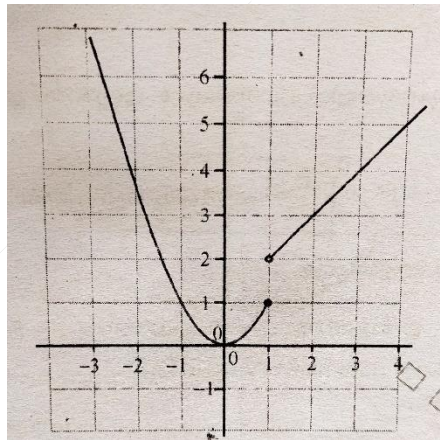


**CHAPTER 13  
LIMITS AND DERIVATIVES**

**DECEMBER 2020**

1. Graph of a function  $f: R \rightarrow R$  is given below:



- i. Find  $\lim_{x \rightarrow 1^-} f(x)$  and  $\lim_{x \rightarrow 1^+} f(x)$ . (2)
- ii. Does the limit  $\lim_{x \rightarrow 1} f(x)$  exist? (1)  
Give reason to your answer.

2. Find the derivative of  $x^2 + 2$  using first principle. (3)

**MARCH 2020**

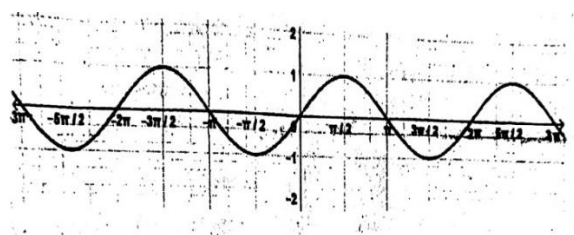
- 3. Find the derivative of  $\cos x$  from first principle. (3)
- 4. i. Derivative of  $f(x) = 1 + x + x^2 + \dots + x^{50}$  at  $x = 1$  is (1)
  - a) 50
  - b) 1250
  - c) 1275
  - d)  $\frac{101}{2}$
- ii.  $\lim_{x \rightarrow 0} f(x)$  if it exists, where (2)
 
$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

**IMPROVEMENT 2019**

- 5. a) If  $\lim_{x \rightarrow 2} \left[ \frac{x^n - 2^n}{x - 2} \right] = 80$ , then  $n = \dots\dots\dots$  (1)
- b) The value of  $\frac{d}{dx} [\sin^2 x + \cos^2 x] = \dots\dots\dots$  (1)
- 6. a) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{e^{3x} - 1}{x} \right)$  (2)
- b) Find the derivative of  $\frac{\sin x - \cos x}{\sin x + \cos x}$  (2)
- c) Find the derivative of  $f(x) - 2x$  with respect to  $x$  from first principle. (2)

**MARCH 2019**

7. Consider the graph of the function  $f(x)$



- a) Identify the function  $f(x)$  (1)
  - i)  $f(x) = \sin x$
  - ii)  $f(x) = \cos x$
  - iii)  $f(x) = \tan x$
  - iv)  $f(x) = \operatorname{cosec} x$
- b) Using function  $f(x)$ , find:

$$\lim_{x \rightarrow 2} \frac{f(x) - f\left(\frac{\pi}{2}\right)}{x - \frac{\pi}{2}} \quad (2)$$

- 8. a) Find the derivative of  $y = x^2$  using the first principle. (3)
- b) Find  $\frac{dy}{dx}$  if  $y = \frac{x}{1 + \tan x}$  (3)

**IMPROVEMENT 2018**

9. Consider the real valued function

$$f(x) = \frac{x-3}{x^2-x-6}$$

- a) Find the domain of  $f(x)$  (2)  
 b) Evaluate  $\lim_{x \rightarrow 3} f(x)$  (1)

10. a) Find the derivative of  $f(x) = x \sin x$  with respect to  $x$ . (1)  
 b) Find the derivative of the function  $y = \sqrt{x}$  with respect to  $x$  by using first principles. (3)

**MARCH 2018**

11. a)  $\lim_{x \rightarrow 2} [x] = \dots\dots\dots$  (1)  
 i) 2      ii) 3  
 iii) 0      iv) does not exist
- b) Evaluate:  $\lim_{x \rightarrow 2} \frac{x^3 - 4x^2 + 4x}{x^2 - 4}$  (3)

12. a) Find the derivative of  $y = \sin x$  from the first principle. (3)  
 b) Find  $\frac{dy}{dx}$ , if  $y = \frac{x^5 - \cos x}{\sin x}$  (3)

**IMPROVEMENT 2017**

13. a) Find  $\lim_{x \rightarrow 2} (x^2 - 2)$  (1)  
 b) Find the derivative of  $\frac{1}{x}$  from the first principles. (3)  
 c) Find the derivative of  $x \sin x$  (2)

OR

- a) Find  $\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$  (2)  
 b) Find the derivative of  $f(x) = \cos x$  from the first principles. (3)

c)  $\frac{d}{dx}(x^n) = \dots\dots\dots$  (1)

**MARCH 2017**

14. a)  $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x} = \dots\dots\dots$   
 i) 0      ii) 1  
 iii) 2      iv) 3 (1)  
 b) Find  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$  (2)  
 c) Find the derivative of  $f(x) = \sin x$  by using the first principle. (3)

**IMPROVEMENT 2016**

15. a)  $\frac{d}{dx}(\tan x) = \dots\dots\dots$  (1)  
 b) Compute:  $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$ . (2)  
 c) Using the first principle, find the derivative of  $\cos x$  (3)

OR

- a)  $\frac{d}{dx}(9 + \sin x) = \dots\dots\dots$  (1)  
 b) Evaluate:  $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$  (2)  
 c) Find  $\frac{dy}{dx}$ , if  $y = \frac{4 + 5 \sin x}{3 + 7 \cos x}$  (3)

**MARCH 2016**

16. a)  $\frac{d}{dx} \left( \frac{x^n}{n} \right) = \dots\dots\dots$  (1)  
 b) Differentiate  $y = \frac{\sin x}{x+1}$  with respect to  $x$  (2)  
 c) Using first principles, find the derivative of  $\cos x$ . (3)

OR

- a)  $\frac{d}{dx}(-\sin x) = \dots\dots\dots$  (1)
- b) Find  $\frac{dy}{dx}$  if  $y = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$ , where a,b are constants. (2)
- c) Using first principles, find the derivative of  $\sin x$  (3)

iii) $\text{Lt}_{x \rightarrow 0} \frac{\sin 8x}{x}$	3) 2
	4) 8

- b) Find the derivative of  $\tan x$  using the first principle. (3)

OR

- a) Match the following: (3)

**IMPROVEMENT 2015**

17. a)  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \dots\dots\dots$  (1)
- i) 1                                      ii) -1
- iii) 0                                      iv) 2
- b) Find  $\text{Lt}_{x \rightarrow 0} \frac{\sin 4x}{\sin 2x}$  (2)
- c) Find the derivative of  $\cos x$  using first principle. (3)

i) $\text{Lt}_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$	1) 0
ii) $\text{Lt}_{x \rightarrow 0} \frac{\sin \pi x}{\cos 2x}$	2) 32
iii) $\text{Lt}_{r \rightarrow 1} \pi r^2$	3) 3.0
	4) $\pi$

OR

- b. If  $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots\dots + \frac{x^2}{2} + x + 1$ , prove that  $f'(1) = 100f'(0)$ . (3)

- a) Derivative of  $x^2 - 2$  at  $x = 10$  is ..... (1)
- i) 10                                      ii) 20
- iii) -10                                      iv) -20

**IMPROVEMENT 2014**

- b) If  $f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$  (1)
- Find  $\text{Lt}_{x \rightarrow 0} f(x)$  and  $\text{Lt}_{x \rightarrow 1} f(x)$ . (2)
- c) If  $xy = c^2$ , prove that  $x^2 \frac{dy}{dx} + c^2 = 0$  (3)

19. a) Evaluate  $\text{Lt}_{x \rightarrow 0} \frac{\sin ax}{bx}$ . (3)

- b) Using first principles, find the derivative of  $\cos x$  (3)

OR

Find the derivative of  $\frac{\cos x}{2x+3}$  w.r.t.  $x$ .

**MARCH 2015**

18. a) Match the following: (3)

i) $\text{Lt}_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$	1) 1
ii) $\text{Lt}_{x \rightarrow 0} \cos 5x$	2) 0

**MARCH 2014**

20. a) Find the derivative of  $\sin x$ , using first principle. (3)

OR

Find the derivative of  $\frac{x^5 - \cos x}{\sin x}$ , using quotient rule. (3)

21. a) The value of  $\lim_{x \rightarrow 0} \frac{\sin 5x}{5x}$  is ..... (1)

b) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$ ,  $ab \neq 0$  (2)

**IMPROVEMENT 2013**

22. If  $f(x) = \begin{cases} a + bx & ; x < 1 \\ 4 & ; x = 1 \\ b - ax & ; x > 1 \end{cases}$  and  $\lim_{x \rightarrow 1} f(x) = f(1)$ , then find the values of a and b. (3)

23. Find the derivative of  $y = \cos \operatorname{csc} x$ , using first principle. (3)

OR

Find the derivative of  $\frac{x+1}{x-1}$ , using first principle. (3)

**MARCH 2013**

24. a) The value of  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$  is ..... (1)

b) Evaluate  $\lim_{x \rightarrow 1} \frac{x^{15} - 1}{x^{10} - 1}$ . (2)

25. Find the derivative of  $y = \sin x$ , using first principle. (3)

OR

Find the derivative of  $\frac{x + \cos x}{x - 1}$  w.r.t  $x$ . (3)

**IMPROVEMENT 2012**

26. Evaluate  $\lim_{x \rightarrow 0} \frac{(x+5)^2 - 25}{x}$  (2)

27. Find the derivative of the following:

i)  $f(x) = \frac{2x+3}{x-1}$  (2)

ii)  $f(x) = \cos(x-5)$  (2)

OR

Compute the derivative of  $\sec x \operatorname{csc} x$  with respect to  $x$  from first principle. (4)

**MARCH 2012**

28. a) Evaluate  $\lim_{x \rightarrow 3} \left( \frac{x^3 - 27}{x^2 - 9} \right)$  (1)

b) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\tan x - \sin x}{\sin^3 x} \right)$  (2)

29. Prove that  $\frac{d}{dx} \left( \frac{\cos x}{1 + \sin x} \right) = \frac{-1}{1 + \sin x}$  (3)

OR

Find the derivative of  $\cot x$ , using first principle. (3)

**IMPROVEMENT 2011**

30. a) Evaluate  $\lim_{x \rightarrow -1} \frac{x^5 + 1}{x + 1}$  (1)

b) Find the derivative of  $\lim_{x \rightarrow -1} x^2 + x + 1$  from first principle. (2)

31. a) Differentiate  $\frac{x^2 \tan x}{1 + x}$  (3)

OR

b) Differentiate  $\frac{x + 2 \cos x}{3x + 4 \sin x}$ . (3)

**MARCH 2011**

32. i) Evaluate  $\lim_{x \rightarrow -1} \frac{x^2 - 5x + 6}{x - 1}$  (3)

ii) Evaluate  $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$  (3)

33. Using first principle, find the derivative of  $f(x) = \frac{1}{x}$  (3)

OR

Using quotient rule, find the derivative of  $f(x) = \cot x$ . (3)

**IMPROVEMENT 2010**

34. Choose the most appropriate answer from those given in the bracket, choose either (a), (b), (c) or (d), (e), (f).

a) If  $\lim_{x \rightarrow 2} \frac{x^p - 2^p}{x - 2} = 192$ , then

$P = [2, 4, 6, 10]$  (1)

b)  $\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - 1}{x} = \dots$

$\left[ 0, 1, \frac{1}{2}, \text{not defined} \right]$  (1)

c)  $\lim_{x \rightarrow 0} \frac{\sin ax}{x \cos bx} = \dots\dots\dots$

$[0, a, b, \text{not defined}]$  (1)

OR

d)  $\lim_{x \rightarrow 0} \frac{\sqrt{z} - 1}{1 - z} = \dots\dots$

$\left[ 0, -\frac{1}{2}, \frac{1}{2}, 1 \right]$  (1)

e)  $\lim_{x \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} - x\right)}{\left(\frac{\pi}{4} - x\right)}$  is

$\left[ 0, 1, \frac{\pi}{4}, \text{not defined} \right]$  (1)

f) If  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = k \neq 0$ , find  $\lim_{x \rightarrow 0} f(x) =$

$[0, 1, k, \text{not defined}]$  (1)

**MARCH 2010**

35. i) Evaluate  $\lim_{x \rightarrow 1} \frac{x^7 - 1}{x^4 - 1}$  (1)

ii) Evaluate  $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$  (2)

36. If  $xy = c^2$ , prove that  $x^2 \frac{dy}{dx} + c^2 = 0$  (3)

OR

Find the derivative of  $\tan x$  from first principle.

**IMPROVEMENT 2009**

37. i) Evaluate  $\lim_{x \rightarrow a} \frac{\sqrt{x} + \sqrt{a}}{x + a}$ . (1)

ii) Evaluate  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$  (2)

iii) Evaluate  $\lim_{x \rightarrow 0} (\cos ecx - cot x)$ . (3)

**MARCH 2009**

38. a) Choose the correct value of

$\lim_{x \rightarrow -1} \left[ \frac{x^{10} + x^5 + 1}{x - 1} \right]$  from the bracket. (1)

$\left[ \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, 0 \right]$

b) Evaluate  $\lim_{x \rightarrow 1} \left[ \frac{\sqrt{1+x} - 1}{x} \right]$  (2)

c) Evaluate  $\lim_{x \rightarrow 0} \left[ \frac{\sqrt{1 - \cos 4x}}{x^2} \right]$  (3)

