$

$\therefore$ area of triangle $= 2772\text{ m}^2$

Also, area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$2772 = \frac{1}{2} \times 154 \times h = 77h$

$\therefore \quad 77h = 2772$

$\therefore \quad h = \frac{2772}{77} = 36$ m

$\therefore$ the length of the perpendicular from the opposite vertex on the side measuring 154 m = 36 m.

Question 9:

Let a $= 13$ cm, B $= 13$ cm and c $= 20$ cm

Now, $s = \frac{1}{2}(a + b + c)$

$= \left(\frac{13 + 13 + 20}{2}\right) \text{Cm} = \frac{46}{2} = 23 \text{cm}$

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{23(23-13)(23-13)(23-20)}$

$= \sqrt{23 \times 10 \times 10 \times 3}$

$= 10\sqrt{69}$

$= 10 \times 8.306 = 83.06 \text{ cm}^2$

$\therefore$ area of an isosceles triangle $= 83.06 \text{ cm}^2$

Question 10:

Let ΔABC be an isosceles triangle and Let $AL \perp BC$.

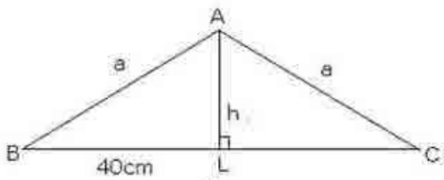
Given that $BC = 80$ cm and Area of $\Delta ABC = 360 \text{ cm}^2$

$\therefore \quad \frac{1}{2} \times BC \times AL = 360 \text{ cm}^2$

$\Rightarrow \quad \frac{1}{2} \times 80 \times h = 360 \text{ cm}^2$

$\Rightarrow \quad 40 \times h = 360 \text{ cm}^2$

$\Rightarrow \quad h = \frac{360}{40} = 9 \text{ cm}$



Now $BL = \frac{1}{2}(BC)$

$= \left(\frac{1}{2} \times 80\right) \text{cm} = 40 \text{ cm}$ and $AL = 9\text{cm}$

$a = \sqrt{BL^2 + AL^2}$

$= \sqrt{(40)^2 + (9)^2} \Rightarrow \sqrt{1600 + 81}$

$\Rightarrow \quad \sqrt{1681} = 41\text{cm}$

$\therefore$ Perimeter $= (41 + 41 + 80) = 162$ cm

Perimeter of the triangle $= 162$ cm.

Question 11:

In an isosceles triangle, the lateral sides are of equal length.
Let the length of lateral side be x cm.

Then, base $= \frac{3}{2} \times x$ cm [given]

(i) Length of each side of the triangle :

Perimeter of an isosceles triangle = 42 cm

$\Rightarrow \quad x + x + \frac{3}{2}x = 42$ cm

$\Rightarrow \quad 2x + 2x + 3x = 84$ cm

$\Rightarrow \quad 7x = 84$

$\Rightarrow \quad x = \frac{84}{7} = 12$ cm

$\therefore$ length of lateral side = 12 cm

And base $= \frac{3}{2}x = \frac{3}{2} \times 12 = 18$ cm

$\therefore$ the length of each side of the triangle = 12 cm, 12 cm and 18 cm.

(ii) Area of the triangle :

Let a = 12 cm, b = 12 cm and c = 18 cm.

Now, $s = \frac{1}{2}(a + b + c)$

$= \left(\frac{12 + 12 + 18}{2}\right)$ cm $= \left(\frac{42}{2}\right)$ cm

$= 21$ cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-12)(21-12)(21-18)}$

$= \sqrt{21 \times 9 \times 9 \times 3}$

$= \sqrt{3 \times 7 \times 9 \times 9 \times 3}$

$= 27\sqrt{7} = 71.42\ cm^2 \quad (\sqrt{7} = 2.64)$

$\therefore$ area of the triangle = 71.42 cm^2.

(iii) Height of the triangle :

Area of a triangle $= \frac{1}{2}$ x base x height

$71.42\ cm^2 = \frac{1}{2} \times 18 \times h$

$\Rightarrow \quad 71.42\ cm^2 = 9 \times h$

$\Rightarrow \quad h = \frac{71.42}{9} = 7.94$ cm

$\therefore$ the height of the triangle = 7.94 cm.

Question 12:

Let a be the length of a side of an equilateral triangle.

$\therefore$ Area of an equilateral triangle $= \frac{\sqrt{3} \times a^2}{4}$ sq units

Area of the equilateral triangle $= 36\sqrt{3}\ cm^2$ [given]

$\Rightarrow \quad \frac{\sqrt{3} \times a^2}{4} = 36 \times \sqrt{3}$

$\Rightarrow \quad a^2 = \frac{36 \times \sqrt{3} \times 4}{\sqrt{3}}$

$\Rightarrow \quad a^2 = 36 \times 4 = 144$

$\therefore \quad a = \sqrt{144} = 12$ cm

Perimeter of an equilateral triangle $= 3 \times a$

Since, a = 12 cm,

Perimeter $= (3 \times 12)$ cm $= 36$ cm

Question 13:

Let a be the length of the side of an equilateral triangle

$\therefore$ Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$ sq units

Area of the equilateral triangle $= 81\sqrt{3}\,cm^2$ [given]

$\Rightarrow \quad 81\sqrt{3}\,cm^2 = \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \quad a^2 = \frac{81\sqrt{3} \times 4}{\sqrt{3}} = 324$

$\Rightarrow \quad a = \sqrt{324} = 18$ cm

Height of an equilateral triangle $= \frac{\sqrt{3}}{2}a$

Since $a = 18$ cm,

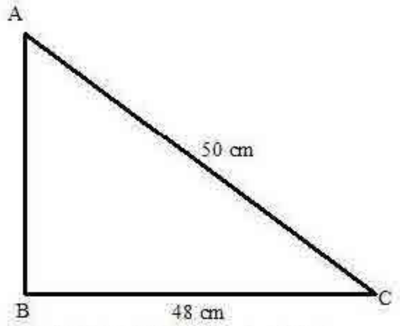
Height of the equilateral triangle $= \frac{\sqrt{3}}{2} \times 18 = 9\sqrt{3}$ cm.

Question 14:

Base of the right triangle is BC = 48 cm

Hypotenuse of the right triangle is AC = 50 cm

Let AB = x cm



By Pythagoras Theorem, we have,

$AC^2 = AB^2 + BC^2$

That is we have

$50^2 = x^2 + 48^2$

$\Rightarrow \quad x^2 = 50^2 - 48^2$

$\Rightarrow \quad x^2 = 2500 - 2304 = 196$

$\Rightarrow \quad x = \sqrt{196} = 14cm$

$\therefore$ Area of the right angle triangle $= \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 48 \times 14$

$= (24 \times 14)cm^2 = 336cm^2$

$\therefore$ Area of the triangle $= 336\,cm^2$

Question 15:

(i) Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

Where a is the side of the equilateral triangle

$\therefore$ $\text{area} = \frac{\sqrt{3}}{4} \times 8^2$

$= \frac{\sqrt{3}}{4} \times 64 \Rightarrow \sqrt{3} \times 16$

$= 1.732 \times 16$

$= 27.712 = 27.71 cm^2$. $\left[\begin{matrix}\text{correct upto 2} \\ \text{decimal places}\end{matrix}\right]$

(ii) Height of an equilateral triangle $= \frac{\sqrt{3}}{2} a$

$= \frac{\sqrt{3}}{2} \times 8$

$= \sqrt{3} \times 4$

$= 1.732 \times 4 = 6.928$

$= 6.93 cm$ $\left[\begin{matrix}\text{Correct upto 2} \\ \text{decimal places}\end{matrix}\right]$

Question 16:

Let a be the side of an equilateral triangle.

$\therefore$ Height of an equilateral triangle $= \frac{\sqrt{3}}{2} a$ units

Height of an equilateral triangle = 9cm [given]

$\Rightarrow \frac{\sqrt{3}}{2} a = 9$

$\Rightarrow a = \frac{9 \times 2}{\sqrt{3}}$

$\Rightarrow = \frac{9 \times 2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$ [Rationalizing the denominator]

$\Rightarrow = \frac{9 \times 2\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$\Rightarrow a = 6\sqrt{3}$

$\Rightarrow \text{base} = 6\sqrt{3}$

Area of the equilateral triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 6\sqrt{3} \times 9$ [$\because$ base $= 6\sqrt{3}$ and height = 9cm]

$= 27\sqrt{3}$

Area of the equilateral triangle $= 27 \times 1.732 = 46.764$

$= 46.76 cm^2$

[Correct to 2 places of decimal]

Question 17:

Let a=50cm, b=20cm and c=50cm.

Let us find s:

$s = \frac{1}{2}(a + b + c)$

$= \left(\frac{50 + 20 + 50}{2}\right) cm = \left(\frac{120}{2}\right) cm$

$= 60 cm$

Now, area of one triangular piece of cloth

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-50)(60-20)(60-50)}$

$= \sqrt{60 \times 10 \times 40 \times 10}$

$= \sqrt{6 \times 10 \times 10 \times 4 \times 10 \times 10}$

$= \sqrt{10 \times 10 \times 10 \times 10 \times 2 \times 2 \times 2 \times 3}$

$= 10 \times 10 \times 2\sqrt{6}$

$= 200\sqrt{6} = 200 \times 2.45 = 490 cm^2$

$\therefore$ area of one piece of cloth $= 490 cm^2$

Now area of 12 pieces $= (12 \times 490) cm^2 = 5880 cm^2$

Question 18:

Let, a = 16 cm, b = 12 and c = 20 cm

Let us now find s:

$s = \frac{1}{2}(a+b+c)$

$= \left(\frac{16+12+20}{2}\right) cm = \left(\frac{48}{2}\right) cm$

$= 24$ cm

Area of one triangular tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-16)(24-12)(24-20)}$

$= \sqrt{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times3}$

$= 2\times2\times2\times2\times2\times3$

$= 96\ cm^2$

$\therefore$ Area of one tile $= 96\ cm^2$

$\Rightarrow$ Area of 16 tiles $= 96\times16 = 1536\ cm^2$

Cost of polishing the tiles per sq.cm = Re.1

Thus, the total cost of polishing all the tiles = Rs. (1×1536)

= Rs. 1536.

Question 19:

Consider the right triangle ABC.

By Pythagoras Theorem, we have,

$BC = \sqrt{AB^2 - AC^2}$

$= \sqrt{17^2 - 15^2}$

$= \sqrt{289-225}$

$= \sqrt{64}$

$= 8$ cm

Perimeter of quad. ABCD = 17 + 9 + 12 + 8 = 46 cm

Area of triangle $\Delta ABC = \frac{1}{2}\times base\times height$

$= \frac{1}{2}\times BC\times AC$

$= \frac{1}{2}\times 8\times 15$

$= 60\ cm^2$

For area of triangle ACD,

Let a = 15 cm, b = 12 cm and c = 9 cm

Therefore, $s = \frac{a+b+c}{2} = \frac{15+12+9}{2} = 18$ cm

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-15)(18-12)(18-9)}$

$= \sqrt{18\times3\times6\times9}$

$= \sqrt{18\times18\times3\times3}$

$= 18\times3 = 54\ cm^2$

Thus the area of quad. ABCD = Area of ΔABC + Area of ΔACD

$= 60 + 54 = 114\ cm^2$.

Question 20:

Perimeter of quad. ABCD = 34 + 29 + 21 + 42 = 126 cm

Area of triangle BCD = $\frac{1}{2} \times 20 \times 21 = 210 \text{ cm}^2$

For area of triangle ABD,
Let a = 42 cm, b = 20 cm and c = 34 cm

Therefore, $s = \frac{42+20+34}{2} = \frac{96}{2} = 48$ cm

Area of ABD $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-42)(48-20)(48-34)}$

$= \sqrt{48 \times 6 \times 28 \times 14}$

$= \sqrt{16 \times 3 \times 3 \times 2 \times 2 \times 14 \times 14}$

$= 4 \times 3 \times 2 \times 14 = 336 \text{ cm}^2$

Area of quad. ABCD = Area ΔABD + Area ΔBCD

Thus the area of quad. ABCD = 336 + 210 = 546 cm^2.

Question 21:

Consider the right triangle ABD.
By Pythagoras Theorem, we have

$AB = \sqrt{BD^2 - AD^2}$

$\therefore AB = \sqrt{26^2 - 24^2}$

$= \sqrt{676 - 576}$

$= \sqrt{100}$

AB = 10 cm

$\Rightarrow$ base = 10 cm

Area of the triangle ABD $= \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow$ Area of $\Delta ABD = \frac{1}{2} \times 10 \times 24$ [$\because$ base = 10 cm, height = 24 cm]

$\Rightarrow$ Area of $\Delta ABD = 120 \text{ cm}^2$

Area of equilateral triangle BCD $= \frac{\sqrt{3}}{4} a^2$

$\Rightarrow$ $= \frac{1.73}{4}(26)^2$ [$a = 26$ cm, $\sqrt{3} = 1.73$]

$\Rightarrow$ $= 292.37 \text{ cm}^2$

Area of quad. ABCD = Area of ΔABD + Area of ΔBCD

= 120 + 292.37

= 412.37 cm^2.

Question 22:

Consider the triangle ABC,

Let a = 26 cm, b = 30 cm and c = 28 cm

$s = \frac{26+30+28}{2} = \frac{84}{2} = 42$ cm

$$\begin{aligned}\text{Area of ABC} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{42(42-26)(42-30)(42-28)}\\ &= \sqrt{42\times16\times12\times14}\\ &= \sqrt{14\times3\times16\times4\times3\times14}\\ &= \sqrt{14\times14\times3\times3\times16\times4}\\ &= 14\times3\times4\times2\\ &= 336\ \text{cm}^2\end{aligned}$$

In a parallelogram , diagonal divides the parallelogram in two equal area therefore

$$\begin{aligned}\therefore\ \text{Area of quad. ABCD} &= \text{Area of }\Delta ABC + \text{Area of }\Delta ACD\\ &= \text{Area of }\Delta ABC\times2\\ &= 336\times2\\ &= 672\ \text{cm}^2.\end{aligned}$$

Question 23:

Consider the triangle ABC,

Let a = 10 cm, b = 16 cm and c = 14 cm

$s = \frac{10+16+14}{2} = \frac{40}{2} = 20$

$$\begin{aligned}\text{Area of ABC} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{20(20-10)(20-16)(20-14)}\\ &= \sqrt{20\times10\times4\times6}\\ &= \sqrt{10\times2\times10\times4\times3\times2}\\ &= \sqrt{10\times10\times4\times2\times2\times3}\\ &= 10\times2\times2\times\sqrt{3}\\ &= 40\sqrt{3}\ \text{cm}^2\end{aligned}$$

In a parallelogram , diagonal divides the parallelogram in two equal area therefore

$$\begin{aligned}\therefore\ \text{Area of quad. ABCD} &= \text{Area of }\Delta ABC + \text{Area of }\Delta ACD\\ &= \text{Area of }\Delta ABC\times2\\ &= 40\sqrt{3}\times2\\ &= 80\sqrt{3}\ \text{cm}^2\\ &= 138.4\text{cm}^2\quad[\because \sqrt{3}=1.73]\end{aligned}$$

Question 24:

$$\begin{aligned}\text{Area of triangle ABD} &= \frac{1}{2}\times\text{base}\times\text{height}\\ &= \frac{1}{2}\times BD\times AL\\ &= \frac{1}{2}\times64\times16.8\\ &= 537.6\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of triangle BCD} &= \frac{1}{2}\times\text{base}\times\text{height}\\ &= \frac{1}{2}\times BD\times CM\\ &= \frac{1}{2}\times64\times13.2\\ &= 422.4\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of quad. ABCD} &= \text{Area of }\Delta ABD + \text{Area of }\Delta BCD\\ &= 537.6 + 422.4 = 960\ \text{cm}^2.\end{aligned}$$