CHAPTER 4

PRINCIPLES OF MATHEMATICAL INDUCTION

DECEMBER 2020

1. Consider the statement:

$$P(n):1+3+3^2+...+3^{n-1}=\frac{3^n-1}{2}.$$

a) Show that P(1) is true. (1)

b) Prove by principle of Mathematical induction that P(n) is true for all $n\in N$. (2)

MARCH 2020

2. For every positive integer n, prove that 7^n-3^n is divisible by 4 using principle of mathematical induction. (4)

IMPROVEMENT 2019

3. Consider the statement

$$P(n): a+ar+ar^2+....+ar^{n-1}=a\frac{(r^n-1)}{(r-1)},\text{where}$$

$n\in N$

a) Write the value of $P(1)$ (1)

b) Write $P(k), k\in N$. (1)

c) By assuming the result obtained in part (b) prove the result is true for $n=k+1$. (2)

MARCH 2019

4. Using principle of mathematical induction, prove that $n(n+1)(n+5)$ is a multiple of 3 for all $n\in N$. (4)

IMPROVEMENT 2018

5. Consider the statement:

$$P(n):1+3+3^2+...+3^{n-1}=\frac{3^n-1}{2}.$$

c) Show that P(1) is true. (1)

d) Prove by principle of Mathematical induction that P(n) is true for all $n\in N$. (3)

[same as Dec 2020, Mar 2018]

MARCH 2018

6. a) If $3^{2n+2}-8n-9$ is divisible by k for all $n\in N$ is true, then which one of the following is a value of k? (1)

i) 8 ii) 6 iii) 3 iv) 12

b) Prove by using the Principle of Mathematical Induction $P(n):1+3+3^2+...+3^{n-1}=\frac{3^n-1}{2}$ is true for all $n\in N$.

IMPROVEMENT 2017

7. Consider the statement: $P(n)$:"7^n-3^n is divisible by 4".

a) Verify the statement for $n=1$ (1)

b) Prove the statement by using the principle of mathematical induction. (3)

MARCH 2017

8. Consider the statement "$10^{2n-1}+1$ is divisible by 11". Verify that $P(1)$ is true and then prove that the statement by using mathematical induction. (4)

IMPROVEMENT 2016

9. Consider the statement:

"$P(n): x^n-y^n$ is divisible by $x-y$".

a) Show that is $P(1)$ true. (1)

b) Using the principle of mathematical inductions verify that $P(n)$ is true for all natural numbers. (3)

MARCH 2016

10. Consider the following statement:

$$P(n): a+ar+ar^2+...+ar^{n-1}=\frac{a(r^n-1)}{r-1}$$

a) Prove that $P(1)$ is true. (1)

b) Hence by using the principle of Mathematical induction, prove that $P(n)$ is true for all natural numbers n. (3)

IMPROVEMENT 2015

11. Consider the statement: $P(n) = 7^n - 3^n$ is divisible by 4.
    a) Show that $P(1)$ is true. (1)
    b) Verify, by the method of Mathematical induction that $P(n)$ is true for all $n \in N$. (3)

MARCH 2015

12. A statement $p(n)$ for a natural number n is given by $p(n) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^n} = 1 - \frac{1}{2^n}$
    a) Verify that $p(1)$ is true. (1)
    b) By assuming that $P(k)$ is true for a natural number k, show that $P(k+1)$ is true. (3)

IMPROVEMENT 2014

13. Using the principal of mathematical induction, prove that $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^n} = 1 - \frac{1}{2^n}$. (4)

MARCH 2014

14. Consider the statement "$3^{2n+2} - 8n - 9$ is divisible by 8".
    a) Verify the statement is true for $n = 1$ (1)
    b) Prove the statement using the principle of mathematical induction for all natural numbers. (3)

IMPROVEMENT 2013

15. Consider the statement

$$P(n): 1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

    a) Verify that $P(n)$ is true. (1)
    b) By mathematical induction show that $P(n)$ is true for all $n \in N$ (3)

MARCH 2013

16. Consider the statement

$$P(n): 1 + 3 + 3^2 + ... + 3^{n-1} = \frac{3^n - 1}{2}$$

    a) Check whether P(1) is true. (1)
    b) If $P(k)$ is true, prove that $P(k+1)$ is also true. (2)
    c) Is $P(n)$ true for all natural numbers n? Justify your answer. (1)

IMPROVEMENT 2012

17. Prove that

$$1.2 + 2.3 + 3.4..... + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

by using the principle of mathematical induction for all $n \in N$ (4)

2012 MARCH

18. Consider the statement, "$n(n+1)(2n+1)$ is divisible by 6".
    a) Verify the statement for $n = 2$. (1)
    b) By assuming that $P(k)$ is true for a natural number k, verify that $P(k+1)$ is true. (3)

IMPROVEMENT 2011

No question from this chapter.

MARCH 2011

19. Consider the statement $P(n)$: "$9^n - 1$ is a multiple of 8", where 'n' is a natural number.
    a) Is P(1) true? (1)
    b) Assuming $P(k)$ is true, show that $P(k+1)$ is true. (3)

IMPROVEMENT 2010

20. a) Which among the following is the least number that will divide $7^{2n} - 4^{2n}$ for every positive integer n?
[4,7,11,33] (1)

b) Prove by mathematical induction,
$\left(\cos\theta + i\sin\theta\right)^n = \left(\cos n\theta + i\sin n\theta\right)$, where $i = \sqrt{-1}$ (3)

MARCH 2010

21. Consider the statement "$7^n - 3^n$ is divisible by 4"
a) Verify the result for n = 2. (1)
b) Prove the statement using mathematical induction. (3)

IMPROVEMENT 2009

[same as March 2010]

22. Let P(n) be the statement :
"$7^n - 3^n$ is divisible by 4".
a) Verify whether the statement is true for n=2. (1)
b) Prove the result by using mathematical induction. (3)

MARCH 2009

23. a) For every positive integer n, $7^n - 3^n$ should be divisible by (2, 3, 4, 8). (1)
c) Prove by principle of mathematical induction that: $2 + 2^2 + 2^3 + 2^4 + ... + 2^n = 2\left(2^n - 1\right)$ (3)

MARCH 2008

24. Consider the statement
$P(n): 1 + 3 + 5 + ... + (2n - 1) = n^2$
a) Verify $P(1)$ is true. (1)
b) Prove $P(n)$ by induction. (2)