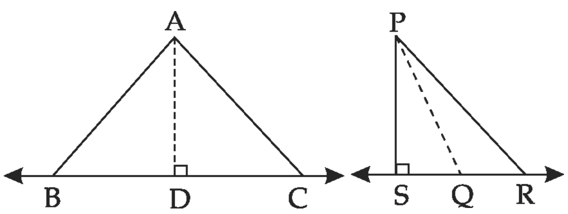
1 Similarity

... INDEX ...

Points to Remember:

Ratio of Areas of Two Triangles

- **Property - 1 : The ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights.**



In ΔABC,

seg AD is the height and seg BC is the base.

In ΔPQR,

seg PS is the height and seg QR is the base.

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$$

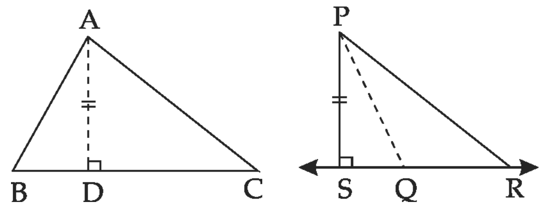
- **To learn the next property, we have to first understand the meaning of Triangles with equal heights.**

  In theorems and problems we will come across three situations where two or more triangles have equal height.

(1) In the following figure,

seg AD and seg PS are the heights of ΔABC and ΔPQR respectively.

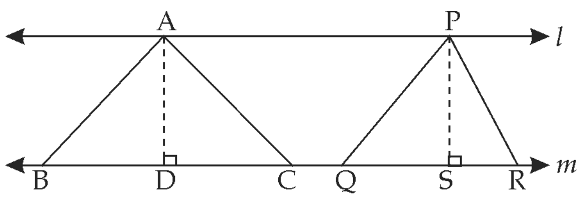
If AD = PS then ΔABC and ΔPQR are said to have equal height.



(2) In the following figure, line $l \parallel$ line m

ΔABC and ΔPQR lie between the same two parallel lines l and m.

$\therefore$ They are said to have equal heights.



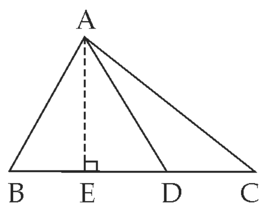
(3) In the following figure,

ΔABD and ΔADC and ΔABC have common vertex A and their bases BD, DC and BC lie on the same line BC.

Also, seg AE $\perp$ line BC.

$\therefore$ seg AE is their common height.

$\therefore$ These three triangles have same height.

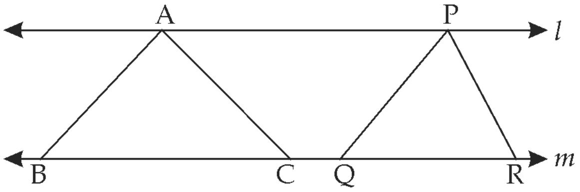


- **Property - 2 : The ratio of areas of two triangles with equal height is equal to the ratio of their corresponding bases.**

(1) In the following figure, ΔABC and ΔPQR lie between the same two parallel lines l and m.

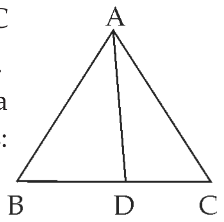
$\therefore$ Their heights are equal.

$\therefore$ $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR}$



(2) In the following figure, ΔABD, ΔADC and ΔABC have common vertex A, and their bases BD, CD and BC lie on the same line BC

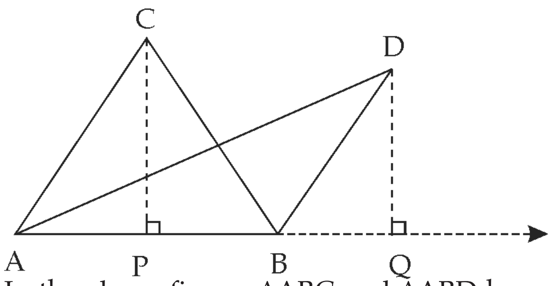
Hence they have equal heights. Considering two triangles at a time we get the following results:



(i) $\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{BD}{CD}$

(ii) $\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{BD}{BC}$ (iii) $\frac{A(\Delta ADC)}{A(\Delta ABC)} = \frac{DC}{BC}$

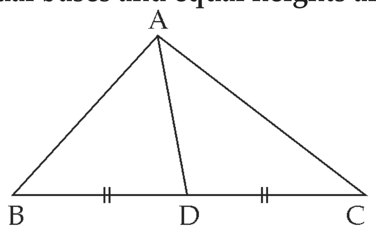
- **Property -3 : The ratio of areas of two triangles having equal bases, is equal to the ratio of their corresponding heights.**



In the above figure, ΔABC and ΔABD have the same base AB.

$\frac{A(\Delta ABC)}{A(\Delta ABD)} = \frac{CP}{DQ}$

- **Property - 4 : Areas of two triangles having equal bases and equal heights are equal.**



In the above figure, ΔABD and ΔACD have common vertex A and their bases BD and CD lie on the same line BC.

$\therefore$ Their heights are equal.

Also, D is the midpoint of seg BC.

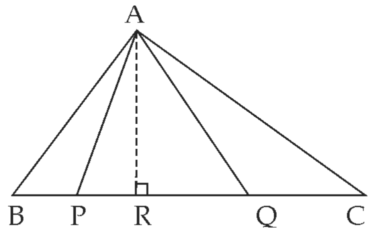
$\therefore$ BD = CD.

$\therefore$ Their bases are equal.

$\therefore$ **A (ΔABD) = A (ΔACD)**

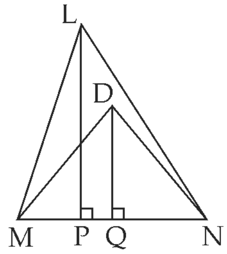
Activity : Fill in the blanks properly.

(i) **(Textbook page no. 3)**



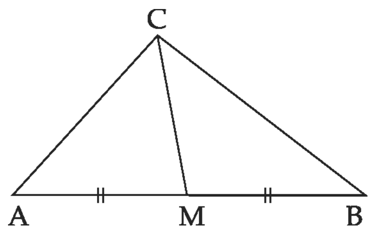
$\frac{A(\Delta ABC)}{A(\Delta APQ)} = \frac{\boxed{BC} \times \boxed{AR}}{\boxed{PQ} \times \boxed{AR}} = \frac{\boxed{BC}}{\boxed{PQ}}$

(ii)



$\frac{A(\Delta LMN)}{A(\Delta DMN)} = \frac{\boxed{MN} \times \boxed{LP}}{\boxed{MN} \times \boxed{DQ}} = \frac{\boxed{LP}}{\boxed{DQ}}$

(iii) Point M is the midpoint of seg AB Seg CM is the median of ΔABC.



$\frac{A(\Delta AMC)}{A(\Delta BMC)} = \frac{\boxed{AM}}{\boxed{BM}}$ ...(Triangles with equal heights)

$= \frac{\boxed{AM}}{\boxed{AM}} = \boxed{1}$

...($\because$ M is the midpoint of AB)

Area of two triangles having equal bases and equal heights are equal.

MASTER KEY QUESTION SET - 1

Practice Set - 1.1 *(Textbook Page No. 5)*

(1) **Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.** *(2 marks)*

Solution :

Let the area, base and height of the first triangle be A_1, b_1 and h_1 respectively.

Let the area, base and height of the second triangle be A_2, b_2 and h_2 respectively.

$b_1 = 9,\ h_1 = 5,\ b_2 = 10$ and $h_2 = 6$ ...(Given)

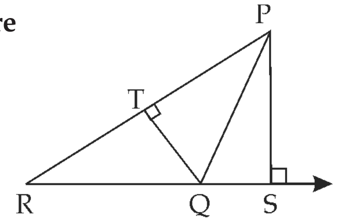
$\frac{A_1}{A_2} = \frac{b_1 \times h_1}{b_2 \times h_2}$ ...(Write the statement of) property I)

$\therefore \quad = \frac{9 \times 5}{10 \times 6}$

$\therefore \quad \frac{A_1}{A_2} = \frac{3}{4}$

$\therefore$ **The ratio of the areas of the triangles is 3 : 4**

(3) **In the adjoining figure seg PS ⊥ ray RQ, seg QT ⊥ seg PR. If RQ = 6, PS = 6 and PR = 12 then find QT.** *(2 marks)*



Solution :

Area of a triangle $= \frac{1}{2} \times$ base $\times$ height

$A(\Delta PQR) = \frac{1}{2} \times RQ \times PS$

$= \frac{1}{2} \times 6 \times 6$

$A(\Delta PQR) = 18$ sq. units

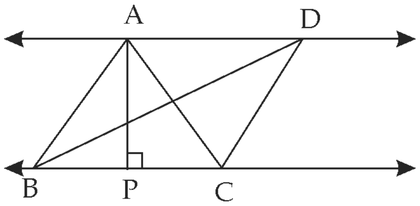
Also, $A(\Delta PQR) = \frac{1}{2} \times PR \times QT$

$\therefore \quad 18 = \frac{1}{2} \times 12 \times QT$

$QT = \frac{18 \times 2}{12}$

$\therefore$ **QT = 3 units**

(4) **In adjoining figure AP ⊥ BC, AD || BC, then find A(ΔABC) : A (ΔBCD)** *(2 marks)*



Solution :

line AD || line BC ...(Given)

$\therefore$ ΔABC and ΔBCD lie between the same two parallel lines AD and BC.

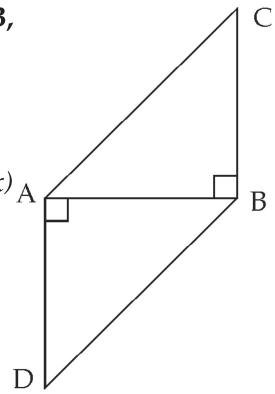
$\therefore$ Their heights are equal.

Also, they have a common base BC

$\therefore$ A(ΔABC) = (ΔBCD) ...(Triangles having equal base and equal height)

$\therefore \quad \mathbf{\frac{A(\Delta ABC)}{A(\Delta BCD)} = 1}$

(2) **In figure BC ⊥ AB, AD ⊥ AB, BC = 4, AD = 8 then find $\frac{A(\Delta ABC)}{A(\Delta ADB)}$** *(1 mark)*



Solution

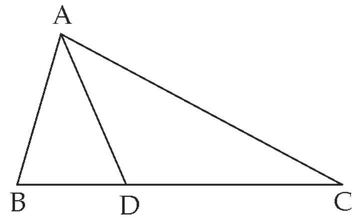
$\frac{A(\Delta ABC)}{A(\Delta ADB)} = \frac{BC}{AD}$ ...(Triangles with common base)

$\therefore \quad \frac{A(\Delta ABC)}{A(\Delta ADB)} = \frac{4}{8}$

$\therefore \quad \mathbf{\frac{A(\Delta ABC)}{A(\Delta ADB)} = \frac{1}{2}}$

Problem Set - 1 *(Textbook Page No. 27)*

(2) **In ΔABC, B-D-C and BD = 7, BC = 20. Then find the following ratio.**



(i) $\mathbf{\frac{A(\Delta ABD)}{A(\Delta ADC)}}$

(ii) $\mathbf{\frac{A(\Delta ABD)}{A(\Delta ABC)}}$ **(iii)** $\mathbf{\frac{A(\Delta ADC)}{A(\Delta ABC)}}$ *(3 marks)*

Solution :

BC = BD + DC ...(B - D - C)

$\therefore$ 20 = 7 + DC

$\therefore$ 20 – 7 = DC

$\therefore$ DC = 13 units.

ΔABD, ΔADC and ΔABC have a common vertex A and their bases BD, DC and BC lie on the same line BC.

$\therefore$ their heights are equal.

(1) $\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{BD}{DC}$ ...(triangles with equal height)

$\mathbf{\frac{A(\Delta ABD)}{A(\Delta ADC)} = \frac{7}{13}}$

(2) $\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{BD}{BC}$ ...(triangles with equal height)

$\mathbf{\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{7}{20}}$

(3) $\frac{A(\Delta ADC)}{A(\Delta ABC)} = \frac{DC}{BC}$ ...(triangles with equal height)

$\mathbf{\frac{A(\Delta ADC)}{A(\Delta ABC)} = \frac{13}{20}}$

(3) Ratio of areas of two triangles with equal height is 2 : 3. If base of smaller triangle is 6 cm then what is the corresponding base of the bigger triangles. *(2 marks)*

Solution :

Let the area and base of the smaller triangle be A_1, and b_1 respectively.

Let the area and base of the bigger triangle be A_2, and b_2 respectively.

Both the triangles have equal height ...(Given)

$\therefore \quad \dfrac{A_1}{A_2} = \dfrac{2}{3}$ and $b_1 = 6$ cm ...(Given)

$\therefore \quad \dfrac{A_1}{A_2} = \dfrac{b_1}{b_2}$ ...(Triangles with equal height)

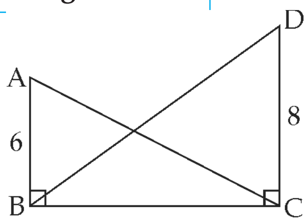
$\therefore \quad \dfrac{2}{3} = \dfrac{6}{b_2}$

$\therefore \quad b_2 = \dfrac{3 \times 6}{2}$

$\therefore \quad b_2 = 9$ cm

$\therefore$ **The base of the bigger triangle is 9 cm.**

(4) In the figure given
$\angle ABC = \angle DCB = 90^o$.
AB = 6, DC = 8.
then $\dfrac{A(\Delta ABC)}{A(\Delta DCB)}$?



(2 marks)

Solution :

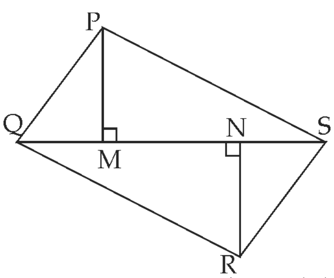
ΔABC and ΔDCB have a common base BC.

$\dfrac{A(\Delta ABC)}{A(\Delta DCB)} = \dfrac{AB}{DC}$ ...(Triangles with equal base)

$\dfrac{A(\Delta ABC)}{A(\Delta DCB)} = \dfrac{6}{8}$

$\therefore \quad \boxed{\dfrac{A(\Delta ABC)}{A(\Delta DCB)} = \dfrac{3}{4}}$

(5) In the adjoining figure,
PM = 10 cm,
$A(\Delta PQS) = 100$ sq cm
$A(\Delta QRS) = 110$ sq cm
then find NR.



(2 marks)

Solution :

ΔPQS and ΔQRS have common base QS.

$\dfrac{A(\Delta PQS)}{A(\Delta QRS)} = \dfrac{PM}{RN}$ ...(triangles with equal base)

$\therefore \quad \dfrac{100}{110} = \dfrac{10}{RN}$

$\therefore \quad RN = \dfrac{10 \times 110}{100}$

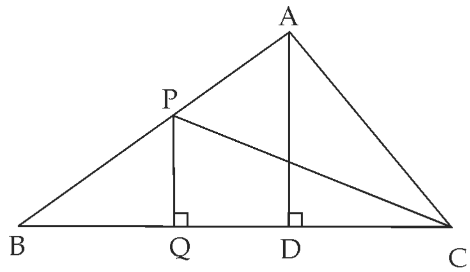
$\therefore \quad$ **RN = 11 cm**

Practice set - 1.1 *(Textbook Page No. 6)*

(5) In the adjoining figure, PQ $\perp$ BC, AD $\perp$ BC, then find the following ratios

(i) $\dfrac{A(\Delta PQB)}{A(\Delta PBC)}$ (ii) $\dfrac{A(\Delta PBC)}{A(\Delta ABC)}$

(iii) $\dfrac{A(\Delta ABC)}{A(\Delta ADC)}$ (iv) $\dfrac{A(\Delta ADC)}{A(\Delta PQC)}$ *(2 marks)*



Solution :

(i) ΔPQB and ΔPBC have a common height PQ.

$\therefore \quad \boxed{\dfrac{A(\Delta PQB)}{A(\Delta PBC)} = \dfrac{BQ}{BC}}$ ...(Triangles with equal height)

(ii) ΔPBC and ΔABC have a common base BC.

$\therefore \quad \boxed{\dfrac{A(\Delta PBC)}{A(\Delta ABC)} = \dfrac{PQ}{AD}}$ ...(Triangles with equal base)

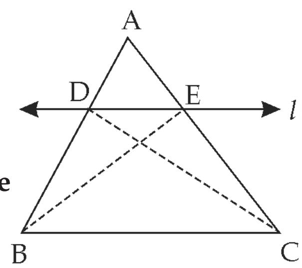
(iii) ΔABC and ΔADC have a common height AD.

$\therefore \quad \boxed{\dfrac{A(\Delta ABC)}{A(\Delta ADC)} = \dfrac{BC}{DC}}$ ...(Triangles with equal height)

(iv) $\boxed{\dfrac{A(\Delta ADC)}{A(\Delta PQC)} = \dfrac{DC \times AD}{QC \times PQ}}$ ...(The ratio of areas of two triangles is equal to the ratio of product of bases and their corresponding heights)

Points to Remember:

- **Basic Proportionality Theorem (B. P. T.)**



Statement : **If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides those sides in the same proportion.**

Given:

In ΔABC,

(i) line $l \parallel$ side BC

(ii) Line l intersects sides AB and AC at points D and E respectively.

A-D-B, A-E-C

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw seg BE and seg CD.

Proof: ΔADE and ΔBDE have a common vertex E and their bases AD and BD lie on the same line AB.

$\therefore$ Their heights are equal .

$\therefore$ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{AD}{DB}$ ...(i)

(Triangles having equal height)

ΔADE and ΔCDE have a common vertex D and their bases AE and EC lie on the same line AC.

$\therefore$ Their heights are equal.

$\therefore$ $\frac{A(\Delta ADE)}{A(\Delta CDE)} = \frac{AE}{CE}$ ...(ii)

(Triangles having equal height)

line DE || side BC ...(Given)

ΔBDE and ΔCDE are between the same two parallel lines DE and BC.

$\therefore$ Their heights are equal.

Also, they have same base DE.

$\therefore$ A(ΔBDE) = A(ΔCDE) ...(iii)

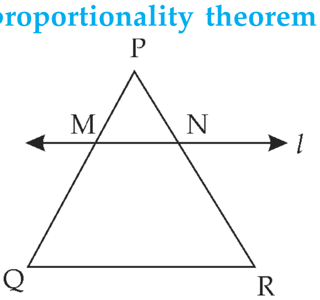
(Areas of two triangles having equal base and equal height are equal)

$\therefore$ $\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{A(\Delta ADE)}{A(\Delta CDE)}$ ...(iv) [From (iii)]

$\therefore$ $\mathbf{\frac{AD}{DB} = \frac{AE}{EC}}$ ...[From (i), (ii) and (iv)]

- **Converse of Basic proportionality theorem**

Statement : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.



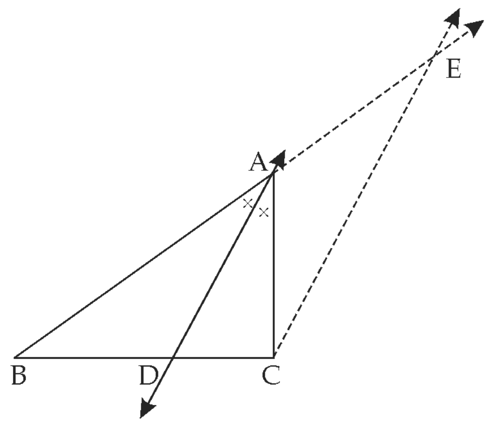
In ΔPQR, line l intersects the side PQ and side PR in the points M and N respectively.

such that $\frac{PM}{MQ} = \frac{PN}{NR}$ and P-M-Q, P-N-R

$\therefore$ **line l || side QR**

- **Property of an Angle Bisector of a Triangle**

Statement : In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides. (4 marks)



Given :

In ΔABC, ray AD is the bisector of $\angle$BAC such that B - D - C.

To Prove : $\frac{BD}{DC} = \frac{AB}{AC}$

Construction : Draw a line passing through C, parallel to line AD and intersecting line BA at point E, B - A - E.

Proof : In ΔBEC,

line AD || side CE ...(Construction)

$\frac{BD}{DC} = \frac{AB}{AE}$ ... (i) (By Basic Proportionately Theorem)

line AD || line CE ... (Construction)

$\therefore$ On transversal BE,

$\angle$BAD $\cong$ $\angle$AEC ...(ii) (Corresponding angle theorem)

Also, on transversal AC,

$\angle$DAC $\cong$ $\angle$ACE ...(iii) (alternate angle theorem)

But, $\angle$BAD $\cong$ $\angle$DAC ...(iv)

($\because$ ray AD bisects $\angle$BAC)

In ΔAEC, $\angle$AEC $\cong$ $\angle$ACE

...[From (ii), (iii) and (iv)]

$\therefore$ seg AC $\cong$ seg AE ...(Converse of Isosceles triangle theorem)

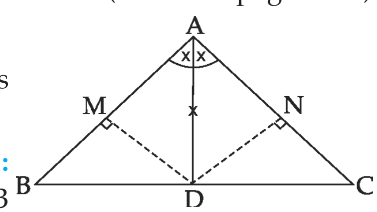
$\therefore$ AC = AE ...(v)

$\therefore$ $\mathbf{\frac{BD}{DC} = \frac{AB}{AC}}$ ...[From (i) and (v)]

For more information:

Write another proof of the theorem.

(Textbook page no. 9)



Given : In ΔABC, bisector $\angle$A insects side BC

Construction : Draw DM $\perp$ AB and DN $\perp$ AC

Proof : ΔABC and ΔADC have a common vertex A and their bases. BD and DC lie on the same line BC.

$\therefore$ their heights are equal.

$\therefore \quad \dfrac{A(\Delta ABD)}{A(\Delta ADC)} = \dfrac{BD}{DC}$ ...(i) (Triangles having equal heights)

Also, $\dfrac{A(\Delta ABD)}{A(\Delta ADC)} = \dfrac{AB \times DM}{AC \times DN}$ ... (ii) (Statmement of property - I)

Every point on the bisector of an angle is equidistant from the sides of the angle

DM = DN ...(iii)

$\therefore \quad \dfrac{A(\Delta ABD)}{A(\Delta ADC)} = \dfrac{AB}{AC}$ ...(iv) [From (ii) and (iii)]

$\therefore \quad \dfrac{BD}{DC} = \dfrac{AB}{AC}$ ...[From (i) and (iv)]

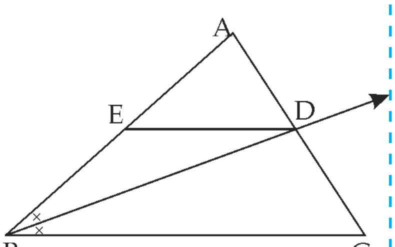
$\therefore \quad \mathbf{\dfrac{AB}{AC} = \dfrac{BD}{DC}}$ ...[From (i) and (iv)]

Converse of angle bisector property

If in ΔABC point D on side BC

such that $\dfrac{AB}{AC} = \dfrac{BD}{DC}$, then ray AD bisects $\angle BAC$ (Textbook page no. 13)

Example :



In ΔABC,

ray BD bisects

$\angle ABC$ A-D-C,

side DE || side BC,

A-E-B

then prove, $\dfrac{AB}{BC} = \dfrac{AE}{EB}$

Proof :

In ΔABC, ray BD bisects $\angle B$.

$\therefore \quad \dfrac{AB}{BC} = \dfrac{AD}{DC}$ ...(i) (angle bisector property of a triangle)

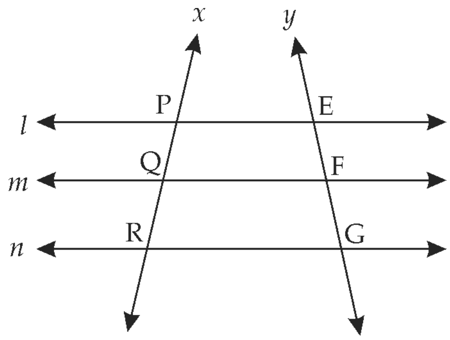
In ΔABC, DE || BC,

$\dfrac{AE}{EB} = \dfrac{AD}{DC}$ ...(ii) (By basic proportionality theorem)

$\therefore \quad \boxed{\mathbf{\dfrac{AB}{BC} = \dfrac{AE}{EB}}}$ ...[From (i) and (ii)]

- **Property of Three Parallel Lines and their transversals**

The ratio of the intercepts made on the transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

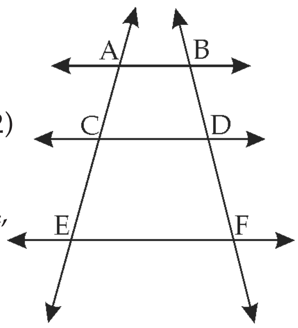


line l || line m || line n and lines l, m and n cut the transversal x in points P, Q and R respectively and lines l, m and n cut the transversal y in point E,F and G respectively.

$\therefore \quad \dfrac{PQ}{QR} = \dfrac{EF}{FG}$

Activity :

(Textbook page no. 12)



In the adjoining figure,

AB || CD || EF. If AC = 5.4,

CE = 9, BD = 7.5, then

find DF.

Solution :

AB || CD || EF

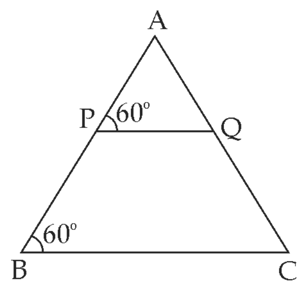
$\dfrac{AC}{\boxed{CE}} = \dfrac{\boxed{BD}}{DF}$ ... (Property of three parallel lines and their transversal)

$\dfrac{5.4}{9} = \dfrac{\boxed{7.5}}{DF}$

$\therefore \quad \boxed{\mathbf{DF = 12.5}}$

Practice Set - 1.2 *(Textbook Page No. 14)*

(4) Measurements of the some angles in the figure are given. Prove that $\dfrac{AP}{PB} = \dfrac{AQ}{QC}$ *(2 marks)*



Solution :

$\angle APQ \cong \angle ABC$ ...(Given)

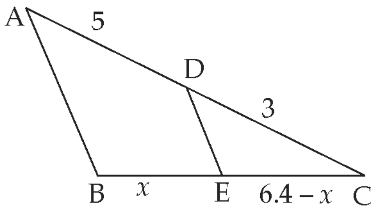
$\therefore$ seg PQ || side BC ...(i) (Corresponding angles test)

In ΔABC, seg PQ || side BC ...[From (i)]

$\therefore \quad \boxed{\mathbf{\dfrac{AP}{PB} = \dfrac{AQ}{QC}}}$ ...(By Basic Proportionality Theorem)

Problem Set - 1 *(Textbook Page No. 28)*

(7) In the figure, A - D - C and B-E-C. Seg DE || side AB. If AD = 5, DC = 3, BC = 6.4 then find BE. *(2 marks)*



Solution :

Let BE = x units ...(Supposition)

BC = BE + CE ...(B - E - C)

$\therefore$ 6.4 = x + CE

$\therefore$ CE = (6.4 – x) units

In ΔABC, seg DE || side AB ...(Given)

$\therefore$ $\frac{AD}{DC} = \frac{BE}{EC}$ ...(By Basic Proportionality theorem)

$\therefore$ $\frac{5}{3} = \frac{x}{6.4 - x}$

$\therefore$ $5(6.4 - x) = 3x$

$\therefore$ $(6.4 \times 5) - 5x = 3x$

$\therefore$ $6.4 \times 5 = 3x + 5x$

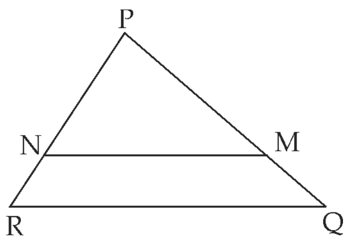
$\therefore$ $\frac{6.4 \times 5}{8} = x$

$\therefore$ $x = 4$

$\therefore$ $\boxed{\textbf{BE = 4 units}}$

Practice Set - 1.2 *(Textbook Page No. 13)*

(2) In ΔPQR, PM = 15, PQ = 25, PR = 20, NR = 8 whether lin NM is parallel to side RQ? Give reason. *(3 marks)*



Solution :

PQ = PM + MQ ...(P - M - Q)

$\therefore$ 25 = 15 + MQ

$\therefore$ MQ = 25 – 15

$\therefore$ MQ = 10

PR = PN + NR ...(P - N - R)

$\therefore$ 20 = PN + 8

$\therefore$ PN = 20 – 8

$\therefore$ PN = 12

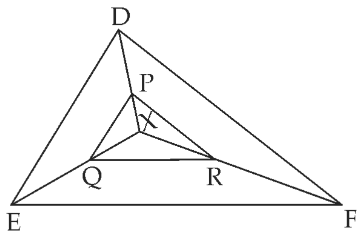
Now, $\frac{PM}{MQ} = \frac{15}{10} = \frac{3}{2}$ ...(i)

and $\frac{PN}{NR} = \frac{12}{8} = \frac{3}{2}$ ...(ii)

$\therefore$ In ΔPRQ, $\frac{PM}{MQ} = \frac{PN}{NR}$ ...[From (i) and (ii)]

$\therefore$ $\boxed{\textbf{seg NM || side QR}}$ ...(Converse of basic proportionality theorem)

(10) In the adjoining figure X is any point in interior of triangle. Point X is joined to vertices of triangle. Seg PQ || DE, seg QR || seg EF. Then fill in the blanks to prove that, seg PR || seg DF. *(3 marks)*



Proof :

In ΔXDE, PQ || DE ...(Given)

$\therefore$ $\frac{XP}{\boxed{PD}} = \frac{\boxed{XQ}}{QE}$ ...(i) (Basic proportionality theorem)

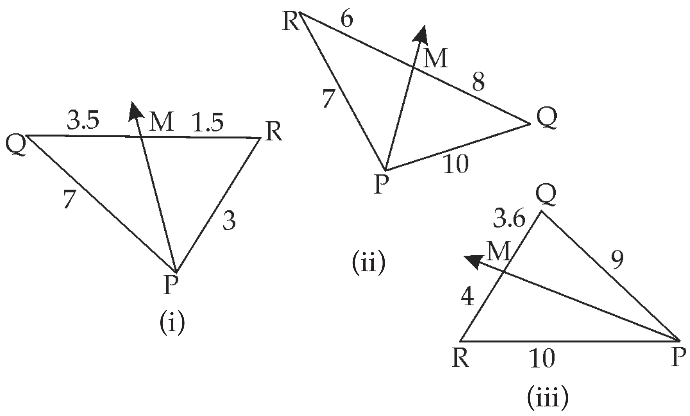
In ΔXEF, seg QR || side EF ... (Given)

$\therefore$ $\frac{\boxed{XQ}}{\boxed{QE}} = \frac{\boxed{XR}}{\boxed{RF}}$ ...(ii) (Basic proportionality theorem)

$\therefore$ $\frac{\boxed{XP}}{\boxed{PD}} = \frac{\boxed{XR}}{\boxed{RF}}$ ...[From (i) and (ii)]

$\therefore$ **seg PR || side DF** ...(Converse of Basic Proportionality theorem)

(1) Given below some triangles and lengths of line segments. Identity in which figures, Ray PM is bisector of $\angle$QPR. *(3 marks)*



Solution :

(i) $\frac{PQ}{PR} = \frac{7}{3}$ ...(i)

$\frac{QM}{RM} = \frac{3.5}{1.5} = \frac{35}{15} = \frac{7}{3}$ ...(ii)

In ΔPQR, $\frac{PQ}{PR} = \frac{QM}{RM}$ ...[From (i) and (ii)]

$\therefore$ **Ray PM bisects $\angle$QPR**

(ii) $\dfrac{PQ}{PR} = \dfrac{10}{7}$ ...(i)

$\dfrac{QM}{RM} = \dfrac{8}{6} = \dfrac{4}{3}$ ...(ii)

In ΔPQR, $\dfrac{PQ}{PR} \neq \dfrac{QM}{RM}$ ...[From (i) and (ii)]

$\therefore$ **Ray PM does not bisect $\angle QPR$**

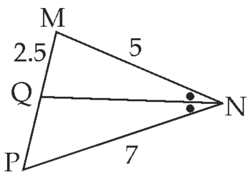
(iii) $\dfrac{PQ}{PR} = \dfrac{9}{10}$ ...(i)

$\dfrac{QM}{RM} = \dfrac{3.6}{4} = \dfrac{36}{40} = \dfrac{9}{10}$ ...(ii)

In ΔPQR, $\dfrac{PQ}{PR} = \dfrac{QM}{RM}$ ...[From (i) and (ii)]

$\therefore$ **Ray PM bisect $\angle QPR$**

(3) In ΔMNP, NQ is bisector of $\angle N$. If MN = 5, PN = 7, MQ = 2.5 then find QP. *(2 marks)*



Solution :

In ΔMNP, NQ bisects $\angle MNP$ ...(Given)

$\dfrac{MN}{PN} = \dfrac{MQ}{PQ}$ ...(Angle bisector property of a triangle)

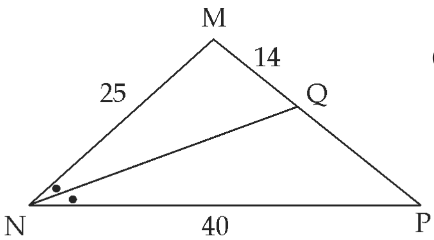
$\therefore$ $\dfrac{5}{7} = \dfrac{2.5}{PQ}$

$\therefore$ $5 \times PQ = 2.5 \times 7$

$\therefore$ $PQ = \dfrac{2.5 \times 7}{5}$

$\therefore$ $\boxed{PQ = 3.5 \text{ units}}$

(6) Find QP using given information in the figure. *(2 marks)*



Solution :

In ΔMNP, NQ bisects $\angle MNP$ ...(Given)

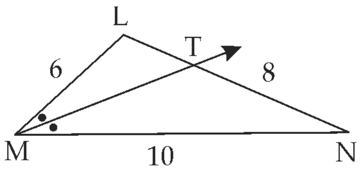
$\therefore$ $\dfrac{MN}{NP} = \dfrac{MQ}{QP}$ ...(Angle bisector property of a triangle)

$\therefore$ $\dfrac{25}{40} = \dfrac{14}{QP}$

$\therefore$ $QP = \dfrac{14 \times 40}{25}$

$\therefore$ $\boxed{QP = 22.4 \text{ units}}$

(8) In ΔLMN, Ray MT bisects $\angle LMN$, if LM = 6, if MN = 10, TN = 8. then find LT. *(2 marks)*



Solution :

In ΔLMN, Ray MT bisects $\angle LMN$ ...(Given)

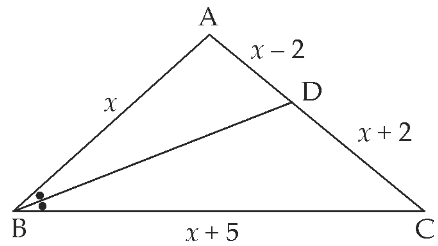
$\therefore$ $\dfrac{LM}{MN} = \dfrac{LT}{TN}$ ...(Angle bisector property of a triangle)

$\therefore$ $\dfrac{6}{10} = \dfrac{LT}{8}$

$\therefore$ $LT = \dfrac{6 \times 8}{10}$

$\therefore$ $\boxed{LT = 4.8 \text{ units}}$

(9) In ΔABC , seg BD bisects $\angle ABC$, if AB = x, BC = $x + 5$, AD = $x - 2$, DC = $x + 2$. Then find the value of x. *(2 marks)*



Solution :

In ΔABC, ray BD bisects $\angle ABC$ ...(Given)

$\therefore$ $\dfrac{AB}{BC} = \dfrac{AD}{DC}$ ...(Angle bisector property of a triangle)

$\therefore$ $\dfrac{x}{x+5} = \dfrac{x-2}{x+2}$

$x(x + 2) = (x + 5)(x - 2)$

$x^2 + 2x = x^2 + 5x - 2x - 10$

$x^2 + 2x = x^2 + 3x - 10$

$x^2 + 2x - x^2 - 3x = -10$

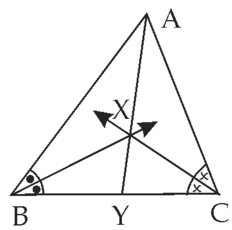
$-x = -10$

$x = 10$

$\therefore$ $\boxed{x = 10}$

Problem Set - 1 *(Textbook Page No. 29)*

(10) In the adjoining figure, bisectors of $\angle B$ and $\angle C$ intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\dfrac{AX}{XY}$. *(3 marks)*



Solution :

In ΔABY, ray BX bisects $\angle ABY$ ...(Given)

$\therefore \quad \frac{AB}{BY} = \frac{AX}{XY}$ ...(i) (Angle bisector property of a triangle)

In ΔACY, ray CX bisects $\angle ACY$ ...(Given)

$\therefore \quad \frac{AC}{CY} = \frac{AX}{XY}$ ...(ii) (Angle bisector property of a triangle)

$\therefore \quad \frac{AB}{BY} = \frac{AC}{CY} = \frac{AX}{XY}$ ...[From (i) and (ii)]

$\therefore \quad \frac{AB + AC}{BY + CY} = \frac{AX}{XY}$ ...(By theorem on equal ratios)

$\therefore \quad \frac{AB + AC}{BC} = \frac{AX}{XY}$ ...($\because$ B - Y - C)

$\therefore \quad \frac{5 + 4}{6} = \frac{AX}{XY}$

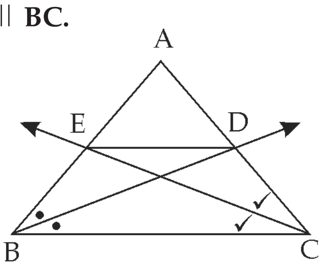
$\therefore \quad \frac{AX}{XY} = \frac{9}{6}$

$\therefore \quad \boxed{\frac{AX}{XY} = \frac{3}{2}}$

Practice Set - 1.2 *(Textbook Page No. 15)*

*(11) **In ΔABC, Ray BD bisects $\angle ABC$ and Ray CE bisects $\angle ACB$. If seg AB $\cong$ seg AC, then prove that ED || BC.** *(3 marks)*

Proof :



In ΔABC, ray BD bisects $\angle ABC$ ...(Given)

$\therefore \quad \frac{AB}{BC} = \frac{AD}{DC}$ ...(i) (Angle bisector property of a triangle)

In ΔABC, ray CE bisects $\angle ACB$ ...(Given)

$\therefore \quad \frac{AC}{BC} = \frac{AE}{BE}$ ...(ii) (Angle bisector property of a triangle)

seg AB $\cong$ seg AC ...(iii) (Given)

$\therefore \quad \frac{AB}{BC} = \frac{AE}{BE}$ ...(iv) [From (ii) and (iv)]

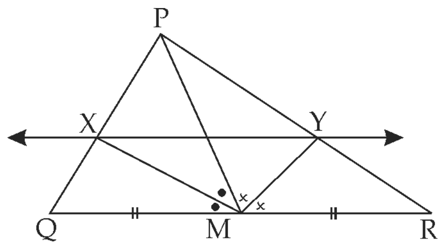
$\therefore \quad$ In ΔABC, $\frac{AD}{DC} = \frac{AE}{BE}$ ...[From (i) and (iv)]

$\therefore \quad$ **seg ED || side BC** ...(Converse of Basic proportionality theorem)

Problem Set - 1 *(Textbook Page No. 28)*

(9) **In ΔPQR, seg PM is a median. Angle bisectors of $\angle PMQ$ and $\angle PMR$ intersect side PQ and side PR in point X and Y respectively. Prove that XY || QR.** *(3 marks)*

(Complete the proof by filling the boxes)



Proof :

In ΔPMQ, ray MX bisects $\angle PMQ$ ...(Given)

$\therefore \quad \frac{\boxed{PM}}{\boxed{MQ}} = \frac{\boxed{PX}}{\boxed{XQ}}$ ...(i) (Angle bisector property of a triangle)

In ΔPMR, ray MY bisects $\angle PMR$ ...(Given)

$\therefore \quad \frac{\boxed{PM}}{\boxed{MR}} = \frac{\boxed{PY}}{\boxed{YR}}$ ...(ii) (Angle bisector property of a triangle)

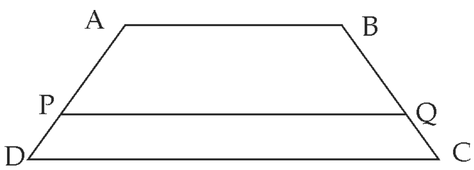
But, $\frac{PM}{MQ} = \frac{PM}{MR}$ ...($\because$ M is midpoint of seg QR, $\therefore$ MQ = MR)

$\frac{PX}{XQ} = \frac{PY}{YR}$ ...[From (i) and (ii)]

$\therefore \quad$ **seg XY || side QR** ...(Converse of Basic proportionality theorem)

Practice set - 1.2 *(Textbook Page No. 14)*

(5) **In trapezium ABCD, side AB || side PQ || side DC. AP = 15, PD = 12, QC = 14. Find BQ.** *(2 marks)*



Solution :

seg AB || seg PQ || seg DC ...(Given)

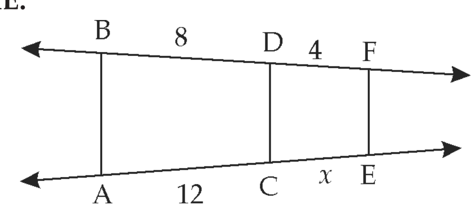
$\therefore \quad \frac{AP}{PD} = \frac{BQ}{QC}$ ...(Property of three parallel lines and their transversals)

$\therefore \quad \frac{15}{12} = \frac{BQ}{14}$

$\therefore \quad BQ = \frac{15 \times 14}{12}$

$\therefore \quad \boxed{\textbf{BQ = 17.5 units}}$

(7) **In the adjoining figure AB || CD || EF. Find x and AE.** *(2 marks)*



Solution :

seg AB || seg CD || seg EF ...(Given)

$\therefore$ $\dfrac{AC}{CE} = \dfrac{BD}{DF}$ ...(Property of three parallel lines and their transversals)

$\therefore$ $\dfrac{12}{x} = \dfrac{8}{4}$

$\therefore$ $x = \dfrac{12 \times 4}{8}$

$\therefore$ $\boxed{x = 6}$

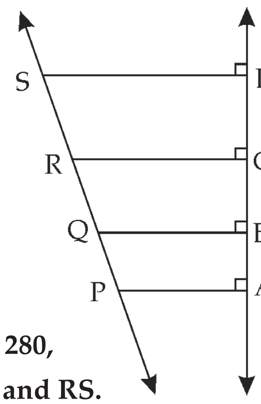
AE = AC + CE ...(A–C–E)

AE = 12 + 6

$\therefore$ $\boxed{\textbf{AE = 18 units}}$

Problem Set - 1 *(Textbook Page No. 28)*

(8) **In the figure given seg PA, seg QB, seg RC and seg SD are perpendicular to line AD. AB = 60, BC = 70, CD = 80 and PS = 280, then find PQ, QR and RS.** *(3 marks)*



Solution :

seg PA $\perp$ line AD
seg QB $\perp$ line AD
seg RC $\perp$ line AD
seg SD $\perp$ line AD
...(Given)

$\therefore$ seg PA || seg QB || seg RC || seg SD ...(If two or more lines are perpendicular to the same line then they are parallel to each other)

$\therefore$ PQ : QR : RS = AB : BC : CD ...(Property of three parallel lines and their transversals)

$\therefore$ PQ : QR : RS = 60 : 70 : 80

$\therefore$ PQ : QR : RS = 6 : 7 : 8

Let the common multiple be x.

$\therefore$ PQ = $6x$, QR = $7x$, RS = $8x$

PS = PQ + QR + RS ...(P-Q-R-S)

$\therefore$ $280 = 6x + 7x + 8x$

$\therefore$ $21x = 280$

$\therefore$ $x = \dfrac{280}{21} = \dfrac{40}{3}$

PQ = $6x$ = $6 \times \dfrac{40}{3}$ = 80 units

QR = $7x$ = $7 \times \dfrac{40}{3} = \dfrac{280}{3}$ units

RS = $8x$ = $8 \times \dfrac{40}{3} = \dfrac{320}{3}$ units

Points to Remember:

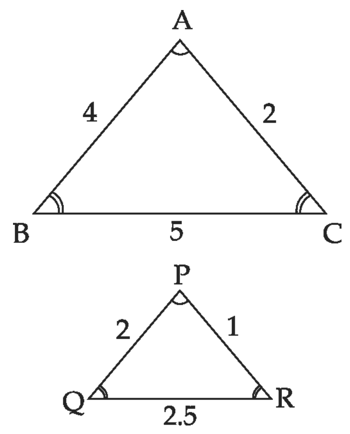
- **Similarity of Triangles**

Definition: For a given one - to - one correspondence between the vertices of two triangles, if

(i) their corresponding angles are congruent,

(ii) their corresponding sides are in proportion,

then the correspondence is known as **similarity** and the triangles are said to be **Similar Triangles**.



In above figure, for correspondence ABC $\leftrightarrow$ PQR

(i) $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$ and

(ii) $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR} = \dfrac{2}{1}$

Hence, ΔABC and ΔPQR are similar triangles.

ΔABC is similar to ΔPQR under ABC $\leftrightarrow$ PQR, this statement is written symbolically as $\Delta ABC \sim \Delta PQR$.

Note

If two triangles are similar, then

(i) their corresponding angles are congruent

(ii) their corresponding sides are in proportion.

If $\Delta ABC \sim \Delta PQR$, then

(i) $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

(ii) $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$

Points to Remember:

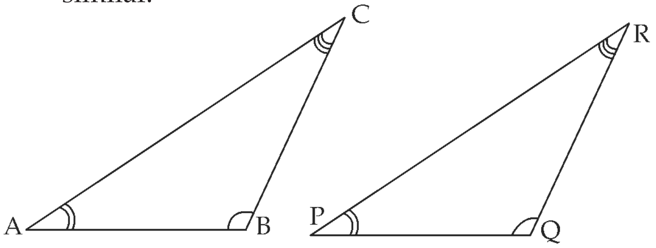
- **Test of Similarity of Triangles**

When two triangles are similar, then three pairs of corresponding angles are congruent and three pairs of corresponding sides are in proportion.

But to prove that two triangles are similar, we select only three conditions taken in proper order. These conditions are called **Tests of similarity**. There are three tests of similarity :

(1) A - A - A test (A - A test) :

For a given one - to - one correspondence between the vertices of two triangles, if the corresponding angles are congruent, then the two triangles are similar.



In the figure, for ABC $\leftrightarrow$ PQR,

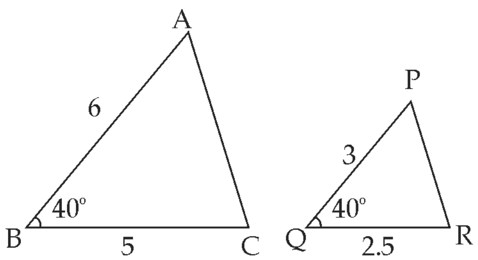
if, $\angle A \cong \angle P$, $\angle B \cong \angle Q$ and $\angle C \cong \angle R$, then $\Delta ABC \sim \Delta PQR$ by A-A-A test of similarity.

We know that sum of measures of three angles of a triangle is 180°. Because of this if two pairs of corresponding angles of two given triangles are congruent then remaining pair is also congruent, and thus the triangles become similar triangles. This is known as A - A test.

A - A Test : For a given one-one correspondence between the vertices of two triangles, if two angles of one triangle are congruent with the corresponding two angles of other triangle, then the two triangles are similar.

(2) S - A - S Test :

For a given one-one correspondence between the vertices of two triangles, if two sides of one triangle are proportional to the corresponding sides of the other triangle and angles included by them are congruent, then the two triangles are similar.



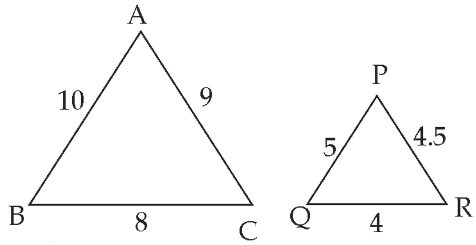
In the above figure, for ABC $\leftrightarrow$ PQR,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2}{1}$, $\angle B \cong \angle Q$,

then $\Delta ABC \sim \Delta PQR$ by S - A - S test of similarity.

(3) S - S - S Test :

For a given one-one correspondence between the vertices of two triangles, if three sides of one triangle are proportional to the three corresponding sides of other triangle, then the two triangles are similar.



In the above figure, for ABC $\leftrightarrow$ PQR,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{2}{1}$

then $\Delta ABC \sim \Delta PQR$

by S - S - S test of similarity.

Properties of similar triangles

(1) $\Delta ABC \sim \Delta ABC$ ...(Reflexivity)

(2) If $\Delta ABC \sim \Delta DEF$ then

$\Delta DEF \sim \Delta ABC$ ...(Symmetry)

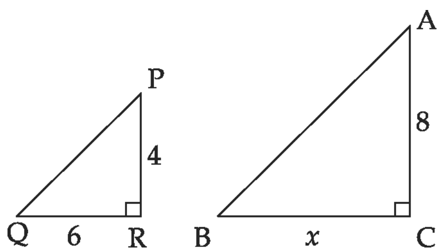
(3) If $\Delta ABC \sim \Delta DEF$ and

$\Delta DEF \sim \Delta GHI$ then $\Delta ABC \sim \Delta GHI$

...(Transitivity)

PRACTICE SET - 1.3 *(Textbook Page No.)*

(3) As shown in adjoining figure, two poles of height 8 m and 4 mare perpendicular to ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of bigger pole at the same time? *(2 marks)*



Solution :

Let PR and AC represent poles of length 4 m and 8 m respectively.

Let QR and BC represent the lengths cast by them of the poles at the same time.

Now $\Delta PQR \sim \Delta ABC$ ...(Shadow reckoning property)

$\therefore \quad \frac{PR}{AC} = \frac{QR}{BC}$ ...(c.s.s.t.)

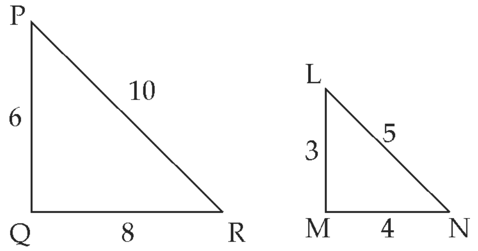
$\therefore \quad \frac{4}{8} = \frac{6}{BC}$ ...(Given)

$\therefore \quad BC = \frac{8 \times 6}{4}$

$\therefore \quad BC = 12$ m

$\therefore$ **Length of the shadow casted by longer pole is 12 m.**

(2) Are the triangles in the figure given similar? *(2 marks)*



Solution :

$\therefore \quad \frac{PQ}{LM} = \frac{6}{3} = \frac{2}{1}$ ....(i)

$\therefore \quad \frac{QR}{MN} = \frac{8}{4} = \frac{2}{1}$ ....(ii)

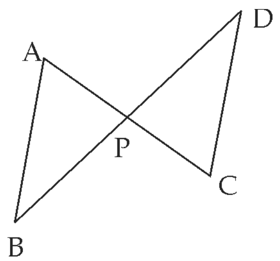
$\therefore \quad \frac{PR}{LN} = \frac{10}{5} = \frac{2}{1}$ ....(iii)

$\therefore$ In ΔPQR and ΔLMN

$\frac{PQ}{LM} = \frac{QR}{MN} = \frac{PR}{LN}$ ...[From (i), (ii) and (iii)]

$\therefore$ $\boxed{\Delta PQR \sim \Delta LMN}$ ...(SSS test for similarity)

(8) In the figure seg AC and seg BD intersects each other at point P and $\frac{AP}{CP} = \frac{BP}{DP}$. Then



Prove that $\Delta ABP \sim \Delta CDP$. *(2 marks)*

Proof :

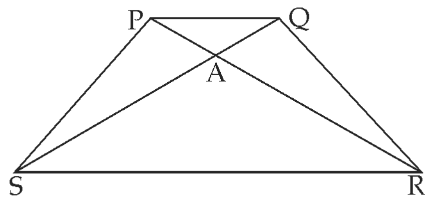
In ΔABP and ΔCDP

$\frac{AP}{CP} = \frac{BP}{DP}$ ....(Given)

$\angle APB \cong \angle CPD$ ....(vertically opposite angles)

$\therefore$ **$\Delta ABP \sim \Delta CDP$** ...(SAS test for similarity)

(5) In trapezium PQRS, side PQ || side SR. AR = 5AP and AS = 5AQ. Prove that : SR = 5PQ *(3 marks)*



Proof :

$AR = 5AP$ ...(Given)

$\therefore \quad \frac{AR}{AP} = \frac{5}{1}$ ...(i)

$AS = 5AQ$ ...(Given)

$\therefore \quad \frac{AS}{AQ} = \frac{5}{1}$ ...(ii)

In ΔASR and ΔAQP,

$\frac{AR}{AP} = \frac{AS}{AQ}$ ...[From (i) and (ii)]

$\angle SAR \cong \angle QAP$ ...(Vertically opposite angles)

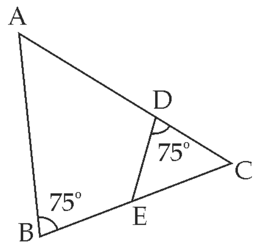
$\therefore \quad \Delta ASR \sim \Delta AQP$ ...(By SAS Test of similarity)

$\therefore \quad \frac{SR}{PQ} = \frac{AR}{AP}$ ...(c.s.s.t.)

$\therefore \quad \frac{SR}{PQ} = \frac{5}{1}$ ...[From (i)]

$\therefore$ **SR = 5 PQ**

(1) In adjoining figure, $\angle ABC = 75^\circ$, $\angle EDC = 75^\circ$ state which two triangles are similar and by which test? Also triangles by a proper one to one correspondence *(2 marks)*



Solution :

In ΔABC and ΔEDC

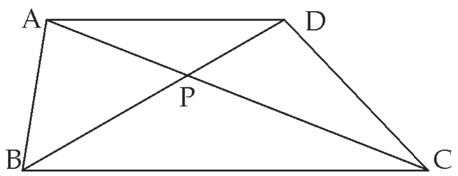
$\angle ABC \cong \angle EDC$ ...(Each 75°)

$\angle C \cong \angle C$ ...(Common angle)

$\therefore$ **$\Delta ABC \sim \Delta EDC$** ...(By AA test for similarity)

Problem Set - 1 *(Textbook Page No. 29)*

(11) In □ABCD, seg AD || seg BC. Diagonal AC and diagonal BD intersect each other in point P. Then show that $\frac{AP}{PD} = \frac{PC}{BP}$ *(3 marks)*



Proof :

seg AD || seg BC ...(Given)

$\therefore$ $\angle PAD \cong \angle PCB$ ...(i) (Alternate angles theorem)

In ΔAPD and ΔCPB

$\angle PAD \cong \angle PCB$ ...[From (i)]

$\angle APD \cong \angle CPB$ ...(vertically opposite angles)

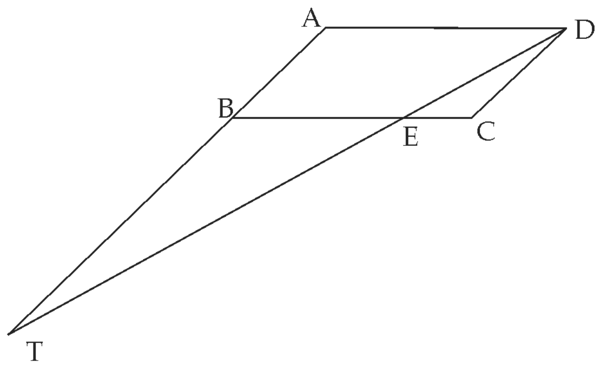
$\therefore$ $\Delta APD \sim \Delta CPB$ ...(By AA test for similarity)

$\therefore$ $\dfrac{AP}{PC} = \dfrac{PD}{BP}$

$\therefore$ $\boxed{\dfrac{AP}{PD} = \dfrac{PC}{BP}}$ ...(Alternendo)

Practice set - 1.3 *(Textbook Page No. 22)*

(7) **□ABCD is a parallelogram. Point E is on side BC, line DE intersects Ray AB in point T. Prove that : DE × BE = CE × TE.** *(3 marks)*



Proof :

□ABCD is a parallelogram ...(Given)

seg AB||seg CD ...(Opposite sides of a parallelogram)

seg AT || seg CD ...(A - B - T)

on transversal TD,

$\therefore$ $\angle ATD \cong \angle CDT$ ...(Alternate angles theorem)

$\therefore$ $\angle BTE \cong \angle CDE$ ...(i) (A - B - T, T - E - D)

In ΔBTE and ΔCDE,

$\angle BTE \cong \angle CDE$ ...[From (i)]

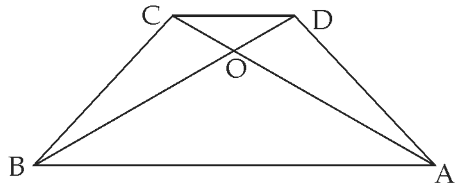
$\angle BET \cong \angle CED$ ...(vertically opposite angles)

$\therefore$ $\Delta BTE \sim \Delta CDE$ ...(By AA test of similarity)

$\therefore$ $\dfrac{BE}{CE} = \dfrac{TE}{DE}$ (c.s.s.t.)

$\therefore$ **DE × BE = CE × TE**

(6) **In trapezium ABCD, side AB || side DC. Diagonals AC and BD intersect in O. If AB = 20, DC = 6, OB = 15. Find OD.** *(3 marks)*



Solution :

side AB || side CD ...(Given)

On transversal AC

$\angle CAB \cong \angle ACD$ (Alternate angles theorem)

$\therefore$ $\angle OAB \cong \angle OCD$ ...(i)

In ΔAOB and ΔCOD

$\angle AOB \cong \angle COD$ ...(vertically opposite angles)

$\angle OAB \cong \angle OCD$ ...[From (i)]

$\therefore$ $\Delta AOB \sim \Delta COD$ ...(By AA test for similarity)

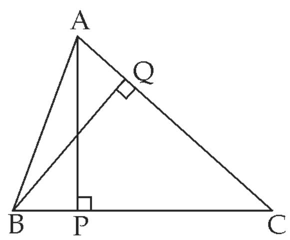
$\therefore$ $\dfrac{AB}{DC} = \dfrac{OB}{OD}$ ...(c.s.s.t.)

$\therefore$ $\dfrac{20}{6} = \dfrac{15}{OD}$

$\therefore$ $OD = \dfrac{15 \times 6}{20}$

$\boxed{\textbf{OD = 4.5 units}}$

(4) **In ΔABC, AP ⊥ BC, BQ ⊥ AC, B-P-C, A-Q-C, then prove that $\Delta CPA \sim \Delta CQB$. If AP = 7, BQ = 8, BC = 12 then find AC.** *(3 marks)*



Proof and Solution :

In ΔCPA and ΔCQB

$\angle C \cong \angle C$ ...(Common angle)

$\angle APC \cong \angle BQC$ ...(Each is 90°)

$\therefore$ $\boldsymbol{\Delta CPA \sim \Delta CQB}$ ...(By AA test for similarity)

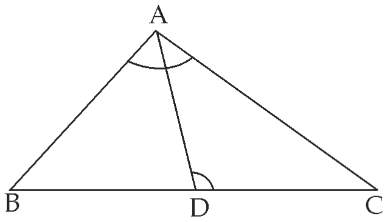
$\therefore$ $\dfrac{AP}{BQ} = \dfrac{AC}{BC}$ ...(c.s.s.t.)

$\therefore$ $\dfrac{7}{8} = \dfrac{AC}{12}$

$\therefore$ $\dfrac{7 \times 12}{8} = AC$

$\boxed{\textbf{AC = 10.5 units}}$

(9) **In the figure, in ΔABC, point D on side BC is such that, $\angle BAC \cong \angle ADC$, then prove that, $CA^2 \equiv CB \times CD$.** *(3 marks)*



Proof :

In ΔABC and ΔDAC,

$\angle BAC \cong \angle ADC$ ...(Given)

$\angle BCA \cong \angle ACD$ ...(Common Angle)

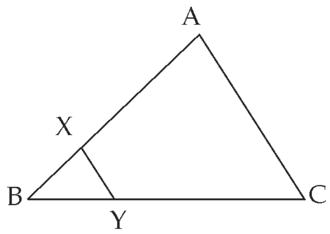
$\therefore$ $\Delta ABC \sim \Delta DAC$ ...(By AA test of similarity)

$\therefore$ $\dfrac{CA}{CD} = \dfrac{CB}{CA}$ ...(c.s.s.t.)

$\mathbf{CA^2 = CB \times CD}$

Problem Set - 1 *(Textbook Page No. 29)*

(12) **In the adjoining figure, XY ‖ seg AC. If 2AX = 3 × BX and XY = 9. Complete the activity to find the value of AC.** *(3 marks)*



Activity :

$2\ AX = 3\ BX \quad \therefore \dfrac{AX}{BX} = \dfrac{\boxed{3}}{\boxed{2}}$

$\therefore$ $\dfrac{AX + BX}{BX} = \dfrac{\boxed{3} + \boxed{2}}{\boxed{2}}$ ...(By componendo)

$\therefore$ $\dfrac{AB}{BX} = \dfrac{\boxed{5}}{\boxed{2}}$ ...(i)

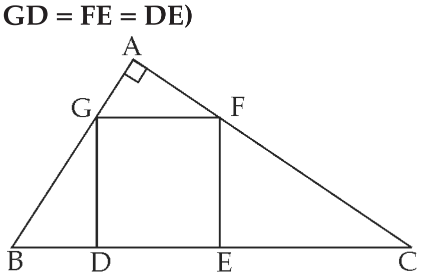
$\therefore$ $\Delta BCA \sim \Delta BYX$ ...(By AA test for similarity)

$\therefore$ $\dfrac{BA}{BX} = \dfrac{AC}{XY}$ ...(c.s.s.t.)

$\therefore$ $\dfrac{\boxed{5}}{\boxed{2}} = \dfrac{AC}{9}$ ...[From (i)]

$\therefore$ $\boxed{\textbf{AC = 22.5 units}}$

***(13)** **In the adjoining figure, □DEFG is a square. In ΔABC, A = 90°. Then prove that DE² = BD × EC (Hint : ΔGBD is similar to ΔCFE. Use GD = FE = DE)** *(4 marks)*



Proof :

□ DEFG is a square.

$\therefore$ DE = EF = GF = DG ...(i) (Sides of a square)

on transversal AB

$\therefore$ seg GF ‖ seg DE ...(Opposite sides of a square are parallel.)

$\therefore$ seg GF ‖ side BC ...(B-D-E-C)

on transversal AC

$\therefore$ $\angle AGF \cong \angle ABC$ ...(ii)

on transversal AC

$\angle AFG \cong \angle ACB$ ...(iii) ...(Corresponding angles theorem)

In ΔAGF and ΔDBG,

$\angle AGF \cong \angle GBD$ ...[From (ii) and A-G-B, B-D-C]

$\angle GAF \cong \angle BDG$ ...(Each is 90°)

$\therefore$ $\Delta AGF \sim \Delta DBG$ ...(iv) (By AA Test for similarity)

In ΔAGF and ΔEFC,

$\angle AFG \cong \angle FCE$ ...[From (iii) and A-F-C, C-E-A]

$\angle GAF \cong \angle FEC$ ...(Each is 90°)

$\therefore$ $\Delta AGF \sim \Delta EFC$ ...(v) (AA Test for similarity)

$\therefore$ $\Delta DBG \sim \Delta EFC$ ...[From (iv) and (v)]

$\therefore$ $\dfrac{BD}{EF} = \dfrac{DG}{EC}$ ...(c.s.s.t.)

$\therefore$ $BD \times EC = EF \times DG$

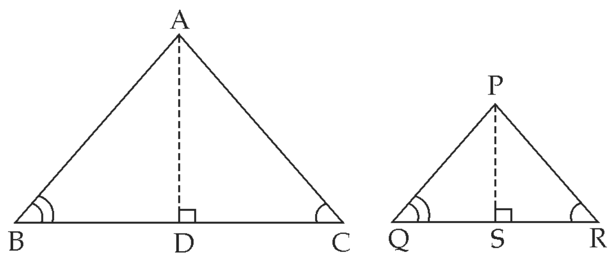
$\therefore$ $BD \times EC = DE \times DE$ ...[From (i)]

$\therefore$ $\mathbf{DE^2 = BD \times EC}$

Points to Remember:

- **Theorem of Areas of Similar Triangles**

Statement : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. *(5 marks)*



Given : $\Delta ABC \sim \Delta PQR$.

To Prove :

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

Construction :

(i) Draw seg AD ⊥ side BC, B - D - C

(ii) Draw seg PS ⊥ side QR, Q - S - R

Proof :

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$$

[The ratio of the areas of two triangles is equal to the ratio of the product of their bases and corresponding height.]

$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$ ...(i)

$\Delta ABC \sim \Delta PQR$ ... (Given)

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$ ...(ii) (c.s.s.t.)

Also, $\angle B \cong \angle Q$ ...(iii) (c.a.s.t.)

In ΔADB and ΔPSQ,

$\angle ADB \cong \angle PSQ$ ...(Each is a right angle)

$\angle B \cong \angle Q$ ...[From (iii)]

$\therefore \Delta ADB \sim \Delta PSQ$ (By A-A test of similarity)

$\therefore \frac{AD}{PS} = \frac{AB}{PQ}$ ...(iv) (c.s.s.t.)

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ ...[From (i), (ii) and (iv)]

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ...(vi)

Similarly we can prove ,

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ ...(vii)

$\therefore \mathbf{\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}}$

...[From (vi) and (vii)]

Practice Set - 1.4 *(Textbook Page No. 25)*

(2) If $\Delta ABC \sim \Delta PQR$ and AB : PQ = 2 : 3, then fill in the blanks. *(2 marks)*

$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{\boxed{PQ^2}} = \frac{2^2}{3^2} = \frac{\boxed{4}}{\boxed{9}}$

(1) Ratio of corresponding sides of two similar triangles is 3 : 5, then find ratio of their area. *(2 marks)*

Solution :

Let the areas of two similar triangles be A_1 and A_2 and their corresponding sides S_1 and S_2 respectively.

$\frac{S_1}{S_2} = \frac{3}{5}$ ...(Given)

Both the triangles are similar ...(Given)

$\therefore \frac{A_1}{A_2} = \frac{{S_1}^2}{{S_2}^2}$ ...(Theorem on areas of similar triangles)

$\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$

$\therefore \frac{A_1}{A_2} = \left(\frac{3}{5}\right)^2$

$\therefore \frac{A_1}{A_2} = \frac{9}{25}$

$\therefore$ $\boxed{\textbf{The ratio of the areas of two similar triangles is 9 : 25}}$

(3) If $\Delta ABC \sim \Delta PQR$, A ($\Delta ABC$) = 80, A ($\Delta PQR$) = 125, then fill in the blanks. *(1 mark)*

$\frac{A(\Delta ABC)}{A(\Delta \boxed{PQR})} = \frac{80}{125} \quad \therefore \frac{AB}{PQ} = \frac{\boxed{4}}{\boxed{5}}$

(4) $\Delta LMN \sim \Delta PQR$, 9 × A($\Delta PQR$) = 16 × A ($\Delta LMN$). If QR = 20, then find MN. *(2 marks)*

Solution :

9 × A(ΔPQR) = 16 × A (ΔLMN) ...(Given)

$\therefore \frac{9}{16} = \frac{A(\Delta LMN)}{A(\Delta PQR)}$

i.e. $\frac{A(\Delta LMN)}{A(\Delta PQR)} = \frac{9}{16}$ ...(i)

In ΔLMN and ΔPQR, ...(Given)

$\frac{A(\Delta LMN)}{A(\Delta PQR)} = \frac{MN^2}{QR^2}$ ...(Theorem on areas of similar triangles)

$\therefore \frac{9}{16} = \frac{MN^2}{20^2}$

$\therefore \frac{3}{4} = \frac{MN}{20}$ ...(Taking square roots)

$\therefore MN = \frac{3 \times 20}{4}$

$\therefore$ MN = 15

$\therefore$ $\boxed{\textbf{MN = 15 units}}$

(5) Areas two similar triangles are 225 sq. cm, 81 sq. cm. If a side of the smaller triangles is 12 cm, then find corresponding side of bigger triangle. *(3 marks)*

Solution :

Let the areas of two similar triangles be A_1 and A_2 and their corresponding sides S_1 and S_2 respectively.

$A_1 = 225$ cm², $A_2 = 81$ cm², $S_2 = 12$ cm ...(Given)

Both the triangles are similar ...(Given)

$\therefore \frac{A_1}{A_2} = \frac{{S_1}^2}{{S_2}^2}$ ...(Theorem on areas of similar triangles)

$\therefore \frac{225}{81} = \frac{{S_1}^2}{(12)^2}$

$\therefore \frac{15}{9} = \frac{S_1}{12}$ ...(Taking square roots)

$\therefore \quad S_1 = \dfrac{15 \times 12}{9}$

$\therefore \quad S_1 = 20$ cm

$\therefore$ $\boxed{\text{The corresponding side of the bigger triangle is 20 cm}}$

(6) ΔABC and ΔDEF are equilateral triangles. A(ΔABC) : A (ΔDEF) = 1 : 2 and AB = 4 find DE. *(3 marks)*

Solution :

ΔABC and ΔDEF are equilateral triangles.

Equilateral triangles are always similar .

$\therefore \quad \Delta ABC \sim \Delta DEF$

$\dfrac{A(\Delta ABC)}{A(\Delta DEF)} = \dfrac{AB^2}{DE^2}$ ...(Theorem on areas of similar triangles)

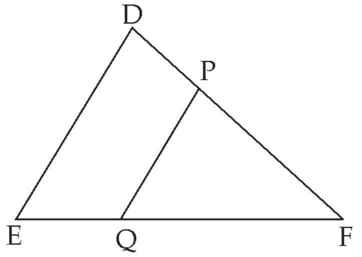
$\therefore \quad \dfrac{1}{2} = \dfrac{4^2}{DE^2}$

$\therefore \quad \dfrac{1}{\sqrt{2}} = \dfrac{4}{DE}$ ...(Taking square roots)

$\therefore \quad DE = 4\sqrt{2}$

$\therefore$ $\boxed{\mathbf{DE = 4\sqrt{2}\ units}}$

(7) In the adjoining figure, seg PQ || seg DE, A(ΔPQF) = 20 sq units. PF = 2 DP, then find A($\square$DPQE) by completing the following activity. *(3 marks)*



Solution :

A(ΔPQF) = 20 sq units.

PF = 2DP

Let us assume DP = x units $\quad \therefore \quad$ PF = $2x$

DF = DP + $\boxed{PF}$ = $\boxed{x}$ + $\boxed{2x}$ = $\boxed{3x}$

In ΔFDE and ΔFPQ,

$\angle FDE = \boxed{\angle FPQ}$ ...(Corresponding angles theorem)

$\angle FED = \boxed{\angle FQP}$ ...(Corresponding angles theorem)

$\therefore$ In $\Delta FDE \sim \Delta FPQ$ ...(By AA test of similarity)

$\therefore \quad \dfrac{A(\Delta FDE)}{A(\Delta FPQ)} = \boxed{\dfrac{DF^2}{PF^2}} = \dfrac{(3x)^2}{(2x)^2} = \dfrac{9}{4}$

$A(\Delta FDE) = \dfrac{9}{4} A(\Delta FPQ) = \dfrac{9}{4} \times \boxed{20} = \boxed{45}$ sq units

$A(\square DPQE) = A(\Delta FDE) - A(\Delta FPQ)$

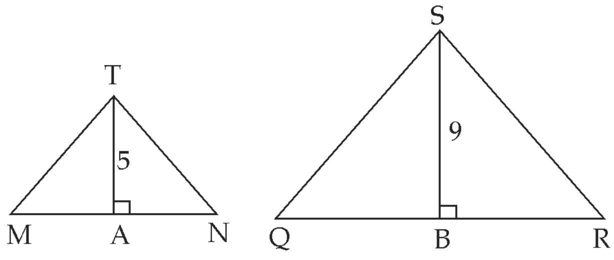
$= \boxed{45} - \boxed{20}$

$= \boxed{25}$ sq units

Problem Set - 1 *(Textbook Page No. 27)*

(6) $\Delta MNT \sim \Delta QRS$: Length of altitude drawn from vertex T is 5 and length of altitude drawn from vertex S is 9. Find $\dfrac{A(\Delta MNT)}{A(\Delta QRS)}$ *(3 marks)*

Solution :



$\Delta MNT \sim \Delta QRS$ ...(Given)

$\therefore \quad \angle M \cong \angle Q$ ...(i) (c.a.s.t.)

In ΔMAT and ΔQBS

$\therefore \quad \angle M \cong \angle Q$ ...[From (i)]

$\angle MAT \cong \angle QBS$ ...(Each 90°)

$\Delta MAT \sim \Delta QBS$ ...(By AA test of similarity)

$\dfrac{TM}{SQ} = \dfrac{TA}{SB}$ ...(ii) (c.s.s.t.)

$\dfrac{A(\Delta TMN)}{A(\Delta SQR)} = \dfrac{TM^2}{SQ^2}$ ...(Theorem on areas of similar triangles)

$\dfrac{A(\Delta TMN)}{A(\Delta SQR)} = \dfrac{TA^2}{SB^2}$ ...[From (ii)]

$\dfrac{A(\Delta TMN)}{A(\Delta SQR)} = \dfrac{TA^2}{SB^2}$ ...[From (ii)]

$\dfrac{A(\Delta TMN)}{A(\Delta SQR)} = \dfrac{5^2}{9^2}$ ...[From (ii)]

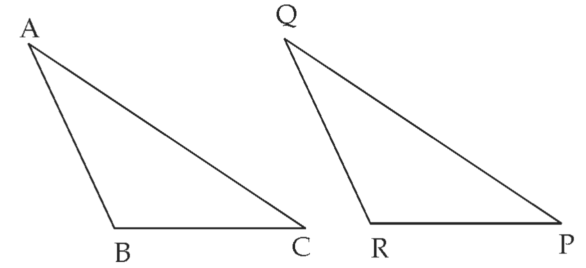
$\boxed{\mathbf{\dfrac{A(\Delta TMN)}{A(\Delta SQR)} = \dfrac{25}{81}}}$ ...[From (i)]

Problem Set - 1 *(Textbook Page No. 27)*

MCQ's

Q. 1. Choose correct alternative for each of the following questions. *(1 mark each)*

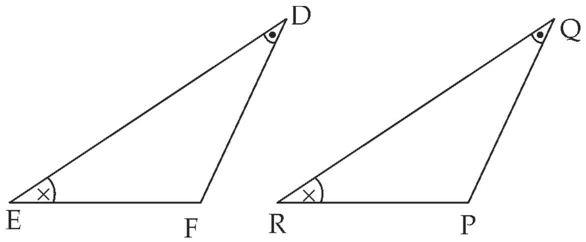
(1) If in ΔABC and ΔPQR for some one-one correspondence if $\dfrac{AB}{QR} = \dfrac{BC}{PR} = \dfrac{CA}{PQ}$ then



(A) ΔPQR ~ ΔABC (B) ΔPQR ~ ΔCAB

(C) ΔCBA ~ ΔPQR (D) ΔBCA ~ ΔPQR

(2) If in ΔDEF and ΔPQR. ∠D ≅ ∠Q, ∠R ≅ ∠E, then which of the following statement is false?



(A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$

(C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

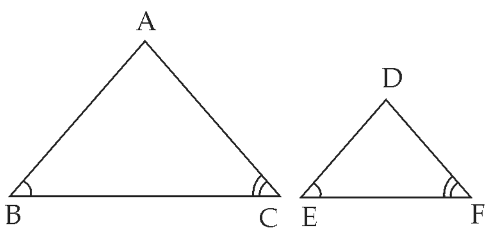
(3) In ΔABC and ΔDEF. ∠B ≅ ∠E, ∠F ≅ ∠C and AB = 3 DE, then which statement regarding two triangles is true?

(A) The triangles are not congruent and not similar.

(B) The triangles are similar but not congruent.

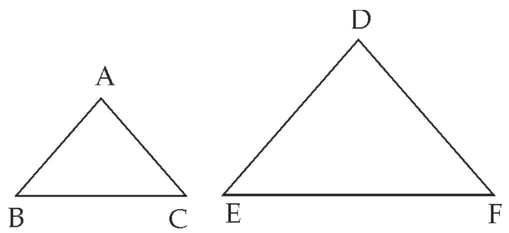
(C) The triangles are congruent and similar.

(D) None of the statements above is true.



(4) ΔABC and ΔDEF both are equilateral triangles. A(ΔABC) : A(ΔDEF) = 1 : 2. If AB = 4, then what is the length of DE ?

(A) $2\sqrt{2}$ (B) 4 (C) 8 (D) $4\sqrt{2}$.



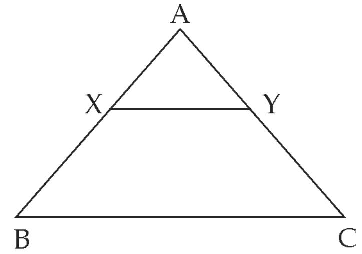
(5) In the figure seg XY || seg BC, then which of the following statement is true?

(A) $\frac{AB}{AC} = \frac{AX}{AY}$

(B) $\frac{AX}{XB} = \frac{AY}{AC}$

(C) $\frac{AX}{YC} = \frac{AY}{XB}$

(D) $\frac{AB}{YC} = \frac{AC}{XB}$



(6) In ΔABC, AB = 3 cm, BC = 2 cm and AC = 2.5 cm. ΔDEF ~ ΔABC, EF = 4 cm. What is the perimeter of ΔDEF?

(A) 30 cm (B) 22.5 cm (C) 15 cm (D) 7.5 cm

(7) The sides of two similar triangles are 4 : 9. What is the ratio of their area?

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

(8) The areas of two similar triangles are 18 cm² and 32 cm² respectively. What is the ratio of their corresponding sides?

(A) 3 : 4 (B) 4 : 3 (C) 9 : 16 (D) 16 : 9

(9) ΔABC ~ ΔPQR, AB = 6 cm, BC = 8 cm, CA = 10 cm and QR = 6 cm. What is the length of side PR?

(A) 8 cm (B) 10 cm (C) 4.5 cm (D) 7.5 cm

(10) In ΔXYZ, ray YM is the bisector of ∠XYZ where XY = YZ and X - M - Z, then which of the relation is true?

(A) XM = MZ (B) XM ≠ MZ

(C) XM > MZ (D) None

(11) In ΔABC, AB = 6 cm, BC = 8 cm and AC = 10 cm. ΔABC is enlarged to ΔPQR such that the largest side is 12.5 cm. What is the length of the smallest side of ΔPQR?

(A) 7.5 cm (B) 9 cm (C) 8 cm (D) 10 cm

(12) In ΔABC, B - D - C and BD = 6 cm, DC = 4 cm. What is the ratio of A (ΔABC) to A (ΔACD)?

(A) 2 : 3 (B) 5 : 2 (C) 3 : 2 (D) 5 : 3

(13) In ΔXYZ, PQ || YZ, X - P - Y and X - Q - Z.

If $\frac{XP}{PY} = \frac{4}{13}$ and XQ = 4.8 cm. What is XZ?

(A) 15.6 cm (B) 20.4 cm

(C) 7.8 cm (D) 10.2 cm

(14) In ΔABC, P is a point on side BC such that BP = 4 cm and PC = 7 cm.

A (ΔAPC) : A (ΔABC) = ...

(A) 11 : 7 (B) 7 : 11 (C) 4 : 7 (D) 7 : 4

(15) In ΔPQR, seg RS is the bisector of $\angle PRQ$, PS = 8, SQ = 6, PR = 20 then QR = ...

(a) 10 (b) 15 (c) 30 (d) 40

(16) In ΔABC, line PQ || side BC, AP = 3, BP = 6, AQ = 5 then the value of CQ is ....

(a) 20 (b) 10 (c) 5 (d) 16

ANSWERS

(1) (B) $\Delta PQR \sim \Delta CAB$ **(2)** (B) $\frac{DE}{PQ} = \frac{EF}{RP}$

(3) (B) Both triangles are similar but not congruent.

(4) (D) $4\sqrt{2}$ **(5)** (A) $\frac{AB}{AC} = \frac{AX}{AY}$

(6) (C) 15 cm **(7)** (D) 16 : 81

(8) (A) 3 : 4 **(9)** (D) 7.5 cm

(10) (A) XM = MZ **(11)** (A) 7.5 cm

(12) (B) 5 : 2 **(13)** (B) 20.4 cm

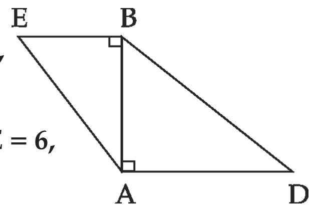
(14) (B) 7 : 11 **(15)** (B) 15 units

(16) (B) 10

PROBLEMS FOR PRACTICE

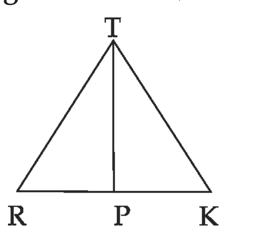
<u>Based on Practice Set 1.1</u>

(1) **In the adjoining figure, seg BE $\perp$ seg AB and seg BA $\perp$ seg AD. If BE = 6, AD = 9, then find A(ΔABE) : A(ΔBAD)** *(1 mark)*



(2) **The ratio of the areas of two triangles with the common base is 6 : 5. Height of the larger triangles is 9 cm. Then find the corresponding height of the smaller triangle.** *(1 mark)*

(3) **In the adjoining figure, RP : PK = 3 : 2, then find the value of**

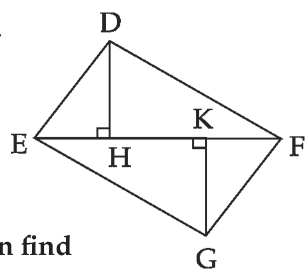


(i) **A(ΔTRP) : A(ΔTPK)**

(ii) **A(ΔTRK) : A(ΔTPK)**

(iii) **A(ΔTRP) : A(ΔTRK** *(3 marks)*

(4) **In the adjoining figure, seg DH $\perp$ seg EF, seg GK $\perp$ seg EF. If DH = 12 cm, GK = 20 cm and A(ΔDEF) = 300°cm², then find**



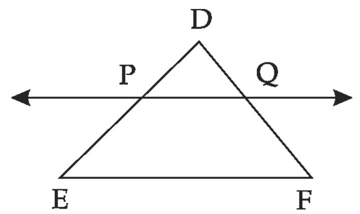
(i) **EF**

(ii) **A(ΔGEF)** *(3 marks)*

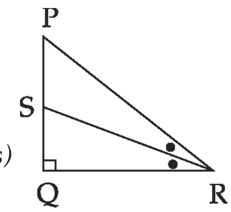
(5) **The ratio of the areas of two triangles with equal height is 3 : 2. The base of the larger triangle is 18 cm. Find the corresponding base of the smaller triangle.** *(2 marks)*

<u>Based on Practice set 1.2</u>

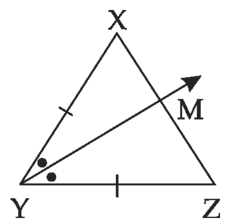
(6) **In ΔDEF, line PQ || side EF. DQ = 1.8, QF = 5.4, PE = 7.2. Find DE.** *(2 marks)*



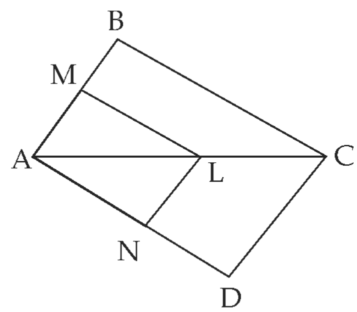
(7) **In ΔPQR, seg RS is bisector of $\angle$PRQ. PS = 6, SQ = 8, PR = 15. Find QR.** *(2 marks)*



(8) **In ΔXYZ, XY = YZ. Ray YM bisects $\angle$XYZ. X-M-Z. Prove that M is midpoint of seg XZ.** *(2 marks)*

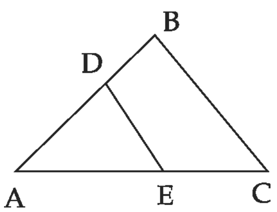


(9) **In the adjoining figure, seg ML || seg BC, seg NL || seg DC. Prove that AM : AB = AN : AD.** *(3 marks)*



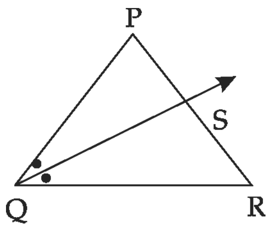
(10) **$\square$ABCD is a trapezium in which AB || DC and its diagonals intersect each other at point O. Show that AO : BO = CO : DO.** *(3 marks)*

(11) **Point D and E are the points on sides AB and AC such athat AB = 5.6, AD = 1.4, AC = 7.2 and AE = 1.8.**

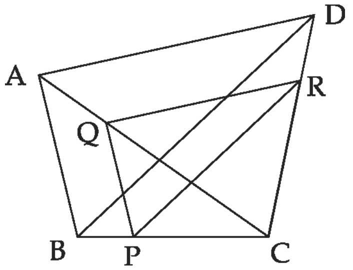


Show that DE || BC. *(3 marks)*

(12) In ΔPQR, ray QS bisects of $\angle PQR$. P-S-R. Show that $\dfrac{A(\Delta PQS)}{A(\Delta QRS)} = \dfrac{PQ}{QR}$. *(3 marks)*



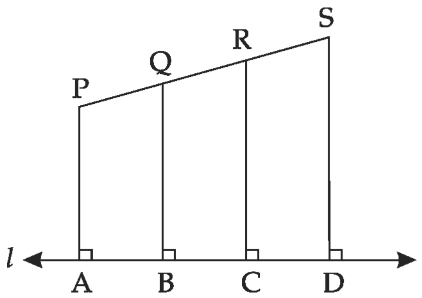
(13) In the adjoining figure, seg PQ || seg AB. Seg PR || seg BD. Prove that QR || AD.



(3 marks)

<u>Based on Practice Set 1.3</u>

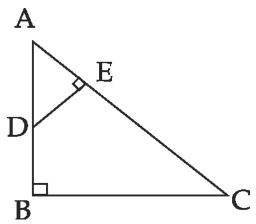
(14) In the adjoining figure, seg PA, seg QB, seg RC and seg SD are $\perp$ to line l AB = 6, BC = 9, CD = 12 and PS = 36, then find PQ, QR and RS. *(3 marks)*



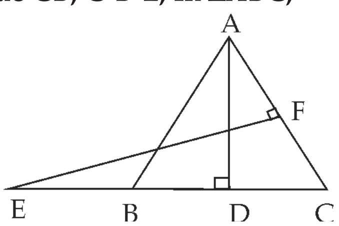
(15) A vertical pole of a length 6 m casts a shadow of 4 m long on the ground. At the same time a tower casts a shadow 28 m long. Find the height of the tower. *(2 marks)*

(16) In ΔABC, AB = 5, BC = 6, AC = 7. $\Delta PQR \sim \Delta ABC$. Perimeter of ΔPQR is 360. Find PQ, QR and PR. *(3 marks)*

(17) In ΔABC, $\angle B = 90^\circ$, seg DE $\perp$ side AC. AD = 6, AB = 12, AC = 18, then find AE. *(3 marks)*



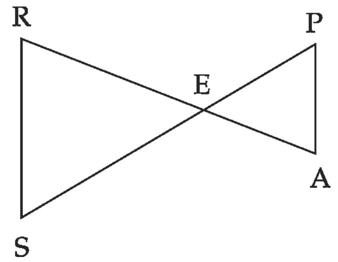
(18) E is a point on side CB, C-B-E, In ΔABC, AB = AC. If seg AD $\perp$ BC, B-D-C and seg EF $\perp$ side AC, A-F-C. Prove that $\Delta ABD \sim \Delta ECF$.



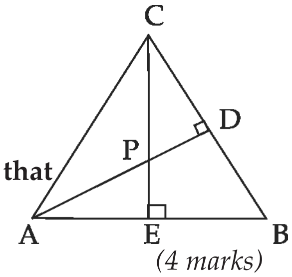
(3 marks)

(19) D is a point on side BC of ΔABC such that, $\angle ADC = \angle BAC$. Show that $AC^2 = BC \times DC$. *(3 marks)*

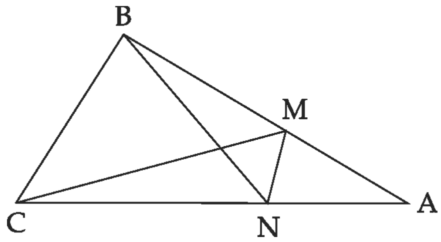
(20) In ΔRES, RE = 15, SE = 10. In ΔPEA, PE = 8, AE = 12. Prove that $\Delta RES \sim \Delta AEP$ *(3 marks)*



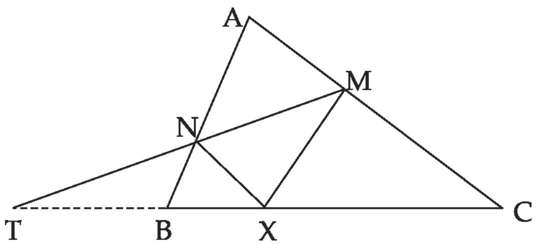
(21) In the adjoining figure, seg CE $\perp$ side AB, seg AD $\perp$ side BC. Prove that
(i) $\Delta AEP \sim \Delta CDP$
(ii) $\Delta AEP \sim \Delta ADB$ *(4 marks)*



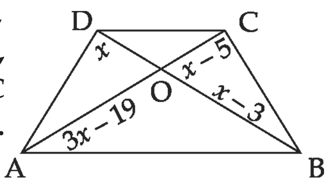
(22) In the adjoining figure, if $\Delta ABN \cong \Delta ACM$, show that $\Delta AMN \sim \Delta ABC$. *(4 marks)*



(23) Let X be any point on side BC of ΔABC. seg XM || side AB and seg XN || side CA. M-N-T, T-B-X. Prove that : $TX^2 = TB.TC$. *(2 marks)*



(24) In the adjoining figure, seg AB || side DC, OD = x OB = $x - 3$, OC = $x - 5$, OA = $3x - 19$. Find the value of x. *(4 marks)*

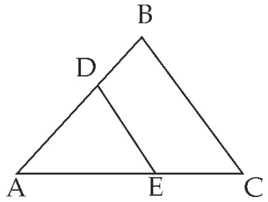


<u>Based on Practice set 1.4</u>

(25) $\Delta DEF \sim \Delta MNK$, If DE = 5 and MN = 6, then find the value of $A(\Delta DEF) : A(\Delta MNK)$ *(2 marks)*

(26) If $\Delta ABC \sim \Delta DEF$ such that the area of ΔABC is 9 cm² and the area of ΔDEF is 16 cm². If BC = 2.1 cm. Find length of EF. *(2 marks)*

(27) **In the adjoining figure, seg DE || side BC. If DE : BC = 3 : 5, then find A(ΔADE) : A(ΔDBCE)** *(3 marks)*



(28) **In ΔABC, PQ is a line segment intesecting AB at point P and AC at point Q. PQ || BC. If PQ divides ΔABC into two equal parts equal in area, find BP : AB.** *(3 marks)*

(29) **In ΔABC, ∠ABC = 90º. ΔPAB, ΔQAC and ΔRBC are the equilateral triangles constructed on sides AB, AC and BC respectively.**

Prove that : A(ΔPAB) + A(ΔRBC) = A(ΔQAC). *(4 marks)*

(30) **In ΔABC, seg DE || side BC. If 2A(ΔADE) = A(□DBCE). Show that BC = $\sqrt{3}$ × DE.** *(4 marks)*

ANSWERS

(1) 2 : 3 (2) 7.5 cm (3) (i) 3 : 2 (ii) 5 : 2 (iii) 3 : 5

(4) (i) 50 cm (ii) 500 cm^2 (5) 12 cm (6) 9.6 units

(7) 20 units (14) PQ = 8, QR = 12, RS = 16 (15) 42 m

(16) 100 units, 120 units, 140 units (17) 4 units

(24) $x = 8, x = 9$ (25) 25 : 36 (26) 2.8 (27) 9 : 16

(28) $\dfrac{\sqrt{2}-1}{\sqrt{2}}$

❖ ❖ ❖

ASSIGNMENT – 1

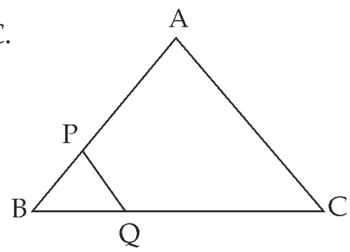
Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the proper alternative answer for the question given below: (2)

(1) Select the appropriate alternative in the adjoing figure, PQ || AC.

BP = 6, PA = 8, BQ = 9, then QC = ...

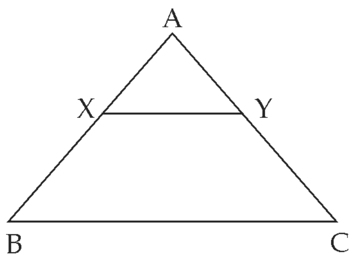
(A) 15 (B) 12 (C) 18 (D) 20



(2) In the figure seg XY || seg BC, then which of the following statement is true?

(A) $\dfrac{AB}{AC} = \dfrac{AX}{AY}$ (B) $\dfrac{AX}{XB} = \dfrac{AY}{AC}$

(C) $\dfrac{AX}{YC} = \dfrac{AY}{XB}$ (D) $\dfrac{AB}{YC} = \dfrac{AC}{XB}$



Q.1. (B) Solve the following questions: (2)

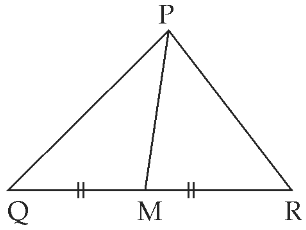
(1) ΔABC ~ ΔPQR A(ΔABC) : A(ΔPQR) = 9 : 16. Find BC : QR.

(2) ΔPQR, seg RS is the bisector of ∠PRQ. PS = 8, SQ = 6, PR = 20, then find QR.

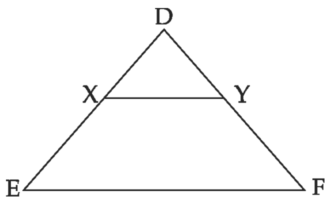
Q.2. Perform any one of the following activities: (2)

(1) In the adjoining figure, seg PM is a median.

Prove that A(ΔPQM) = A(ΔPRM)



(2) In the adjoining figure, DX = 4, DE = 8, FY = 6, OF = 12. Complete the following activity to prove that seg XY || seg EF.



Proof : XE = $\square - \square = \square - \square = \square$ ...[D – X – E]

DY = $\square - \square = \square - \square = \square$ ...[D – Y – F]

$\frac{DX}{XE} = \frac{\square}{\square} = \square$ ...(i)

$\frac{DY}{XE} = \frac{\square}{\square} = \square$ ...(ii)

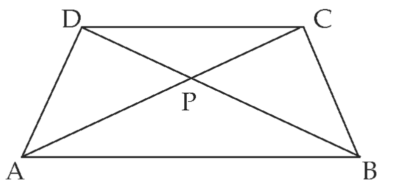
In ΔDEF, $\frac{\square}{\square} = \frac{\square}{\square}$ ...[From (i) and (ii)]

∴ Seg XY || seg EF ...[By ....]

Q.3. Attempt any Two of the following: **(6)**

(1) In □ABCD, seg AB || seg CD. Diagonals AC and BD intersect each other at point P.

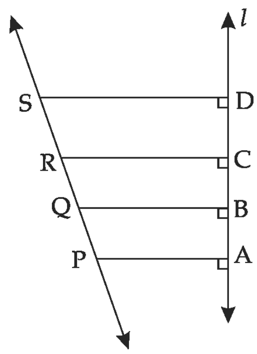
Prove : $\frac{\mathbf{A(\Delta ABP)}}{\mathbf{A(\Delta CPD)}} = \frac{\mathbf{AB^2}}{\mathbf{CD^2}}$



AB || CD ...(Given)

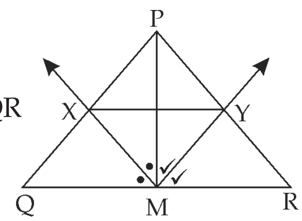
(2) D is a point on side BC of ΔABC such that ∠ADC = ∠BAC. Show that $AC^2 = BC \times DC$.

(3) In the adjoining figure, seg PA, seg QB seg RC and seg SD are ⊥ line *l*. AB = 60, BC = 70, CD = 80. If PS = 280 then find PQ, QR, RS.

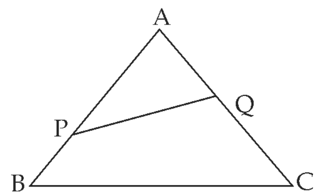


Q.4. Attempt any two of the following: **(8)**

(1) In ΔPQR, ray MX and ray MY bisect ∠PMQ and ∠PMR respectively. P – X – Q, P – Y – R. Seg PM is a median, prove that seg XY || seg QR



(2) In the adjoining figure, in ΔABC, A – P – B and A – Q – C prove that $\frac{A(\Delta APQ)}{A(\Delta ABC)} = \frac{AP \times AQ}{AB \times AC}$



(3) Prove: In a triangle the angle bisector divides the side opposite to the angle in the ratio of the remaining sides.

2 Pythagoras Theorem

... INDEX ...

Points to Remember:

Theorem : 1

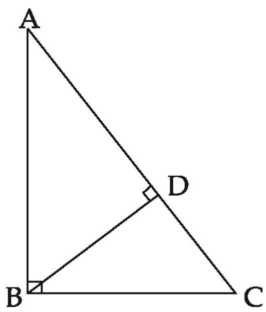
- **Similarity and Right Angled Triangles :**

  'In a right angled triangle, if the altitude is drawn from the vertex of the right angle to the hypotenuse, then the two triangles formed are similar to the original triangle and to each other'.

Given :

(1) In ΔABC, $\angle ABC = 90^\circ$

(2) seg $BD \perp$ hypotenuse AC, $A - D - C$



To Prove :

$\Delta ABC \sim \Delta ADB \sim \Delta BDC$

Proof :

In ΔABC and ΔADB,

$\angle ABC \cong \angle ADB$ ...(Each is a right angle)

$\angle A \cong \angle A$ ...(Common angle)

$\therefore$ $\Delta ABC \sim \Delta ADB$ ...(i) (By AA Test of similarity)

In ΔABC and ΔBDC,

$\angle ABC \cong \angle BDC$ ...(Each is a right angle)

$\angle C \cong \angle C$ ...(Common angle)

$\therefore$ $\Delta ABC \sim \Delta BDC$ ...(ii) (By AA Test of similarity)

$\therefore$ $\Delta ABC \sim \Delta ADB \sim \Delta BDC$ ...[From (i) and (ii)]

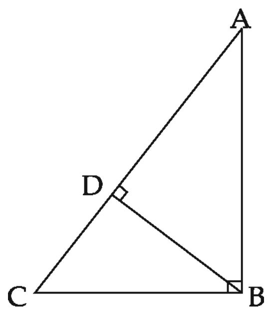
- **Theorem of Geometric Mean :**

  'In a right angled triangle, the length of perpendicular segment drawn on the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided'.

Given :

(1) In ΔABC, $\angle ABC = 90^\circ$

(2) seg $BD \perp$ hypotenuse AC, $A - D - C$



To Prove :

$BD^2 = AD \times CD$

Proof :

In ΔABC, $\angle ABC = 90^\circ$ ...(Given)

seg $BD \perp$ hypotenuse AC ...(Given)

$\therefore$ $\Delta ADB \sim \Delta BDC$ ...(Similarity in right angled triangles)

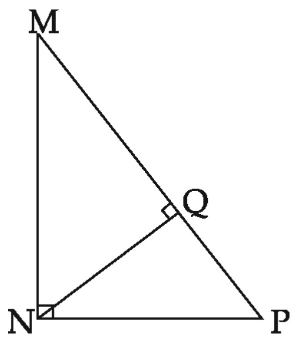
$\therefore$ $\dfrac{AD}{BD} = \dfrac{BD}{CD}$ ...(c.s.s.t)

$\therefore$ $BD^2 = AD \times CD$

MASTER KEY QUESTION SET - 2

Practice Set - 2.1 *(Textbook Page No. 38)*

(2) In the adjoining figure, $\angle MNP = 90^\circ$ seg $NQ \perp$ seg MP, $MQ = 9$, $QP = 4$. Find NQ. *(2 marks)*



Solution :

In ΔMNP, $\angle MNP = 90^\circ$ ...(Given)

seg $NQ \perp$ hypotenuse MP ...(Given)

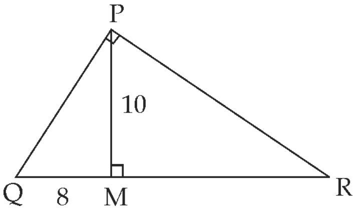
$\therefore$ $NQ^2 = MQ \times PQ$ ...(Theorem of Geometric mean)

$\therefore$ $NQ^2 = 9 \times 4$

$\therefore$ $NQ^2 = 36$

$\therefore$ $\boxed{\textbf{NQ = 6 units}}$ ...(Taking square roots)

(3) **In the adjoining figure, $\angle QPR = 90^\circ$, seg $PM \perp$ hypotenuse QR, Q - M - R. If PM = 10, QM = 8 then find QR.** *(2 marks)*



Solution :

In ΔQPR, $\angle QPR = 90^\circ$ ...(Given)

seg PM $\perp$ hypotenuse QR ...(Given)

$\therefore$ $PM^2 = QM \times RM$ ...(Theorem of Geometric mean)

$\therefore$ $10^2 = 8 \times RM$

$\therefore$ $RM = \frac{100}{8}$

$\therefore$ RM = 12.5 units

QR = QM + RM ...(Q - M - R)

$\therefore$ QR = 8 + 12.5

$\therefore$ $\boxed{\textbf{QR = 20.5 units}}$

Points to Remember:

Theorem : 2

- **Theorem of Pythagoras :**

Statement : 'In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides'.

Given :

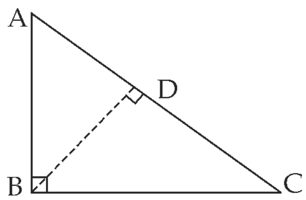
In ΔABC, $m\angle ABC = 90^\circ$

To Prove :

$AC^2 = AB^2 + BC^2$

Construction :

Draw seg $BD \perp$ side AC such that A - D - C



Proof :

In ΔABC, $\angle ABC = 90^\circ$ ...(Given)

seg BD $\perp$ hypothenuse AC ...(Construction)

$\therefore$ $\Delta ABC \sim \Delta ADB \sim \Delta BDC$ ...(i)(Similarity in right angled triangles)

$\Delta ABC \sim \Delta ADB$ ...(From (i)]

$\therefore$ $\frac{AB}{AD} = \frac{AC}{AB}$ ...(c.s.s.t)

$\therefore$ $AB^2 = AC \times AD$ ...(ii)

$\Delta ABC \sim \Delta BDC$ ...(From (i)]

$\therefore$ $\frac{BC}{DC} = \frac{AC}{BC}$ ...(c.s.s.t)

$\therefore$ $BC^2 = AC \times DC$ ...(iii)

Adding (ii) and (iii) we get,

$AB^2 + BC^2 = AC \times AD + AC \times DC$

$\therefore$ $AB^2 + BC^2 = AC\,(AD + DC)$

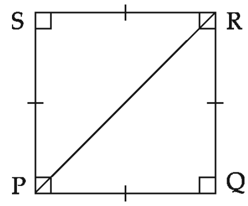
$\therefore$ $AB^2 + BC^2 = AC \times AC$ [$\because$ A - D - C]

$\therefore$ $AB^2 + BC^2 = AC^2$

$\therefore$ $\mathbf{AC^2 = AB^2 + BC^2}$

Practice Set - 2.1 *(Textbook Page No. 39)*

(6) **Find the side and perimeter of a square whose diagonal is 10 cm.** *(2 marks)*



Given :

(1) $\square$PQRS is a square

(2) PR = 10 cm

To Find :

(a) Side PQ

(b) Perimeter of $\square$PQRS

Solution :

$\square$PQRS is a square ...(Given)

$\therefore$ PQ = QR ...(i) (Sides of a square)

In ΔPQR, $\angle Q = 90^\circ$ ...(Angle of a square)

$\therefore$ $PR^2 = PQ^2 + QR^2$ ...(Pythagoras theorem)

$\therefore$ $10^2 = PQ^2 + PQ^2$ ...[From (i)]

$\therefore$ $2PQ^2 = 100$

$\therefore$ $PQ^2 = \frac{100}{2}$

$\therefore$ $PQ^2 = 50$

$\therefore$ $PQ = \sqrt{25 \times 2}$

$\therefore$ $\boxed{\mathbf{PQ = 5\sqrt{2}\ cm}}$

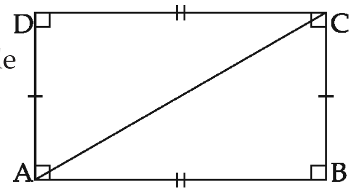
$\therefore$ Perimeter of $\square$PQRS $= 4 \times$ side

$= 4 \times PQ$

$= 4 \times 5\sqrt{2}$

$\therefore$ $\boxed{\textbf{Perimeter of } \square\textbf{PQRS} = \mathbf{20\sqrt{2}\ cm}}$

(8) **Length and breadth of a rectangle are 35 cm and 12 cm respectively. Find length of its diagonal.** *(2 marks)*



Given :

(1) $\square$ABCD is a rectangle

(2) AB = 35 cm,

BC = 12 cm

To Find :

diagonal AC

Solution :

In ΔABC, $\angle ABC = 90^\circ$ ...(Angle of a rectangle)

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(By Pythagoras theorem)

$\therefore$ $= 35^2 + 12^2$

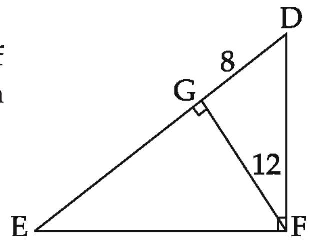
$\therefore$ $= 1225 + 144$

$\therefore$ $= 1369$

$\therefore$ $AC = 37$ cm ...(Taking square roots)

Length of the diagonal of the rectangle is 37 cm

(7) **In the adjoining figure, $\angle DFE = 90^\circ$, $FG \perp ED$ if $GD = 8$, $FG = 12$ then find (i) EG (ii) FD (iii) EF.** *(3 marks)*



Solution :

(i) In ΔDFE, $\angle DFE = 90^\circ$ ...(Given)

$\therefore$ seg $FG \perp$ hypotenuse DE ...(Given)

$FG^2 = DG \times EG$ ...(Theorem of Geometric mean)

$\therefore$ $12^2 = 8 \times EG$

$\therefore$ $EG = \dfrac{12 \times 12}{8}$

$\therefore$ **EG = 18 units** ...(i)

(ii) In ΔFGD, $\angle FGD = 90^\circ$ ...(Given)

$\therefore$ $FD^2 = FG^2 + GD^2$ ...(By Pythagoras theorem)

$\therefore$ $= 12^2 + 8^2$

$\therefore$ $= 144 + 64$

$\therefore$ $FD^2 = 208$

$\therefore$ $FD = \sqrt{208}$ ...(Taking square roots)

$\therefore$ $FD = \sqrt{16 \times 13}$

$\therefore$ **FD = $4\sqrt{13}$ units**

(iii) In ΔFGE, $m\angle FGE = 90^\circ$ ...(Given)

$\therefore$ $EF^2 = EG^2 + FG^2$ ...(By Pythagoras theorem)

$\therefore$ $= 18^2 + 12^2$ ...[From (i)]

$\therefore$ $= 324 + 144$

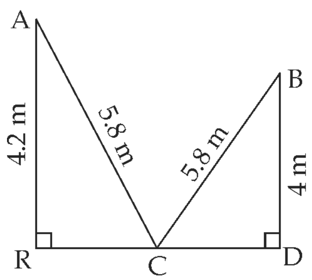
$\therefore$ $EF^2 = 468$

$\therefore$ $EF = \sqrt{468}$ (Taking square roots)

$\therefore$ $EF = \sqrt{36 \times 13}$

$\therefore$ **EF = $6\sqrt{13}$ units**

***(10)** **Walls of two buildings on either side of a street are parallel to each other. A ladder 5.8 m long is placed on the street such that its top just reaches the window of a building at the height if 4 m. On turning the ladder over to the other side of the street, its top touches the window of the other building at a heitht 4.2 m. Find the width of the street.** *(3 marks)*



Solution :

Let RD represents the width of the street.

BD represents the first building.

AR represents the second building

CA and CB are two different positions of the same ladder from point C.

$AR = 4.2$ m, $BD = 4$ m, $AC = BC = 5.8$ m, $RD = ?$

In ΔARC, $\angle R = 90^\circ$ ...(Given)

$\therefore$ $AC^2 = AR^2 + CR^2$ ...(By Pythagoras theorem)

$\therefore$ $(5.8)^2 = (4.2)^2 + CR^2$

$\therefore$ $CR^2 = (5.8)^2 - (4.2)^2$

$\therefore$ $CR^2 = (5.8 + 4.2)(5.8 - 4.2)$

$\therefore$ $CR^2 = 10 \times 1.6$

$\therefore$ $CR^2 = 16$

$\therefore$ $CR = 4$ m ...(Taking square root)

In ΔBDC, $\angle D = 90^\circ$ ...(Given)

$\therefore$ $BC^2 = CD^2 + BD^2$ ...(By Pythagoras theorem)

$\therefore$ $5.8^2 = CD^2 + 4^2$

$\therefore$ $CD^2 = (5.8)^2 - 4^2$

$\therefore$ $CD^2 = (5.8 + 4)(5.8 - 4)$

$\therefore$ $CD^2 = 9.8 \times 1.8$

$\therefore$ $CD^2 = \dfrac{98}{10} \times \dfrac{18}{10}$

$\therefore$ $CD^2 = \dfrac{98 \times 18}{100}$

$\therefore$ $CD^2 = \dfrac{98 \times 2 \times 9}{100}$

$\therefore$ $CD^2 = \dfrac{196 \times 9}{100}$

$\therefore$ $CD = \dfrac{14 \times 3}{10}$ ...(Taking square roots)

$\therefore$ $CD = \dfrac{42}{10}$

$\therefore$ $CD = 4.2$ m

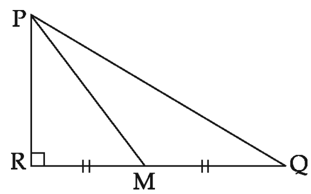
$RD = RC + CD$ ...(R – C – D)

$= 4 + 4.2$

$RD = 8.2$ m

$\therefore$ **Width of the street is 8.2 m**

***(9)** **In the adjoining figure, M is the midpoint of QR. $\angle PRQ = 90^\circ$. Prove that, $PQ^2 = 4PM^2 - 3PR^2$** *(4 marks)*



Proof :

In ΔPRQ, $\angle PRQ = 90^\circ$ ...(Given)

$\therefore$ $PQ^2 = PR^2 + QR^2$ ...(i) (By Pythagoras theorem)

$\therefore$ $QR = 2RM$ ...(ii) (M is the midpoint of seg QR)

$\therefore$ $PQ^2 = PR^2 + (2\ RM)^2$ ...[From (i) and (ii)]

$\therefore$ $PQ^2 = PR^2 + 4\ RM^2$ ...(iii)

In ΔPRM, $\angle PRM = 90^\circ$

$\therefore$ $PM^2 = PR^2 + RM^2$ ...(Pythagoras theorem)

$\therefore$ $RM^2 = PM^2 - PR^2$ ...(iv)

$\therefore$ $PQ^2 = PR^2 + 4\ (PM^2 - PR^2)$ ...[From (iii) and (iv)]

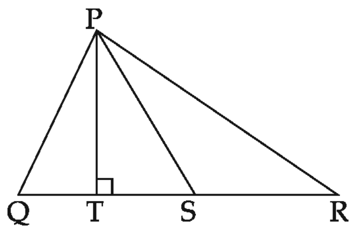
$\therefore$ $PQ^2 = PR^2 + 4\ PM^2 - 4\ PR^2$

$\therefore$ $\boxed{PQ^2 = 4\ PM^2 - 3\ PR^2}$

Practice Set - 2.2 *(Textbook Page No. 43)*

(3) **In ΔPQR seg PS is median of ΔPQR. and $PT \perp QR$,**

(4 marks)



Prove that :

(i) $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

(ii) $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$

Proof :

$QS = RS = \frac{QR}{2}$ ...(i) (S is the midpoint of seg QR)

In ΔPTS, $\angle PTS = 90^\circ$ ...(Given)

$\therefore$ $PS^2 = PT^2 + ST^2$ ...(Pythagoras theorem)

$\therefore$ $PT^2 = PS^2 - ST^2$ ...(ii)

(i) In ΔPTR, $\angle PTR = 90^\circ$ ...(Given)

$\therefore$ $PR^2 = PT^2 + TR^2$ ...(By Pythagoras theorem)

$\therefore$ $PR^2 = PS^2 - ST^2 + (RS + ST)^2$ ...[From (ii), R - S - T]

$= PS^2 - ST^2 + RS^2 + 2 \times RS \times ST + ST^2$

$= PS^2 + 2RS \times ST + RS^2$

$= PS^2 + 2 \times \frac{QR}{2} \times ST + \left(\frac{QR}{2}\right)^2$ ...[From (i)]

$\therefore$ $\boxed{PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2}$

(ii) In ΔPTQ, $\angle PTQ = 90^\circ$ ...(Given)

$\therefore$ $PQ^2 = PT^2 + QT^2$ ...(By Pythagoras theorem)

$\therefore$ $PQ^2 = PS^2 - ST^2 + (QS - ST)^2$ ...[From (ii), Q - T - S]

$PQ^2 = PS^2 - ST^2 + QS^2 - 2\ QS \times ST + ST^2$

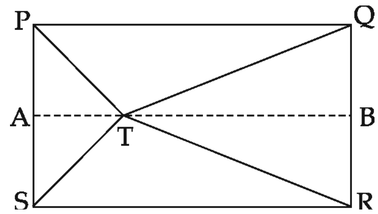
$PQ^2 = PS^2 - 2QS \times ST + QS^2$

$PQ^2 = PS^2 - 2 \times \frac{QR}{2} \times ST + \left(\frac{QR}{2}\right)^2$ ...[From (i)]

$\therefore$ $\boxed{PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2}$

*(5) **In adjoining figure, point T is in the interior of rectangle PQRS.**

Prove that, $TS^2 + TQ^2 = TP^2 + TR^2$ *(5 marks)*



To Prove :

$TS^2 + TQ^2 = TP^2 + TR^2$

Construction :

Draw a line parallel to side SR, through point T, intersecting sides PS and QR at point A and B respectively.

Proof :

$\square$PQRS is a rectangle ...(Given)

$\therefore$ $\angle SPQ = \angle PSR = \angle SRQ = \angle PQR = 90^\circ$ ...(i) (Angles of a rectangle)

seg PQ || seg SR ...(ii) (Opposite sides of rectangle are parallel)

But, seg AB || seg SR ...(iii) [Construction]

$\therefore$ seg PQ || seg SR || seg AB ...(iv) [From (i), (ii) & (iii)]

In $\square$ABRS, seg AB || seg SR ...[From (iii)]

seg AS || seg BR ...(Opposite sides of rectangle are parallel and P - A - S, Q - B - R)

In $\square$ABRS is parallelogram ...(By definition)

$\therefore$ AS = BR ...(v) (Opposite sides of parallelogram are equal)

Similarly by proving $\square$ABQP is a parallelogram we get,

AP = BQ ...(vi)

seg PQ || seg AB ...[From (ii)]

on transversal PS,

$\angle QPS = \angle BAS = 90^\circ$ ...(vii)

Similarly we can prove,

seg BA $\perp$ seg PS ...(viii)

$\therefore$ In ΔTAS, $\angle TAS = 90^\circ$ ...[From (vii)]

$\therefore$ $TS^2 = TA^2 + AS^2 = 90^\circ$ ...(ix) (By Pythagoras theorem)

$\therefore$ In ΔTBQ, $\angle$TBQ = 90° ...[From (viii)]

$\therefore$ $TQ^2 = TB^2 + BQ^2 = 90°$ ...(x) (By Pythagoras theorem)

Adding (ix) and (x),

$TS^2 + TQ^2 = TA^2 + AS^2 + TB^2 + BQ^2$ ...(xi)

In ΔTAP, $\angle$TAP = 90° ...[From (vii)]

$\therefore$ $TP^2 = TA^2 + AP^2$ ...(xii) (By Pythagoras theorem)

In ΔTBR, $\angle$TBR = 90°

$\therefore$ $TR^2 = TB^2 + BR^2$ ...(xiii) (By Pythagoras theorem)

Adding (xii) and (xiii),

$TP^2 + TR^2 = TA^2 + AP^2 + TB^2 + BR^2$

$TP^2 + TR^2 = TA^2 + BQ^2 + TB^2 + AS^2$ ...(xiv) ...[From (vi) and (v)

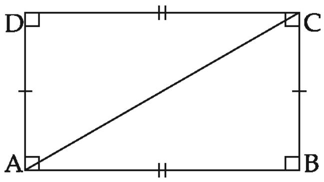
$\therefore$ $\boxed{\mathbf{TS^2 + TQ^2 = TP^2 + TR^2}}$ ...[From (xi) and (xiv)

Problem Set - 2 *(Textbook Pg No. 44)*

(2) Solve the following:

(iii) Find the length a diagonal of a rectangle having side 11 cm and 60 cm. *(2 marks)*

Solution :



In rectangle ABCD, AB = 60 cm, BC = 11 cm.

In ΔABC, $\angle$ABC = 90° ...(Angle of a rectangle)

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(By Pythagoras theorem)

$AC^2 = (60)^2 + (11)^2$

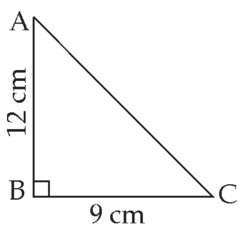
$= 3600 + 121$

$AC^2 = 3721$

$\therefore$ AC = 61 cm ...(Taking square roots)

$\therefore$ **The length of the diagonal of the rectangle is 61 cm**

(iv) Find the length of the hypotenuse of a right angled triangle if remaining sides are 9 cm and 12 cm. *(2 marks)*



Solution :

In ΔABC, $\angle$ABC = 90° ...(Given)

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(By Pythagoras theorem)

$AC^2 = (9)^2 + (12)^2$

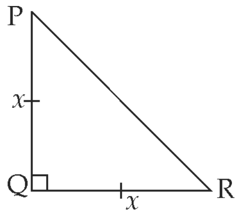
$= 81 + 144$

$AC^2 = 225$

$\therefore$ AC = 15 cm ...(Taking square roots)

$\therefore$ **length of the hypotenuse is 15 cm**

(v) A side of and isosceles right angled triangle is x. Find its hypotenuse.. *(2 marks)*



Solution :

In ΔPQR, $\angle$PQR = 90° ...(Given)

$\therefore$ $PR^2 = PQ^2 + QR^2$ ...(By Pythagoras theorem)

$PR^2 = x^2 + x^2$

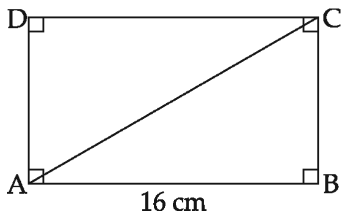
$= 2x^2$

$\therefore$ $PR = \sqrt{2}x$ ...(Taking square roots)

$\therefore$ **The length of the hypothenuse is $\sqrt{2}\,x$ units**

(4) Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq.cm. *(3 marks)*

Given :



(1) □ABCD is a rectangle

(2) AB = 16 cm

(3) A(□ABCD) = 192 sq. cm.

To find :

AC

Solution :

□ABCD is a rectangle

A(□ABCD) = length × breadth

$\therefore$ 192 = AB × BC

$\therefore$ 192 = 16 × BC

$\frac{192}{16}$ = BC

$\therefore$ BC = 12 cm

In ΔABC, $\angle$ABC = 90° ...(Angle of a rectangle)

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(By Pythagoras theorem)

$AC^2 = (16)^2 + (12)^2$

$= 256 + 144$

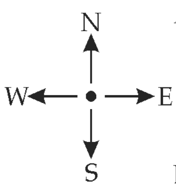
$\therefore$ $AC^2 = 400$

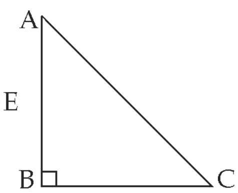
$\therefore$ AC = 20 cm ...(Taking square roots)

$\therefore$ **length of the diagonal is 20 cm**

(10) Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After 2 hours distance between them was $15\sqrt{2}$ km. Find their speed per hour. *(3 marks)*

Solution :





B represents starting point of journey.

BA is the distance

travelled by Prasad in North direction.

BC is the distance travelled by Pranali in east direction.

AC is the distance between Pranali and Prasad after two hours.

Let the speed of each one be x km/hr.

$\therefore$ Distance travelled by each one hour is $2x$ km.

i.e. AB = BC = $2x$ km

In ΔABC, $\angle B = 90^\circ$ ...(Line joining adjacent direction are $\perp$ to each other)

$\therefore$ $AB^2 + BC^2 = AC^2$ ...(By Pythagoras theorem)

$\therefore$ $(2x)^2 + (2x)^2 = (15\sqrt{2})^2$

$\therefore$ $4x^2 + 4x^2 = 225 \times 2$

$\therefore$ $8x^2 = 225 \times 2$

$\therefore$ $x^2 = \dfrac{225 \times 2}{8}$

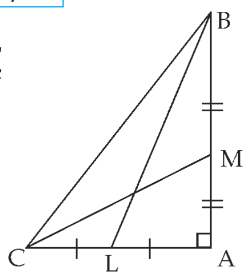
$\therefore$ $x^2 = \dfrac{225}{4}$

$\therefore$ $x = \dfrac{15}{2}$ ...(Taking square roots)

$\therefore$ $x = 7.5$

$\therefore$ $\boxed{\textbf{Speed of each one is 7.5 km / hr}}$

***(11)** **In ΔABC, $\angle BAC = 90^\circ$, seg BL and seg CM are medians of ΔABC, prove that $4(BL^2 + CM^2) = 5\,BC^2$.** *(5 marks)*



To Prove :

$4(BL^2 + CM^2) = 5BC^2$ ...(Given)

Proof :

In ΔBAC, $\angle BAC = 90^\circ$ ...(Given)

$\therefore$ $BC^2 = AB^2 + AC^2$ ...(i) (By Pythagoras theorem)

In ΔBAL, $\angle BAC = 90^\circ$ ...(Given)

$\therefore$ $BL^2 = AB^2 + AL^2$ ...(ii) (By Pythagoras theorem)

In ΔCAM, $\angle CAM = 90^\circ$ ...(Given)

$\therefore$ $CM^2 = AC^2 + AM^2$ ...(iii) (By Pythagoras theorem)

Adding (ii) and (iii),

$BL^2 + CM^2 = AB^2 + AL^2 + AC^2 + AM^2$

$\therefore$ $BL^2 + CM^2 = AB^2 + AC^2 + AL^2 + AM^2$

$\therefore$ $BL^2 + CM^2 = BC^2 + AL^2 + AM^2$ [From (i)]

$\therefore$ $BL^2 + CM^2 = BC^2 + \left(\dfrac{1}{2}AC\right)^2 + \left(\dfrac{1}{2}AB\right)^2$ [$\because$ L and M are the midpoint of sides AC and AB respectively]

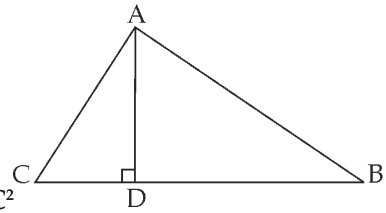
$\therefore$ $BL^2 + CM^2 = BC^2 + \dfrac{AC^2}{4} + \dfrac{AB^2}{4}$

$\therefore$ $4(BL^2 + CM^2) = 4BC^2 + AC^2 + AB^2$ (Multiplying throughout by 4)

$\therefore$ $4(BL^2 + CM^2) = 4BC^2 + BC^2$ ...[From (i)]

$\therefore$ $\boxed{\mathbf{4(BL^2 + CM^2) = 5BC^2}}$

(13) **In ΔABC, seg AD $\perp$ seg BC, DB = 3CD. Prove that : $2AB^2 = 2AC^2 + BC^2$**



(4 marks)

To Prove :

$2AB^2 = 2AC^2 + BC^2$

Proof :

DB = 3CD ...(i) (Given)

In ΔADB, $\angle ADB = 90^\circ$ (Given)

$\therefore$ $AB^2 = AD^2 + DB^2$ (By Pythagoras theorem)

$\therefore$ $AB^2 = AD^2 + (3CD)^2$ [From (i)]

$\therefore$ $AB^2 = AD^2 + 9CD^2$ ...(ii)

In ΔADC, $\angle ADC = 90^\circ$ ...(Given)

$\therefore$ $AC^2 = AD^2 + CD^2$ (By Pythagoras theorem)

$\therefore$ $AD^2 = AC^2 - CD^2$ ...(iii)

$AB^2 = AC^2 - CD^2 + 9CD^2$ [From (ii) and (iii)]

$\therefore$ $AB^2 = AC^2 + 8CD^2$ ...(iv)

But BC = CD + DB ...[C - D - B]

$\therefore$ BC = CD + 3CD ...[From (i)]

$\therefore$ BC = 4CD

$\therefore$ $CD = \dfrac{BC}{4}$ ...(v)

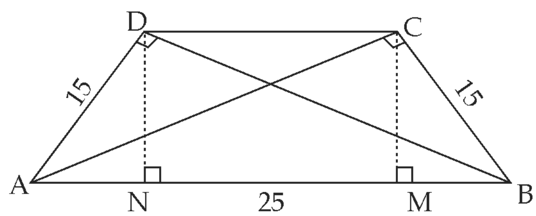
$\therefore$ $AB^2 = AC^2 + 8\left(\dfrac{BC}{4}\right)^2$ ...[From (iv) and (v)]

$\therefore$ $AB^2 = AC^2 + 8 \times \dfrac{BC^2}{16}$

$\therefore$ $AB^2 = AC^2 + \dfrac{BC^2}{2}$ ...[From (iv) and (v)]

$\therefore$ $\boxed{\mathbf{2AB^2 = 2AC^2 + BC^2}}$ (Multiplying throughout by 2)

(15) **In trapesium ABCD, seg AB || seg DC. seg BD $\perp$ seg AD, seg AC $\perp$ seg BC. If AD = 15, BC = 15 and AB = 25, then find A ($\square$ABCD)** *(5 marks)*



Construction :

Draw seg CM $\perp$ side AB, (A - M - B)

Draw seg DN $\perp$ side AB, (A - N - B)

Solution :

In ΔABC, $\angle ACB = 90^\circ$ ...(Given)

$\therefore$ $AC^2 + BC^2 = AB^2$ ...(By Pythagoras theorem)

$\therefore$ $AC^2 + (15)^2 = (25)^2$

$AC^2 + (25)^2 - (15)^2$

$AC^2 = 625 - 225 = 400$

AC = 20 units ...(Taking square roots)

$A(\Delta ABC) = \frac{1}{2} \times AB \times CM$ ...(i)

Also, $A(\Delta ABC) = \frac{1}{2} \times AC \times BC$ ...(ii)

$\therefore$ $\frac{1}{2} \times AB \times CM = \frac{1}{2} \times AC \times BC$

$\therefore$ $25 \times CM = 20 \times 15$

$\therefore$ $CM = \frac{20 \times 15}{25}$

$\therefore$ CM = 12 units ...(iii)

In ΔBMC, $\angle BMC = 90^\circ$ ...(Construction)

$\therefore$ $BC^2 = CM^2 + BM^2$ ...(By Pythagoras theorem)

$\therefore$ $15^2 = 12^2 + BM^2$

$BM^2 = 15^2 - 12^2$

$\therefore$ $BM^2 = 225 - 144$

$\therefore$ $BM^2 = 81$

$\therefore$ BM = 9 units ...(iv) (Taking square roots)

$\therefore$ CM = DN ...(v) (Prependicular distance between the same two parallel lines are equal)

In ΔBMC and ΔAND

$\angle BMC \cong \angle AND$ ...(Each 90^0)

Hyp. BC $\cong$ Hyp. AD ...(Given)

seg CM $\cong$ seg DN ...[From (v)]

$\therefore$ $\Delta BMC \cong \Delta AND$ ...(Hypotenuse side test)

$\therefore$ seg BM $\cong$ seg AN ...(vi) ...(c.s.s.t.)

$\therefore$ AN = 9 units ...(vii) ...[From (iv) and (vi)]

AB = AN + MN + BM ...(A - N- M - B)

$\therefore$ 25 = 9 + MN + 9

MN = 25 – 18 = 7 units ....(viii)

In $\square$CMND, seg MN || seg CD ...(Given, A - N - M - B)

seg CM || seg DN ...(Perpendiculars drawn to the same line are parallel)

$\therefore$ $\square$CMND is a parallelogram ...(Definition)

$\therefore$ CD = MN ...(Opposite sides of parallelogram are equal)

$\therefore$ CD = 7 units ...[From (viii)]

$\therefore$ $A(\text{trapezium } ABCD) = \frac{1}{2}(AB + CD) \times CM$

$= \frac{1}{2}(25 + 7) \times 12$

$= \frac{1}{2}(32) \times 12 = 192$

$\therefore$ $\boxed{\text{A (trapezium ABCD) = 192 square units}}$

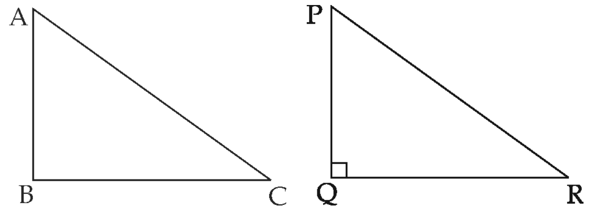
Points to Remember:

- **Converse of Pythagoras theorem :**

  Statement : In a triangle, if the square of one side is equal to the sum of the squares of remaining two sides, then the angle opposite to the first side is a right angle and the triangle is right angled triangle.

  Given : In ΔABC, $AC^2 = AB^2 + BC^2$

  To prove : $\angle ABC = 90^\circ$



  Construcion: Draw ΔPQR such that $AB = PQ$, $BC = QR$ and $\angle PQR = 90^\circ$

  Proof : In ΔABC and ΔPQR

  $PR^2 = PQ^2 + QR^2$ ...(By Pythagores theorem)

  $= AB^2 + BC^2$ ...(Construction)

  $= AC^2$ ...(Givem)

  $\therefore$ $PR^2 = AC^2$

  PR = AC ...(taking square roots)

  $\Delta ABC \cong \Delta PQR$ ...(SSS test)

  $\angle ABC = \angle PQR = 90^\circ$

- **Pythagorean Triplet:**

  In a triplet of natural numbers, if the square of the largest number is equal to the sum of the squares of the remaining two numbers. Then the triplet is called Pythagorean triplet.

Example: In the triplet (11, 60, 61)

$11^2 = 121$; $60^2 = 3600$; $61^2 = 3721$

121 + 3600 = 3721

The square of the largest number is equal of the sum of the squares of the other two number.

Formula for the Pythagorean triplet:

If a, b, c are natural numbers and a > b, then $[(a^2 + b^2), (a^2 - b^2), 2ab]$ is pythagorean triplet.

$\therefore$ $(a^2 + b^2)^2 = a^4 + 2a^2b^2 + b^4$ ...(i)

$(a^2 - b^2)^2 = a^4 - 2a^2b^2 + b^4$ ...(ii)

$(2ab)^2 = 4a^2b^2$ ...(iii)

by (i), (ii) and (iii),

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\therefore$ $[(a^2 + b^2), (a^2 - b^2), (2ab)]$ is a Pythagorean triplet.

Example: For $a = 5$ and $b = 3$

$a^2 + b^2 = 25 + 9 = 34$

$a^2 - b^2 = 25 - 9 = 16$

$2ab = 30$

$\therefore$ (16, 30, 34) is a Pythagorean triplet

Practice Set - 2.1 *(Textbook Page No. 38)*

(1) Which of the following are Pythagorean triplets? Justify.

(i) 3, 5, 4 *(1 mark)*

$5^2 = 25$ ...(i)

$3^2 + 4^2 = 9 + 16$

$\therefore$ $3^2 + 4^2 = 25$ ...(ii)

From (i) and (ii)

$5^2 = 3^2 + 4^2$

$\therefore$ **3, 5, 4 is a Pythagorean triplet.**

(ii) 4, 9, 12 *(1 mark)*

$12^2 = 144$ ...(i)

$4^2 + 9^2 = 16 + 81$

$\therefore$ $4^2 + 9^2 = 97$ ...(ii)

From (i) and (ii)

$12^2 \neq 4^2 + 9^2$

$\therefore$ **4, 9, 12 is not a Pythagorean triplet.**

(iii) 5, 12, 13 *(1 mark)*

$13^2 = 169$ ...(i)

$5^2 + 12^2 = 25 + 144$

$\therefore$ $5^2 + 12^2 = 169$ ...(ii)

$13^2 = 5^2 + 12^2$ ...[From (i) and (ii)]

$\therefore$ **5, 12, 13 is a Pythagorean triplet.**

(iv) 24, 70, 74 *(1 mark)*

$74^2 = 5476$ ...(i)

$24^2 + 70^2 = 576 + 4900$

$\therefore$ $24^2 + 70^2 = 5476$ ...(ii)

$74^2 = 24^2 + 70^2$ ...[From (i) and (ii)]

$\therefore$ **24, 70, 74 is a Pythagorean triplet.**

(v) 10, 24, 27 *(1 mark)*

$27^2 = 729$ ...(i)

$10^2 + 24^2 = 100 + 576$

$\therefore$ $10^2 + 24^2 = 676$ ...(ii)

From (i) and (ii)

$27^2 \neq 10^2 + 24^2$

$\therefore$ **10, 24, 27 is not a Pythagorean triplet.**

(vi) 11, 60, 61 *(1 mark)*

$61^2 = 3721$ ...(i)

$60^2 + 11^2 = 3600 + 121$

$\therefore$ $60^2 + 11^2 = 3721$ ...(ii)

From (i) and (ii)

$61^2 = 60^2 + 11^2$

$\therefore$ **11, 60, 61 is a Pythagorean triplet.**

Problem Set - 2 *(Textbook Pg No. 44)*

(2) Solve the following

(ii) Do sides 7 cm, 24 cm, 25 cm from a right angled triangle? Give reason. *(1 mark)*

Solution :

$25^2 = 625$ ...(i)

$7^2 + 24^2 = 49 + 576$

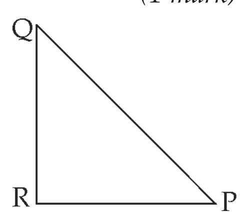
$7^2 + 24^2 = 625$ ...(ii)

$\therefore$ $25^2 = 7^2 + 4^2$ ...[From (i) and (ii)]

$\therefore$ **By converse of Pythagoras theorem, given triangle is a right angled triangle.**

(vi) In ΔPQR, $PQ = \sqrt{8}$, $QR = \sqrt{5}$, $PR = \sqrt{3}$. Is ΔPQR a right angle ? If yes, which angle is of 90°? *(1 mark)*

Solution :



$PQ^2 = \left(\sqrt{8}\right)^2 = 8$ ...(i)

$PR^2 + QR^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2$

$\therefore$ $PR^2 + QR^2 = 3 + 5$

$\therefore$ $PR^2 + QR^2 = 8$ ...(ii)

$\therefore$ $PQ^2 = PR^2 + QR^2$ ...[From (i) and (ii)]

Yes, ΔPQR is a right angled triangled.

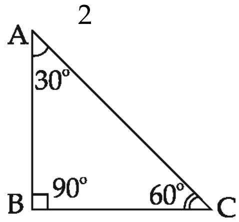
$\therefore$ $\angle R = 90°$ ...(Converse of Pythagoras theorem)

$\therefore$ $\boxed{\angle R = 90°}$

Points to Remember:

- **Theorem of 30° - 60° - 90° triangle.**

  If the angles of a triangle are 30°, 60° and 90°, then the side opposite to 30° is half of the hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse.



In ΔABC,

m $\angle A = 30^\circ$

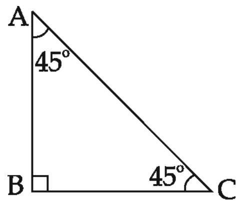
m $\angle C = 60^\circ$ and

m $\angle B = 90^\circ$

$\therefore$ **$BC = \frac{1}{2}$ AC and $AB = \frac{\sqrt{3}}{2}$ AC**

- **Theorem of 45° - 45° - 90° triangle.**

If the angles of a triangle are 45°, 45° and 90°, then the length of the perpendicular sides are $\frac{1}{\sqrt{2}}$ times the hypotenuse.



In ΔABC,

m $\angle A = 45^\circ$

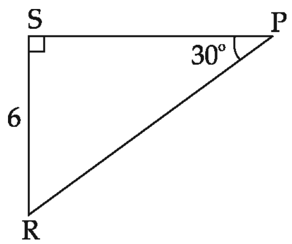
m $\angle C = 45^\circ$ and

m $\angle B = 90^\circ$

$\therefore$ **$AB = BC = \frac{1}{\sqrt{2}}$ AC.**

Practice Set - 2.1 *(Textbook Page No. 39)*

(4) In adjoining figure, find RP and PS using the information given in ΔPSR, find RP and PS. *(2 marks)*



Solution :

In ΔPSR, $\angle S = 90^\circ$ ...(Given)

$\angle P = 30^\circ$ ...(Given)

$\therefore$ $\angle R = 60^\circ$ ...(Sum of all angles of a triangle is 180°)

$\therefore$ ΔPSR is 30° - 60° - 90° triangle

By 30° - 60° - 90° triangle theorem,

$RS = \frac{1}{2} PR$ ...(side opposite to 30°)

$\therefore$ $6 = \frac{1}{2} \times PR$

$\therefore$ $\boxed{\textbf{PR = 12 units}}$

$PS = \frac{\sqrt{3}}{2} \times PR$ ...(side opposite to 60°)

$PS = \frac{\sqrt{3}}{2} \times 12$

$\therefore$ $\boxed{\textbf{PS} = 6\sqrt{3} \textbf{ units}}$

Problem Set - 2 *(Textbook Pg No. 44)*

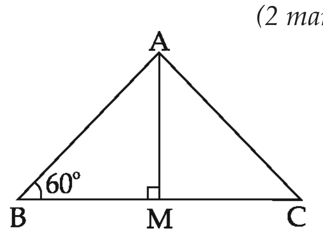
(2) Solve the following

(i) Find the height of an equilateral triangle having side 2*a*. *(2 marks)*

Given :

(i) ΔABC is an equilateral triangle.

(ii) $AB = 2a$



Construction: seg AM $\perp$ side BC, B - M - C

To find : AM.

Solution :

In ΔAMB, $\angle AMB = 90^\circ$ ...(Given)

$\angle B = 60^\circ$ ...(angle of an equilateral triangle)

$\angle BAM = 30^\circ$ ...(Sum of all angles of a triangle is 180°)

$\therefore$ ΔAMB is 30° - 60° - 90° triangle

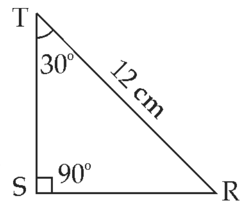
By 30° – 60° – 90° triangle theorem,

$AM = \frac{\sqrt{3}}{2} \times AB$ ...(Side opposite to 60°)

$\therefore$ $AM = \frac{\sqrt{3}}{2} \times 2a$

$\therefore$ $\boxed{\textbf{AM} = \sqrt{3}\,\textbf{a}}$

(3) In ΔRST, $\angle S = 90^\circ$, $\angle T = 30^\circ$, RT = 12 cm. Find RS and ST. *(2 marks)*



Solution :

In ΔPSR, $\angle S = 90^\circ$ ...(Given)

$\angle T = 30^\circ$ ...(Given)

$\therefore$ $\angle R = 60^\circ$ ...(Sum of all angles of a triangle is 180°)

$\therefore$ ΔPSR is 30° - 60° - 90° triangle

By 30° - 60° - 90° triangle theorem,

$RS = \frac{1}{2} \times RT$

$\therefore$ $RS = \frac{1}{2} \times 12$ ...(side opposite to 30°)

$\therefore$ $\boxed{\textbf{RS = 6 cm}}$

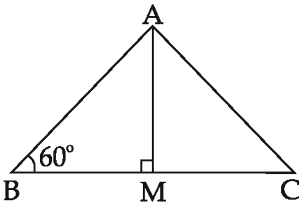
$ST = \frac{\sqrt{3}}{2} \times RT$ ...(side opposite to 60°)

$ST = \frac{\sqrt{3}}{2} \times 12$

$\therefore$ $\boxed{\textbf{ST} = 6\sqrt{3} \textbf{ cm}}$

***(5) Find the length if the side and perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.**

(2 marks)



Given :

(i) ΔABC is an equilateral triangle.

(ii) seg $AM \perp$ side BC, B - M - C

(iii) $AM = \sqrt{3}$ cm

To find : (i) AB (ii) Perimeter of ΔABC.

Solution :

In ΔAMB, $\angle AMB = 90^\circ$ ...(Given)

$\angle B = 60^\circ$ ...(Angle of an equilateral triangle)

$\therefore$ $\angle BAM = 30^\circ$ ...(Sum of all angles of a trangle is 180°)

$\therefore$ ΔAMB is 30° - 60° - 90° triangle

By 30° - 60° - 90° triangle theorem,

$AM = \dfrac{\sqrt{3}}{2} \times AB$ ...(Side opposite to 60°)

$\therefore$ $AB = \dfrac{2}{\sqrt{3}} \times AM$

$\therefore$ $AB = \sqrt{3} \times \dfrac{2}{\sqrt{3}}$

$\therefore$ $\boxed{AB = 2 \text{ cm}}$

$\therefore$ Perimeter of ABC $= 3 \times AB$

$= 3 \times 2 = 6$cm

$\therefore$ $\boxed{\text{Perimeter of } \Delta ABC = 6 \text{ cm}}$

(7) ΔABC is an equilateral triangle. Point P is on base BC such that $PC = \dfrac{1}{3}$ BC, if AB = 6 cm, find AP.

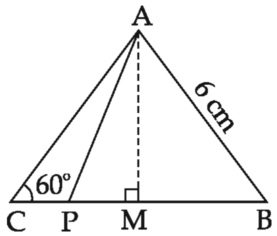
(5 marks)

Given :

(i) ΔABC is an equilateral triangle.

(ii) AB = 6 cm

(iii) $PC = \dfrac{1}{3} BC$



To find : AP

Construction : Draw seg $AM \perp BC$, C - M - B

Solution :

$AB = BC = AC$ ...(Side of an equilateral triangle)

$\therefore$ $BC = AC = 6$ cm ...(i) ($\because$ AB = 6 cm, given)

$PC = \dfrac{1}{3} BC$ ...(Given)

$\therefore$ $PC = \dfrac{1}{3} \times 6 = 2$ cm ...(ii)

In ΔAMC, $\angle C = 60^\circ$ ...(Angle of an equilateral triangle)

$\angle AMC = 90^\circ$ ...(Construction)

$\therefore$ $\angle CAM = 30^\circ$ ...(Sum of all angles of a triangle is 180°)

$\therefore$ ΔAMC is 30° - 60° - 90° triangle

By 30° - 60° - 90° triangle theorem,

$\therefore$ $CM = \dfrac{1}{2} AC$ ...(side opposite to 30°)

$\therefore$ $CM = \dfrac{1}{2} \times 6 \text{ cm} = 3$ cm ...(iii)

$AM = \dfrac{\sqrt{3}}{2} \times AC$ ...(side opposite to 60°)

$\therefore$ $AM = \dfrac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}$ ...(iv)

$PM = CM - CP$ ...(C - P - M)

$\therefore$ $PM = 3 - 2$

$\therefore$ $PM = 1$ cm

$\therefore$ In ΔAMP, $\angle AMP = 90^\circ$ triangle ...(Construction)

$AP^2 = AM^2 + PM^2$ ...(By Pythagoras theorem)

$= (3\sqrt{3})^2 + 1$

$AP^2 = 27 + 1$

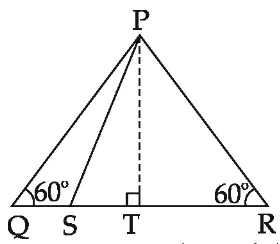
$\therefore$ $AP^2 = 28$

$\therefore$ $AP = \sqrt{4 \times 7}$

$\therefore$ $AP = 2\sqrt{7}$ ...(Taking square roots)

$\therefore$ $\boxed{AP = 2\sqrt{7} \text{ cm}}$

***(16) In the adjoining figure, ΔPQR is an equilateral triangle. Point S is on seg QR such that $QS = \dfrac{1}{3}$ QR. Prove that $9\,PS^2 = 7\,PQ^2$**



(5 marks)

To prove : $9PS^2 = 7PQ^2$

Construction :

Draw seg $PT \perp$ side QR, (Q - S - T - R)

Proof : ΔPQR is an equilateral triangle (Given)

Solution :

$PQ = QR = PR$ ...(i) [sides of an equilateral triangle]

In ΔPTS, $\angle PTS = 90^\circ$ ...(Construction)

$\therefore$ $PS^2 = PT^2 + ST^2$ ...(ii) (By Pythagoras theorem)

In ΔPTQ,

$\angle PTQ = 90^\circ$ ...(Construction)

$\angle PQT = 60^\circ$ ...(angle of an equilated triangle)

$\angle QPT = 30^\circ$ ...(remaining angle)

$\therefore$ ΔPTQ is a $30^\circ - 60^\circ - 90^\circ$ triangle

By 30° – 60° – 90° triangle theorem,

$\therefore$ PT $= \dfrac{\sqrt{3}}{2}$ PQ ...(iii) (side opposite to 60°)

$\therefore$ QT $= \dfrac{1}{2}$ PQ ...(iv) (Side opposite to 30°)

ST = QT – QS ...(Q – S – T)

$\therefore$ ST $= \dfrac{1}{2}$ PQ $- \dfrac{1}{3}$ QR ...[From (iv) and given]

$\therefore$ ST $= \dfrac{1}{2}$ PQ $- \dfrac{1}{3}$ PQ ...[From (i)]

$\therefore$ ST $= \dfrac{3PQ - 2PQ}{6}$

$\therefore$ ST $= \dfrac{1}{6}$ PQ ...(v)

$\therefore$ PS$^2 = \left(\dfrac{\sqrt{3}}{2} PQ\right)^2 + \left(\dfrac{1}{6} PQ\right)^2$ ...[From (ii) (iii) and (v)]

$\therefore$ PS$^2 = \dfrac{3PQ^2}{4} + \dfrac{PQ^2}{36}$

$\therefore$ PS$^2 = \dfrac{27PQ^2 + PQ^2}{36}$

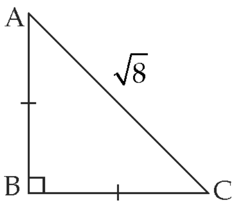
$\therefore$ PS$^2 = \dfrac{28PQ^2}{36}$

$\therefore$ PS$^2 = \dfrac{7}{9}$ PQ^2

$\therefore$ **9 PS^2 = 7 PQ^2**

Practice Set - 2.1 *(Textbook Page No. 39)*

(5) **For finding AB and BC with the help of information in adjoining figure, complete the following activity.** *(2 marks)*



Solution :

AB = BC ...(Side opposite to congruent angle)

$\therefore$ $\angle$BAC = $\boxed{45^\circ}$

$\therefore$ AB = BC $= \boxed{\dfrac{1}{\sqrt{2}}} \times$ AC

$= \boxed{\dfrac{1}{\sqrt{2}}} \times \sqrt{8}$

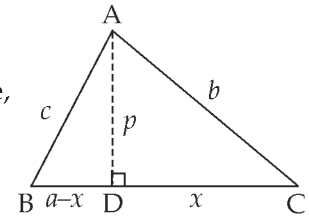
$= \boxed{\dfrac{1}{\sqrt{2}}} \times 2\sqrt{2}$

$\therefore$ **AB = BC = 2 units**

Points to Remember:

Application of Pythagoras Theorem

(1) In acute angled ΔABC, $\angle$C is an acute angle, seg AD $\perp$ side BC, B - D - C.



Prove that :

AB^2 = BC^2 + AC^2 – 2BC · BD

Proof :

In AB = c, AC = b, AD = p, BC = a and DC = x

$\therefore$ BD = $a - x$

In ΔADB,

$c^2 = (a - x)^2 + p^2$ ...(By Pythagoras theorem)

$\therefore$ $c^2 = a^2 - 2ax + x^2 + p^2$ ...(i)

In ΔADC,

$b^2 = p^2 + x^2$ ...(By Pythagoras theorem)

$\therefore$ $p^2 = b^2 - x^2$ ...(ii)

Substituting (ii) in (i)

$c^2 = a^2 - 2ax + x^2 + b^2 - x^2$

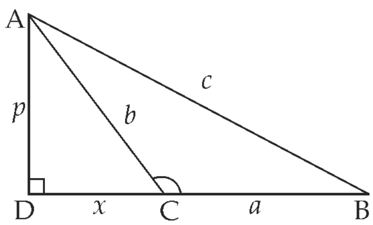
$\therefore$ $c^2 = a^2 + b^2 - 2ax$

$\therefore$ AB^2 = BC^2 + AC^2 – 2BC × DC

(2) In ΔABC, $\angle$ACB > 90°, seg AD $\perp$ seg BC

Prove that :

AB^2 = BC^2 + AC^2 + 2BC × CD



Proof :

Let AD = p, AC = b, AB = c, BC = a and DC = x

$\therefore$ DB = $a + x$

In ΔADB,

$c^2 = (a + x)^2 + p^2$ ...(By Pythagoras theorem)

$\therefore$ $c^2 = a^2 + 2ax + x^2 + p^2$ ...(i)

Similarly, In ΔADC,

$b^2 = x^2 + p^2$

$\therefore$ $p^2 = b^2 - x^2$ ...(ii)

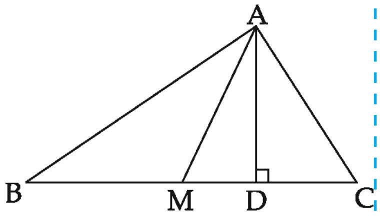
Substituting (ii) in (i),

$c^2 = a^2 + 2ax + x^2 + b^2 - x^2$

$\therefore$ $= a^2 + 2ax + b^2$

$\therefore$ AB^2 = BC^2 + AC^2 + 2BC × CD

(3) The Apollonius Theorem.



Theorem : **In ΔABC, it M is a midpoint of BC. then $AB^2 + AC^2 = 2AM^2 + 2BM^2$.**

Given : In ΔABC, M is midpoint of side BC.

To prove : $AB^2 + AC^2 = 2AM^2 + 2BM^2$

Construction : Draw seg AD $\perp$ seg BC

Proof : If seg AM is not perpendicular to side BC. Then $\angle AMB$ and $\angle AMC$ are either acute angle or obtuse angle.

According to figure, $\angle AMB$ is on obtuse angle and $\angle AMC$ is an acute angle. According examples (1) and (2) above.

$AB^2 = AM^2 + MB^2 + 2BM \times MD$ ...(i)

and $AC^2 = AM^2 + MC^2 - 2MC \times MD$

$\therefore$ $AC^2 = AM^2 + MB^2 - 2BM \times MD$ ...(ii)

($\because BM = MC$)

adding (i) and (ii) we get,

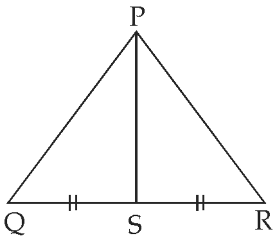
$AB^2 + AC^2 = 2AM^2 + 2MB^2$

This is called Apollonius theorem.

Practice Set - 2.2 *(Textbook Page No. 43)*

(1) In ΔPQR, point S is the midpoint of side QR. If PQ = 11, PR = 17, PS = 13 then find QR.

(2 marks)



Solution :

In ΔPQR, PS is the median ...(By Definition)

$\therefore$ $PQ^2 + PR^2 = 2PS^2 + 2QS^2$ ...(Apollonius theorem)

$\therefore$ $11^2 + 17^2 = 2 \times (13^2 + QS^2)$

$\therefore$ $121 + 289 = 2(169 + QS^2)$

$\therefore$ $\frac{410}{2} = 169 + QS^2$

$\therefore$ $205 - 169 = QS^2$

$\therefore$ $QS^2 = 36$

$\therefore$ $QS = 6$ units ...(Taking square roots)

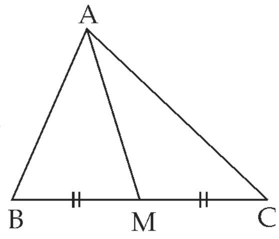
$QR = 2\,QS$ ...($\because$ S is the midpoint of seg QR, given)

$\therefore$ $QR = 2 \times 6$

$\therefore$ $\boxed{QR = 12 \text{ units}}$

(4) In ΔABC, point M is midpoint of side BC. If $AB^2 + AC^2 = 290$ cm² and AM = 8 cm, find BC.

(2 marks)



Solution :

In ΔABC, seg AM is median ...(Given)

$\therefore$ $AB^2 + AC^2 = 2AM^2 + 2BM^2$ ...(Apollonius theorem)

$\therefore$ $290 = 2(8^2 + BM^2)$

$\therefore$ $\frac{290}{2} = 64 + BM^2$

$\therefore$ $145 - 64 = BM^2$

$\therefore$ $BM^2 = 81$

$\therefore$ $BM = 9$ units ...(Taking square roots)

$BC = 2\,BM$ ...($\because$ M is the midpoint of seg BC)

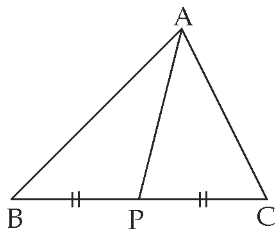
$\therefore$ $BC = 2 \times 9$

$\therefore$ $\boxed{BC = 18 \text{ cm}}$

Problem Set - 2 *(Textbook Pg No. 44)*

(6) In ΔABC, seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$. Find AP. *(3 marks)*

Solution :



In ΔABC, seg AP is the median on side BC

$\therefore$ $BP = \frac{1}{2} \times BC$

$\therefore$ $BP = \frac{1}{2} \times 18$

$\therefore$ $BP = 9$ units ...(i)

In ΔABC, seg AP is median ...(Given)

$\therefore$ $AB^2 + AC^2 = 2BP^2 + 2AP^2$ ...(Apollonius theorem)

$\therefore$ $260 = 2(9^2 + AP^2)$ ...[From (i)]

$\therefore$ $\frac{260}{2} = 81 + AP^2$

$\therefore$ $AP^2 = 130 - 81$

$\therefore$ $AP^2 = 49$

$\therefore$ $\boxed{AP = 7 \text{ units}}$ ...(Taking square roots)

Practice Set - 2.2 *(Textbook Page No. 43)*

(2) In ΔABC, AB = 10, AC = 7, BC = 9. Find the length of the median drawn from point C to side AB.

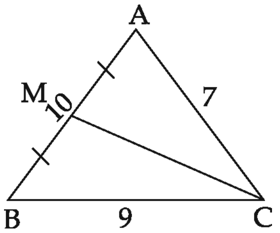
Given :

(i) In ΔABC,

seg CM is a median

(ii) AB = 10, AC = 7,

BC = 9



To Find : CM. *(3 marks)*

Solution :

$\therefore$ $BM = \frac{1}{2} \times AB$ ...(M is the midpoint of seg AB)

$\therefore$ $BM = \frac{1}{2} \times 10$

$\therefore$ BM = 5 units ...(i)

In ΔABC, CM is the median ...(Given)

$\therefore$ $AC^2 + BC^2 = 2CM^2 + 2BM^2$

...(Apollonius theorem)

$\therefore$ $7^2 + 9^2 = 2(CM^2 + 5^2)$

$\therefore$ $49 + 81 = 2(CM^2 + 25)$

$\therefore$ $\frac{130}{2} = CM^2 + 25$

$\therefore$ $CM^2 = 65 - 25$

$\therefore$ $CM^2 = 40$

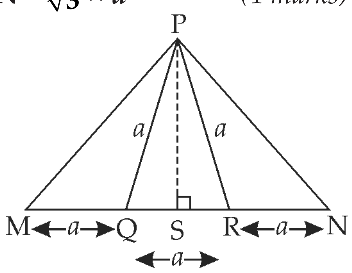
$\therefore$ $CM = \sqrt{40}$ ...(Taking square roots)

$\therefore$ $CM = \sqrt{4 \times 10}$

$\therefore$ $\boxed{\mathbf{CM = 2\sqrt{10}\ units}}$

Problem Set - 2 *(Textbook Pg No. 44)*

(8) From the information given in the figure, Prove that : $PM = PN = \sqrt{3} \times a$ *(4 marks)*



Proof :

MQ = QR = RN = a ...(Given)

Point Q is the midpoint of seg MR ...(i)

$\therefore$ In ΔPMR, seg PQ is a median

...[From (i), Definition]

$\therefore$ $PM^2 + PR^2 = 2PQ^2 + 2QM^2$

...(Apollonius theorem)

$\therefore$ $PM^2 + a^2 = 2a^2 + 2a^2$

$\therefore$ $PM^2 = 4a^2 - a^2$

$\therefore$ $PM^2 = 3a^2$

$\therefore$ $\boxed{\mathbf{PM = \sqrt{3}\ a}}$ ...(Taking square roots)

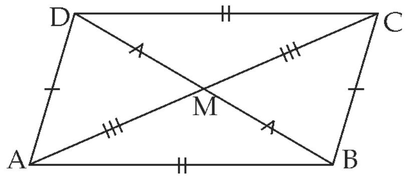
Similarly we can prove, $PN = \sqrt{3}a$

$\therefore$ $\boxed{PM = PN = \sqrt{3}a}$

(9) Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Given :

In $\square$ABCD, is a parallelogram Diagonals AC and BD intersect each other at point M.



To Prove :

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$ *(5 mark)*

Proof :

$\square$ABCD is a parallelogram ...(Given)

$AM = CM = \frac{1}{2} AC$ ...(i)

$BM = DM = \frac{1}{2} BD$ ...(ii)

[Diagonals of a parallelogram bisect each other]

$\therefore$ In ΔABC, seg BM is a median ...[From (i)]

$\therefore$ $AB^2 + BC^2 = 2BM^2 + 2AM^2$ ...(iii) (By Apollonius theorem)

$\therefore$ In ΔADC, seg DM is a median ...[From (i)]

$\therefore$ $CD^2 + AD^2 = 2DM^2 + 2AM^2$ ...(iv) (By Apollonius theorem)

Adding (iii) and (iv)

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 2BM^2 + 2AM^2 + 2DM^2 + 2AM^2$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 2BM^2 + 2DM^2 + 4AM^2$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 2BM^2 + 2BM^2 + 4AM^2$

...[From (ii)]

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4BM^2 + 4AM^2$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4\,[BM^2 + AM^2]$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4\left[\left(\frac{1}{2}BD\right)^2 + \left(\frac{1}{2}AC\right)^2\right]$

...[From (i) and (ii)]

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = 4\left[\frac{1}{4}BD^2 + \frac{1}{4}AC^2\right]$

$\therefore$ $= 4 \times \frac{1}{4}\,[BD^2 + AC^2]$

$\therefore$ $AB^2 + BC^2 + CD^2 + AD^2 = BD^2 + AC^2$

$\therefore$ $\boxed{\mathbf{BD^2 + AC^2 = AB^2 + BC^2 + CD^2 + AD^2}}$

(12) Sum of squares of adjacent sides of a parallelogram is 130 cm² and length of one of its diagonal is 14 cm. Find length of the other diagonal.

Given :

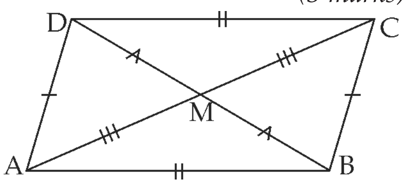
(i) □ABCD is a parallelogram

(ii) $AB^2 + BC^2 = 130 \text{ cm}^2$

(iii) AC = 14 cm

To find : BD *(3 marks)*

Solution :



□ABCD is a parallelogram ...(Given)

$\therefore$ $BM = \frac{1}{2}BD$ ...(i)

$\therefore$ $AM = \frac{1}{2}AC$ ...(ii)

[Diagonals of a parallelogram bisect each other]

$\therefore$ $AM = \frac{1}{2} \times 14 = 7\text{cm}$

In ΔABC, seg BM is the a median ...[From (ii) and Definition]

$\therefore$ $AB^2 + BC^2 = 2AM^2 + 2BM^2$...(Apollonius theorem)

$130 = 2(7^2 + BM^2)$

$\therefore$ $\frac{130}{2} = 49 + BM^2$

$\therefore$ $65 - 49 = BM^2$

$\therefore$ $BM^2 = 16$

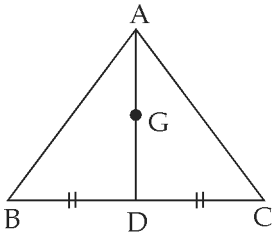
$\therefore$ $BM = 4$ cm ...(Taking square roots)

$\therefore$ $\frac{1}{2}BD = 4\text{cm}$ ...[From (i)]

$\therefore$ $\boxed{BD = 8 \text{ cm}}$

***(14) In an isosceles triangle, length of each congruent side is 13 cm and length of the base is 10 cm. Find the distance between vertex opposite to base and centroid.**

Given :



(i) In ΔABC, is an isosceles triangle

(ii) AB = AC = 13 cm, BC = 10 cm

(iii) seg AD is the median

(iv) G is the centroid of ΔABC

To find : AG *(3 marks)*

Solution :

$BD = \frac{1}{2}BC$ ...(Median bisects opposite side)

$\therefore$ $BD = \frac{1}{2} \times 10 = 5$ cm ...(i)

In ΔABC, seg AD is a median ...(Definition)

$\therefore$ $AB^2 + AC^2 = 2AD^2 + 2BD^2$...(Apollonius theorem)

$\therefore$ $13^2 + 13^2 = 2(AD^2 + 5^2)$ ...[From (i) and given]

$\therefore$ $169 + 169 = 2(AD^2 + 25)$

$\therefore$ $\frac{338}{2} = AD^2 + 25$

$\therefore$ $169 - 25 = AD^2$

$\therefore$ $144 = AD^2$

$\therefore$ $AD = 12$ cm ...(Taking square roots)

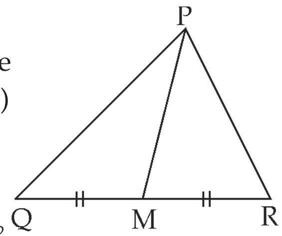
$AG = \frac{2}{3}AD$ ...(Centroid divides each median in the ratio 3 : 1)

$\therefore$ $AG = \frac{2}{3} \times 12$

$\therefore$ $\boxed{AG = 8 \text{ cm}}$

***(17) Seg PM is a median of ΔPQR. If PQ = 40, PR = 42 and PM = 29, find QR.** *(3 marks)*

Solution :



In ΔPQR, seg PM is the median ...(Given)

$\therefore$ $PQ^2 + PR^2 = 2PM^2 + 2QM^2$ ...(Appollonius theorem)

$\therefore$ $40^2 + 42^2 = 2(29)^2 + 2(QM)^2$

$\therefore$ $(40)^2 + (42)^2 = 2(29^2 + QM^2)$

$\therefore$ $1600 + 1764 = 2(841 + QM^2)$

$\therefore$ $\frac{3364}{2} = 841 + QM^2$

$\therefore$ $1682 - 841 = QM^2$

$\therefore$ $QM^2 = 841$

$\therefore$ $QM = 29$ ...(Taking square roots)

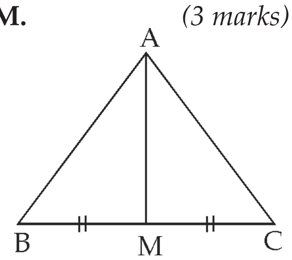
$QR = 2QM$ ...(M is midpoint of seg QR)

$\therefore$ $QR = 2 \times 29$

$\therefore$ $\boxed{QR = 58 \text{ units}}$

(18) Seg AM is a median of ΔABC. If AB = 22, AC = 34, BC = 24, find AM. *(3 marks)*

Solution :



In ΔABC

$\therefore$ $BM = \frac{1}{2}BC$ ...(M is the midpoint of BC)

$\therefore$ $BM = \frac{1}{2} \times 24 = 12$ units ...(i)

In ΔABC, seg AM is the median ...(Given)

$\therefore$ $AB^2 + AC^2 = 2AM^2 + 2BM^2$ ...(By Apollonius theorem)

$\therefore$ $22^2 + 34^2 = 2(AM^2 + BM^2)$

$\therefore$ $484 + 1156 = 2\,(AM^2 + 12^2)$

$\therefore$ $\dfrac{1640}{2} = AM^2 + 144$

$\therefore$ $820 - 144 = AM^2$

$AM^2 = 676$

$\therefore$ $\boxed{\text{AM} = 26 \text{ units}}$ ...(Taking square roots)

Problem Set - 2 *(Textbook Page No. 43)*

MCQ's

Choose the correct alternative for each of the following. *(1 mark each)*

(1) Out of the following which is the Pythagorean triplet?

(A) (1, 5, 10) (B) (3, 4, 5) (C) (2, 2, 2) (D) (5, 5, 2)

(2) In a right angled triangle, if sum of the squares of the sides making right angle is 169 then what is the length of the hypotenuse?

(A) 15 (B) 13 (C) 5 (D) 12

(3) Out of the dates given below which date constitutes a Pythagorean triplet?

(A) 15/08/17 (B) 16/08/16

(C) 03/05/17 (D) 04/09/15

(4) If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle.

(A) Obtuse angled triangle

(B) Acute angled triangle

(C) Right angled triangle

(D) Equilateral triangle

(5) Find perimeter of a square if its diagonal is $10\sqrt{2}$ cm.

(A) 10 cm (B) $40\sqrt{2}$ (C) 20 cm (D) 40 cm

(6) Altitude on the hypotenuse of a right angle triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.

(A) 9 cm (B) 4 cm (C) 6 cm (D) 18 cm

(7) Height and base of a right angled triangle are 24 cm and 18 cm, find the length of its hypotenus.

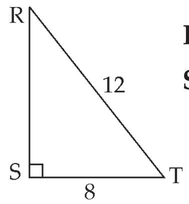
(A) 24 cm (B) 30 cm (C) 15 cm (D) 18 cm

(8) In ΔABC AB = $6\sqrt{3}$ cm, AC = 12 cm, BC = 6 cm. Find measure of $\angle A$.

(A) 30° (B) 60° (C) 90° (D) 45°

Additional MCQ's

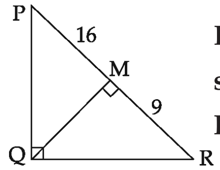
(9)



In ΔRST, $\angle S = 90°$, RT = 12 m, ST = 8 m then RS = ...

(A) $10\sqrt{8}$ m (B) $5\sqrt{4}$ m (C) $4\sqrt{5}$ m (D) 5m

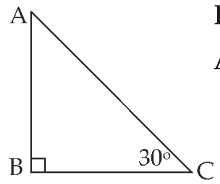
(10)



In ΔPQR, $\angle PQR = 90°$, seg QM $\perp$ hyp PR, PM = 16 and RM = 9 then QM = ...

(A) 12 (B) 25 (C) 7 (D) 16 × 9

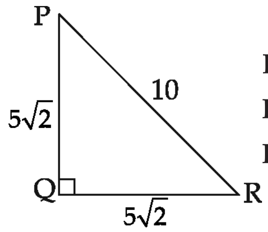
(11)



In ΔABC, $\angle B = 90°$, $\angle C = 30°$, AB = 6cm then AC = ...

(A) $3\sqrt{3}$ cm (B) $2\sqrt{3}$ cm (C) $12\sqrt{3}$ cm (D) 12cm

(12)



In ΔPQR, $\angle Q = 90°$, PQ = QR = $5\sqrt{2}$ PR = 10 then $\angle P$ ....

(A) 30° (B) 45°

(C) 60° (D) Data not sufficient

(13) Which of the following is a Pythagorean triplet?

(a) 60, 61, 11 (b) 40, 41, 42

(c) 11, 12, 15 (d) 9, 15, 17

(14) In ΔQSR, m$\angle Q$ = 45°, m$\angle S$ = 90° and SR = 4, find QS.

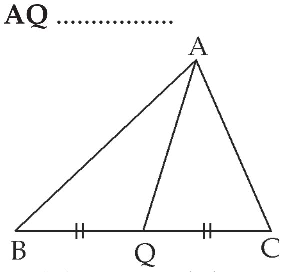
(A) 3 (B) 4 (C) 5 (D) 6

(15) Appollonius theorem is a theorem relating the length of .... of a triangle.

(A) Altitude (B) Angle bisector

(C) Perpendicular bisector (D) Median and sides

(16) In the adjoining figure, $AB^2 + AC^2 = 122$, BC = 10, then find AQ ....



(A) 3 (B) 6 (C) 12 (D) 36

(17) **In ΔPQR, $m\angle PQR = 90°$, seg $QS \perp$ hyp PR then.**

(A) $QS^2 = PS \times RS$ (B) $PS^2 = QS \times PR$

(C) $PR^2 = QS \times PS$ (D) $PR^2 = QS^2 \times PS^2$

(18) **In which of the following quadrilateral sum of squares of all sides is equal to the sum of squares of diagonals?**

(A) Parallelogram (B) Rhombus

(C) Square (D) (A), (B) and (C)

ANSWERS

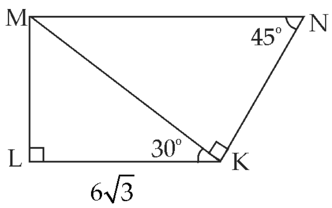
(1)	(B) (3, 4, 5)	(2)	(B) 13
(3)	(A) 15/08/17	(4)	(C) Right angled
(5)	(D) 40cm	(6)	(C) 6cm
(7)	(B) 30cm	(8)	(A) 30°
(9)	(C) $4\sqrt{5}$ m	(10)	(A) 12
(11)	(D) 12 cm	(12)	(B) 45°
(13)	(A) (60, 61, 11)	(14)	(B) 4
(15)	(D) Median and sides	(16)	(B) 6
(17)	(A) $QS^2 = PS \times RS$	(18)	(D) A, B and C

PROBLEMS FOR PRACTICE

Based on Practice Set 2.1

(1) **In ΔXYZ, $\angle Y = 90°$, $\angle Z = a°$, $\angle X = (a + 30°)$. If $XZ = 24$, find XY and YZ.** *(3 marks)*

(2) **In the adjoining figure, $\angle L = \angle MKN = 90°$, $\angle MKL = 30°$ and $\angle MNK = 45°$. If $KL = 6\sqrt{3}$, then find MK, ML, KN, MN and perimeter of $\square MNKL$.** *(3 marks)*



(3) **Sides of triangles are given below. Determine which of them are right angled triangle.** *(2 marks)*

(i) 8, 15, 17 **(ii) 20, 30, 40**

(iii) 11, 12, 15 **(iv) 20, 16, 12**

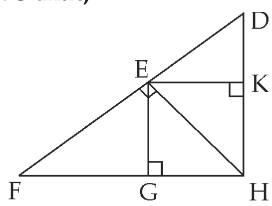
(4) **A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.** *(2 marks)*

(5) **E is a point on hypotenuse DF of ΔDFH, such that seg $HE \perp$ seg DF, seg $EG \perp$ seg FH and seg $EK \perp$ seg DH. Prove that,**

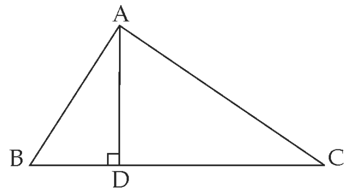
(i) $EG^2 = FG \times EK$

(ii) $EK^2 = DK \times EG$

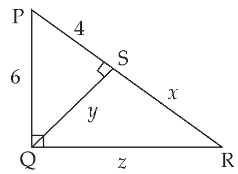
(3 marks)



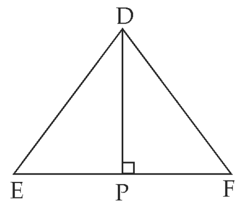
(6) **In adjoining figure, seg $AD \perp$ side BC, B-D-C. Prove that $AB^2 + CD^2 = BD^2 + AC^2$** *(3 marks)*



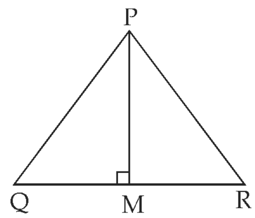
(7) **In the adjoining figure, $\angle PQR = 90°$ seg $QS \perp$ side PR, $PS = 4$, $PQ = 6$. Find x, y and z.** *(3 marks)*



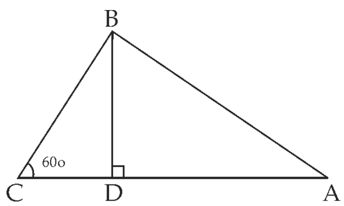
(8) **ΔDEF is an equilateral triangle. seg $DP \perp$ side EF, E-P-F. Prove that : $DP^2 = 3EP^2$** *(3 marks)*



(9) **ΔPQR is an equilateral triangle, seg $PM \perp$ side QR, Q-M-R. Prove that : $PQ^2 = 4QM^2$** *(3 marks)*



(10) **In the adjoining figure, seg $BD \perp$ side AC, C-D-A. Prove that : $AB^2 = BC^2 + AC^2 - BC.AC$** *(3 marks)*



Based on Practice Set 2.2

(11) **In ΔPQR, M is the midpoint of side QR. If $PQ = 11$, $PR = 17$ and $QR = 12$, then find PM.** *(2 marks)*

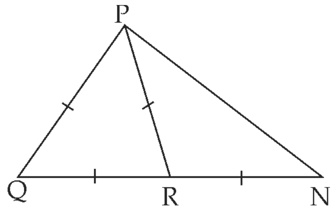
(12) **In ΔABC, AP is a median. If AP = 7, $AB^2 + AC^2 = 260$, find BC.** *(2 marks)*

(13) **In ΔABC, $AB^2 + AC^2 = 122$ and BC = 10. Find the length of the median on side BC.** *(2 marks)*

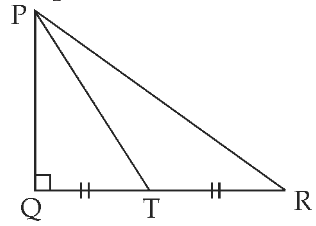
(14) **Adjacent sides of a parallelogram are 11 cm and 17 cm. If the length of one of its diagonals is 26 cm, find the length of the other.** *(3 marks)*

(15) **If 'O' is any point in the interior of rectangle ABCD, then prove that : $OB^2 + OD^2 = OA^2 + OC^2$**

(16) **In the adjoining figure, ΔPQR is an equilateral triangle. QR = RN. Prove that $PN^2 = 3PR^2$** *(3 marks)*



(17) **In the adjoining figure, $\angle PQR = 90^o$. T is the midpoint of side QR. Prove that $PR^2 = 4PT^2 - 3PQ^2$.** *(3 marks)*



ANSWERS

(1) XY = 12, YZ = $12\sqrt{3}$

(2) MK = 12, ML = 6, KN = 12, MN = 12 $\sqrt{2}$, Perimeter of □MNKL = $6(3 + 2\sqrt{2} + \sqrt{3})$

(3) (i) and (iv) are right angled triangle

(4) 6*m* (7) $x = 5$, $y = 2\sqrt{5}$, $z = 3\sqrt{5}$ (11) 13

(12) 18 (13) 6 (14) 12 cm

❖ ❖ ❖

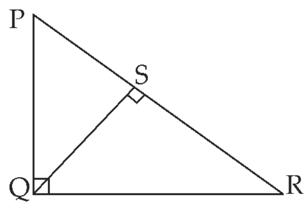
ASSIGNMENT – 2

Time : 1 Hr. **Marks : 20**

Q.1. (A) Solve the following sub questions: **(2)**

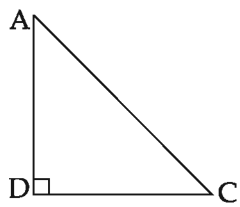
(1) Is 28, 21 and 35 a pythagorean triplet?

(2) In ΔPQR, $\angle PQR = 90°$ seg QS ⊥ hypotenuse PR, PS = 16, RS = 9. Find QS



Q.1. (B) Solve any one of the following questions: **(2)**

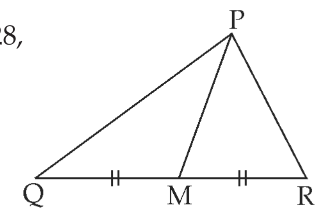
(1) In ΔADC, $\angle ADC = 90°$
$\angle C = 45°$, AC = $8\sqrt{2}$ cm.
Find AD.



(2) In ΔXYZ, $\angle Y = 90°$, $\angle Z = a°$, $\angle X = (a + 30)°$. Find $\angle X$

Q.2. Solve the any one of following sub questions: **(2)**

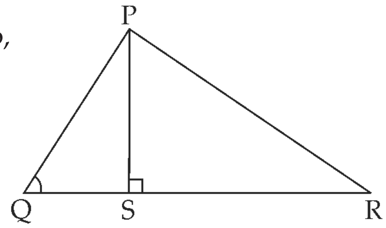
(1) In ΔPQR, seg PM is a median. PM = 10 and $PQ^2 + PR^2 = 328$, then find QR



(2) In m and n are two distinct numbers then prove that $m^2 - n^2$, $2mn$ and $m^2 + n^2$ is a pythagorean triplet.

Q.3. Solve the following sub questions: (any two) **(6)**

(1) In the adjoining figure, seg PS ⊥ side QR. If PQ = a, PR = b, QS = c and RS = d then complete the following activity to prove that $(a + b)(a - b) = (c + d)(c - d)$



Proof: In ΔPSQ, ∠PSQ = 90° ...(Given)

$\therefore \quad \boxed{\quad}^2 = PS^2 + \boxed{\quad}^2$ ...(Pythagoras theorem)

$\therefore \quad PS^2 = \boxed{\quad}^2 - \boxed{\quad}^2$ ...(i)

In ΔPSR, ∠PSR = 90° ...(Given)

$\therefore \quad \boxed{\quad}^2 = PS^2 + \boxed{\quad}^2$ ...(Pythagoras theorem)

$\therefore \quad PS^2 = \boxed{\quad}^2 - \boxed{\quad}^2$ ...(ii)

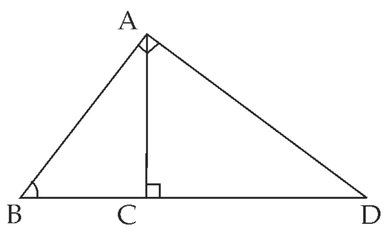
$\therefore \quad \boxed{\quad}^2 - \boxed{\quad}^2 = \boxed{\quad}^2 - \boxed{\quad}^2$ ...[From (i) and (ii)]

$\therefore \quad a^2 - c^2 = b^2 - d^2$ ...(Substitution)

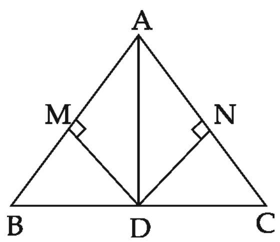
$\therefore \quad a^2 - b^2 = c^2 - d^2$

$\boxed{\quad} \times \boxed{\quad} = \boxed{\quad} \times \boxed{\quad}$

(2) In ΔABD, ∠BAD = 90° seg AC ⊥ hypo BD, B – C – D. Show that (i) $AB^2 = BC : BD$ (ii) $AD^2 = BD : CD$



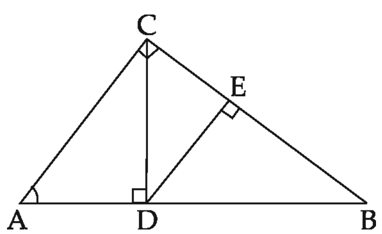
(3) In the adjoining figure AD bisects ∠BAC, B – D – C. AD ⊥ BC, DM ⊥ AB. A – M – B. DN ⊥ AC. A – N – C. Proove that AM × MB = AN × NC



Q.4. Solve the following sub questions: (any 2) **(8)**

(1) State and prove 'Pythagoras theorem'

(2) In ΔACB, ∠ACB = 90° seg CD ⊥ side AB, A – D – B, seg DE ⊥ side CB. Show that $CD^2 \times AC = AD \times AB \times DE$.



(3) In on equilateral triangle ABC, the side BC is triseccted at D. Prove that: $9\,AD^2 = 7\,AB^2$

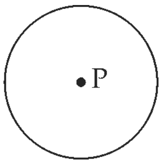
❖ ❖ ❖

3 Circle

... INDEX ...

Points to Remember:

- **Circle :** The set of all points equidistant from a fixed point in a plane is called **circle**.

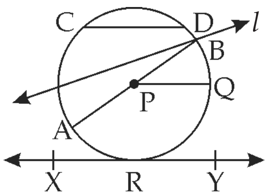


The fixed point is called the **centre** of the circle.

In the above figure, point P is the centre of the circle.

Basic Terms used in circle

- **Radius :** Distance between centre of a circle and any point on the circle is called the **radius**.



Seg PQ, Seg PA and Seg PB are radii.

- **Chord :** A segment whose end points lie on a circle is called the **chord**.

Seg CD and Seg AB are chords.

- **Diameter :** A chord which passes through the centre of the circle is called the **diameter**.

Seg AB is a diameter.

Length of the diameter is double the radius.

- **Tangent :** A line in the plane of a circle which touches the circle exactly in only one point is called **tangent** of the circle.

The point at which the tangent touches the circle is called the **point of contact**.

Line XY is tangent to the circle and point R is the point of the contact.

- **Secant :** A line which intersects the circle in two distinct points is called the **secant**.

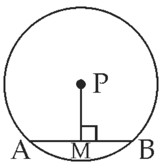
Line *l* is a secant.

Basic concepts related to circles.

(1) 'A Perpendicular segment drawn from the centre of a circle to the chord bisects the chord.'

Given : (1) A circle with centre P.

(2) Seg PM $\perp$ chord AB, A–M–B.

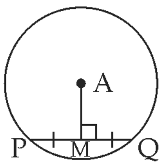


Conclusion : AM = BM.

(2) 'A segment joining centre of a circle and the midpoint of the chord is perpendicular to the chord.'

Given : (1) A circle with centre A.

(2) Point M is midpoint of chord PQ.

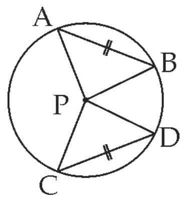


Conclusion: Seg AM $\perp$ chord PQ.

(3) 'In a circle (or in congruent circles), congruent chords subtend congruent angles at the centre.'

Given : (1) A circle with centre P

(2) Chord AB ≅ chord CD

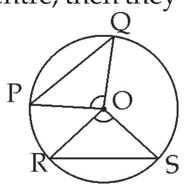


Conclusion: ∠APB ≅ ∠CPD

(4) 'In a circle (or in congruent circles) if two of more chords subtend congruent angles at the centre, then they are congruent.'

Given : (1) A circle with centre O.

(2) ∠POQ ≅ ∠ROS



Conclusion: Chord PQ ≅ chord RS

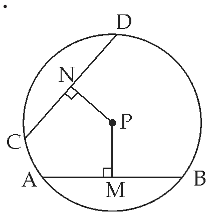
(5) 'In a circle (or in congruent circles), congruent chords are equidistant from the centre.'

Given : (1) A circle with centre P.

(2) Seg PM ⊥ chord AB, A–M–B

(3) Seg PN ⊥ chord CD, C–N–D

(4) Chord AB ≅ chord CD



Conclusion: PM = PN

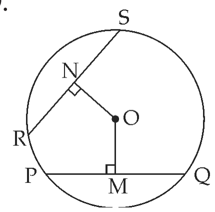
(6) 'In a circle (or in congruent circles), chords which are equidistant from the centre are congruent.'

Given : (1) A circle with centre O.

(2) Seg OM ⊥ chord PQ, P–M–Q

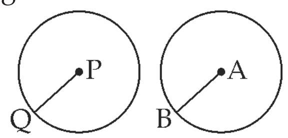
(3) Seg ON ⊥ chord RS, R–N–S

(4) OM = ON



Conclusion: Chord PQ ≅ chord RS

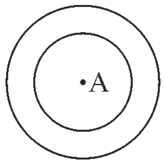
(7) **Congruent circles :** Two or more circles are said to be congruent circles if their radii are equal.



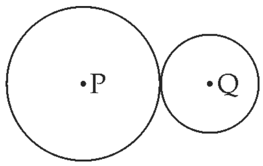
In the above figure, Radius PQ ≅ Radius AB.

∴ Both circles are congruent.

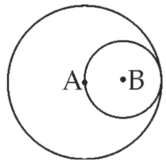
(8) **Concentric circles :** Two or more circles with same centre but different radii are called concentric circles.



(9) **Touching circles :** Two circles in the same plane having only one point in common are called touching circles.

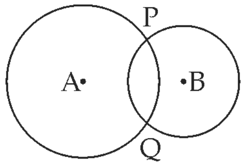


Externally touching circles



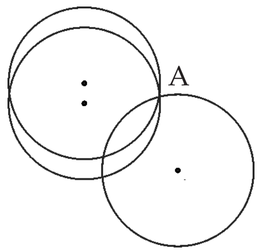
Internally touching circles

(10) **Intersecting circles :** Two circles which have exactly two points in common are called intersecting circles.



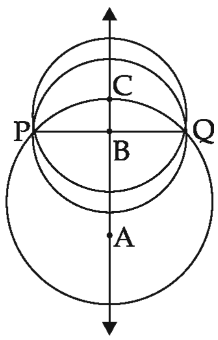
(11) Circles passing through one point. Two points and Three points

(i) We shall first consider one point. There are infinite number of circles passing through a point.



(ii) **Circles passing through two distinct points.**

There can be infinite number of circles passing through two distinct points.



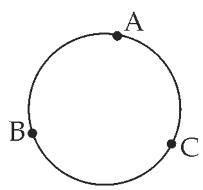
Note : Centres of all these circles will pass through a line containing perpendicular bisector of the segment joining these two points.

(iii) **Circles passing through three points.**

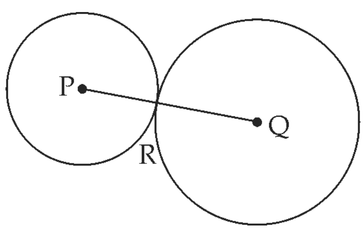
Here, arises to cases.

Case (I): When three points are non collinear.

We can draw exactly one circle passing through three non-collinear points.

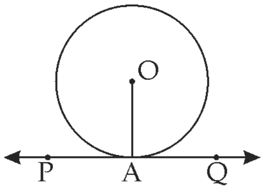


Case (II): When three points are collinear.



We can not draw any circle passing through three collinear points.

(12) **Tangent theorem :** A tangent at any point of a circle is perpendicular to the radius, through the point of contact.



In above figure,

line PQ is a tangent at A and

Seg AO is the radius through

the point of contact A.

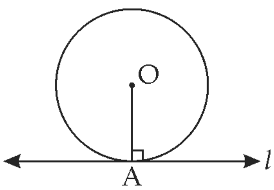
$\therefore$ **seg OA $\perp$ line PQ.**

(13) **Converse of tangent theorem :**

A line perpendicular to a radius of a circle at its outer end is a tangent to the circle.

In adjoining figure, line l is perpendicular to radius OA at its outer end A.

$\therefore$ line l is a tangent.



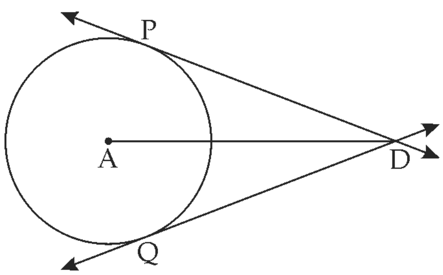
(14) **Tangent segment Theorem :**

Statement : The lengths of the two tangent segments to a circle drawn from an external point are equal.

Given : (i) A circle with centre A.

(ii) D is a point in the exterior of the circle.

(iii) Points P and Q are the points of contact of the two tangents from D to the circle.



In ΔPAD and ΔQAD,

Seg PA $\cong$ $\boxed{\text{AQ}}$ ...(radii of the same circle)

Seg AD $\cong$ Seg AD ...(Common side)

$\angle$APD $\cong$ $\angle$AQD = 90° ...(by Tangent theorem)

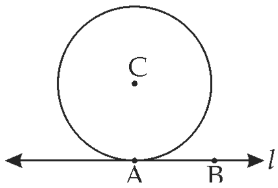
$\therefore$ ΔPAD $\cong$ ΔQAD ...(by Hypotenuse side test)

$\therefore$ Seg DP $\cong$ Seg DQ ...(c.s.c.t.)

MASTER KEY QUESTION SET - 3

Practice Set - 3.1 *(Textbook Page No. 55)*

(1) In the adjoining figure, the radius of a circle with centre C is 6 cm, Line AB is a tangent at A? Answer the following questions.



(i) What is the measure of $\angle$CAB? Why?

(ii) What is the distance of point C from line AB? Why?

(iii) d(A, B) = 6 cm, find d(B, C).

(iv) What the measure of $\angle$ABC? Why? . *(3 marks)*

Solution :

(i) Radius CA $\perp$ Line AB ...(Tangent Theorem)

$\therefore$ m$\angle$CAB = 90° ... (i)

(ii) d(C, A) = 6 cm ...(Radius of circle)

$\therefore$ Distance of point C from line AB is 6 cm.

(iii) In ΔCAB, $\angle$CAB = 90° ...[From (i)]

$\therefore$ $BC^2 = AC^2 + AB^2$ ...(By Pythagoras theorem)

$= 6^2 + 6^2$

$BC^2 = 36 + 36 = 72$

$\therefore$ BC $= 6\sqrt{2}$ cm ...(Taking square roots)

(iv) In ΔABC, $\angle$A = 90° ...[From (i)]

AC = AB ...(Given)

$\therefore$ $\angle$ACB = $\angle$ABC ... (ii) (converse of isosceles triangle theorem)

In ΔCAB,

$\angle$ABC + $\angle$ACB + $\angle$CAB + = 180°

...(Sum of all angles of a triangle is 180°)

$\therefore$ $\angle$ABC +$\angle$ABC + 90 = 180 ...[From (i) and (ii)]

$\therefore$ 2$\angle$ABC = 180 – 90

$\therefore$ 2$\angle$ABC = 90

$\therefore$ $\boxed{\angle ABC = 45^\circ}$

(4) What is the distance between two parallel tangents of a circle having radius 4.5 cm. Justify your answer. *(2 marks)*

Given :

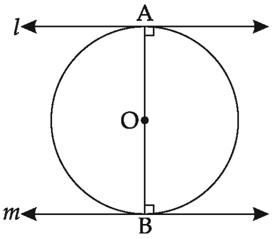
(i) A circle with centre O and radius 4.5 cm.

(ii) Line l is tangent to the circle at point A

(iii) Line m is tangent to the circle at point B

(iv) **Line $l \parallel$ line m**

To find : Distance between line l and line m



Solution :

seg OA $\perp$ line l ...(i)
seg OB $\perp$ line m ...(ii) } (Tangent Theorem)

line $l \parallel$ line m ...(iii) ...(Given)

$\therefore$ A-O-B ...[From (i), (ii) and (iii)]

$\therefore$ AB = AO + BO

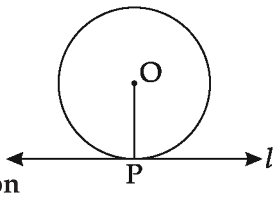
$\therefore$ AB = 4.5 + 4.5

$\therefore$ AB = 9 cm

$\therefore$ **Distance between line l and line m is 9 cm.**

Problem Set - 3 *(Textbook Pg No. 83)*

(2) **Line l touches the circle with centre O at P ; radius of the circle is 9 cm. Answer the following.**



(i) **Find d(O, P) = write reason**

(ii) **d(O, Q) = 8 cm. Where does the point Q lie?**

(iii) **d(O,R) = 15 How many such 'R' contained in line l. What is the distance of those points from 'R'?** *(3 marks)*

Solution :

(i) Radius of the circle is 9 cm.

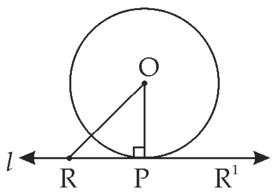
$\therefore$ d(O, P) = 9 cm

(ii) d(O, Q) = 8 cm

d(Q, O) < Radius

$\therefore$ Point Q lies in the interior of the circle.

(iii) Point can have two different positions on line l as shown in the adjoining figure.



In ΔOPR, $\angle$OPR = 90° ...(Tangent Theorem)

$\therefore$ $OR^2 = OP^2 + PR^2$ ...(Pythagoras theorem)

$\therefore$ $15^2 = 9^2 + PR^2$

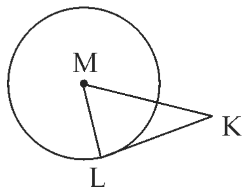
$\therefore$ $PR^2 = 225 - 81 = 144$

$\therefore$ $PR^2 = 144$

$\therefore$ **PR = 12 units** ...(Taking square roots)

$\therefore$ Two such 'R' contained in line l

(3) **In the adjoining figure, M is the centre of the circle and seg KL is a tangent segment. If MK = 12, KL = $6\sqrt{3}$, then**



(i) **Find radius of the circle.**

(ii) **Find measure of $\angle$K and $\angle$M.** *(3 marks)*

Solution :

(i) In ΔMLK, $\angle$MLK = 90^0 ...(Tangent and radius $\perp$ at the point of contact) (i)

$\therefore$ $MK^2 = ML^2 + LK^2$ ...(Pythagoras theorem)

$\therefore$ $12^2 = ML^2 + \left(6\sqrt{3}\right)^2$

$\therefore$ $144 = ML^2 + 108$

$\therefore$ $ML^2 = 144 - 108$

$\therefore$ $ML^2 = 36$

$\therefore$ ML = 6 units ...(ii) (Taking square roots)

$\therefore$ **Radius of the circle is 6 units.**

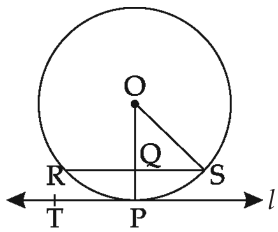
(ii) In ΔMLK, $\angle$MLK = 90° ...[From (i)]

$ML = \dfrac{1}{2}$ MK ...[From (ii) and given]

$\therefore$ $\boxed{\angle K = 30^\circ}$ ...(Converse of 30^0-60°-90° theorem)

$\therefore$ $\boxed{\angle M = 60^\circ}$ ...(Sum of all angles of a triangle is 180°)

(9) **In the adjoining figure, line l touches the circle at P. O is the centre. Q is the mid point of radius OP. Chord RS $\parallel$ line l. RS = 12, find radius of the circle.** *(2 marks)*



Solution :

Take a point T on line l as shown in the figure.

$\angle$OPT = 90° ... (i) (Tangent Theorem)

chord RS $\parallel$ line l ...(Given)

$\therefore$ $\angle$OPT $\cong$ $\angle$OQR ... (ii) (Corresponding angles theorm)

$\therefore$ $\angle$OQR = 90° ... (iii) [From (i) and (ii)]

$\therefore$ seg OQ $\perp$ chord RS ... [From (iii)]

$\therefore$ $QR = \dfrac{1}{2}$ RS ...(Perpendicular drawn from the centre of the circle to the chord bisects the chord.)

$\therefore$ $QR = \dfrac{1}{2} \times 12$

$\therefore$ QR = 6 units

Let the radius of the circle be x units.

$\therefore$ OR = OP = x units ...(Radii of the same circle)

$\therefore$ OQ = $\frac{1}{2}$ OP ($\because$ Q is midpoint of seg OP)

$\therefore$ OQ = $\frac{1}{2} \times x$

$\therefore$ OQ = $\frac{x}{2}$

In ΔOQR,

$\angle$OQR = 90° ... [From (iii)]

$\therefore$ OR² = OQ² + QR² ...(By Pythagoras theorem)

$\therefore \quad x^2 = \left(\frac{x}{2}\right)^2 + (6)^2$

$\therefore \quad x^2 = \frac{x^2}{4} + 36$

$\therefore \quad 4x^2 = x^2 + 144$ ...(Multiplying throughout by 4)

$\therefore \quad 4x^2 - x^2 = 144$

$\therefore \quad 3x^2 = 144$

$\therefore \quad x^2 = \frac{144}{3}$

$\therefore \quad x^2 = 48$

$\therefore \quad x = \sqrt{48}$ ...(taking square roots)

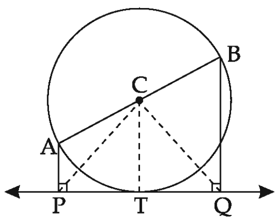
$\therefore \quad x = \sqrt{16 \times 3}$

$\therefore \quad x = 4\sqrt{3}$

$\therefore$ OR = OP = $4\sqrt{3}$ units

$\therefore$ $\boxed{\text{Radius of the circle is } 4\sqrt{3} \text{ units.}}$

(10) In the adjoining figure, seg AB is a diameter of a circle with centre C. Line PQ is a tangent, it touches the circle at T. segs AP and BQ are perpendiculars to line PQ. Prove seg CP ≅ seg CQ. *(2 marks)*



Construction :

Draw seg CT, seg CP and seg CQ.

Proof :

seg AP ⊥ line PQ ...(i) (Given)

seg CT ⊥ line PQ ...(ii) (Tangent Theorem

seg BQ ⊥ line PQ ...(iii) (Given)

$\therefore$ seg AP || seg CT || seg BQ ...[Perpendiculares drawn to the same line are parallel to each other from (i), (ii) and (iii)]

On transversals AB and PQ,

$\frac{PT}{QT} = \frac{AC}{BC}$ ...(iv) [Property of intercepts made by three parallel lines]

But, AC = BC ...(Radii of the same circle)

$\therefore \quad \frac{AC}{BC} = 1$ ...(v)

$\therefore \quad \frac{PT}{QT} = 1$ ...(vi) ... [From (iv) and (v)]

$\therefore$ PT = QT ...(vi)

In ΔPTC and ΔQTC,

seg CT ≅ seg CT ...(Common side)

$\angle$CTP ≅ $\angle$CTQ ...[Each is 90° from (ii)]

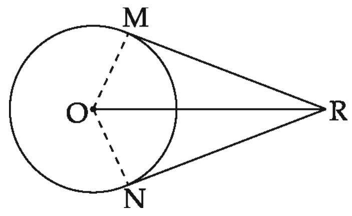
seg PT ≅ seg QT ... [From (vi)]

$\therefore$ ΔPTC ≅ ΔQTC ...(By SAS test of congruency)

$\therefore$ seg CP ≅ seg CQ ...(c.s.c.t.)

Practice Set - 3.1 *(Textbook Page No. 55)*

(2) In the adjoining figure, O is the centre of the circle. From point R, Seg RM and RN are tangent segments, touch the circle at M, N. (O,R) = 10 cm, radius of the circle = 5 cm, then find *(2 marks)*



(i) the length of each tangent segment?

(ii) Measure of $\angle$MRO?

(iii) Measure of $\angle$MRN

Construction : Draw seg OM and seg ON

Solution :

Radius OM ⊥ tangent RM ... (i) (Tangent Theorem)

In ΔOMR, $\angle$OMR = 90° [From (i)]

$\therefore \quad OR^2 = OM^2 + RM^2$

$\therefore \quad 10^2 = 5^2 + RM^2$

$\therefore \quad RM^2 = 100 - 25$

$\therefore \quad RM^2 = 75$

$\therefore \quad RM = 5\sqrt{3}$ cm ... (ii) (Taking square roots)

$\therefore$ MR = RN (Tangent segments Theorem)

$\therefore$ $\boxed{RM = 5\sqrt{3} \text{ cm}}$ ... [From (ii)]

In ΔOMR, $\angle$OMR = 90° [From (i)]

$\therefore \quad OM = \frac{1}{2} OR$ (Given)

$\therefore$ $\boxed{\angle MRO = 30°}$ ... (iii) (Converse of 30° – 60° – 90°)

Similarly we can prove,

$\angle$NRO = 30° ... (iv)

$\angle$MRN = $\angle$MRO + $\angle$NRO ...(Angle addition property)

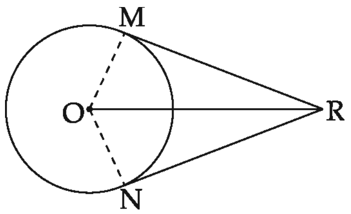
= 30° + 30°

$\therefore$ $\boxed{\angle MRN = 60°}$

(3) In the figure, Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR divides ∠MRN as well as ∠MON.

(2 marks)

Construction : Draw seg OM and seg ON



Proof :

In ΔOMR and ΔONR,

(i) ∠OMR = ∠ONR = 90° ...(Tangent Theorem)

(ii) seg OR ≅ seg OR (Common side)

(iii) seg OM ≅ seg ON (Radii of same circle)

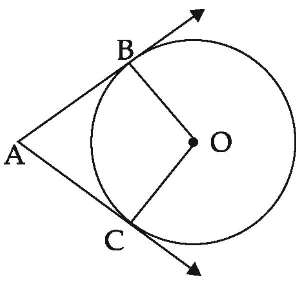
∴ ΔOMR ≅ ΔONR [Hypotenuse – side test]

∴ ∠MOR ≅ ∠NOR ... (i)
∴ ∠MRO ≅ ∠NRO ... (ii) } [c.a.c.t.]

∴ seg OR bisects ∠MRN and ∠MON ... [From (i) and (ii)]

Problem Set - 3 *(Textbook Pg No. 83)*

(4) In the adjoining figure, O is the centre of the circle. Seg AB, seg AC are tangent segments. Radius of the circle is r and l(AB) = r, Prove □ABOC is a square.



(2 marks)

Proof :

seg AB ≅ seg AC ...(i) (Tangent segment Theorem)

l(AB) = r ...(ii) ...(Given)

l(AB) = l(AC) = r ...(iii) [From (i) and (ii)]

l(OB) = l(OC) = r ...(iv) (Radii of the same circle)

In □ABOC, seg AB ≅ seg AC ≅ seg OB ≅ seg OC ... [From (iii) and (iv)]

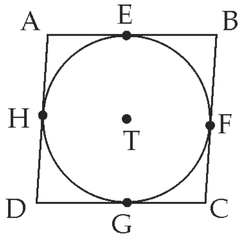
∴ □ABOC is a rhombus ...(v) (Definition)

But, ∠OBA = 90° ...(vi) (Tangent Theorem

In rhombus ABOC, ∠OBA = 90° ... [From (v) and (vi)]

∴ □ABOC is a square (Definition)

(5) In the adjoining figure, □ABCD is a parallelogoam. It circumscribs the circle with centre T. Point E, F, G, H are touching points. AE = 4.5, EB = 5.5, find AD.



(3 marks)

Solution :

AE = AH = 4.5 ...(i)
BE = BF = 5.5 ...(ii)
Let, DH = DG = x ...(iii)
CG = CF = y ...(iv)
} (Tangent segment of Theorem and supportion)

□ABCD is a parallelogram ...(Given)

∴ AB = CD ...[Opposite sides of parallelogram are equal]

∴ AE + BE = DG + CG ...[A-E-B and D-G-C]

∴ $4.5 + 5.5 = x + y$ ...[From (i), (ii), (iii) and (iv)]

∴ $x + y = 10$ ...(v)

∴ AD = BC ...[Opposite sides of parallelogram are equal]

∴ AH + DH = BF + CF ...[A-H-D and B-F-C]

∴ $4.5 + x = 5.5 + y$ ...[From (i), (ii), (iii) and (iv)]

∴ $x - y = 5.5 - 4.5$

∴ $x - y = 1$ ...(vi)

Adding (v) and (vi),

$x + y + x - y = 10 + 1$

∴ $2x = 11$

∴ $x = \frac{11}{2}$

∴ $x = 5.5$

AD = AH + DH ...(A-H-D)

∴ AD = 4.5 + x ...[From (iii)]

∴ AD = 4.5 + 5.5 ...[From (iii)]

∴ AD = 10 units

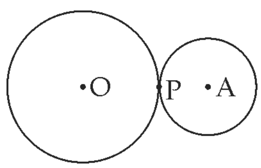
Points to Remember:

- **Theorem**

  If two circles are touching circles, then the common point lies on the line joining their centres.

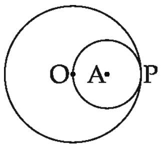
- **Externally touching circles :**

  In the adjoining figure, two circles with centres O and A are touching externally at point P.



$\therefore$ **O - P - A**

- **Internally touching circles :**

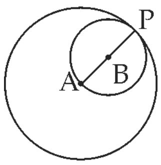


In the adjoining figure, two circles with centres O and A are touching internally at point P.

$\therefore$ **O - A - P**

Practice Set - 3.2 *(Textbook Page No. 58)*

(1) Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres. *(2 marks)*



Given :

(i) Two circles with centers A and B touch each other internally at point P.

(ii) Radius of circle with centre A is 4.8 cm.

(iii) Radius of circle with centre B is 3.5 cm.

To Find : AB

Solution :

AP = 4.8 cm, BP = 3.5 cm ...(Given)

A-B-P ...(When two circles touch each other, the point of contact lies on the line joining the centres.)

$\therefore$ AP = AB + BP

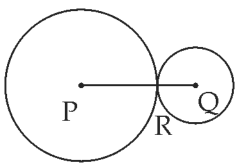
$\therefore$ 4.8 = AB + 3.5

$\therefore$ AB = 4.8 – 3.5

$\therefore$ AB = 1.3 cm

$\therefore$ $\boxed{\textbf{AB = 1.3 cm}}$

(2) Two circles having radii 5.5 cm, 4.2 cm touch each other externally. Find distance between their centres? *(2 marks)*



Given :

(i) Two circles with centres P and Q touch each other externally at point R.

(ii) Radius of circle with centre P is 5.5 cm

(iii) Radius of circle with centre Q is 4.2 cm

To Find : PQ, QR

Solution :

PR = 5.5 cm, QR = 4.2 cm ...(Given)

P-R-Q ...(When two circles touch each other, the point of contact lies on the line joining the two centres.)

PQ = PR + QR

$\therefore$ PQ = 5.5 + 4.2

$\therefore$ PQ = 9.7 cm

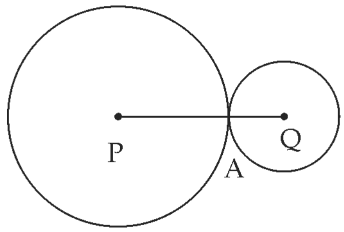
$\therefore$ $\boxed{\textbf{PQ = 9.7 cm}}$

(3) If radii of two circles are 4 cm, 2.8 cm. Draw figures of circles touching each other, (i) externally (ii) internally. *(2 marks)*

Solution :

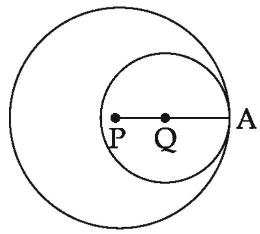
Case (i): Externally touching circles.

PA = 4 cm, QA = 2.8 cm



Case (ii) Internally touching circles.

PA = 4 cm, QA = 2.8 cm

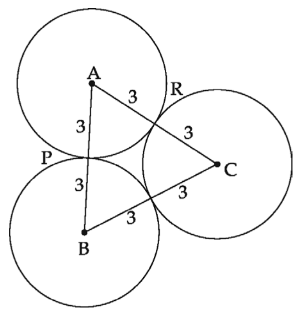


Problem Set - 3 *(Textbook Pg No. 83)*

(11) Draw circles with centres A, B, C each of radius 3 cm. Such that each circle touches the remaining 2 circles. *(2 marks)*

Solution :

Draw an equilateral triangle ABC with each side measuring 6 cm. Taking A as centre draw a circle with radius 3 cm. Repeat same thing taking B and C as centres.

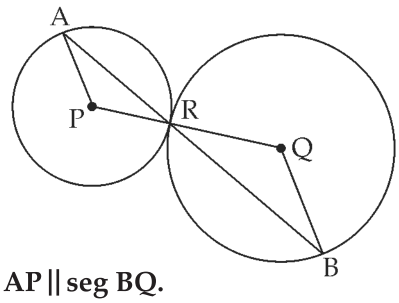


A-P-B
B-Q-C
C-R-A

} ...(For two touching circles point of contact lies on the line joining the centres.)

Practice Set - 3.2 *(Textbook Page No. 58)*

(4) In the adjoining figure, the circles with centres P and Q touch each other at R. A line passing through R meets the circles at A and B respectively then



Prove that :

(i) Seg AP || seg BQ.

(ii) ΔAPR ~ ΔRQB.

(ii) Find $\angle$RQB, if $\angle$PAR = 35° *(3 marks)*

Proof and Solution :

P-R-Q ...(When two circles touch each other, then point of contact lies on the line joining the two centres.)

$\therefore$ $\angle PRA \cong \angle QRB$ ...(i) [Vertically opposite angles]

In ΔPRA, seg PA $\cong$ seg PR ...(Radii of same circle)

$\therefore$ $\angle PRA \cong \angle PAR$ ...(ii) [Isosceles triangle theorem]

In ΔQRB, seg QR $\cong$ seg QB ...(Radii of same circle)

$\therefore$ $\angle QRB \cong \angle QBR$ ...(iii) [Isosceles triangle theorem]

$\therefore$ $\angle PRA \cong \angle PAR \cong \angle QRB \cong \angle QBR$ ...(iv) [From (i), (ii) and (iii)]

$\therefore$ $\angle PAR \cong \angle QBR$ ...[From (iv)]

Seg AP || seg BQ ...(Alternate angles test)

In ΔAPR and ΔRQB,

(i) $\angle PAR \cong \angle QRB$ ...[From (iv)]

(ii) $\angle PRA \cong \angle QBR$ ...[From (iv)]

$\therefore$ $\Delta APR \sim \Delta RQB$ ...(By AA test of similarity)

$\therefore$ $\angle PAR = 35^\circ$ ...(v) (Given)

$\therefore$ $\angle QRB = \angle QBR = 35^\circ$ ...(vi) [From (iv) and (v)]

In ΔQRB

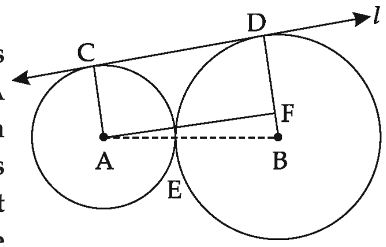
$\angle RQB + \angle QRB + \angle QBR = 180^\circ$ ...(Sum of all angles of a Δ is 180°)

$\therefore$ $\angle RQB + 35^\circ + 35^\circ = 180^\circ$ ...[From (vi)]

$\therefore$ $\angle RQB = 180^\circ - 70^\circ$

$\therefore$ $\boxed{\angle RQB = 110^\circ}$

(5) In the adjoining figure, the circles with centres A and B touch each other at E Line *l* is a common tangent that touches the circle at C and D respectively.



Find length of seg CD if the radii of the circles are 4 cm, 6 cm? *(3 marks)*

Construction :

Draw seg AC and seg BD

Draw seg AF $\perp$ seg BD, B-F-D.

Solution :

A-E-B ... (i) [When two circles touch each other, the point of contact lies on the line joining the centres.]

$\therefore$ AC = AE = 4 cm ...(ii) [Given, radii of same circle]

$\therefore$ BD = BE = 6 cm ...(iii) [Given, radii of same circle]

AB = AE + BE [A-E-B, from (i)]

$\therefore$ AB = 4 + 6 [From (ii) and (iii)]

$\therefore$ AB = 10 cm ...(iv)

In $\square$ACDF, $\angle ACD = 90^\circ$, $\angle FDC = 90^\circ$ [Tangent and radius are $\perp$ to each other at the point of contact]

$\angle AFD = 90^\circ$ ...(Construction)

$\angle FAC = 90^\circ$ ...(Remaining angle)

$\therefore$ $\square$ACDF is a rectangle. ...(Definition)

$\therefore$ CD = AF ...(v), FD = AC ...(vi) [Opposite sides of rectangle are equal]

$\therefore$ FD = 4 cm ...(vii) ...[From (ii) and (vi)]

BD = BF + FD ...[B-F-D]

$\therefore$ 6 = BF + 4 ...[From (iii) and (vii)]

$\therefore$ BF = 6 – 4 = 2 cm ...(viii)

In ΔAFB, $\angle AFB = 90^\circ$ ...(Construction)

$\therefore$ $AB = AF^2 + BF^2$ ...(By Pythagoras theorem)

$\therefore$ $10^2 = AF^2 + 2^2$

$\therefore$ $AF^2 = 100 - 4$

$\therefore$ $AF^2 = 96$

$\therefore$ $AF = \sqrt{96}$ ...(Taking square roots)

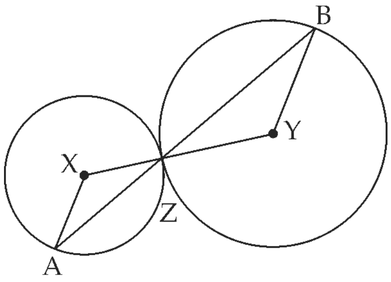
$\therefore$ $AF = \sqrt{16 \times 6}$

$\therefore$ $AF = 4\sqrt{6}$ cm

$\therefore$ $CD = 4\sqrt{6}$ cm ...[From (v)]

Problem Set - 3 *(Textbook Pg No. 83)*

(7) **In the adjoining fig. circles with centres X, Y touch each other at Z. A secant passing through Z meets the circles at A and B respectively.**



Prove that, Radius XA || radius YB.

Fill in the blanks and complete the proof. *(3 marks)*

Construction :

Draw segments XZ and seg YZ

Proof :

By theorem of touching circles,

points X, Z, Y are collinear points

∠XZA ≅ ∠BZY ...(Vertically Opposite angles)

Let ∠XZA = ∠BZY = a ...(i)

seg XA ≅ seg XZ Radii of the same circle

∴ ∠XAZ = ∠XZA = a ...(ii) (Isosceles triangle theorem)

seg YB ≅ seg YZ ∵ Radii of the same circle

∴ ∠BZY = ∠YBZ = a ...(iii) (Isosceles triangle theorem)

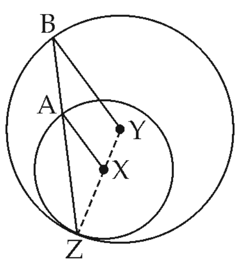
m∠XAZ = m∠YBZ = a ...[From (i), (ii) and (iii)]

∴ Radius XA || radius YB

...Converse of alternate angles test

(8) **In the adjoining fig., circles with centres X, Y touch at Z internally. Chord BZ of bigger circle, intersects inner circle at A. Prove : seg AX || seg BY.** *(3 marks)*

Construction :



Draw seg ZY.

Proof :

Y-X-Z ..,(Theorem of touching circle)

In ΔXAZ, seg XA ≅ seg XZ

...(Radii of the same circle)

∴ ∠XAZ ≅ ∠XZA (Isosceles triangle theorem)

∴ ∠XAZ ≅ ∠YZB ...(i) [Y-X-Z, Z-A-B]

In ΔYBZ, seg YB ≅ seg YZ ...(Radii of same circle)

∴ ∠YBZ ≅ ∠YZB ...(ii) (Isosceles triangle theorem)

∴ ∠XAZ ≅ ∠YBZ ...[From (i) and (ii)]

∴ seg AX || seg BY...(Corresponding angles test)

Points to Remember:

- **Arc of a circle :** A part of a circle is called an arc of a circle.

In the adjoining figure, points A and B divide circle into two arcs, viz arc AXB and arc AYB.

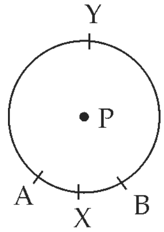


Fig . I

- **Measure of an arc:** Measure of an arc equals to its corresponding central angle.

m(arc PXQ) = m ∠POQ

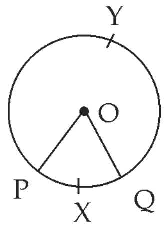


Fig . II

- **Types of arc:**

On the basis of measure of arc, arcs can be classified into three types:

(1) **Minor arc:** An arc whose measure is less than 180° is called the minor arc.

Note : Minor arcs are often named using two letters via in fig I and fig. II, arc AXB and arc PXQ can be named arc AB and arc PQ respectively.

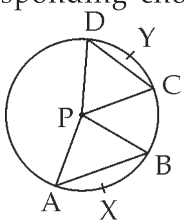
(2) **Major arc:** An arc measuring more than 180° is called the major arc.

Measure of major arc = 360° – Measure of minor arc.

(3) **Semicircle:** An arc whose measure is 180° is called the semicircle.

Note: A diameter divides a circle into two semicircles.

Theorem : In a circle (or in congruent circles), congruent arcs have corresponding chords congruent.



Given : (1) A circle with centre P.

(2) arc AXB ≅ arc CYD

To prove : chord AB ≅ chord CD

Proof :

arc AXB ≅ arc CYD ...(i) [Given]

m(arc AXB) = m∠APB ...(ii)

m(arc CYD) = m∠CPD ...(iii)

} [Definition of measure of a minor arc]

$\therefore$ m$\angle$APB = m$\angle$CPD ...(iv)

[From (i), (ii) and (iii)]

In ΔAPB and ΔCPD,

(i) seg PA $\cong$ seg PC (Radii of a circle)

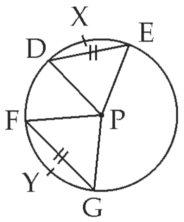
(ii) $\angle$APB $\cong$ $\angle$CPD [From (iv)]

(iii) Seg PB $\cong$ Seg PD Radii of same circle

$\therefore$ ΔAPB $\cong$ ΔCPD ...(by SAS Test of congruence)

$\therefore$ chord AB $\cong$ chord CD ...(c.s.c.t.)

- **Theorem:** In a circle (or in congruent circles), congruent chords have their corresponding minor arcs congruent.



Given: (i) A circle with centre P.

(ii) chord DE $\cong$ chord FG

To prove : arc DXE $\cong$ arc FYG

Proof:

(1) In ΔDPE and ΔFPG,

(i) side DP $\cong$ side FP
(ii) side PE $\cong$ side PG } ...(Radii of the same circle)

(iii) side DE $\cong$ side FG ...(Given)

(2) $\therefore$ ΔDPE $\cong$ ΔFPG ...(SSS Test)

(3) $\therefore$ $\angle$DPE $\cong$ $\angle$FPG ...(i) [c.a.c.t.]

(4) m (arc DXE) = m$\angle$DPE ...(ii)
(5) m (arc FYG) = m$\angle$FPG ...(iii) } [Definition of measure of a minor arc]

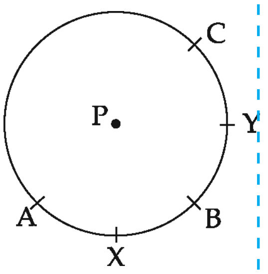
(6) $\therefore$ m$\angle$DPE = m$\angle$FPG ...[From (i), (ii) and (iii)]

i.e. $\angle$DPE $\cong$ $\angle$FPG

$\therefore$ arc DXE $\cong$ arc FYG

Arc addition Property:

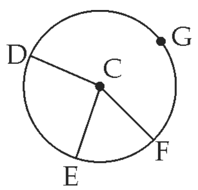
In the adjoining figure points A, X, B, Y and C are on the same circle and arc AXB and arc BYC has exactly one end point common.



$\therefore$ m(arc AXB) + m(arc BYC) = m(arc ABC)

Practice Set - 3.3 *(Textbook Page No. 63)*

(1) In the adjoining figure, G, D, E, F are concyclic points of a circle with centre C. $\angle$ECF = 70°, m(arc DGF) = 200° find (i) m arc DE (iii) m(arc DEF). *(2 marks)*



Solution :

$\therefore$ m(arc EF) = m$\angle$ECF (Definition of measure of a minor arc)

$\therefore$ m(arc EF) = 70° ...(i)

m(arc DE) + m(arc EF) + m(arc DGF) = 360° ...(Measure of a circle is 360°)

$\therefore$ m(arc DE) + 70° + 200° = 360° ...[From (i) and given]

$\therefore$ m(arc DE) = 360° – 270°

$\therefore$ $\boxed{\textbf{m(arc DE) = 90°}}$

m(arc DEF) + m(arc DE) + m(arc EF) ...(Arc addition property)

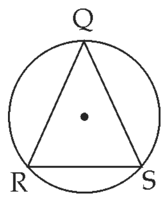
$\therefore$ m(arc DEF) = 90° + 70° ...[From (i) and given]

$\therefore$ $\boxed{\textbf{m(arc DEF) = 160°}}$

(2) In the adjoining figure, ΔQRS is an equilateral triangle.

Prove (1) arc RS $\cong$ arc QS $\cong$ arc QR

(2) m(arc QRS) = 240°. *(3 marks)*



Proof :

ΔQRS is an equilateral triangle. ...(Given)

$\therefore$ chord QR $\cong$ chord RS $\cong$ chord QS ...(Sides of equilateral Δ are equal)

arc RS $\cong$ arc QS $\cong$ arc QR ...(i)

(In circle, congruent chords have corresponding minor arcs are congruent]

Let m (arc RS) = m (arc QS) = m (arc QR) = x ...(ii) [From (i) and supposition]

$\therefore$ m (arc RS) + m (arc QS) + m (arc QR) = 360° ...(Measure of a circle is 360°)

$\therefore$ $x + x + x = 360°$

$\therefore$ $3x = 360°$

$\therefore$ $x = 120°$

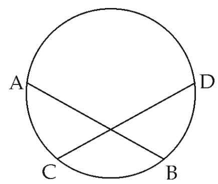
$\therefore$ m (arc RS) = m (arc QS) + m (arc QR) = 120° ...(iii)

$\therefore$ m (arc QRS) = m (arc QR) + m (arc RS) ...(Arc addition property)

$\therefore$ m (arc QRS) = 120° + 120° [From (iii)]

$\therefore$ $\boxed{\textbf{m(arc QRS) = 240°}}$

(3) In the adjoining figure, chord AB = chord CD, prove that, arc AC = arc BD *(2 marks)*



Proof :

chord AB $\cong$ chord CD ...(Gven)

arc ACB $\cong$ arc DBC

[In a circle, congruent chords have their corresponding minor arcs are congruent] m (arc ACB) = m (arc DBC)

m (arc AC) + m (arc CB) = m (arc CB) + m (arc BD)

...(Arc addition property)

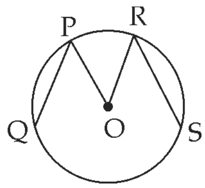
m (arc AC) = m (arc BD)

arc AC ≅ arc BD

Problem Set - 3 *(Textbook Pg No. 87)*

(14) In a circle with centre 'O',chord PQ ≅ chord RS.

If m∠POR = 70° and m(arc RS) = 80°, then find.



(1) m (arc PR)

(2) m (arc QSR)

(3) m (arc QS) *(3 marks)*

Solution :

m (arc PR) = m∠POR

...(Definition of measure of minor arc)

∴ $\boxed{\textbf{m (arc PR) = 70°}}$ ...(i)

chord PQ ≅ chord RS ...(Given)

(arc PQ) ≅ (arc RS)

...(In a circle, congruent chords have corresponding minor arcs congruent)

∴ **m (arc PQ) = 80°** ...(ii)

m (arc PR) + m (arc RS) + m (arc PQ) + m (arc QS) = 360° ...(Measure of circle)

∴ 70° + 80° + 80° + m (arc QS) = 360°

∴ m (arc QS) = 360° – 230°

∴ $\boxed{\textbf{m (arc QS) = 130°}}$ ...(iii)

m (arc QSR) = m (arc QS) + m (arc SR)

...(Arc addition property)

∴ m (arc QSR) = 130° + 80°

∴ $\boxed{\textbf{m (arc QSR) = 210°}}$

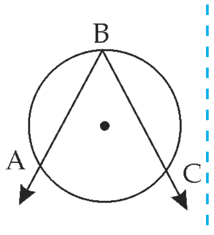
Points to Remember:

- **Inscribed angle :**

An angle is said to be an inscribed angle, if

(i) the vertex is on the circle

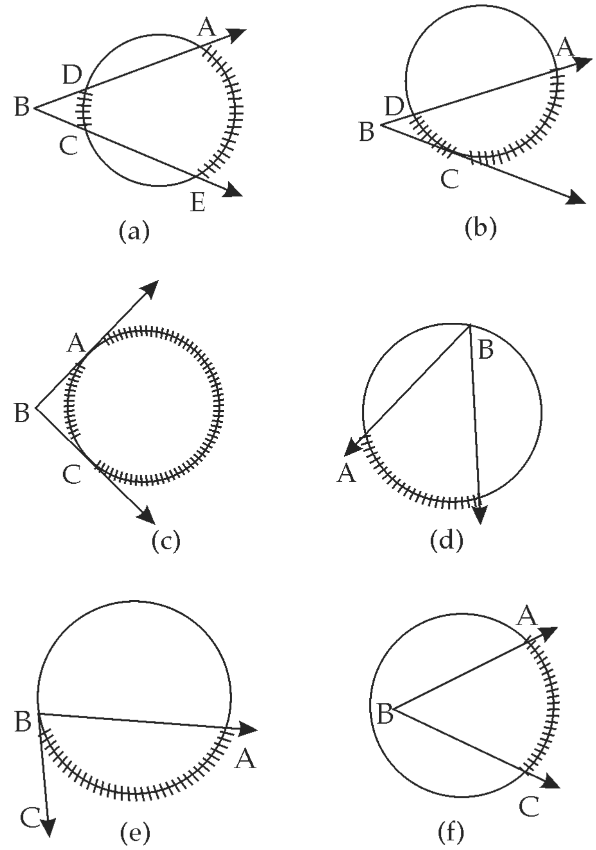
(ii) both the arms are secants. In the adjoining figure, ∠ABC is an inscribed angle, because vertex B lies on the circle and both the arms BA and BC are secants.



In other words, ∠ABC is inscribed in arc ABC.

- **Intercepted arc :**

Given an arc of the circle and an angle, if each side of the angle contains an end point of the arc and all other points of the arc except the end points lie in the interior of the angle, then the arc is said to be intercepted by the angle.



(i) In figures (a), (b) and (c), ∠ABC has its vertex B outside the circle and intercepts two arcs.

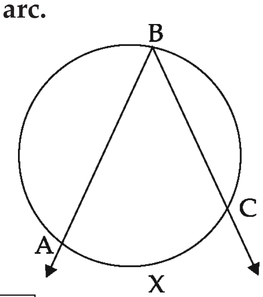
(ii) In figures (d) and (e), ∠ABC has its vertex on the circle and intercepts only one arc.

(iii) In figure (f), ∠ABC has its vertex B inside the circle and intercepts only one arc.

- **Inscribed Angle Theorem**

The measure of an inscribed angle is half of the measure of its intercepted arc.

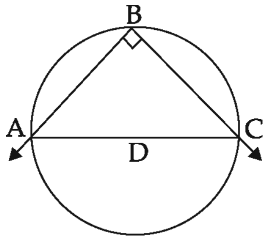
In the adjoining figure, ∠ABC is an inscribed angle and arc AXC is the intercepted arc.



∴ $\boxed{\textbf{m } \angle ABC = \frac{1}{2} \textbf{ m (arc AXC)}}$

➩ Corollary - 1 :

An angle inscribed in a semicircle is a right angle.

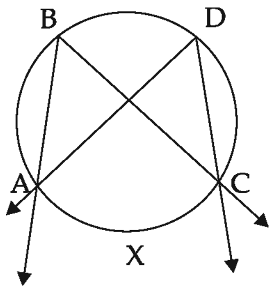


In the adjoining figure, $\angle ABC$ is inscribed in the semicircle ABC.

$\therefore$ $\boxed{m\angle ABC = 90°}$

➩ Corollary - 2 :

Angles inscribed in the same arc are congruent.



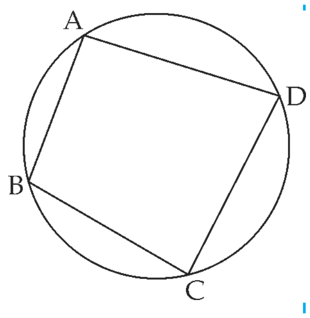
In the adjoining figure, $\angle ABC$ and $\angle ADC$, both are inscribed in the same arc ABC.

$\therefore$ $\boxed{\angle ABC \cong \angle ADC}$

Note

Inscribed angles, $\angle ABC$ and $\angle ADC$ both intercept the same arc AC. $\therefore$ $\angle ABC = \angle ADC$

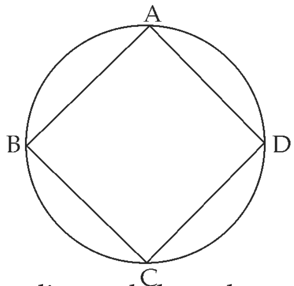
- **Cyclic Quadrilateral :**



A quadrilateral whose all four vertices lie on a circle is called cyclic quadrilateral.

In the adjoining figure, □ABCD is cyclic, as all the four vertices A, B, C and D, lie on a circle.

- **Theorem**



Statement:

The opposite angles of a cyclic quadrilateral are supplementary.

Given : □ABCD is a cyclic quadrilateral.

To Prove :

$m\angle ABC + m\angle ADC = 180°$

$m\angle BAD + m\angle BCD = 180°$

Proof:

$m\angle ABC = \frac{1}{2} m(\text{arc } ADC)$ ...(i)

$m\angle ADC = \frac{1}{2} m(\text{arc } ABC)$ ...(ii)

(Inscribed angle theorem)

Adding (i) and (ii), we get

$m\angle ABC + m\angle ADC$

$= \frac{1}{2} m(\text{arc } ADC) + \frac{1}{2} m(\text{arc } ABC)$

$\therefore$ $m\angle ABC + m\angle ADC$

$= \frac{1}{2}[m(\text{arc } ADC) + m(\text{arc } ABC)]$

$\therefore$ $m\angle ABC + m\angle ADC = \frac{1}{2} \times 360$

...(Measure of a circle is 360°)

$\therefore$ $m\angle ABC + m\angle ADC = 180°$ ...(iii)

In □ABCD,

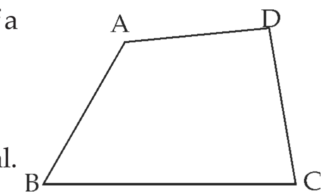
$m\angle BAD + m\angle BCD + m\angle ABC + m\angle ADC = 360°$

(Sum of measure of angles of a quadrilateral is 360°)

$\therefore$ $m\angle BAD + m\angle BCD + 180° = 360°$ ...[From (iii)]

$\therefore$ $m\angle BAD + m\angle BCD = 180°$

- **Converse of cyclic quadrilateral theorem :**



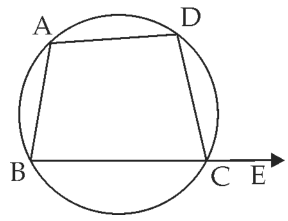
If the opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

In □ABCD, if $m\angle A + m\angle C = 180°$

or $m\angle B + m\angle D = 180°$

then, □ABCD is a cyclic quadrilateral.

- **Corollary :**



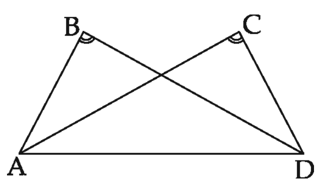
An exterior angle of cyclic quadrilateral is congruent to the angle opposite to adjacent interior angle.

In the adjoining figure, □ABCD is cyclic

$\angle DCE$ is an exterior angle.

$\therefore$ $\angle DCE \cong \angle BAD$

- If end points of a segment forms congruent angles on the same side of the segment, then the vertices of those angles and end points of the segment are concyclic.



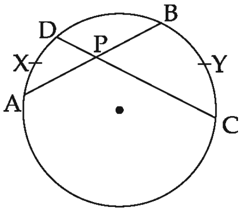
$\angle ABD \cong \angle ACD$

$\therefore$ Points A, B, C and D are concyclic.

Note: Concyclic means points lying on the same circle.

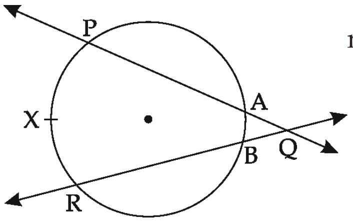
- **Two special properties**

(i)



$m\angle APD = \frac{1}{2}$ [m(arc AXD) + m(arc BYC)]

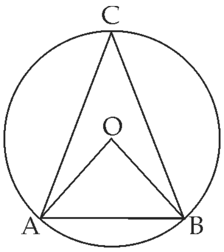
(ii)



$m\angle PQR = \frac{1}{2}$ [m(arc PXR) – m(arc AB)]

Practice Set - 3.4 *(Textbook Page No. 73)*

(1) In the adjoining figure, point O is the centre of the circle. Length of chord AB is equal to the radius of the circle. Find (i) ∠AOB (ii) ∠ACB (iii) m (arc AB) (iv) m (arc ACB) *(3 marks)*



Solution :

OA = OB ...(i) (Radii of the same circle)

AB = Radius of the given circle ...(ii) (Given)

∴ AB = OA = OB ...[From (i) and (ii)]

∴ ΔOAB is an equilateral triangle ...(By definition)

∴ $\boxed{\angle AOB = 60}$ ... (iii) (Measure of an angle of an equilateral triangle)

m (arc AB) = m∠AOB ...(Definition of measure of minor arc)

∴ $\boxed{m(\text{arc } AB) = 60^\circ}$ ... (iv) [From (iii)]

$\angle ACB = \frac{1}{2}$ (arc AB) ...(Inscribed angle theorem)

$\angle ACB = \frac{1}{2} \times 60 = 30^\circ$ ... (v) [From (iv)]

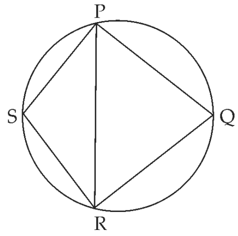
m (arc ACB) + m (arc AB) = 360° ...(Measure of a circle is 360°)

m (arc ACB) + 60° = 360° ...[From (v)]

m (arc ACB) = 360° – 60°

∴ $\boxed{m(\text{arc } ACB) = 300^\circ}$

(2) In the adjoining figure, □ PQRS is a cyclic quadrilateral, side PQ ≅ side RQ. ∠PSR = 110°. Find (i) ∠PQR (ii) m (arc PQR) (iii) m (arc QR) (iv) ∠PRQ *(3 marks)*



Solution :

□PQRS is a cyclic quadrilateral (Given)

∴ ∠PQR + ∠PSR = 180° ...(Cyclic quadrilateral theorem)

∴ ∠PQR + 110° = 180°

∴ ∠PQR = 180° – 110° ...(i)

$\boxed{m\angle PQR = 70^\circ}$

$\angle PSR = \frac{1}{2}$ m(arc PQR) ...(Inscribed angle theorem)

∴ $110^\circ = \frac{1}{2}$ m(arc PQR) ... (v) [From (iv)]

∴ $\boxed{m(\text{arc } PQR) = 220^\circ}$ ...(ii)

∴ In ΔPQR, side PQ ≅ side RQ ...(Given)

∴ ∠QPR ≅ ∠QPR ...(iii) (Isosceles triangle theorem)

In ΔPQR,

∠PQR + ∠PRQ + ∠QPR = 180°(Sum of all angles of a triangle is 180°)

∴ 70° + ∠QPR + ∠QPR = 180° ...(From (i) and (ii)]

∴ 2∠QPR = 180° – 70°

∴ 2∠QPR = 110°

∴ ∠QPR = 55° ...(iv)

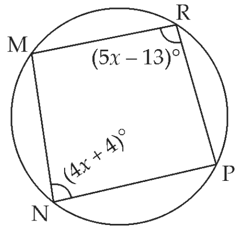
$\angle QPR = \frac{1}{2}$ m(arc QR) ...(Inscribed angle theorem)

∴ $55 = \frac{1}{2} \times$ m(arc QR) ...[From (iv)]

∴ $\boxed{m(\text{arc } QR) = 110^\circ}$ ...[From (iii) and (iv)]

∴ $\boxed{\angle PRQ = 55^\circ}$

(3) In cyclic □MRPN, ∠R = (5x – 13)° and ∠N = (4x + 4)°. Find the measure of ∠R and ∠N. *(2 marks)*



Solution :

□MRPN is a cyclic quadrilateral ...(Given)

∴ ∠R + ∠N = 180° ...(Cyclic quadrilateral theorem)

∴ $5x - 13 + 4x + 4 = 180$

∴ $9x - 9 = 180$

$\therefore \quad 9x = 180 + 9$

$\therefore \quad 9x = 189$

$\therefore \quad x = \dfrac{189}{9}$

$\therefore \quad x = 21$

$\therefore \quad m\angle R = 5x - 13 = 5 \times 21 - 13 = 105 - 13 = 92°$

$\therefore \quad m\angle N = 4x + 4 = 4 \times 21 + 4 = 84 + 4 = 88°$

(5) Prove that any rectangle is a cyclic quadrilateral. *(2 marks)*

Given : □PQRS is a rectangle

To Prove : □PQRS is a cyclic quadrilateral

Proof : □PQRS is a rectangle ...(Given)

$\therefore \quad \angle P = \angle Q = \angle R = \angle S = 90°$ ...(i) (Angles of rectangle)

$\angle P + \angle R = 90° + 90°$ [From (i)]

$\therefore \quad \angle P + \angle R = 180°$

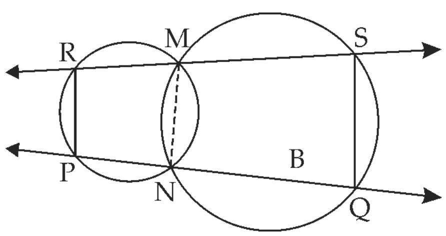
i.e. opposite angles of □PQRS are supplementary

□PQRS is cyclic quadrilateral. (converse of cyclic quadrilateral theorem)

Problem Set - 3 *(Textbook Pg No. 83)*

(23) In the adjoining figure, two circles intersect each other at points M and N. Secants drawn from points M and N intersect circels at point R, S, P and Q as shown in the figure. *(3 marks)*

To Prove : seg PR || segQS



Construction : Draw seg MN.

Proof : □PNMR is a cyclic quadrilateral ...(Definition)

$\angle MNQ$ is an exterior angle of □PNMR ...(Definition)

$\therefore \quad \angle MNQ \cong \angle PRM$ ...(i)

□MNQS is cyclic quadrilateral ...(Definition)

$\therefore \quad \angle MNQ + \angle MSQ = 180°$ ...(ii) (Opposite angles of a cyclic quadrilateral are supplementary)

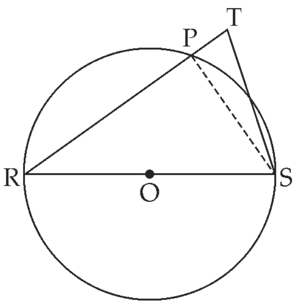
$\therefore \quad \angle PRM + \angle MSQ = 180°$ ...[From (i) and (ii)]

$\therefore \quad \angle PRS + \angle RSQ = 180°$ ...(R - M - S)

$\therefore \quad$ seg PR || segQS ...(Interior angles test)

Practice Set - 3.4 *(Textbook Page No. 73)*

(4) In the adjoining figure, seg RS is the diameter of the circle with centre 'O'. Point T is in the exterior of the circle. Prove that $\angle RTS$ is an acute angle. *(3 marks)*



Construction : Draw seg PS

Proof :

seg RS is the diameter of the circle. ...(Given)

$\therefore \quad \angle RPS = 90°$ ...(i) (Diameter subtends a right angle at any point of the circle)

$\angle RPS$ is an exterior angle of ΔPTS ...(Definition)

$\therefore \quad \angle RPS > \angle PTS$ (Exterior angle theorem)

i.e. $\angle PTS < \angle RPS$

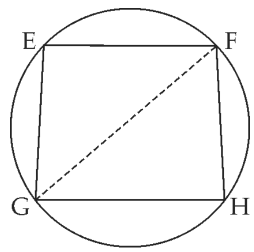
$\therefore \quad \angle PTS < 90°$ [From (i)]

i.e. $\angle RTS < 90°$ (R – P – T)

i.e. $\angle RTS$ is an acute angle.

Problem Set - 3 *(Textbook Pg No. 83)*

(17) In the adjoining diagram, chord EF || chord GH. Prove that chord EG ≅ chord FH. [Complete the following for the proof] *(2 marks)*



Proof : Draw seg GF

$\angle EFG = \angle FGH$ ...(i) $\boxed{\text{(Alternate angles theorem)}}$

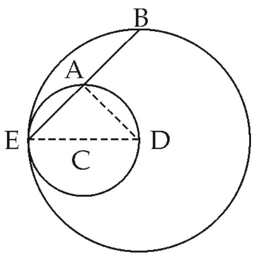
$\angle EFG = \boxed{\dfrac{1}{2}\ m(\text{arc } EG)}$ ...(ii) (Inscribed angle theorem)

$\angle FGH = \boxed{\dfrac{1}{2}\ m(\text{arc } FH)}$ ...(iii) (Inscribed angle theorem)

$\therefore \quad m(\text{arc } EG) = \boxed{m(\text{arc } FH)}$ ...[From (i), (ii) and (iii)]

$\therefore \quad$ chord EG ≅ chord FH ...(In a circle, congruent arcs have their corresponding chords congruent)

(19) **A circle with centre C touches the circle with centre D internally in the point E. Point D lies on the smaller circle. Chord EB of the external circle intersects internal circle at point A. Prove that seg EA ≅ seg AB.**



(3 marks)

Construction : Draw seg DE and seg DA

Proof : E – C – D ...(Theorem on touching circles)

Seg DE is the diameter. (Definition)

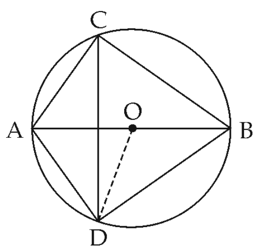
$\angle DAE = 90^\circ$ ...(i) (Diameter subtends a right angle at any point of circle other than its end points)

Consider circle with centre D.

$\therefore$ seg DA $\perp$ chord BE ...[From (i)]

$\therefore$ **seg EA ≅ seg AB** ...(Perpendicular drawn from the centre to the chord bisects the chord)

(20) **In the adjoining figure, seg AB is a diameter of a circle with centre O. Bisector of inscribed ∠ACB intersects circle at point D.** *(3 marks)*



Prove that: seg AD ≅ seg BD

Proof : Draw seg OD.

$\angle ACB = \boxed{90^\circ}$ ($\because$ Angle inscribed in a semicircle)

$\angle DCB = \boxed{45^\circ}$ ($\because$ CD bisects $\angle ACB$)

m(arc DB) = $\boxed{90^\circ}$ ...(Inscribed angle theorem)

$\angle DOB = \boxed{90^\circ}$ ...(i) (Definition of measure of an arc)

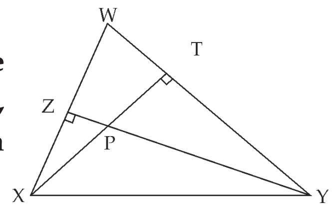
seg OA ≅ seg OB ... (ii) $\boxed{\text{(Radii of same circle)}}$

$\therefore$ seg OD is ...$\boxed{\text{Perpendicular bisector}}$ of seg AB [From (i) and (ii)]

$\therefore$ **seg AD ≅ seg BD** ... ($\because$Perpendicular bisector theorem)

Practice Set - 3.4 *(Textbook Page No. 73)*

(6) **In the adjoining figure, seg YZ and seg XT are altitudes of ΔWXY, which intersect each other at point P.** *(3 marks)*



To Prove:

(i) □WZPT is a cyclic quadrilateral

(ii) Points X, Z, T and Y are concyclic points.

Proof :

$\angle XTW = 90^\circ$ ...(i) (Given)

$\angle YZW = 90^\circ$ ...(ii) (Given)

$\angle XTW + \angle YZW = 90^\circ + 90^\circ$ ...(Adding (i) and (ii))

$\therefore$ $\angle PTW + \angle PZW = 180^\circ$ (X – P – T, Y – P – Z)

□WZPT is a cyclic quadrilateral ...(Convers of cyclic quadrilateral theorem)

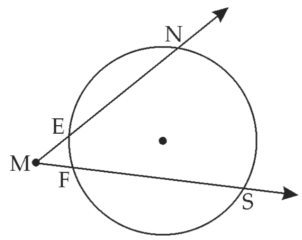
$\angle XZY = \angle XTY = 90^\circ$ ...(Given)

$\therefore$ $\angle XZY = \angle XTY$

$\therefore$ seg XY subtends congruent angle at points Z and T which are on the same side of line XY.

$\therefore$ **Point X, Z, T and Y are concyclic points.**

(7) **In the adjoining figure, m(arc NS) = 125°, m(arc EF) = 37°. Find m∠NMS.** *(1 mark)*



Solution :

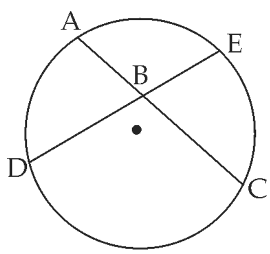
$$m\angle NMS = \frac{1}{2}[m(\text{arc NS}) - m(\text{arc EF})]$$

$$= \frac{1}{2}(125^\circ - 37^\circ)$$

$$= \frac{1}{2} \times 88$$

$\therefore$ $\boxed{m\angle NMS = 44^\circ}$

(8) **In the adjoining figure, chord AC and and chord DE intersect at point B. If ∠ABE = 108° and m(arc AE) = 95°, then find m(arc DC).**



(1 mark)

Solution :

$$m\angle ABE = \frac{1}{2}[m(\text{arc DC}) + m(\text{arc AE})]$$

$\therefore \quad 108 = \frac{1}{2}$ [m(arc DC) + 95°]

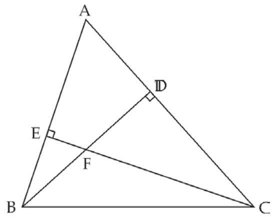
$\therefore \quad 216 =$ m(arc DC) + 95°

$\therefore$ m(arc DC) = 216 – 95°

$\therefore$ $\boxed{\text{m(arc DC)} = 121°}$

Problem Set - 3 *(Textbook Pg No. 83)*

(25) In the adjoining diagram, seg BD and seg CE are altitudes.



To Provet that:

(i) ☐ AEFD is cyclic quadrilateral

(ii) Points B, E, D, C are con-cyclic points.

[Complete the following for the proof] *(3 marks)*

Proof : ∠AEF = 90° ...(i) (Given)

∠ADF = 90° ...(ii) (Given)

Adding (i) and (ii),

∠AEF + ∠ADF = 90° + 90°

∠AEF + ∠ADF = 180°

☐AEFD is cyclic (Converse of cyclic quadrilateral theorem)

∠BEC = ∠BDC = 90° ...(Given)

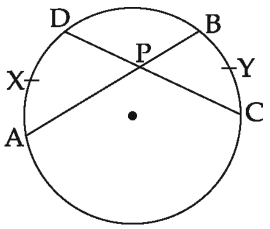
$\therefore$ ∠BEC = ∠BDC

$\therefore$ Seg BC subtends congruent angles at points E and D which are on the same side of line BC.

$\therefore$ **Points B, E, D and C are con-cyclic.**

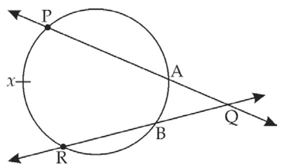
Points to Remember:

(1)



m∠APD = $\frac{1}{2}$ [m(arc AXD) + m (arc BYC)]

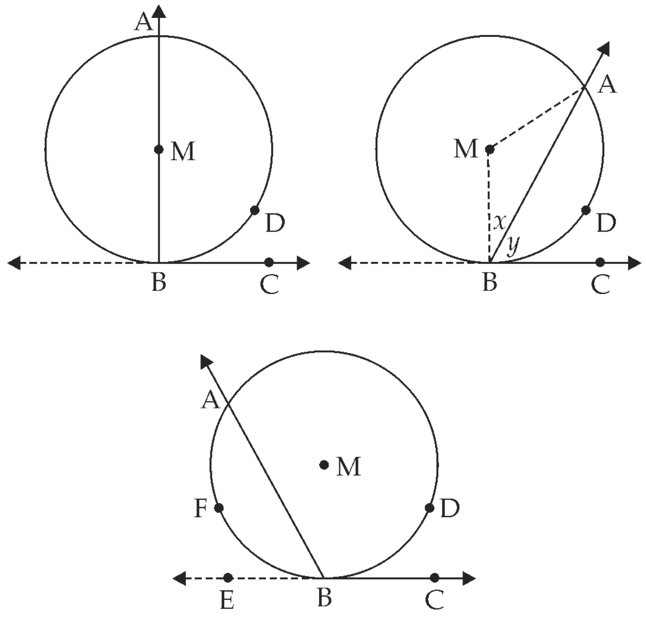
m∠APC = $\frac{1}{2}$ [m(arc DB) + m (arc AC)]



m∠PQR = $\frac{1}{2}$ [m(arc PXR) – m (arc AB)]

- **Tangent Secant Theorem**

If an angle with its vertex on the circle whose one side touches the circle and the other intersects the circle in two points, then the measure of the angle is half the measure of its intercepted arc.



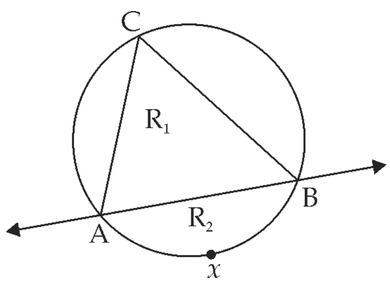
In the above three figures,

∠ABC has its vertex B on the circle, line BC is tangent to circle at B and ray BA is a secant.

$\therefore$ m∠ABC = $\frac{1}{2}$ m(arc ADB)

- **Segment of a circle :** A secant divides the circular region into two parts. Each part is called a segment of the circle.
- **Alternate segment :** Each of the two segments formed by the secant of a circle is called alternate segment in relation with the other.
- **Angle formed in a segment :** An angle inscribed in the arc of a segment is called an angle formed in that segment.

In the adjoining figure, secant AB divides the circular region into two segments R_1 and R_2.



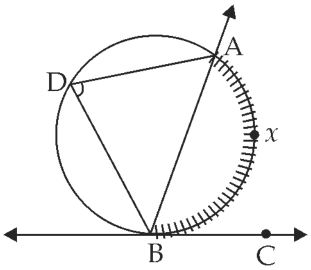
R_1 and R_2 are alternate segments in relation with each other.

∠ACB is inscribed in arc ACB of segment R_1.

$\therefore$ ∠ACB is an angle formed in segment R_1.

- **Angles in Alternate Segment**

If a line touches a circle and from the point of contact a chord is drawn, then the angles which this chord makes with the given line is equal respectively to the angle formed in the corresponding alternate segment.



In the above figure,

m $\angle$ABC = $\frac{1}{2}$ m (arc AXB) ...(i)

...(Tangent secant theorem)

m $\angle$ADB = $\frac{1}{2}$ m (arc AXB) ...(ii)

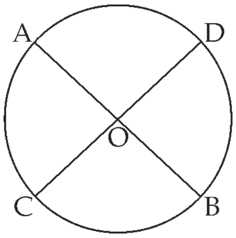
...(Inscribed angle theorem)

$\therefore$ m $\angle$ABC = m $\angle$ADB ...[From (i) and (ii)]

$\therefore$ $\boxed{\angle ABC \cong \angle ADB}$

- **Theorem**

If two secants of a circle intersect inside or outside the circle then the area of the rectangle formed by the two line segments corresponding to one secant is equal in area to the rectangle formed by the two line segments corresponding to the other.



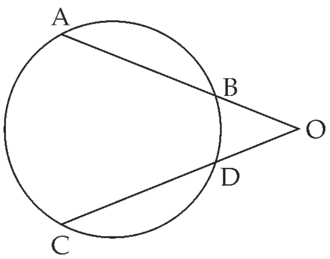
In the adjoining figure,

chords AB and CD **intersect** each

other at point O **inside** the circle.

$\therefore$ OA × OB = OC × OD

In the adjoining figure, chords AB and CD **intersect** each other at point P outside the circle.



$\therefore$ OA × OB = OC × OD

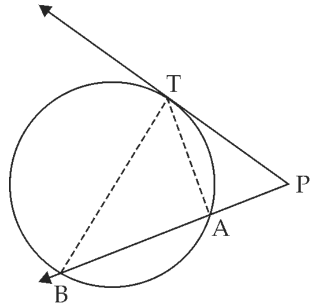
- **Tangent Secant Segment Theorem**

Statement: If a secant and a tangent of a circle intersect in a point outside the circle, then the area of the rectangle formed by the two line segments corresponding to the secant is equal to the area of the square formed by the line segment corresponding to the other tangent.

Given :

(i) line PAB is a secant intersecting the circle at points A and B.

(ii) line PT is a tangent to the circle at point T.



To Prove: PA × PB = PT^2

Construction: Draw seg BT and seg AT.

Proof :

In ΔPTA and ΔPBT,

$\angle$TPA $\cong$ $\angle$BPT (Common angle)

$\angle$PTA $\cong$ $\angle$PBT (Angles in alternate segment)

$\therefore$ ΔPTA ~ ΔPBT (By AA test of similarity)

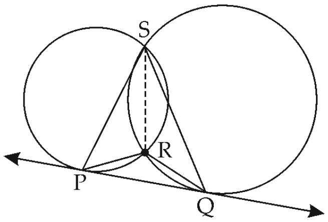
$\therefore$ $\frac{PT}{PB} = \frac{PA}{PT}$

(Corresponding sides of similar triangles)

$\therefore$ $\boxed{PA \times PB = PT^2}$

Problem Set - 3 *(Textbook Pg No. 83)*

(22) **In the adjoining figure, two circles intersect each other in point R and point S. Line PQ is a common tangent touching circle at points P and Q.** *(2 marks)*



Prove that: $\angle$PRQ + $\angle$PSQ = 180°

Construction: Draw seg RS

Proof: $\angle$PSR $\cong$ $\angle$RPQ ...(i)
$\angle$QSR $\cong$ $\angle$RQP ...(ii) } (Angles in alternate segments)

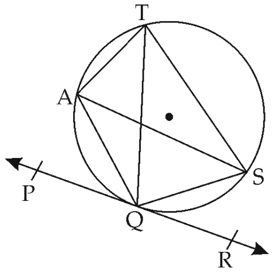
In ΔPRQ,

$\angle$PRQ + $\angle$RPQ + $\angle$RQP = 180° ...(sum of angles of a triangle is 180°)

$\therefore$ $\angle$PRQ + $\angle$PSR + $\angle$QSR = 180° ...[From (i) and (ii)]

$\therefore$ $\angle$PRQ + $\angle$PSQ = 180° ...(Angle addition property)

(13) **In the adjoining figure, line PR touches the circle at the point Q. Using the information given in the diagram, answer the following questions.**



(i) What is the sum of ∠TAQ and ∠TSQ?
(ii) Write names of angles congruent to ∠AQP.
(iii) Write names of angles congruent to ∠QTS.
(iv) If ∠TAQ = 65°, then find ∠TQS and arc TS.
(v) It ∠AQP = 42° and ∠SQR = 58°, then find ∠ATS.

Solution : *(4 marks)*

(i) □TAQS is a cyclic quadrilateral ...(Definition)

∴ $\boxed{\angle TAQ + \angle TSQ = 180°}$ ...(Cyclic quadrilateral theorem)

(ii) ∠ATQ ≅ ∠AQP
∠ASQ ≅ ∠AQP } (Angles in alternate segments)

(iii) ∠QAS ≅ ∠QTS ...(Angles inscribed in the same arc)

∠RQS ≅ ∠QTS ...(Angles in alternate segment)

(iv) ∠TQS ≅ ∠TAS ...(Angles inscribed in the same arc)

∠TAS = 65° ...(Given)

∴ $\boxed{\angle TQS = 65°}$

$\angle TQS = \frac{1}{2}$ m(arc TS) ...(Inscribed angle theorem)

∴ $65° = \frac{1}{2}$ m(arc TS)

∴ m(arc TS) = 65° × 2

∴ $\boxed{m(\text{arc } TS) = 130°}$

(v) ∠ATQ ≅ ∠AQP ...(Angles in alternate segments)

∴ ∠ATQ = 42° ...(i)

∠STQ ≅ ∠SQR ...(Angles in alternate segments)

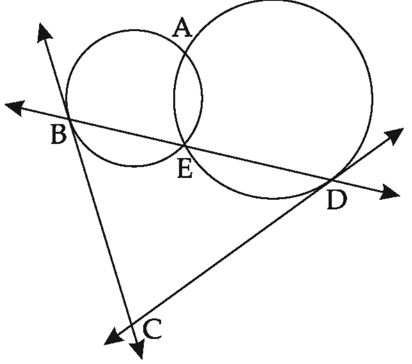
∴ ∠STQ = 58° ...(ii)

∠ATS = ∠ATQ + ∠STQ ...(Angle addition property)

∠ATS = 42° + 58° ...[From (i) and (ii)]

∴ $\boxed{\angle ATS = 100°}$

(24) In the adjoining figure, two circles intersect each other at points A and E. Their common secant through E intersects the circle at points B and D. The tangents of the circles at point B and D intersect each other at point C. Prove that □ABCD is cyclic.



Proof: ∠BAE = ∠EBC ...(i)
∠DAE ≅ ∠EDC ...(ii) } (Angles in alternate segments)

In ΔBCD,

∠BCD + ∠DBC + ∠BDC = 180° ...(Sum of angle of atriangle is 180°)

∴ ∠BCD + ∠EBC + ∠EDC = 180° ...(B–E–D)

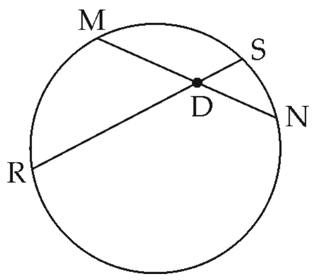
∠BCD + ∠BAE + ∠DAE = 180° ...[From (i) & (ii)]

∠BCD + ∠BAD = 180° ...(Angle addition property)

∴ □ABCD is a cyclic ...(Converse of cyclic quadrilateral Theorem)

Practice Set - 3.5 *(Textbook Page No. 82)*

(2) In the adjoining figure, chord MN and RS intersect each other at point D.



(i) If RD = 15, DS = 4, MD = 8 then DN = ?

(ii) If RS = 18, MD = 9, DN = 8, then find DS. *(3 marks)*

Solution :

(i) Chord MN and Chord RS intersect each other in the interior of the circle at point D.

∴ DM × DN = DR × DS ...(Theorem of internal division of chords)

∴ 8 × DN = 15 × 4

∴ $DN = \frac{15 \times 4}{8}$

∴ $\boxed{DN = 7.5 \text{ units}}$

(ii) Let DS = x ...(i) (Supposition)

RS = DR + DS ...(R – D – S))

∴ 18 = DR + x

∴ DR = $(18 - x)$ ...(ii)

Chord MN and Chord RS intersect each other in the interior of the circle at point D.

∴ DM × DN = DR × DS (Theorem of internal division of chords)

∴ $9 \times 8 = x(18 - x)$

∴ $72 = 18x - x^2$

∴ $x^2 - 18x + 72 = 0$

∴ $x^2 - 12x - 6x + 72 = 0$

∴ $x(x - 12) - 6(x - 12) = 0$

∴ $(x - 12)(x - 6) = 0$

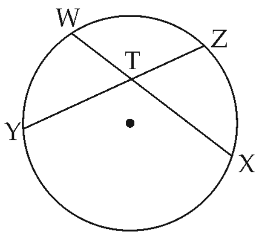
∴ $x - 12 = 0$ or $x - 6 = 0$

∴ $x = 12$ or $x = 6$

∴ $\boxed{DS = 12 \text{ units}}$ or $\boxed{DS = 6 \text{ units}}$

Problem Set - 3 *(Textbook Pg No. 87)*

(15) In the adjoining figure, m(arc WY) = 44, m(arc ZX) = 68 then,



(i) Find m∠ZTX and m(arc WZ)

(ii) If l(WT) = 4.8, l(TX) = 8, l(YT) = 6.4, then find l(TZ)

(iii) If l(WX) = 12.8, l(YT) = 6, l(TX) = 6.4, then find l(WT) *(3 marks)*

Solution:

(i) $m\angle ZTX = \frac{1}{2}$ [m (arc WY) + m(arc ZX)]

$= \frac{1}{2}[44 + 68]$

$= \frac{1}{2} \times 112$

$\boxed{m\angle ZTX = 56°}$

(ii) WT × TX = TZ × YT ...(Theorem on internal division of chords)

∴ 4.8 × 8 = TZ × 6.4

∴ $TZ = \frac{4.8 \times 8}{6.4}$

∴ $\boxed{TZ = 6 \text{ units}}$

(iii) WX = WT + TX ...(W – T – X)

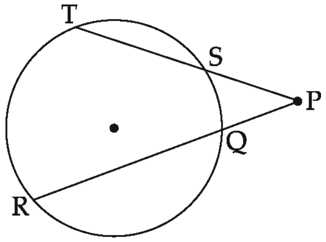
∴ 12.8 = WT + 6.4

∴ WT = 12.8 – 6.4

∴ $\boxed{WT = 6.4 \text{ units}}$

Practice Set - 3.5 *(Textbook Page No. 82)*

(4) In the adjoining figure, if PQ = 6, QR = 10, PS = 8, then find TS. *(2 marks)*



Solution :

PR = PQ + QR (P – Q – R)

∴ PR = 6 + 10

∴ PR = 16 units ...(i)

Secants PST and PQR intersect each other in the exterior of the circle at point P.

∴ PT × PS = PR × PQ ...(Theorem of external division of chords)

∴ PT × 8 = 16 × 6 ...[From (i) and given]

∴ $PT = \frac{16 \times 6}{8}$

∴ PT = 12 units ...(ii)

∴ PT = PS + TS ...(P – S – T)

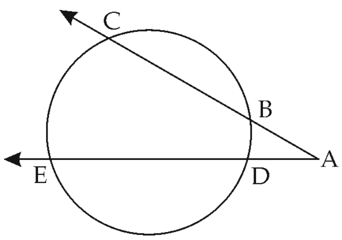
∴ 12 = 8 + TS

∴ TS = 12 – 8

∴ $\boxed{TS = 4 \text{ units}}$

Problem Set - 3 *(Textbook Pg No. 88)*

(16) In the adjoining figure,



(i) It m(arc CE) = 54°, m(arc BD) = 23°, then find ∠CAE

(ii) If AB = 4.2, BC = 5.4 AE = 12, then find AD

(iii) If AB = 3.6, AC = 9.0, AD = 5.4, then find AE. *(3 marks)*

Solution:

(i) $\angle CAE = \frac{1}{2}$ [m (arc CE) – m(arc BD)]

$= \frac{1}{2}(54 - 23)$

$= \frac{1}{2} \times 31$

$\boxed{\angle CAE = 15.5°}$

(ii) AD × AE = AB × AC ...(Theorem of external division of chords)

∴ AD × 12 = 4.2 × 9.6

∴ $AD = \frac{4.2 \times 9.6}{12}$

| AC = AB + BC = 4.2 + 5.4 = 9.6

∴ $\boxed{AD = 3.36 \text{ units}}$

(iii) AE × AD = AC × AB ...(Theorem of external division of chords)

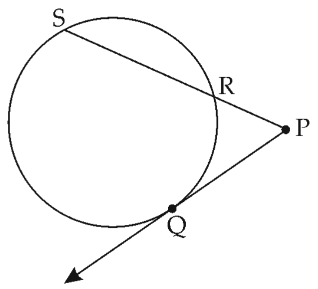
∴ AE × 5.4 = 9 × 3.6

∴ $AE = \frac{9 \times 3.6}{5.4}$

∴ $\boxed{AE = 6 \text{ units}}$

Practice Set - 3.5 *(Textbook Page No. 82)*

(1) In the adjoining figure, point Q is the point of contact. If PQ = 12, PR = 8, then find PS and RS. *(2 marks)*



Solution :

Tangent PQ and secant PRS intersect each other at point P.

$\therefore$ $PQ^2 = PR \times PS$ ...(Tangent secant segments theorem)

$\therefore$ $12^2 = 8 \times PS$

$\therefore$ $\dfrac{12 \times 12}{8} = PS$

$\therefore$ $\boxed{\textbf{PS = 18 units}}$

PS = PR + RS ..(P – R – S)

$\therefore$ 18 = 8 + RS

$\therefore$ RS = 18 –8

$\therefore$ $\boxed{\textbf{RS = 10 units}}$

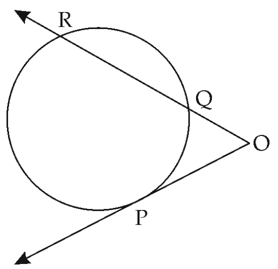
Problem Set - 3 *(Textbook Pg No. 88)*

(18) In the adjoining diagram,

(i) m(arc PR) = 140° m∠POR = 36°. Find m(arc PQ)

(ii) If OP = 7.2, OQ = 3.2, then find QR

(iii) If OP = 7.2, OR = 16.2 then find QR *(3 marks)*



Solution:

(i) $\angle POR = \dfrac{1}{2}$ [marc PR – marc PQ]

$\therefore$ $36° = \dfrac{1}{2}$ [140° – marc PQ]

$\therefore$ 36 × 2 = 140° – marc PQ

$\therefore$ m(arc PQ) = 140° – 72°

$\therefore$ $\boxed{\textbf{m arc PQ = 68°}}$

(ii) $OP^2 = OQ \times OR$ ...(Tangent secant segment theorem)

$\therefore$ $7.2 \times 7.2 = 3.2 \times OR$

$\therefore$ $OR = \dfrac{7.2 \times 7.2}{3.2}$

$\therefore$ $\boxed{\textbf{OR = 16.2 units}}$

OR = OQ + QR ...(O – Q – R)

$\therefore$ 16.2 = 3.2 + QR

$\therefore$ QR = 16.2 – 3.2

$\therefore$ QR = 13 units

(iii) $OP^2 = OQ \times OR$ ...(Tangent secant segment theorem)

$\therefore$ $7.2 \times 7.2 = OQ \times 16.2$

$\therefore$ $OQ = \dfrac{7.2 \times 7.2}{16.2}$

$\therefore$ OQ = 3.2

OQ + QR = OR ...(O – Q – R)

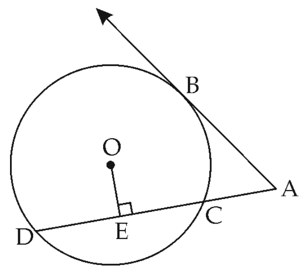
$\therefore$ 3.2 + QR = 16.2

$\therefore$ QR = 16.2 – 3.2

$\therefore$ QR = 13 units

Practice Set - 3.5 *(Textbook Page No. 82)*

(3) In the adjoining figure, point B is the point of contact and point O is the centre of the circle. Seg OE ⊥ Seg AD, if AB = 12, AC = 8, then find (i) AD (ii) DC and (iii) DE *(3 marks)*



Solution :

Tangent AB and Secant ACD intersect at point A.

$\therefore$ $AB^2 = AC \times AD$ ...(Tangent secant segments theorem)

$\therefore$ $12^2 = 8 \times AD$

$\therefore$ $\dfrac{12 \times 12}{8} = AD$

$\therefore$ $\boxed{\textbf{AD = 18 units}}$

$\therefore$ AD = AC + DC ..[A – C – D]

$\therefore$ 18 = 8 + DC

$\therefore$ DC = 18 – 8

$\therefore$ $\boxed{\textbf{DC = 10 units}}$

$\therefore$ Seg DE ⊥ chord CD ...(Given)

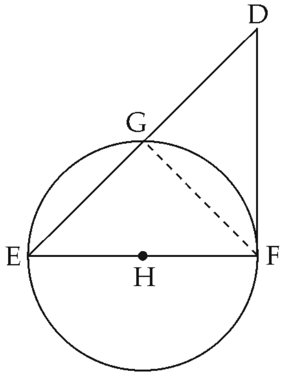
$\therefore$ $OE = \dfrac{1}{2} + CD$ (Perpendicular drawn from the centre of the circle to the chord bisects the chord)

$\therefore$ $DE = \dfrac{1}{2} \times 10$ [From (i)]

$\therefore$ $\boxed{\textbf{DE = 5 units}}$

(5) In the adjoining figure, seg EF is the diameter of the circle with centre H. Line DF is tangent at point F. If r is the radius of the circle, then prove that DE × GE = $4r^2$

(3 marks)



To prove: **DE × GE - $4r^2$**

Construction : Draw seg GF

Proof : $\angle EGF = 90°$ ...(i) (Diameter subtends a right angle at any point on the circle)

In ΔEFD,

$\angle EFD = 90°$ ...(Tangent and radius are perpendicular at the point of contact)

seg FG ⊥ hypotenuse ED ...[From (i)]

∴ $\Delta EFD \sim \Delta EGF \sim \Delta FGD$ ...(ii) (similarity in right angled triangles)

∴ $\Delta EFD \sim \Delta EGF$ ...[From (ii)]

∴ $\dfrac{EF}{GE} = \dfrac{DE}{EF}$ ...(c.s.c.t.)

∴ $DE \times GE = EF^2$

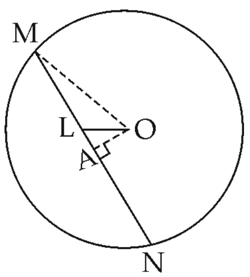
∴ $DE \times GE = (2r)^2$ ...(diameter is twice the radius)

∴ $DE \times GE = 4r^2$

Problem Set - 3 *(Textbook Pg No. 89)*

(21) In the adjoining figure, seg MN is a chord of a circle with centre O. l(MN) = 25. Point L on chord MN such that l(ML) = 9 and l(OL) = 5, then find radius of the circle.

(3 marks)



Construction:

Draw seg OM and seg OA ⊥ chord MN (M-A-N)

Solution:

Seg OA ⊥ chord MN ...(Construction)

∴ $MA = \dfrac{1}{2} \times MN$ ...(Perpendicular drawn from the centre to the chord bisects the chord

∴ $MA = \dfrac{1}{2} \times 25 = 12.5$ units ...(i)

$MA = ML + LA$ ...(M–L–A)

∴ $12.5 = 9 + LA$ ...[From (i) and given]

∴ $12.5 - 9 = LA$

∴ $LA = 3.5$ units ...(ii)

In ΔOAL, $\angle OAL = 90°$ ...(Construction)

∴ $OL^2 = OA^2 + LA^2$ ...(Pythagoras theorem)

∴ $5^2 = OA^2 + (3.5)^2$ ...[From (ii) and given]

∴ $OA^2 = 25 - 12.25$

∴ $OA^2 = 12.75$ ...(iii)

In ΔOAM, $\angle OAM = 90°$ ...(Construction)

∴ $OM^2 = OA^2 + MA^2$ ...(Pythagoras theorem)

∴ $OM^2 = 12.75 + (12.5)^2$ ...[From (iii) and given]

∴ $OM^2 = 12.75 + 156.25$

∴ $OM^2 = 169$

∴ $OM = 13$ units ...(Taking square roots)

∴ $\boxed{\text{Radius of the circle is 13 units}}$

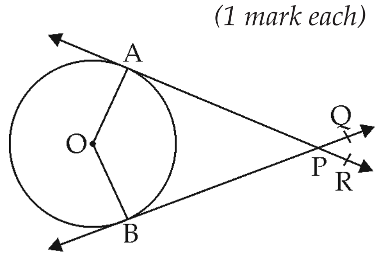
Problem Set - 3 *(Textbook Pg No. 89)*

MCQ's

(1) Choose the correct alternative for each of the following. *(1 mark each)*

(1) $\angle QPR = 60°$

∴ $\angle AOB$ = ...



(A) 60° (B) 90° (C) 120° (D) Can not be found

(2) Angle between external end point of radius and tangent is ....

(A) 90° (B) Acute angle

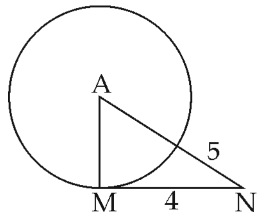
(C) Obtuse angle (D) Can not say

(3) Point P is on the circle. AB is diameter of the circle, $\angle APB$ is

(a) Reflex angle (b) Acute angle

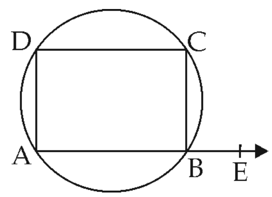
(c) Right angle (d) Obtuse angle

(4) MN is tangent at M and AM is radius. Find AM.



(A) 6 (b) 3 (c) $3\sqrt{3}$ (d) 1

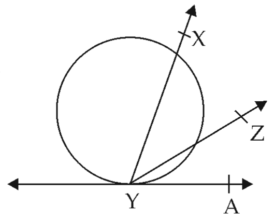
(5) $\angle ADC = 80°$, then $\angle CBE$ = ?



(A) 100° (B) 10° (C) 80° (D) 280°

(6) **$\angle XYZ = 40°$, $\angle AYZ = 20°$, line AY is tangent at point Y.**

$\therefore$ m(arc XY) = ...

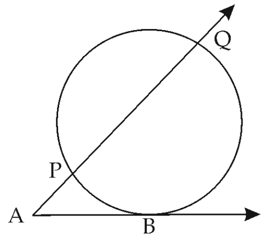


(A) 80° (B) 40° (C) 60° (D) 120°

(7) **AB is tangent at B.**

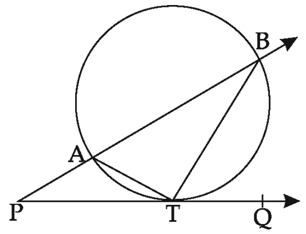
AB = 12, AP = 6

$\therefore$ PQ = ...



(A) 18 (B) 6 (C) 12 (D) 20

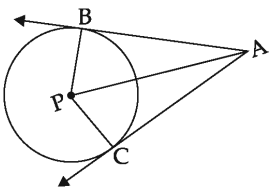
(8) **Line PT is tangent at point T. Which of the following is true?**



(A) $\angle ABT \cong \angle APT$ (B) $\angle ABT \cong \angle BAT$

(C) $\angle BAT \cong \angle BTQ$ (D) None of the (A), (B), (C)

(9) **A circle with centre P. Line AB and line AC are tangents from point A at points B and C respectively. Which of the following is/are true?**

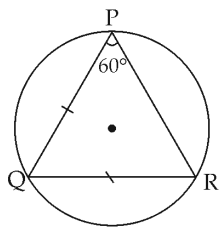


(A) $\angle BPA \cong \angle CPA$ (B) $\angle BAP \cong \angle CAP$

(C) $\angle PBA \cong \angle PCA$ (D) All three (A), (B), (C)

(10) **In adjoining figure, PQ = QR. $\angle P = 60°$**

$\therefore$ m(arc PR) = ...



(A) 120° (B) 60° (C) 90° (D) 240°

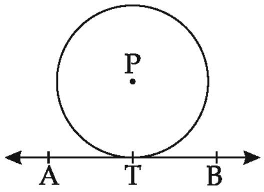
ANSWERS

(1)	(C) 120°	(2)	(A) 90°
(3)	(C) Right angle	(4)	(B) 3
(5)	(C) 80°	(6)	(D) 120°
(7)	(A) 18	(8)	(C) $\angle BAT \cong \angle BTQ$
(9)	(D) All three (A), (B), (C)	(10)	(A) 120°

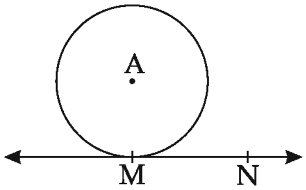
PROBLEMS FOR PRACTICE

Based on Practice Set 3.1

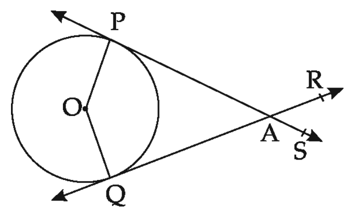
(1) In the adjoining figure, point P is the centre of the circle and line AB is the tangent to the circle at T. The radius of the circle is 6 cm. Find PB if $\angle TPB = 60°$. *(2 marks)*



(2) In the adjoining figure, point A is the centre of the circle AN = 10 cm. Line NM is tangent at M. MN = 5cm. Find the radius. *(2 marks)*

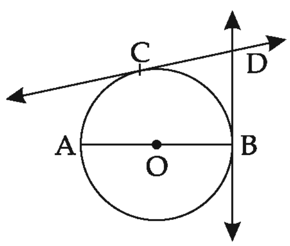


(3) A circle with centre. O Point A is in the exterior of the circle. Line AP and line AQ are tangents at point P and point Q respectively P–A–S, Q–A–R $\angle PAR = 130°$. Find $\angle AOP$. *(2 marks)*



(4) Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$. *(2 marks)*

(5) In the adjoining figure, O is the centre and seg AB is a diameter. At point C on the circle, the tangent CD is drawn. Line BD is tangent at B. Prove that seg OD || seg AC. *(2 marks)*



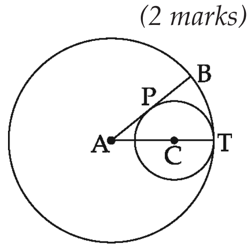
Based on Practice Set 3.2

(6) If two circles of radii 5 cm and 3 cm touch externally. Find the distance between them. *(1 mark)*

(7) Find the distance between two internally touching circles whose radii are 10 cm and 2 cm. *(1 marks)*

(8) The circles which are not congruent touch externally. The sum of their areas is 130π cm^2 and distance between their centres is 14 cm. Find the radii of the two circles. *(2 marks)*

(9) In the adjoining figure circles with centres A and C touch internally at point T. Line AB is tangent to the smaller

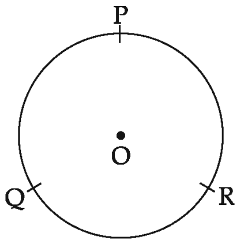


circle at point P. Point B lies on the bigger circle. Radii are 16 cm and 6 cm. Find AP. *(2 marks)*

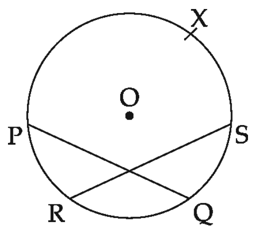
(10) The radii of two circles are 25 cm and 9 cm. The distance between their centres is 34 cm. Find the length of the common tangent segment to these circles. *(2 marks)*

Based on Practice Set 3.3

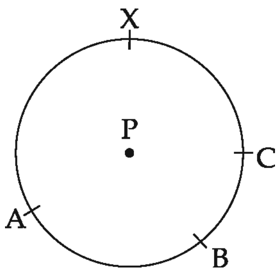
(11) In the adjoining figure, A circle with centre 'O' arc PQ = arc QR = arc PR. Find measure of each of above arcs. *(2 marks)*



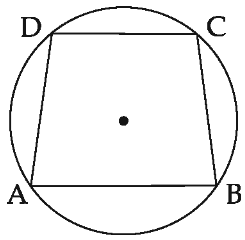
(12) A circle with centre 'O' chord PQ ≅ chord RS. m(arc PXQ) = 260°. Then find m(arc RXS). *(1 marks)*



(13) A circle with centre P. arc AB = arc BC and arc AXC = 2 arc AB. Find measure of arc AB, arc BC and arc AXC. Prove chord AB ≅ chord BC. *(3 marks)*

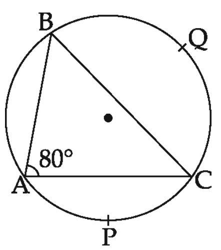


(14) In the adjoining figure, chord AD ≅ chord BC. m(arc ADC) = 100°, m(arc CD) = 60°. Find m(arc AB) and m(arc BC). *(3 marks)*

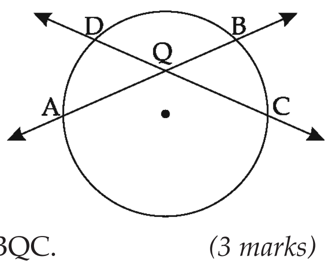


Based on Practice Set 3.4

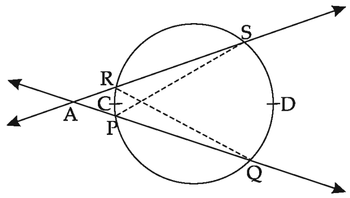
(15) If m(arc APC) = 60° and ∠BAC = 80°. Find (a) ∠ABC (b) m(arc BQC). *(2 marks)*



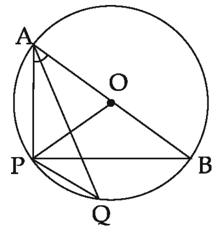
(16) Chords AB and CD of a circle intersect in point Q in the interior of a circle of as shown in the figure. If m(arc AD) = 20°, and m(arc BC) = 36°, then find ∠BQC. *(3 marks)*



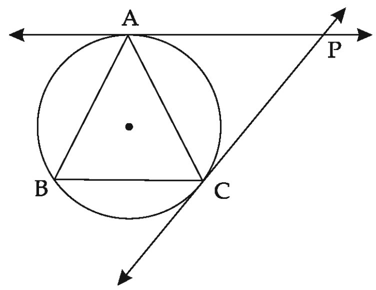
(17) Secants containing chords RS and PQ of a circle intersect each other in point A in the exterior of a circle. If m(arc PCR) = 26° and m(arc QDS) = 48°, then find (1) ∠AQR (2) ∠SPQ (3) ∠RAQ. *(3 marks)*



(18) In the adjoining figure, O is the centre of the circle. Find the value of ∠ABP if ∠POB = 90° *(2 marks)*

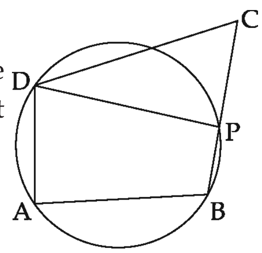


(19) Tangents drawn at points A and C of a circle intersect each other in point P. If ∠APC = 50°, then find ∠ABC. *(2 marks)*

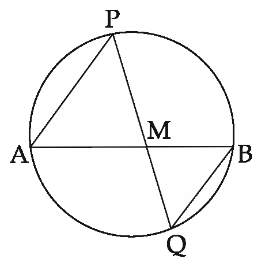


(20) If two consecutive angles of cyclic quadrilateral are congruent, then prove that one pair of opposite sides is congruent and other is parallel. *(4 marks)*

(21) □ABCD is a parallelogram. Side BC intersects circle at point P. Prove that DC = DP. *(3 marks)*

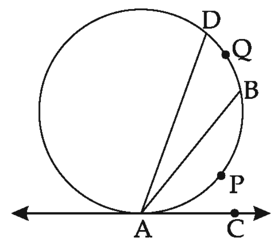


(22) In the adjoining figure, chord PQ and chord AB intersect at point M. If PM = AM, then prove that BM = QM. *(2 marks)*

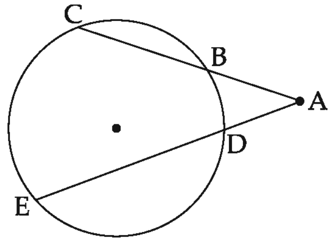


Based on Practice Set 3.5

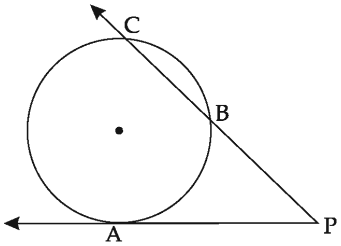
(23) Seg AB and seg AD are the chords of the circle. C is a point on tangent of the circle at point A. If m(arc APB) = 80° and ∠BAD = 30°. Then find (i) ∠BAC (ii) m(arc BQD). *(3 marks)*



(24) Secant AC and secant AE intersects in point A. Points of intersections of the circle and secants are B and D respectively. If CB = 5, AB = 7, EA = 20. Determine ED – AD. *(3 marks)*

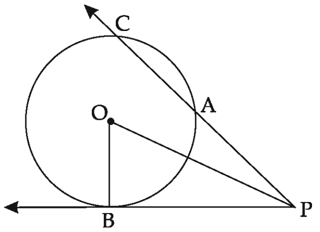


(25) In the adjoining figure line PA is tangent at point A. Line PBC is a secant. If AP = 15 and BP = 10, find BC. *(2 marks)*



(26) □ABCD is a rectangle. Taking AD as a diameter, a semicircle AXD is drawn which intersects the diagonal BD at X. If AB = 12 cm, AD = 9 cm, then find values of BD and BX. *(3 marks)*

(27) In the adjoining figure, point O is the centre of the circle. Line PB is a tangent and line PAC is a secant. Find PA × PC if OP = 25 and radius is 7. *(3 marks)*



ANSWERS

(1) 12 **(2)** $5\sqrt{3}$ **(3)** 65° **(6)** 8 cm **(7)** 8 cm
(8) 11 cm, 3 cm **(9)** 8 units **(10)** 30 units
(11) 120° **(12)** 260°
(13) m(arc AB) = 90°, m(arc BC = 90° m(arc AXC) = 180°
(14) m(arc AD) = m(arc BC) = 100° **(15)** 30°, 160°
(16) 30.5° **(17)** 13°, 24°, 11° **(18)** 45° **(19)** 65°
(23) (i) 40° (ii) 60° **(24)** 11.6 units **(25)** 12.3 units
(26) BD = 15, BX = 9.6 **(27)** 576 units

❖ ❖ ❖

ASSIGNMENT – 3

Time : 1 Hr. **Marks : 20**

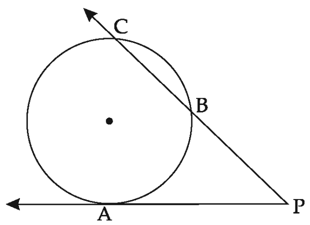
Q.1. (A) Solve the following sub questions: **(2)**

(1) Radius of a circle with centre 'O' is 5 cm, OA = 4 cm, OB = 5.5 cm. Find the position of point A and B with respect to circle.

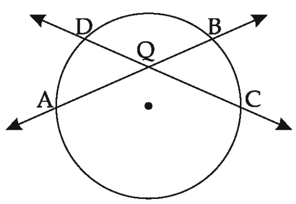
(2) Two circles with diameters 6 cm and 9 cm touch each other externally. Find the distance between their centres.

Q.1. (B) Solve the following any one questions: **(2)**

(1) Line PA is a tangent at point A. Line PBC is a secant AP = 15, BP = 10, find BC.



(2) Secants AB and CD are intersecting in point Q. m(arc AD) = 25° and m(arc BC) = 36°, then find: ∠BQC



Q.2. Perform any on of the activities **(2)**

(1) Measure of a major arc of a circle is four times the measure of corresponding minor arc. Complete the following acitivity to find the measure of each arc.

Sol. Let the measure of minor arc be x.

$\therefore$ Measure of major arc $= \boxed{\quad\quad}$

$\therefore x + \boxed{\quad\quad} = 360°$ (Measure of a circle is 360°)

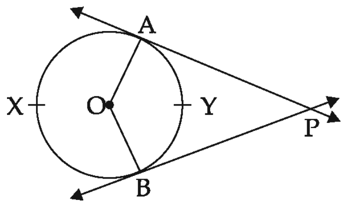
$\therefore \boxed{\quad} \; x = 360°$

$\therefore x = \boxed{\quad\quad}$

$\therefore$ Measure of minor arc $= x =$ ....

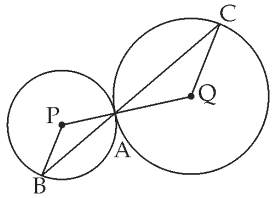
$\therefore$ Measure of major arc $= \boxed{\quad} = \boxed{\quad} \times x =$ ....

(2) Line PA and line PB are tangents to the circle at points A and B. If $\angle APB = 60°$, then find m(arc AXB).

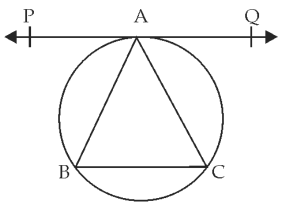


Q.3. Solve the following sub questions: (any two) **(6)**

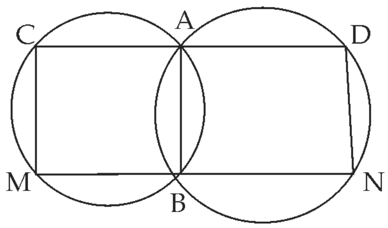
(1) Two circles with centres P and Q touch each other at point A. $\angle BPA = 60°$. Find $m\angle QCA$ and $m\angle CQP$.



(2) Line PQ is a tangent to the circle at point A. arc AB $\cong$ arc AC. Complete the following activity to prove ΔABC as isosceles triangle.

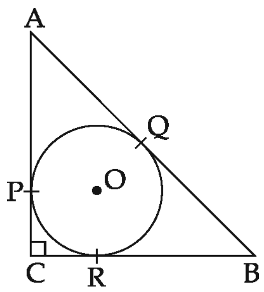


(3) Two circles intersect each other in points A and B. Secants through A and B intersect circles in C, D and M, N as shown in the figure. Prove that: CM || DN

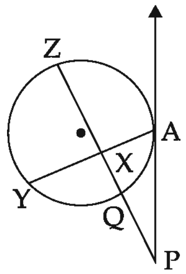


Q.4. Solve any two of the following questions: **(8)**

(1) A circle with centre 'O' is incircle of ΔABC. $\angle ACB = 90°$. Radius of the circle is r. Prove that : $2r = a + b - c$.



(2) In the adjoining figure line PA is a tangent to the circle at point A. Secant PQZ intersects chord AY in point X, such that AP = PX = XY. If PQ = 1 and QZ = 8. Find AX.



(3) In the adjoining figure line PA is a tangent to the circle at

4 Geometric Constructions

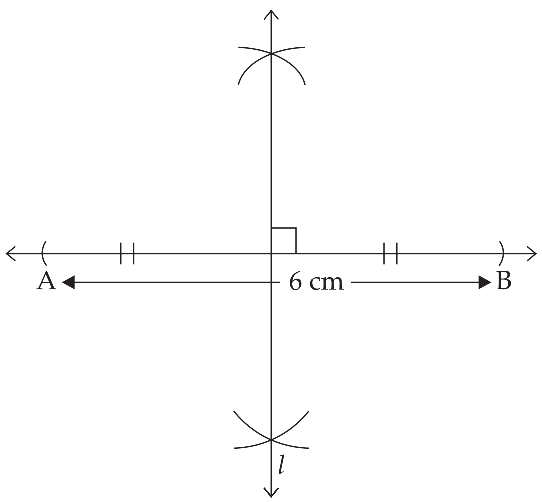
... INDEX ...

Points to Remember:

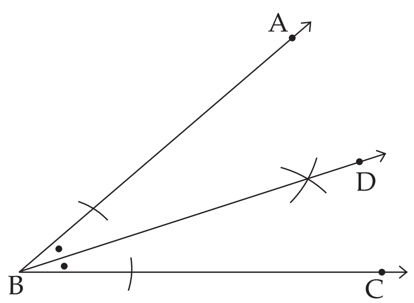
- **Construction of various geometrical figures is a very important part of the study of geometry for understanding the concepts learnt in theoretical geometry.**

Basic Constructions

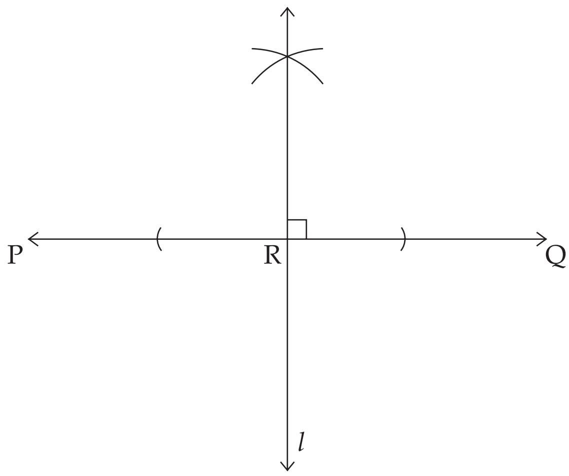
(i) To draw a perpendicular bisector of a given line segment.



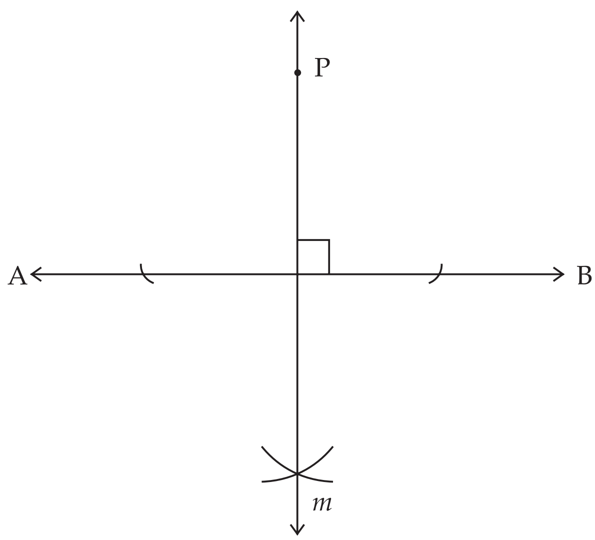
(ii) To draw an angle bisector of a given angle.



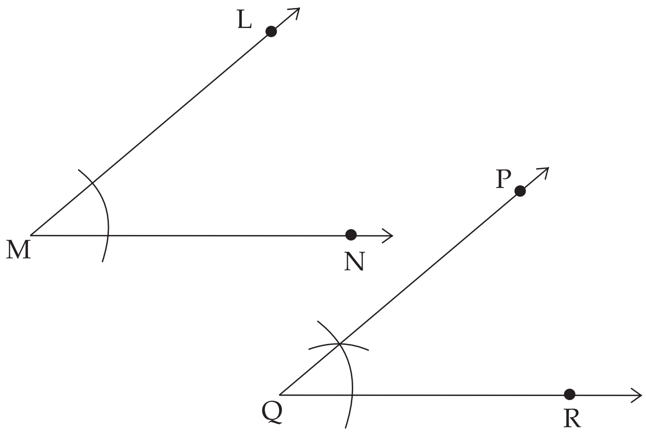
(iii) To draw a perpendicular to a line at a given point on it.



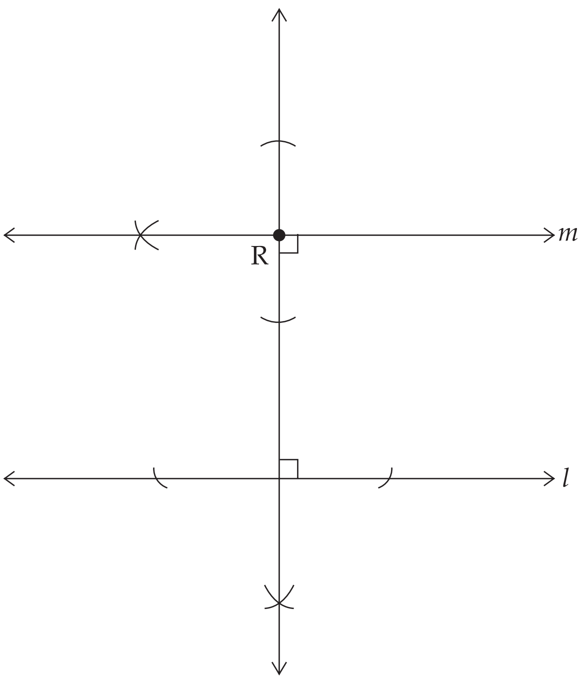
(iv) To draw a perpendicular to a given line from a point outside it.



(v) To draw an angle congruent to a given angle, using scale and compass only.



(vi) To draw a line parallel to a given line through a point outside it.



- **Similar Triangles**

Construction of similar triangle to a given triangle: For a given one-to-one correspondence between the vertices of two triangles, if their corresponding angles are congruent and corresponding sides are in proportion, then these two triangles are called '**Similar triangles**'.

Using these properties, we should construct similar triangles to the given triangle.

Here, we shall see two types of constructions as discussed below.

(A) Both triangles do not have any angle in common

Example: $\Delta ABC \sim \Delta XYZ$. AB = 8 cm, BC = 6 cm and AC = 10 cm. AB : XY = 2 : 1 Construct ΔXYZ.

Solution: $\Delta ABC \sim \Delta XYZ$ ....(Given)

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} \quad \text{....(i) (c.s.s.t.)}$$

$$\frac{AB}{XY} = \frac{2}{1} \quad \text{....(ii) (Given)}$$

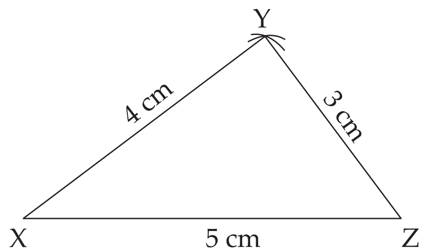
$$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{2}{1} \quad \text{....[From (i) and (ii)]}$$

$$\therefore \frac{8}{XY} = \frac{6}{YZ} = \frac{10}{XZ} = \frac{2}{1}$$

$$\therefore \frac{8}{XY} = \frac{2}{1};\ \frac{6}{YZ} = \frac{2}{1};\ \frac{10}{XZ} = \frac{2}{1}$$

$$\therefore XY = \frac{8}{2};\ YZ = \frac{6}{2};\ XZ = \frac{10}{2}$$

$\therefore$ XY = 4 cm, YZ = 3 cm, XZ = 5 cm



ΔXYZ is the required triangle.

MASTER KEY QUESTION SET - 4

Practice Set - 4.1 *(Textbook Page No. 96)*

(1) $\Delta ABC \sim \Delta LMN$, In ΔABC AB = 5.5 cm, BC = 6 cm and CA = 4.5 cm. Construct ΔABC and ΔLMN, such that $\frac{BC}{MN} = \frac{5}{4}$. *(4 marks)*

Solution :

$\Delta ABC \sim \Delta LMN$ (Given)

$$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN} \quad \text{.... (i) (c.s.s.t.)}$$

$$\frac{BC}{MN} = \frac{5}{4} \quad \text{..... (ii) (Given)}$$

$$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN} = \frac{5}{4} \quad \text{[from (i) and (ii)]}$$

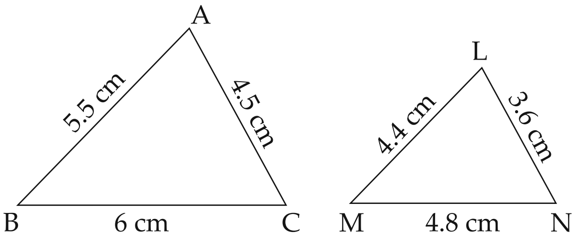
$$\therefore \frac{AB}{LM} = \frac{5}{4},\ \frac{BC}{MN} = \frac{5}{4},\ \frac{AC}{LN} = \frac{5}{4}$$

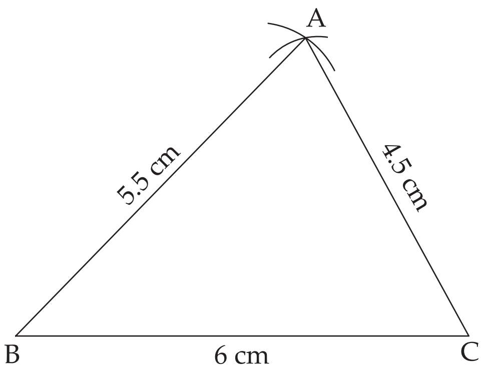
$$\therefore \frac{5.5}{LM} = \frac{5}{4},\ \frac{6}{MN} = \frac{5}{4},\ \frac{4.5}{LN} = \frac{5}{4}$$

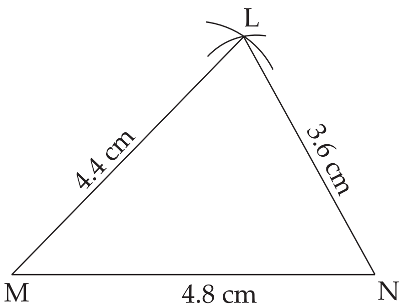
$$\therefore LM = \frac{5.5 \times 4}{5};\ MN = \frac{6 \times 4}{5};\ LN = \frac{4.5 \times 4}{5}$$

$\therefore$ LM = 4.4 cm, MN = 4.8 cm, LN = 3.6 cm

Analytical figure:







ΔLMN is the required triangle similar to the ΔABC.

(3) **ΔRST ~ ΔXYZ, In ΔRST, RS = 4.5 cm, $\angle$RST = 40°, ST = 5.7cm. Construct Δ RST and ΔXYZ, such that $\frac{RS}{XY} = \frac{3}{5}$** *(4 marks)*

Solution :

ΔRST ~ ΔXYZ (Given)

$\therefore$ $\angle$RST $\cong$ $\angle$XYZ (c.a.s.t.)

$\therefore$ $\angle$XYZ = 40° ($\because$ $\angle$RST = 40°, given)

$\frac{RS}{XY} = \frac{ST}{YZ}$ ... (i) (c.s.s.t.)

But, $\frac{RS}{XY} = \frac{3}{5}$ ... (ii) (given)

$\therefore$ $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{3}{5}$ [from (i) and (ii)]

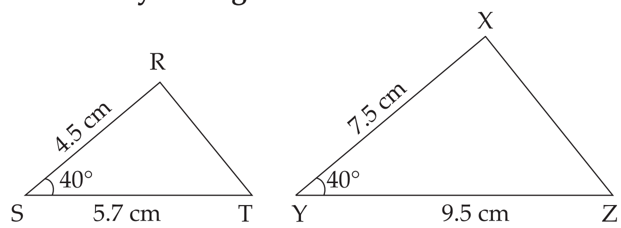
$\therefore$ $\frac{RS}{XY} = \frac{3}{5}$; $\frac{ST}{YZ} = \frac{3}{5}$

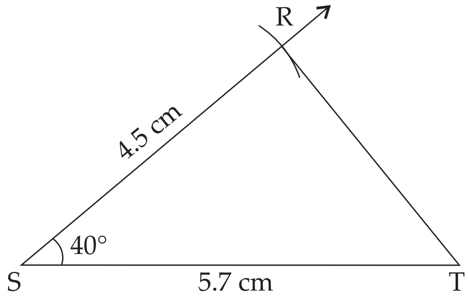
$\therefore$ $\frac{4.5}{XY} = \frac{3}{5}$ $\qquad$ $\frac{5.7}{YZ} = \frac{3}{5}$

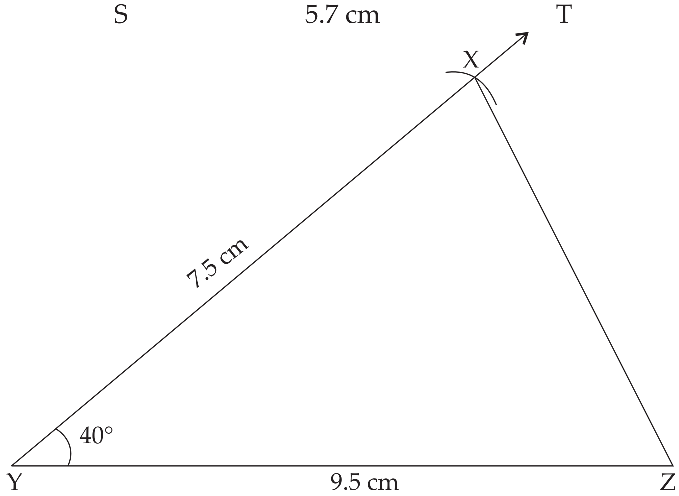
$\therefore$ XY = $\frac{4.5 \times 5}{3}$ $\qquad$ $\therefore$ YZ = $\frac{5.7 \times 5}{3}$

$\therefore$ XY = 7.5 cm $\qquad$ $\therefore$ YZ = 9.5 cm

Analytical figure:







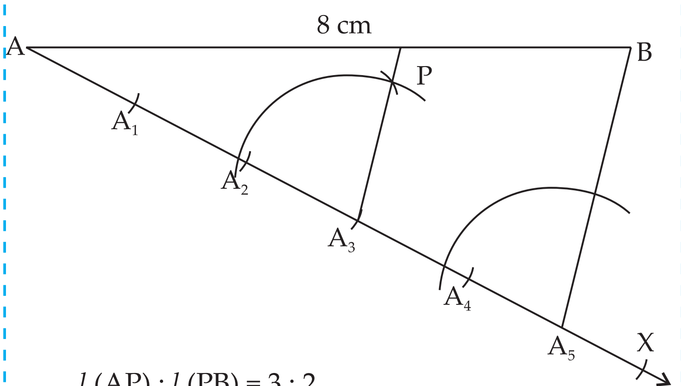
ΔXYZ is the required triangle similar to the ΔRST.

Points to Remember:

- **Basic Construction**

To divide a line segment in the given ratio.

Example: Divide seg AB of length 8 cm in the ratio 3 : 2



l (AP) : l (PB) = 3 : 2

Steps of construction:

(1) Draw seg AB of length 8 cm. Draw ray AX on either side of seg AB.

(2) Make five equal parts $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$ on ray AX.

(3) Draw seg BA_5.

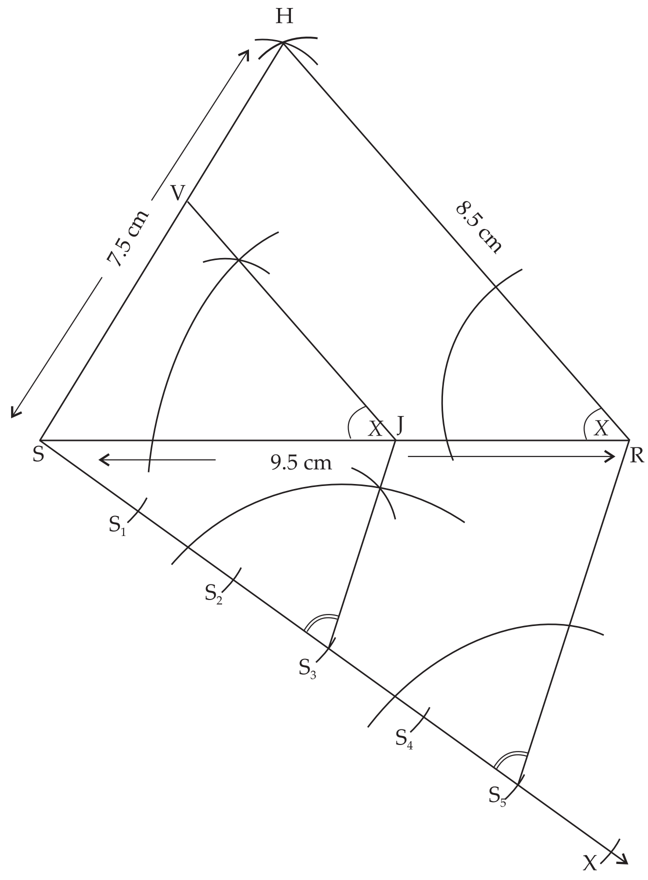
(4) Draw angle ≅ to $\angle AA_5B$ at vertex A_3.

Let its arm intersect seg AB at P. Point P divides seg AB in the ratio AP : PB; i.e., 3 : 2.

(B) To construct similar triangle when one pair of angle is common.

Example: Construct ΔSVJ, if ΔSHR ~ ΔSVJ, SH = 7.5 cm, HR = 8.5 cm, SR = 9.5 cm, SV : SH = 3 : 5

[Note : In this case, we need not do any calculation, we have to use the basic construction that we did in the previous question.]



Steps of construction:

(1) Draw seg SR as base [Note: Base should be taken that side in which vertex of common angle is contained.]

(2) On either side of seg SR, draw ray SX and on ray SX make five equal parts $SS_1 = S_1S_2 = S_2S_3 = S_3S_4 = S_4S_5$. [As bigger triangle has to be divided in five equal parts.]

(3) Draw seg RS_5. Draw an angle ≅ to $\angle SS_5R$ at vertex S_3. Let it's arm intersect side SR at point J.

(4) Draw ∠SJV ≅ ∠SRH, intersecting side SH at point 'V'.

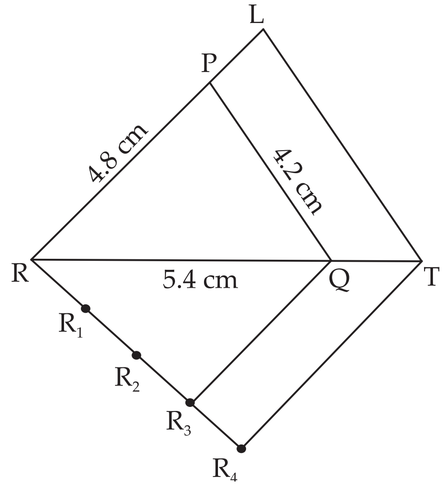
ΔSVJ is the required triangle.

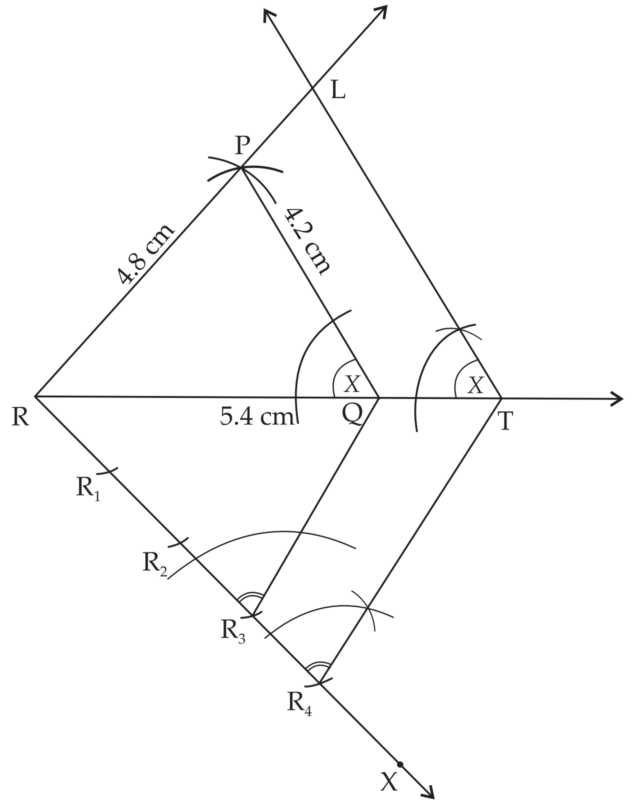
Practice Set - 4.1 *(Textbook Page No. 96)*

(2) ΔPQR ~ ΔLTR, In ΔPQR, PQ = 4.2 cm, QR = 5.4 cm, PR = 4.8 cm. Construct ΔPQR and ΔLTR, such that $\frac{PQ}{LT} = \frac{3}{4}$. *(4 marks)*

Solution :

Analytical figure:



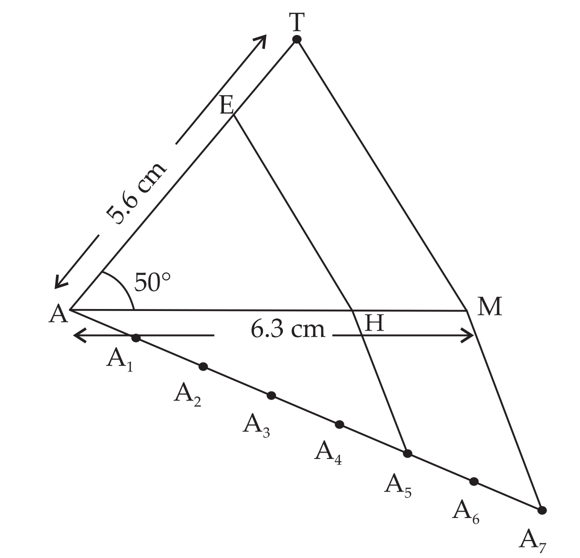


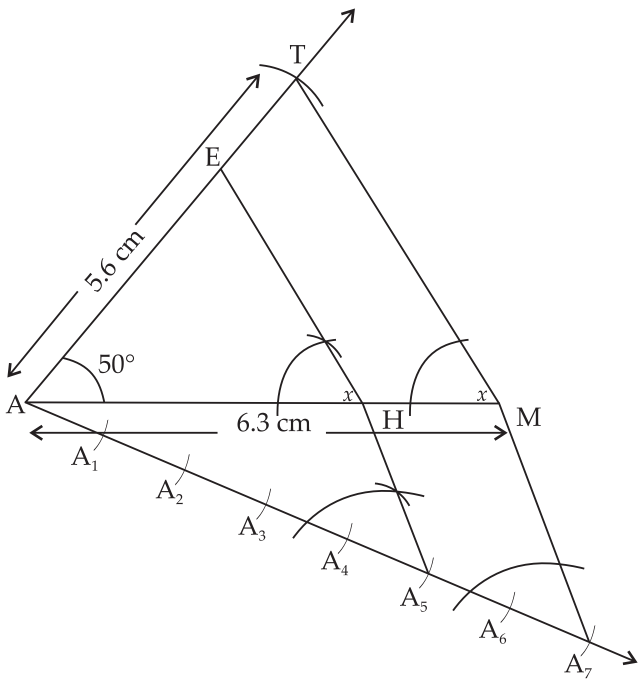
ΔLTR is the required triangle similar to the ΔPQR.

(4) **ΔATM ~ ΔAHE. In ΔAMT, AM = 6.3 cm, $\angle$TAM = 50°, and AT = 5.6 cm. $\frac{AM}{AH} = \frac{7}{5}$. Construct ΔAHE.** *(4 marks)*

Solution :

Analytical figure:





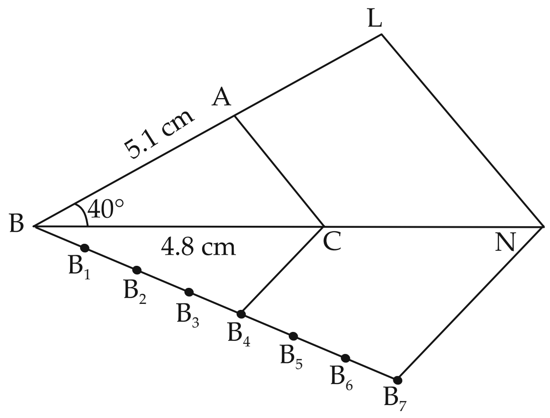
ΔAHE is the required triangle similar to the ΔAMT.

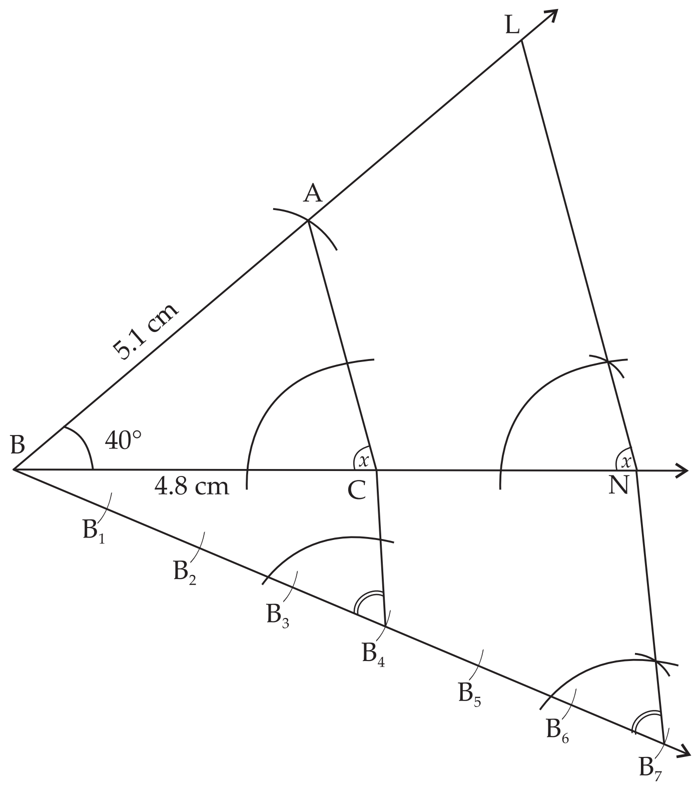
Problem Set - 4 *(Textbook Pg No. 99)*

(7) **ΔABC ~ ΔLBN. In ΔABC, AB = 5.1 cm, $\angle$B = 40°, BC = 4.8 cm, $\frac{AC}{LN} = \frac{4}{7}$. Construct ΔABC and ΔLBN.** *(4 marks)*

Solution :

Analytical figure:



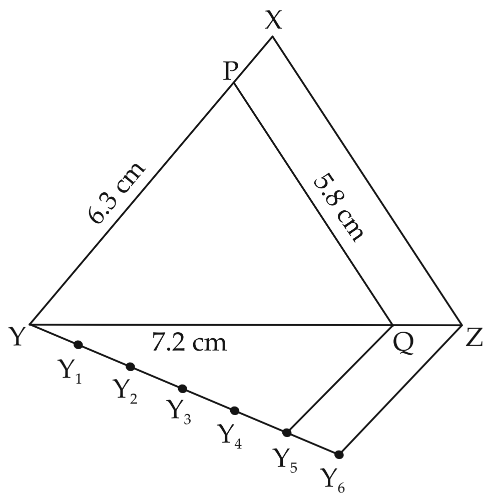


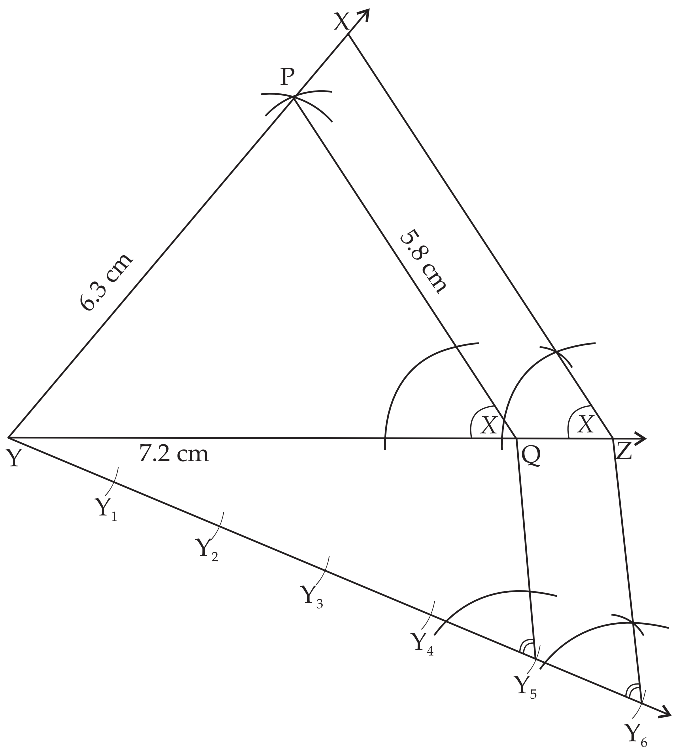
ΔLBN is the required triangle similar to the ΔABC.

(8) **Construct ΔPYQ such that PY = 6.3 cm, YQ = 7.2 cm, PQ = 5.8 cm. If $\frac{YZ}{YQ} = \frac{6}{5}$, then construct ΔXYZ similar to ΔPYQ.** *(4 marks)*

Solution :

Analytical figure:





ΔXYZ is the required triangle similar to the ΔPYQ.

Points to Remember:

(II) Construction of a tangent to the circle.

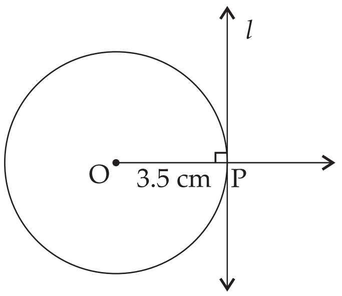
(A) Construction of tangent to a circle at a point on the circle using the centre of the circle.

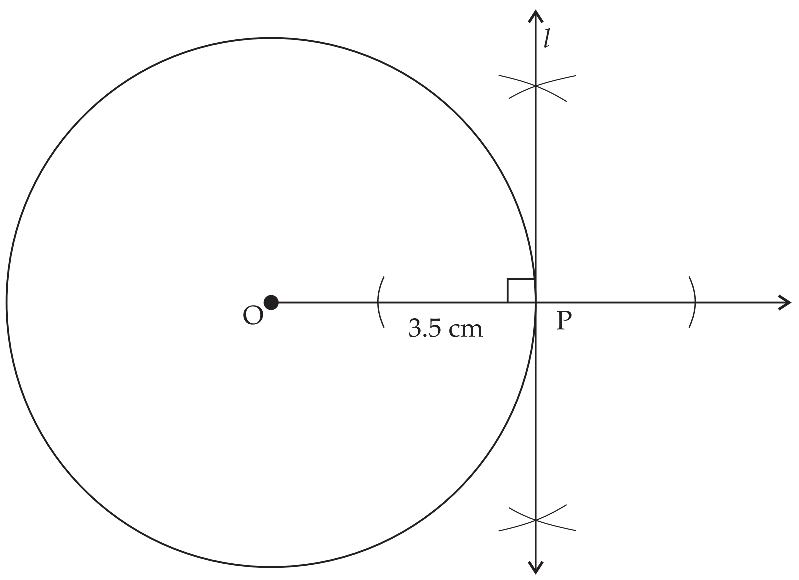
The tangent of a circle is perpendicular to the radius at its outer end. We use the same property to do this construction.

Example: To construct a tangent to a circle of radius 3.5 cm at a point P on it.

Solution :

Analytical figure:





Steps of construction:

(1) Draw circle with given radius with centre 'O'.

(2) Take a point 'P' on the circle.

(3) Draw ray OP.

(4) Draw perpendicular to ray OP at point P. Name line as *l*.

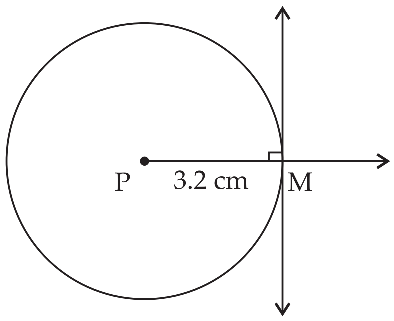
Line '*l*' is tangent to the circle (as perpendicular at outer end of radius is tangent.)

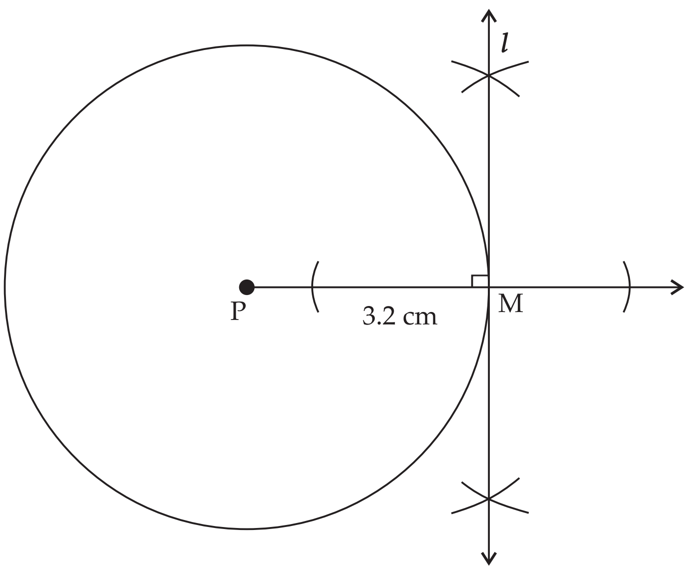
Practice Set - 4.2 *(Textbook Page No. 98)*

(1) Construct a tangent to a circle with centre P and radius 3.2 cm at any point M on it. *(2 marks)*

Solution :

Analytical figure



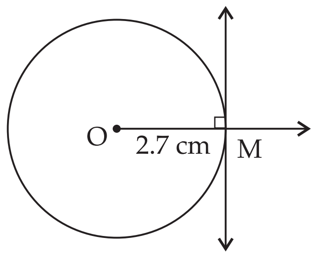


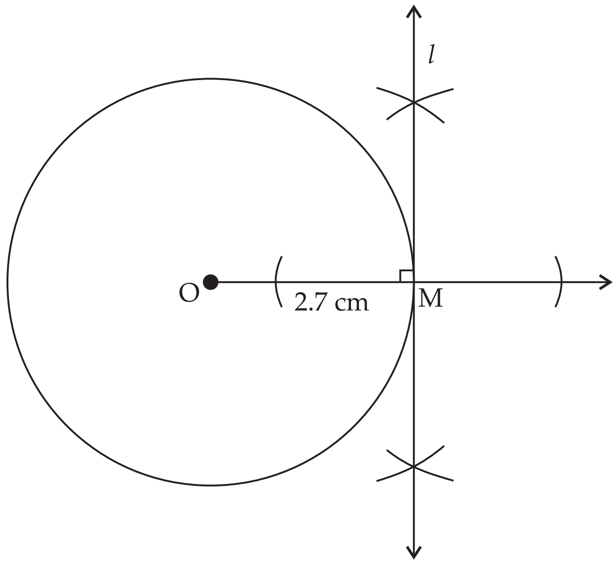
line *l* is the required tangent to the circle passing through point M on the circle.

(2) Draw a circle of radius 2.7 cm. Draw a tangent to the circle at any point on it. *(2 marks)*

Solution :

Analytical figure:





line *l* is the required tangent to the circle passing through point M on the circle.

(4) Draw a circle of radius 3.3 cm. Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q. Write your observation about the tangents. *(3 marks)*

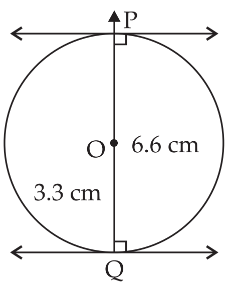
Solution : Analytical figure:

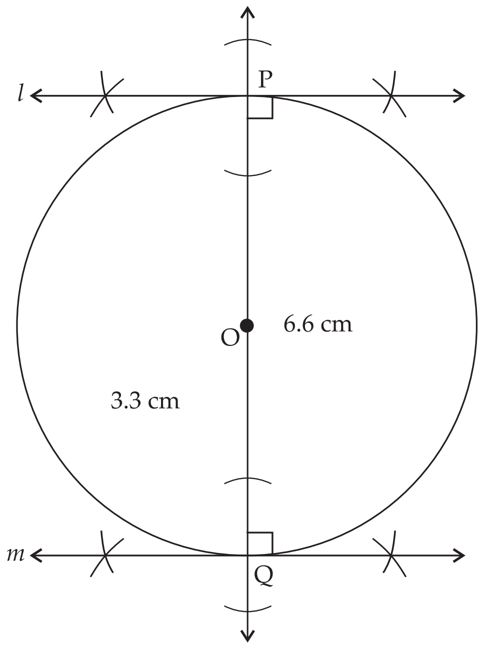
Radius = 3.3 cm (Given)

Chord = 6.6 cm (Given)

∴ Chord is twice of radius.

∴ Chord PQ is a diameter.





line l and m are the required tangents to the circle at point P and point Q.

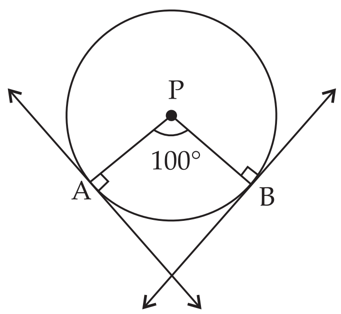
Tangents at the end points of a diameter are parallel to each other.

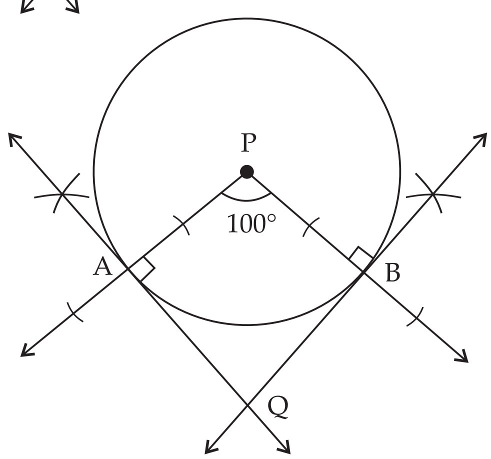
Problem Set - 4 *(Textbook Page No. 99)*

(5) Draw a circle with centre P. Draw an arc AB of 100° measure. Draw tangents to the circle at points A and point B. *(3 marks)*

Solution :

Analytical figure:





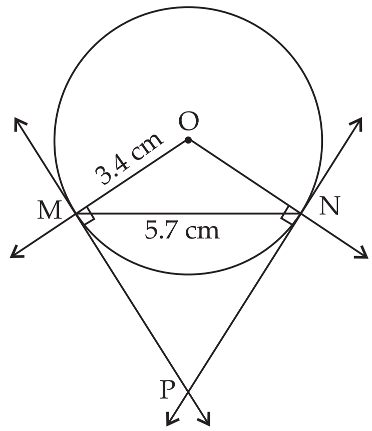
Line AQ and line BQ are tangents to the circle at points A and B respectively.

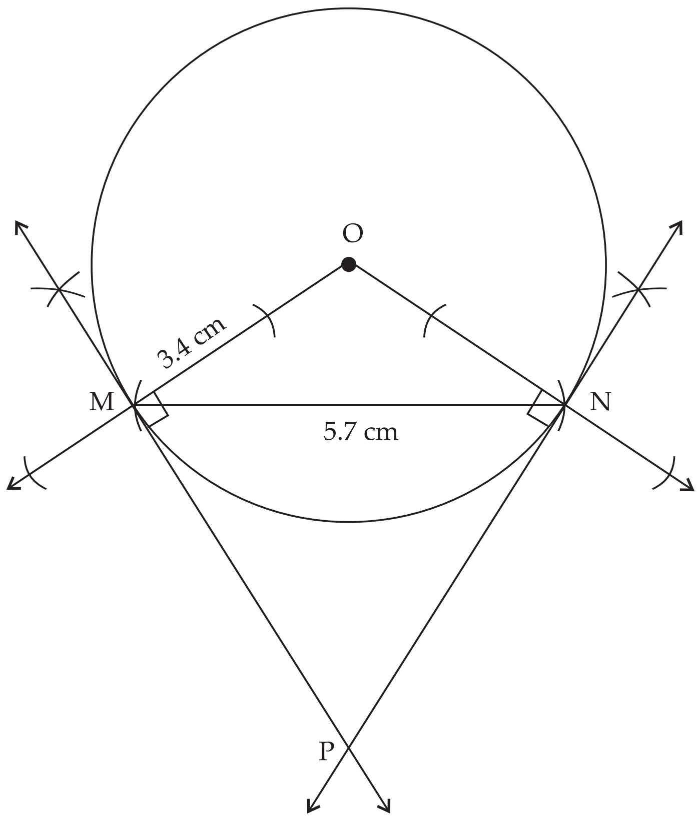
Practice Set - 4.2 *(Textbook Page No. 99)*

(5) Draw a circle with radius 3.4 cm. Draw a chord MN of length 5.7cm in it. Construct tangent at point M and N to the circle. *(3 marks)*

Solution :

Analytical figure:





Line MP and line NP are required tangents to the circle at point M and point N respectively.

Points to Remember:

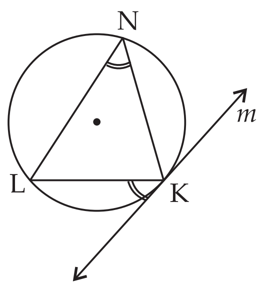
(B) Construction of a tangent to the circle from a point on the circle without using the centre.

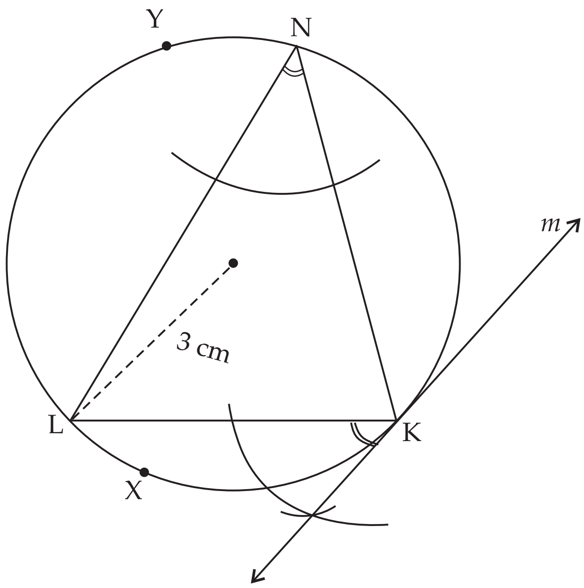
If a line drawn through an end point of a chord of a circle and an angle formed by it with the chord is equal to the angle subtended by the chord in the corresponding alternate segment, then the line is a tangent to the circle.

Example: Draw a circle of radius 3 cm. Take any point K on it. Draw a tangent to the circle at K without using centre of the circle.

Solution:

Analytical figure:





Line '*m*' is the tangent to the circle

Steps of construction:

(1) Draw a circle with radius 3 cm.

(2) Take a point K on the circle. Draw chord KL.

(3) Take a point N in the alternate arc of arc KXL.

(4) Draw seg LN and seg KN to form ∠LNK.

(5) Draw an angle congruent to ∠LNK at vertex K, taking LK as one side.

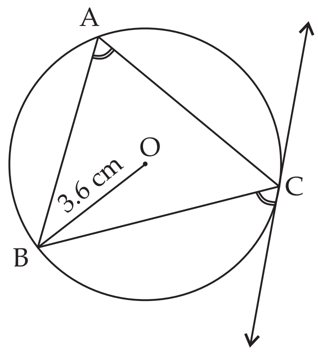
(6) The line *m* is the required tangent.

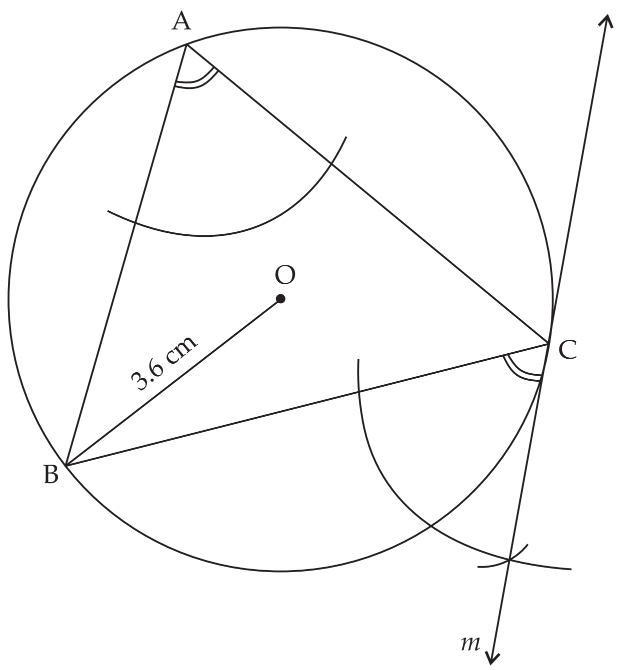
Practice Set - 4.2 *(Textbook Page No. 98)*

(3) Draw a circle of radius 3.6 cm. Draw a tangent to the circle at any point on it without using the centre. *(2 marks)*

Solution :

Analytical figure:





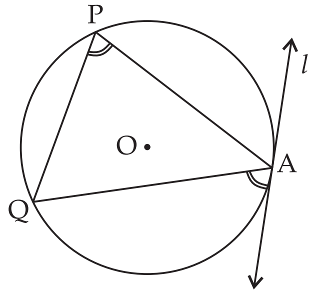
Line *m* is the required tangent to the circle at point C.

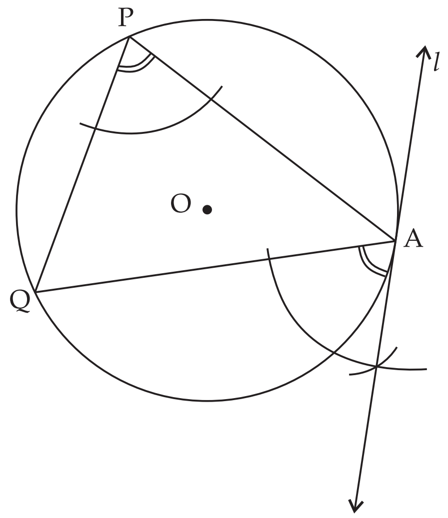
Problem Set - 4 *(Textbook Page No. 99)*

(3) Draw any circle. Take any point A on it and construct tangent at A without using the centre of the circle. *(2 marks)*

Solution :

Analytical figure:





Line *l* is the required tangent to the circle at point A.

Points to Remember:

(C) Construction of tangents to a circle from a point outside the circle.

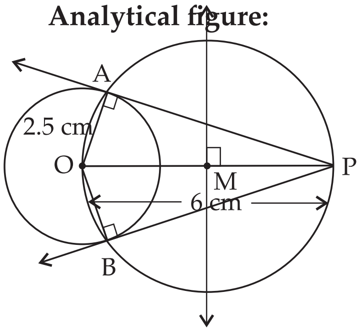
The angle inscribed in a semicircle is a right angle, using this property we shall draw a tangent to a circle from a point outside it.

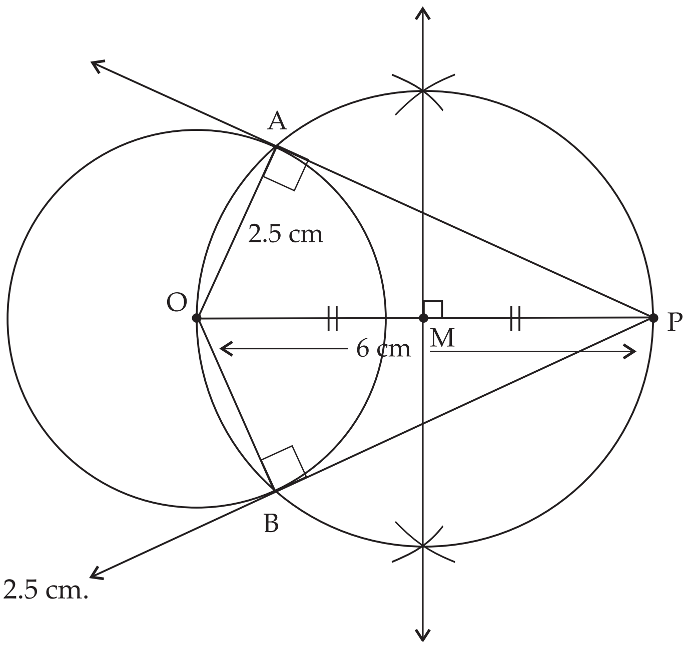
Note: We can draw two tangents from a point outside the circle.

Example: Draw tangent to the circle of radius 2.5 cm from a point 'P' at a distance 6 cm from the centre.

Solution:

Analytical figure:





Steps of construction:

(1) Draw a circle with centre 'O' and radius 2.5 cm.

(2) Take a point 'P' such that OP = 6 cm.

(3) Draw perpendicular bisector of seg OP and obtain midpoint M of seg OP.

(4) Taking 'M' as centre and MO as radius draw a circle, intersecting the circle at points A and B.

(5) Draw ray PA and ray PB.

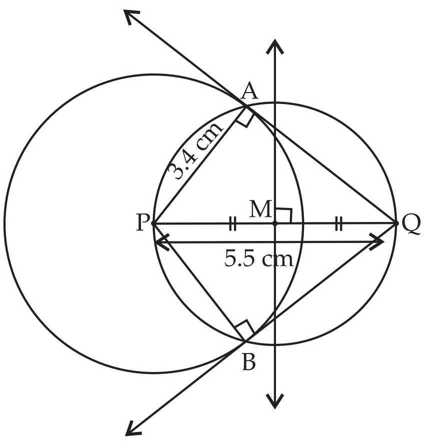
(6) Line PA and line PB are tangents to the circle at points A and B respectively from point 'P'.

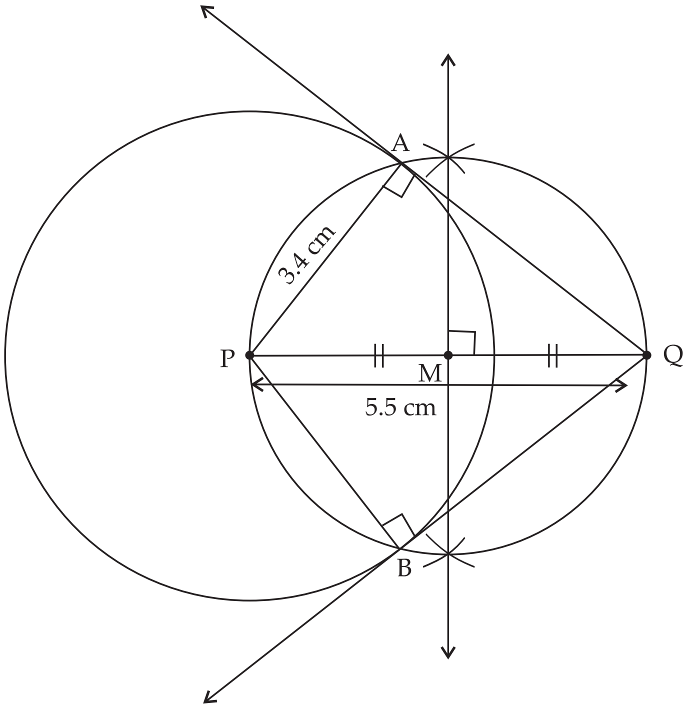
Practice Set - 4.2 *(Textbook Page No. 99)*

(6) Draw a circle with centre P and radius 3.4 cm. Take point Q at a distance 5.5 cm from the centre. Construct tangents to the circle from point Q. *(3 marks)*

Solution :

Analytical figure:



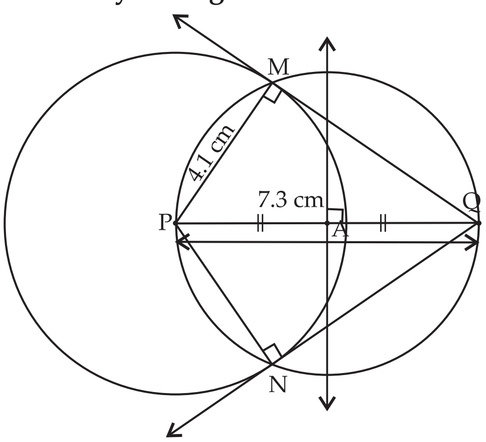


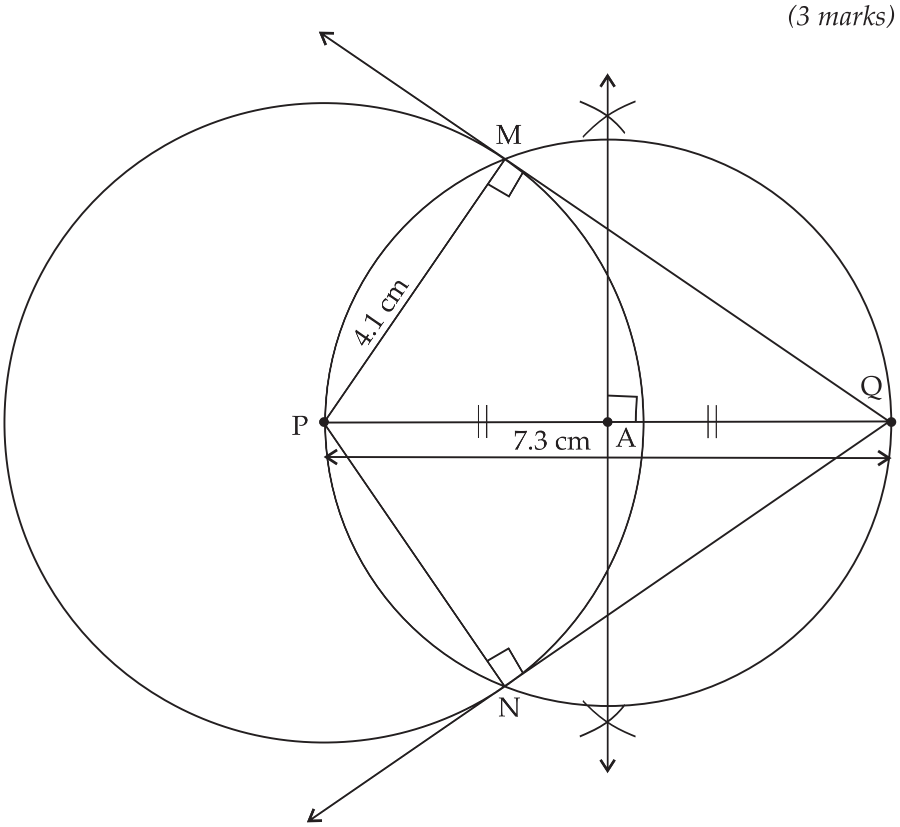
line MQ and line NQ are the required tangents to the circle from point Q.

(7) Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre. *(3 marks)*

Solution :

Analytical figure:





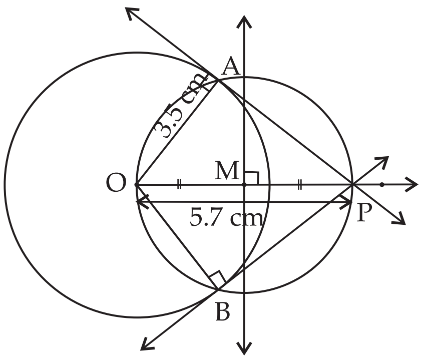
Line MQ and line NQ are the required tangents to the circle from point Q.

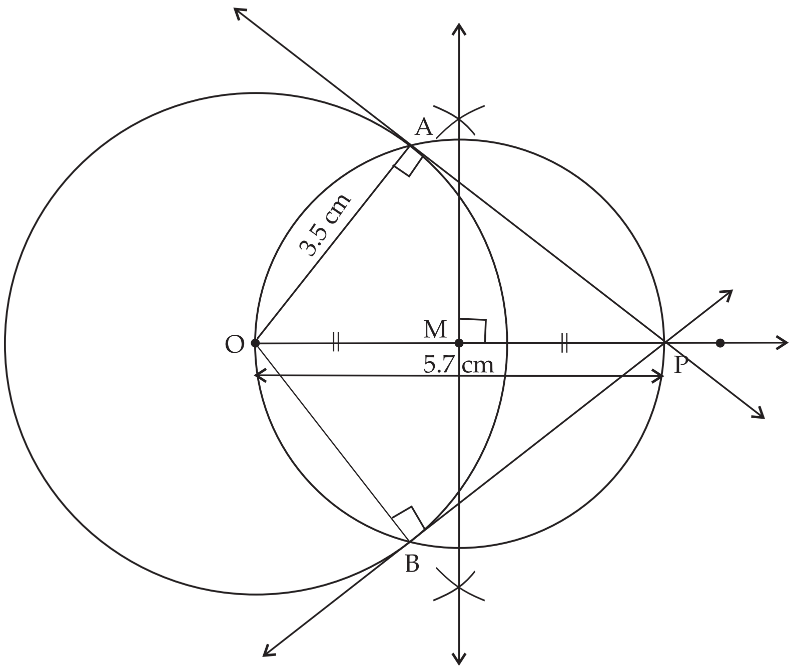
Problem Set - 4 *(Textbook Pg No. 99)*

(2) **Draw a circle with centre O and radius 3.5 cm. Take a point P at a distance 5.7 cm from the centre. Draw tangents to the circle from point P.** *(3 marks)*

Solution :

Analytical figure:

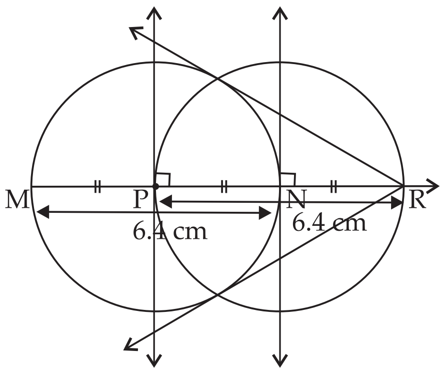


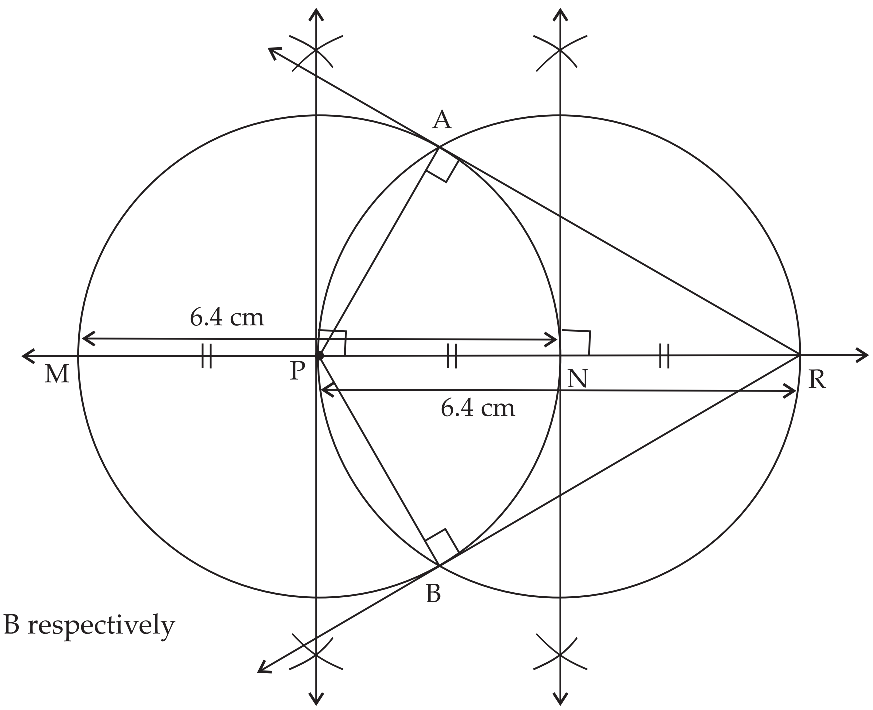


(4) **Draw a circle of diameter 6.4 cm. Take a point R at a distance equal to its diameter from the centre. Draw tangents from point R.** *(3 marks)*

Solution :

Analytical figure:



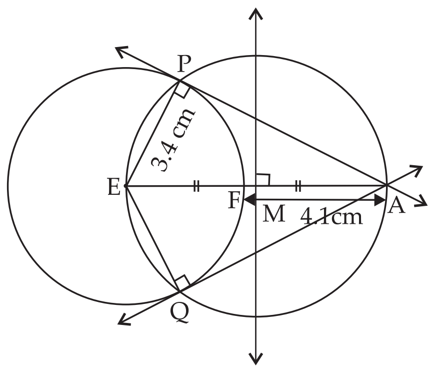


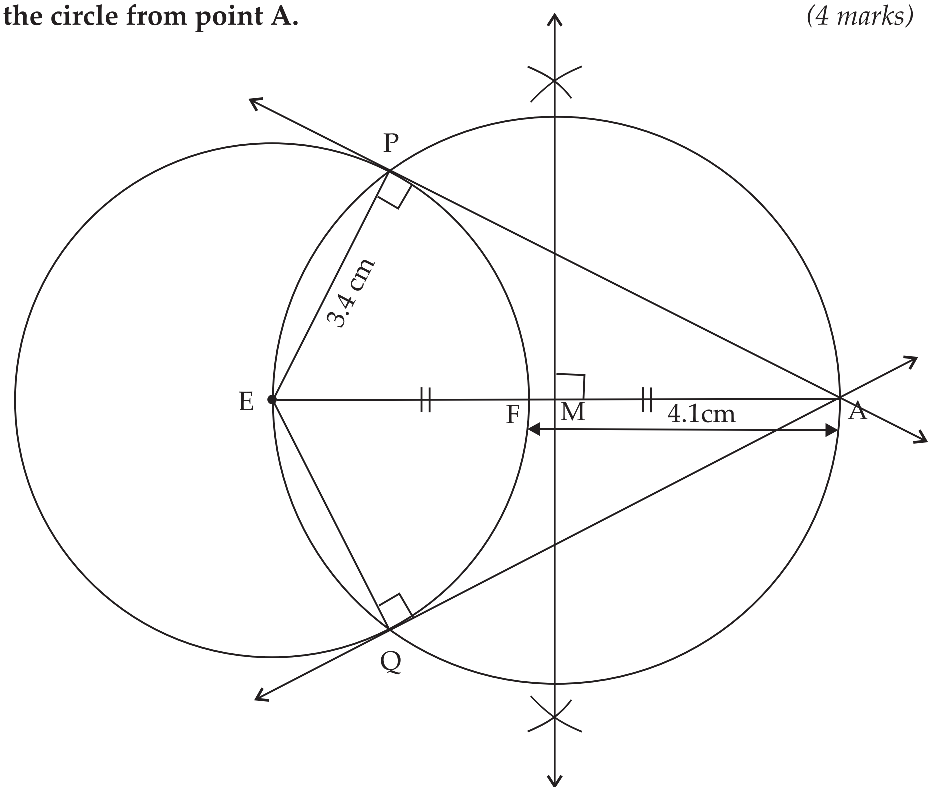
line RA and line RB are the required tangents to the circle at points A and B respectively from point R.

(6) **Draw a circle of radius 3.4 cm and centre E. Take a point F on the circle. Take another point A such that E-F-A and FA - 4.1 cm. Draw tangents to the circle from point A.** *(4 marks)*

Solution :

Analytical figure:





Line AP and line AQ are the required tangents from point A to the circle with centre E.

Problem Set - 4 *(Textbook Pg No. 99)*

MCQ's

(1) **Select correct alternative for each of the following questions.** *(1 mark each)*

*(1) The number of tangents that can be drawn to a circle at a point on the circle is ....

(A) 3 (B) 2 (C) 1 (D) 0

*(2) The maximum number of tangents that can be drawn to a circle from a point outside it is ....

(A) 2 (B) 1

(C) one and only one (D) 0

*(3) $\Delta ABC \sim \Delta PQR$, $\frac{AB}{PQ} = \frac{7}{5}$ then ....

(A) ΔABC is a bigger

(B) ΔPQR is bigger

(C) Both triangles will be equal

(D) Can not be decided.

Additional MCQ's

(4) .... number of tangent/s can be drawn from a point inside the circle.

(A) 0 (B) 1 (C) 2 (D) Infinite

(5) The lengths of the two tangent segments drawn to a circle from an external point are ....

(A) Equal (B) Unequal

(C) Infinite (D) Can't say

(6) Tangents drawn at the endpoints of a diameter of a circle are ....

(A) Equal (B) Perpendicular

(C) Parallel (D) Intersecting each other

(7) In $\Delta ABC \sim \Delta PQR$, AB : PQ = 2 : 3. If BC = 4, then QR = ...

(A) 4 (B) 6 (C) 9 (D) 8

(8) If AB : BC = 3 : 5, then how many equal parts seg AC divided to get point B ....

(a) 3 (b) 5 (c) 8 (d) Can't say

ANSWERS

(1) (C) 1 (2) (A) 2 (3) (A) ΔABC is a bigger (4) (A) 0

(5) (A) Equal (6) (C) Parallel (7) (B) 6 (8) (C) 8

PROBLEMS FOR PRACTICE

Based on Practice Set 4.1

(1) Draw a line segment PQ = 8 cm. Take a point R on it such that l (PR) : l (RQ) = 3 : 2. *(2 marks)*

(2) l(AB) : l(BC) = 3 : 2. Draw seg AB, if l(AB) = 7.2 cm. *(2 marks)*

(3) Δ XYZ ~ Δ ABC, $\angle$X= 40°, $\angle$Y = 80°, XY = 6 cm. Draw Δ ABC, if AB : XY = 3 : 2 *(3 marks)*

(4) Draw ΔABC with side BC = 6 cm, AB = 5 cm, and $\angle$ABC = 60°. Also, construct ΔXYZ whose sides are $\frac{3}{4}$ of the corresponding sides of ΔXYZ. *(4 marks)*

(5) ΔPQR ~ ΔABC, PQ = 3 cm, QR = 4 cm, PR = 5 cm. A(ΔPQR) : A (ΔABC) = 1 : 4. Construct both triangles *(4 marks)*

(6) ΔPQR ~ ΔPEF, m $\angle$P = 70°, PQ = 5 cm, PR = 3.5 cm. Construct ΔPEF, if PQ : PE = 5 : 7. *(4 marks)*

(7) ΔPQR ~ ΔPAB, m $\angle$P = 60°, PQ = 6 cm, PR = 4 cm. Construct ΔPAB, if PQ : PA = 3 : 2. *(4 marks)*

(8) ΔAMT ~ ΔAHE, construct ΔAMT such that MA = 6.3 cm, $\angle$MAT = 120°, AT = 4.9 cm and $\frac{MA}{HA} = \frac{7}{5}$, then construct ΔAHE. *(4 marks)*

Based on Practice Set 4.2

(9) Draw a tangent to a circle of radius 3 cm and centre 'O' at any point 'K' on the circle. *(3 marks)*

(10) Draw a circle with centre 'P' and radius 2.6 cm. Draw a chord AB of length 3.8 cm. Draw tangent to the circle through points A and B. *(3 marks)*

(11) Draw a circle with radius 3.4 cm. Draw tangent to the circle, passing through point B on the circle, without using centre. *(2 marks)*

(12) Construct a circle with centre 'O' and radius 4.3 cm. Draw a chord AB of length 5.6 cm. Construct the tangents to the circle at point A and B without using centre. *(3 marks)*

(13) Draw a circle with centre M and diameter 6 cm. Draw a tangent to the circle from a point N at distance of 9 cm from the centre. *(3 marks)*

(14) Draw a circle with 'O' as centre and radius 3.8 cm. Take two points P and Q such that $\angle$POQ = 120° Draw tangents at P and Q without using centre. *(3 marks)*

(15) Draw a circle with 'O' as centre and radius 4 cm. Take a point P at a distance of 7.5 cm from 'O'. Draw tangents to the circle through the point P. *(4 marks)*

❖ ❖ ❖

ASSIGNMENT – 4

Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the proper alternative answer for the question given below. **(2)**

(1) The number of tangents that can be drawn to a circle at a point on the circle is ....

(A) 3 (B) 2 (C) 1 (D) 0

(2) The lengths of the two tangent segments drawn to a circle from an external point are ....

(A) Equal (B) Unequal (C) Infinite (D) Can't say

Q.1. (B) Attempt any Two of the following: **(4)**

(1) Draw seg PQ = 8 cm. Divide it in the ratio 3 : 5. (2) Draw $\angle$ABC = 120° and bisect it

(3) Draw seg AB of length 6.3 cm and bisect it.

Q.2. Attempt any Two of the following: **(4)**

(1) Draw ΔDEF, EF = 5 cm, $\angle$D = 40°, $\angle$F =50°

(2) Construct a circle with centre 'O' and radius 3.5 cm. Take a point P on it, draw a tangent passing through point P.

(3) ΔABC ~ ΔXYZ, AB : XY = 3 : 5. BC = 9 cm, AC = 4.5 cm. Find YZ and XZ.

Q.3. Attempt any Two of the following: **(6)**

(1) Draw a circle with centre 'P' and suitable radius. Draw chord AB of length 5 cm. Draw tangents at points A and B without using centres.

(2) Draw a circle with centre 'O'. Take two points P and Q on the circle with such that $\angle$AOB = 120°. Draw tangents at points A and B.

(3) ΔDEF ~ ΔPQR, $\angle$D = 40°, $\angle$F = 60°, DF = 6 cm, DE : PQ = 3 : 4. Construct only ΔPQR.

Q.4. Attempt any one of the following: **(4)**

(1) Draw a circle with centre 'A' and radius 3.5 cm. Take a point B such that d (A, B) = 8 cm. Draw tangents to the circle passing through point B.

(2) ΔABC ~ ΔAEF, AB : AE = 5 : 2. AB = 6 cm, BC = 7.5 cm AC = 5 cm. Construct ΔAEF and ΔABC.

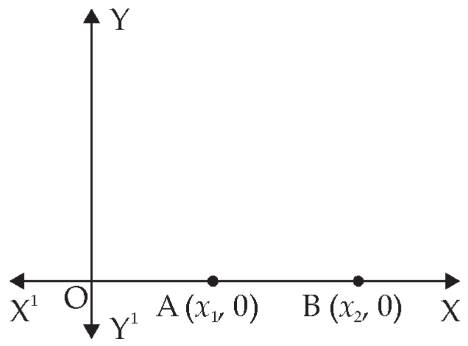
5 Co-ordinate Geometry

... INDEX ...

Points to Remember:

(1) To find distance any two points on an axis.

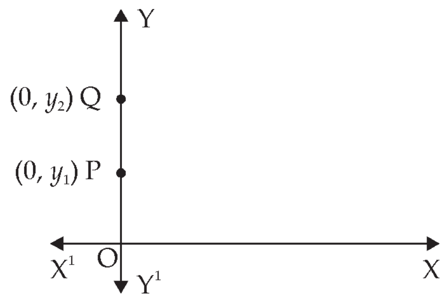
(i) To find distance between two points on X-axis.



In the above figure, points A(x_1, 0) and B(x_2, 0) are on X-axis such that, $x_2 > x_1$

$\therefore$ d(A, B) = $x_2 - x_1$

(ii) To find distance between two points on Y-axis.

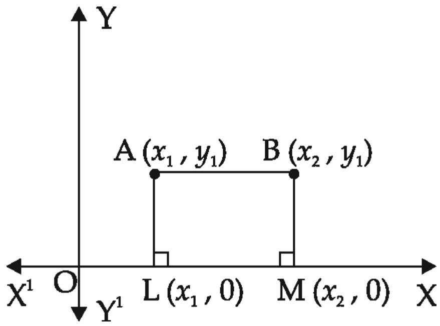


In the above figure, points P(0, y_1) and Q(0, y_2) are on Y-axis such that, $y_2 > y_1$

$\therefore$ d(P, Q) = $y_2 - y_1$

(2) To find the distance between two points if the segment joining these point is parallel to any axis in the XY plane.

(i)



In the figure, seg AB is parallel to X-axis.

$\therefore$ y co-ordinates of points A and B are equal.

Draw seg AL and seg BM perpendicular to X-axis.

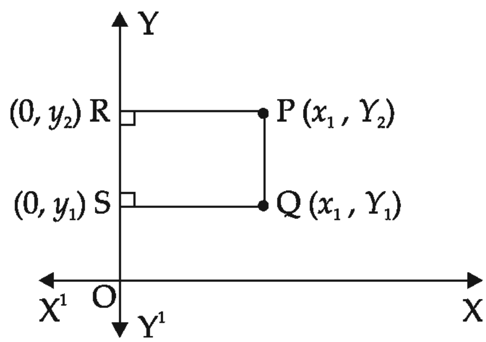
$\therefore$ $\square$ABML is a rectangle.

$\therefore$ AB = LM

But, LM = $x_2 - x_1$

d(A, B) = $x_2 - x_1$

(ii)



In the figure, seg PQ is parallel to Y-axis.

$\therefore$ x co-ordinates of points P and Q are equal.

Draw seg PR and seg QS perpendicular to Y-axis.

$\therefore$ $\square$ABML is a rectangle.

$\therefore$ PQ = RS

But, RS = $y_2 - y_1$

d(P, Q) = $y_2 - y_1$

- **Distance Formula :**

If A (x_1, y_1) and (x_2, y_2) are two points, then distance between these points is given by the following formula :

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or

$$d(A, B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Note : If P (x, y) is a point, then its distance from the origin is given by

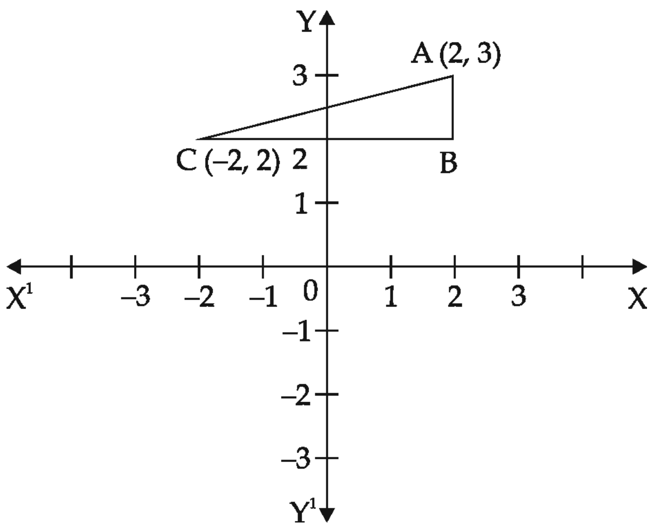
$$d(O, P) = \sqrt{x^2 + y^2}$$

Activity - I (Textbook page no. 102)

In the figure given below, seg AB || y - axis, seg CB || x - axis.

Coordinates of points A and C are given.

To find AC, fill in the following boxes



According to Pythagoras theorem,

$(AB)^2 + (BC)^2 = \boxed{AC^2}$

First find coordinates of B to find lengths AB and BC.

CB || X - axis $\therefore$ y coordinate of B = $\boxed{2}$

BA || Y - axis $\therefore$ x coordinate of B = $\boxed{2}$

$\therefore$ AB = $\boxed{3} - \boxed{2} = \boxed{1}$

BC = $\boxed{2} - \boxed{-2} = \boxed{4}$

$\therefore$ $AC^2 = \boxed{(1)^2} + \boxed{4^2} = \boxed{1 + 16} = \boxed{17}$

$\therefore$ AC = $\boxed{\sqrt{17}}$

MASTER KEY QUESTION SET - 5

Practice Set - 5.1 *(Textbook Page No. 107)*

(1) Find the distance between each of the following pairs of the points. *(2 marks each)*

(i) A(2, 3), B(4, 1)

Solution :

A(2, 3) = (x_1, y_1)

B(4, 1) = (x_2, y_2)

By distance formula,

$$\begin{aligned} d(A, B) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-2)^2 + (1-3)^2} \\ &= \sqrt{(2)^2 + (-2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= \sqrt{4 \times 2} \end{aligned}$$

$\therefore$ $\boxed{\textbf{d(A, B) = } 2\sqrt{2} \textbf{ units}}$

(ii) P(–5, 7), Q(–1, 3)

Solution :

P(–5, 7) = (x_1, y_1)

Q(–1, 3) = (x_2, y_2)

By distance formula,

$$\begin{aligned} d(P, Q) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-1-(-5)]^2 + (3-7)^2} \\ &= \sqrt{(-1+5)^2 + (-4)^2} \\ &= \sqrt{(4)^2 + 16} \\ &= \sqrt{16+16} \\ &= \sqrt{32} \\ &= \sqrt{16 \times 2} \end{aligned}$$

$\therefore$ $\boxed{\textbf{d(P, Q) = } 4\sqrt{2} \textbf{ units}}$

(iii) R(0, –3), S(0, $\frac{5}{2}$)

Solution :

R(0, –3) = (x_1, y_1)

S (0, $\frac{5}{2}$) = (x_2, y_2)

By distance formula,

$$d(R, S) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-0)^2 + \left(\frac{5}{2} - (-3)\right)^2}$$

$$= \sqrt{(0)^2 + \left(\frac{5}{2} + 3\right)^2}$$

$$= \sqrt{0 + \left(\frac{11}{2}\right)^2}$$

$$= \sqrt{\frac{121}{4}}$$

$\therefore$ $\boxed{\mathbf{d(R, S) = \frac{11}{2}\ units}}$

(iv) L(5, –8), M (–7, –3)

Solution :

L(5, –8) = (x_1, y_1)

M(–7, –3) = (x_2, y_2)

By distance formula,

$$d(L, M) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-7-5)^2 + [-3-(-8)]^2}$$

$$= \sqrt{(-12)^2 + (-3+8)^2}$$

$$= \sqrt{(-12)^2 + 5^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$\therefore$ $\boxed{\mathbf{d(L, M) = 13\ units}}$

(v) T(–3, 6), R(9, –10)

Solution :

T(–3, 6) = (x_1, y_1)

R(9, –10) = (x_2, y_2)

By distance formula,

$$d(T, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[9-(-3)]^2 + (-10-6)^2}$$

$$= \sqrt{(9+3)^2 + (-16)^2}$$

$$= \sqrt{(12)^2 + 256}$$

$$= \sqrt{144 + 256}$$

$$= \sqrt{400}$$

$\therefore$ $\boxed{\mathbf{d(T, R) = 20\ units}}$

(vi) W$\left(\frac{-7}{2}, 4\right)$, X(11, 4)

Solution :

W$\left(\frac{-7}{2}, 4\right)$ = (x_1, y_1)

X(11, 4) = (x_2, y_2)

By distance formula,

$$d(W, X) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left[11 - \left(\frac{-7}{2}\right)\right]^2 + (4-4)^2}$$

$$= \sqrt{\left(11 + \frac{7}{2}\right)^2 + 0^2}$$

$$= \sqrt{\left(\frac{22+7}{2}\right)^2 + 0}$$

$$= \sqrt{\left(\frac{29}{2}\right)^2}$$

$\therefore$ $\boxed{\mathbf{d(W, X) = \frac{29}{2}\ units}}$

Problem Set - 5 *(Textbook Pg No. 122)*

(6) Find the distance between the following pairs of points. *(2 marks each)*

(i) A(a, 0), B(0, a)

Solution :

Let A(a, 0) = (x_1, y_1)

B(0, a) = (x_2, y_2)

By distance formula,

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-a)^2 + (a-0)^2}$$

$$= \sqrt{(-a)^2 + a^2}$$

$$= \sqrt{a^2 + a^2}$$

$$= \sqrt{2a^2}$$

$\therefore$ $\boxed{\mathbf{d(A, B) = \sqrt{2}a\ units}}$

(ii) P(–6, –3), Q(–1, 9)

Solution :

Let P(–6, –3) = (x_1, y_1)

Q(–1, 9) = (x_2, y_2)

By distance formula,

$$\begin{aligned} d(P, Q) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-1-(-6)]^2 + [-9-(-3)]^2} \\ &= \sqrt{(-1+6)^2 + (9+3)^2} \\ &= \sqrt{(5)^2 + (12)^2} \\ &= \sqrt{25+144} \\ &= \sqrt{169} \end{aligned}$$

$\therefore$ $\boxed{\textbf{d(P, Q) = 13 units}}$

(iii) R(–3*a*, *a*), S(*a*, –2*a*)

Solution :

Let R(–3a, a) = (x_1, y_1)

S(a, –2a) = (x_2, y_2)

By distance formula,

$$\begin{aligned} d(R, S) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[a-(-3a)]^2 + (-2a-a)^2} \\ &= \sqrt{(a+3a)^2 + (-3a)^2} \\ &= \sqrt{(4a)^2 + (-3a)^2} \\ &= \sqrt{16a^2 + 9a^2} \\ &= \sqrt{25a^2} \end{aligned}$$

$\therefore$ $\boxed{\textbf{d(R, S) = 5}a\textbf{ units}}$

Practice Set - 5.1 *(Textbook Page No. 107)*

(3) Find the point on X-axis which is equidistant from A(–3, 4) and B(1, –4). *(2 marks)*

Solution :

Let P(x, 0) be a point on X axis which is equidistant from A(–3, 4) and B(1, –4).

$\therefore$ d(P, A) = d(P, B)

By distance formula,

$$\sqrt{[x-(-3)]^2 + (0-4)^2} = \sqrt{(x-1)^2 + [0-(-4)]^2}$$

$$\therefore \sqrt{(x+3)^2 + (-4)^2} = \sqrt{(x-1)^2 + (4)^2}$$

Squaring both the sides we get,

$$\begin{aligned} (x+3)^2 + 16 &= (x-1)^2 + 16 \\ \therefore\ x^2 + 6x + 9 &= x^2 - 2x + 1 \\ \therefore\ x^2 + 6x - x^2 + 2x &= 1 - 9 \\ \therefore\ 8x &= -8 \\ \therefore\ x &= -1 \end{aligned}$$

$\therefore$ $\boxed{\textbf{P(–1, 0) is the required point.}}$

Problem Set - 5 *(Textbook Pg No. 122)*

(5) Find a point on X-axis which is equidistant from P(2, –5) and Q(–2, 9).

(2 marks)

Solution :

Let A(a, 0) be a point equidistant from P(2, –5) and Q(–2, 9).

$\therefore$ d(P, A) = d(Q, A)

Using distance formula,

$$\sqrt{(a-2)^2 + [0-(-5)]^2} = \sqrt{[a-(-2)]^2 + (0-9)^2}$$

Squaring both the sides we get,

$$\begin{aligned} (a-2)^2 + 5^2 &= (a+2)^2 + (-9)^2 \\ \therefore\ a^2 - 4a + 4 + 25 &= a^2 + 4a + 4 + 81 \\ \therefore\ a^2 - 4a - a^2 - 4a &= 81 - 25 \\ \therefore\ -8a &= 56 \\ \therefore\ a &= \frac{56}{-8} \\ \therefore\ a &= -7 \end{aligned}$$

$\therefore$ **(–7, 0) is a point on X-axis equidistant from P(2, –5) and Q(–2, 9).**

Practice Set - 5.1 *(Textbook Page No. 107)*

(4) Verify that points P(–2, 2), Q(2, 2) and R(2, 7) are vertices of a right angled triangle. *(3 marks)*

Solution :

P(–2, 2), Q(2, 2) and R(2, 7) be the vertices of a triangle

Using distance formula,

$$\begin{aligned} d(P, Q) &= \sqrt{(-2-2)^2 + (2-2)^2} \\ &= \sqrt{(-4)^2 + 0^2} \\ &= \sqrt{16} \end{aligned}$$

$\therefore$ d(P, Q) = 4 units

i.e. PQ = 4 units

$$\begin{aligned} d(Q, R) &= \sqrt{(2-2)^2 + (2-7)^2} \\ &= \sqrt{0^2 + (-5)^2} \\ d(Q, R) &= \sqrt{25} \end{aligned}$$

$\therefore$ QR = 5 units

$$\begin{aligned} d(P, R) &= \sqrt{(-2-2)^2 + (2-7)^2} \\ &= \sqrt{(-4)^2 + (-5)^2} \\ &= \sqrt{16+25} \end{aligned}$$

$\therefore$ d(P, R) = $\sqrt{41}$ units

i.e. PR = $\sqrt{41}$ units

$PR^2 = 41$ ...(i)

$PQ^2 + QR^2 = 4^2 + 5^2$

$\therefore$ $PQ^2 + QR^2 = 16 + 25 = 41$ ...(ii)

$\therefore$ $PR^2 = PQ^2 + QR^2$ ...[From (i) and (ii)]

$\therefore$ **ΔPQR is a right angled triangle**

...(Converse of Pythagoras theorem)

(6) A(–4, –7), B(–1, 2), C(8, 5) and D(5, –4) are the vertices of rhombus ABCD. *(3 marks)*

Solution :

A(–4, –7), B(–1, 2), C(8, 5) and D(5, –4) are the vertical it a quadrilateral

By distance formula,

$$d(A, B) = \sqrt{[-4-(-1)]^2 + (-7-2)^2}$$

$$= \sqrt{(-3)^2 + (-9)^2}$$

$$= \sqrt{9+81}$$

$\therefore$ $d(A, B) = \sqrt{90}$ units ...(i)

$$d(B, C) = \sqrt{(-1-8)^2 + (2-5)^2}$$

$$= \sqrt{(-9)^2 + (-3)^2}$$

$$= \sqrt{81+9}$$

$\therefore$ $d(B, C) = \sqrt{90}$ units ...(ii)

$$d(C, D) = \sqrt{(8-5)^2 + [5-(-4)]^2}$$

$$= \sqrt{(3)^2 + (5+4)^2}$$

$$= \sqrt{9+81}$$

$\therefore$ $d(C, D) = \sqrt{90}$ units ...(iii)

$$d(A, D) = \sqrt{(-4-5)^2 + [-7-(-4)]^2}$$

$$= \sqrt{(-9)^2 + (-3)^2}$$

$$= \sqrt{81+9}$$

$\therefore$ $d(A, D) = \sqrt{90}$ units ...(iv)

$\therefore$ AB = BC = CD = AD

...[From (i), (ii), (iii) and (iv)]

$\therefore$ **□ABCD is a rhombus.** ...(By Definition)

(7) Find x if distance between points L(x, 7) and M(1, 15) is 10. *(2 marks)*

Solution :

L(x, 7) and M(1, 15)

By distance formula,

$$d(L, M) = \sqrt{(x-1)^2 + (7-15)^2}$$

$$\therefore\ 10 = \sqrt{(x-1)^2 + (-8)^2}$$

Squaring both the sides we get,

$100 = (x-1)^2 + 64$

$\therefore$ $100 - 64 = (x-1)^2$

$\therefore$ $(x-1)^2 = 36$

$\therefore$ $x - 1 = \pm 6$ ...(Taking square roots)

$\therefore$ $x - 1 = 6$ or $x - 1 = -6$

$\therefore$ $x = 6 + 1$ or $x = -6 + 1$

$\therefore$ $x = 7$ or $x = -5$

$\therefore$ $\boxed{x = 7 \text{ or } x = -5}$

(8) Show that the points A(1, 2), B(1, 6) and C(1 + $2\sqrt{3}$, 4) are the vertices of an equilateral triangle. *(3 marks)*

Solution :

A(1, 2), B(1, 6) and C(1 + $2\sqrt{3}$, 4) be the vetices of triangle

Using distance formula,

$$d(A, B) = \sqrt{(1-1)^2 + (2-6)^2}$$

$$= \sqrt{0^2 + (-4)^2}$$

$$= \sqrt{0+16}$$

$$= \sqrt{16}$$

$\therefore$ $d(A, B) = 4$ units ...(i)

$$d(B, C) = \sqrt{(1+2\sqrt{3}-1)^2 + (4-6)^2}$$

$$= \sqrt{\left(2\sqrt{3}\right)^2 + (-2)^2}$$

$$= \sqrt{12+4}$$

$\therefore$ $d(B, C) = \sqrt{16}$

$d(B, C) = 4$ units ...(ii)

$$d(A, C) = \sqrt{(1+2\sqrt{3}-1)^2 + (4-2)^2}$$

$$= \sqrt{(2\sqrt{3})^2 + (2)^2}$$

$$= \sqrt{12+4}$$

$$= \sqrt{16}$$

$\therefore$ $d(A, C) = 4$ units ...(iii)

$\therefore$ AB = BC = AC ...[From (i), (ii) and (iii)]

$\therefore$ **ΔABC is an equilateral triangle**

...(By Definition)

Problem Set - 5 *(Textbook Pg No. 123)*

(8) In the following examples, can the segment joining the given points form a triangle? If triangle is formed, state the type of the triagle cosidering sides of the triangle. *(3 marks each)*

(i) L(6, 4), M(–5, –3), N(–6, 8)

Solution :

Let L(6, 4), M(–5, –3), N(–6, 8) be the given points

Using distance formula,

$$d(L, M) = \sqrt{[6-(-5)]^2 + [4-(-3)]^2}$$
$$= \sqrt{(6+5)^2 + (4+3)^2}$$
$$= \sqrt{11^2 + 7^2}$$
$$= \sqrt{121 + 49}$$
$$\therefore\ d(L, M) = \sqrt{170} \text{ units} \quad ...(i)$$
$$d(M, N) = \sqrt{[-6-(-5)]^2 + [8-(-3)]^2}$$
$$= \sqrt{(-6+5)^2 + (8+3)^2}$$
$$= \sqrt{(-1)^2 + (11)^2}$$
$$\therefore\ d(M, N) = \sqrt{122} \text{ units} \quad ...(ii)$$
$$d(L, N) = \sqrt{[6-(-6)]^2 + (4-8)^2}$$
$$= \sqrt{(6+6)^2 + (-4)^2}$$
$$= \sqrt{12^2 + 16^2}$$
$$= \sqrt{144 + 16}$$
$$\therefore\ d(L, N) = \sqrt{160} \text{ units} \quad ...(iii)$$

$\therefore$ d(L, M) $\neq$ d(L, N) + d(M, N)

$\therefore$ Points L, M and N are non-collinear points.

$\therefore$ We can construct a triangle using above points.

As none of the sides of triangle are equal, it is a scalene triangle.

(ii) P(–2, –6), Q(–4, –2), R(–5, 0)

Solution :

Let P(–2, –6), Q(–4, –2), R(–5, 0) be the given points

Using distance formula,

$$d(P, Q) = \sqrt{[-4-(-2)]^2 + [-2-(-6)]^2}$$
$$= \sqrt{(-4+2)^2 + (-2+6)^2}$$
$$= \sqrt{(-2)^2 + (4)^2}$$
$$= \sqrt{4 + 16}$$
$$= \sqrt{20}$$
$$\therefore\ d(P, Q) = 2\sqrt{5} \text{ units} \quad ...(i)$$
$$d(Q, R) = \sqrt{[-5-(-4)]^2 + [0-(-2)]^2}$$
$$= \sqrt{(-5+4)^2 + (0+2)^2}$$
$$= \sqrt{(-1)^2 + (2)^2}$$
$$= \sqrt{1 + 4}$$
$$\therefore\ d(Q, R) = \sqrt{5} \text{ units} \quad ...(ii)$$
$$d(P, R) = \sqrt{[-2-(-5)]^2 + (-6-0)^2}$$
$$= \sqrt{(-2+5)^2 + (-6)^2}$$
$$= \sqrt{(3)^2 + 36}$$
$$= \sqrt{9 + 36}$$
$$d(P, R) = \sqrt{45} \text{ units} \quad ...(iii)$$

$\therefore$ d(P, R) $\neq$ d(P, Q) + d(Q, R)

$\therefore$ Points P, Q and R are non-collinear points.

$\therefore$ We can construct a triangle using above points.

As none of the sides of triangle are equal, triangle is a scalene triangle.

(iii) $A\left(\sqrt{2}, \sqrt{2}\right), B\left(-\sqrt{2}, -\sqrt{2}\right), C\left(-\sqrt{6}, \sqrt{6}\right)$

Solution :

Let $A\left(\sqrt{2}, \sqrt{2}\right)$, $B\left(-\sqrt{2}, -\sqrt{2}\right)$, $C\left(-\sqrt{6}, \sqrt{6}\right)$ be the given points

By distance formula,

$$d(A, B) = \sqrt{\left[\sqrt{2}-(-\sqrt{2})\right]^2 + \left[\sqrt{2}-(-\sqrt{2})\right]^2}$$
$$= \sqrt{\left[\sqrt{2}+\sqrt{2}\right]^2 + \left[\sqrt{2}+\sqrt{2}\right]^2}$$
$$= \sqrt{\left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2}$$
$$= \sqrt{8 + 8}$$
$$= \sqrt{16}$$
$$\therefore\ d(A, B) = 4 \text{ units} \quad ...(i)$$
$$d(B, C) = \sqrt{\left[-\sqrt{2}-(-\sqrt{6})\right]^2 + \left[-\sqrt{2}-\sqrt{6}\right]^2}$$
$$= \sqrt{\left(-\sqrt{2}+\sqrt{6}\right)^2 + (-1)^2\left(\sqrt{2}+\sqrt{6}\right)^2}$$
$$= \sqrt{2 - 2\sqrt{12} + 6 + 2 + 2\sqrt{12} + 6}$$
$$= \sqrt{16}$$
$$\therefore\ d(B, C) = 4 \text{ units} \quad ...(ii)$$
$$d(A, C) = \sqrt{\left[\sqrt{2}-(-\sqrt{6})\right]^2 + \left(\sqrt{2}-\sqrt{6}\right)^2}$$
$$= \sqrt{\left(\sqrt{2}+\sqrt{6}\right)^2 + \left(\sqrt{2}-\sqrt{6}\right)^2}$$
$$= \sqrt{2 + 2\sqrt{12} + 6 + 2 - 2\sqrt{12} + 6}$$
$$= \sqrt{16}$$

d(A, C) = 4 units ...(iii)

$\therefore$ d(AB) $\neq$ d(BC) + d(AC)

$\therefore$ Points A, B and C are non-collinear points.

$\therefore$ We can construct a triangle using above three points.

AB = BC = AC ...[From (i), (ii) and (iii)

$\therefore$ **ΔABC is an equilateral triangle.**

Problem Set - 5.1 *(Textbook Pg No. 122)*

(15) Show that A(4, –1), B(6, 0), C(7, –2) and D(5, –3) are vertices of a square. *(4 marks)*

Solution :

A(4, –1), B(6, 0), C(7, –2) and D(5, –3) be the vertices of a quadrilateral

Using distance formula,

$$d(A, B) = \sqrt{(4-6)^2 + (-1-0)^2}$$
$$= \sqrt{(-2)^2 + (-1)^2}$$
$$= \sqrt{4+1}$$
$$\therefore \ d(A, B) = \sqrt{5} \text{ units} \quad \text{...(i)}$$

$$d(B, C) = \sqrt{(6-7)^2 + [0-(-2)]^2}$$
$$= \sqrt{(-1)^2 + (2)^2}$$
$$= \sqrt{1+4}$$
$$\therefore \ d(B, C) = \sqrt{5} \text{ units} \quad \text{...(ii)}$$

$$d(C, D) = \sqrt{(7-5)^2 + [-2-(-3)]^2}$$
$$= \sqrt{(2)^2 + (-2+3)^2}$$
$$= \sqrt{4+(1)^2}$$
$$= \sqrt{4+1}$$
$$\therefore \ d(C, D) = \sqrt{5} \text{ units} \quad \text{...(iii)}$$

$$d(A, D) = \sqrt{(4-5)^2 + [-1-(-3)]^2}$$
$$= \sqrt{(-1)^2 + (-1+3)^2}$$
$$= \sqrt{1+(2)^2}$$
$$= \sqrt{1+4}$$
$$\therefore \ d(A, D) = \sqrt{5} \text{ units} \quad \text{...(iv)}$$

In □ABCD,

AB = BC = CD = AD...[From (i), (ii), (iii) and (iv)]

∴ □ABCD is a rhombus ...(v) [Definition]

Now, we shall find length of each diagonal.

Using distance formula,

$$d(A, C) = \sqrt{(4-7)^2 + [-1-(-2)]^2}$$
$$= \sqrt{(-3)^2 + (-1+2)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$
$$\therefore \ d(A, C) = \sqrt{10} \text{ units} \quad \text{...(vi)}$$

$$d(B, D) = \sqrt{(6-5)^2 + [0-(-3)]^2}$$
$$= \sqrt{(1)^2 + (0+3)^2}$$
$$= \sqrt{1+9}$$
$$= \sqrt{10}$$
$$\therefore \ d(B, D) = \sqrt{10} \text{ units} \quad \text{...(vii)}$$

In rhombus ABCD,

diagonal AC ≅ diagonal BD

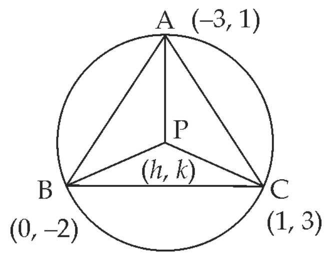
...[From (v), (vi) and (vii)]

∴ **□ABCD is a square**

...(A rhombus is a square if its diagonals are congruent)

(7) Find the coordinates of circumcentre of a triangle whose vertices are (–3, 1), (0, –2) and (1, 3) *(4 marks)*

Solution :



Let A(–3, 1), B(0, –2) and C(1, 3) and circum centre be p (h, k) be the vertices of a triangle

∴ PA = PB = PC ...(i) (Radii of same circle)

PA = PB ...[From (i)]

Using distance formula,

$$\sqrt{[h-(-3)]^2 + (k-1)^2} = \sqrt{(h-0)^2 [k-(-2)]^2}$$
$$\therefore \ \sqrt{(h+3)^2 + (k-1)^2} = \sqrt{h^2 + (k+2)^2}$$

Squaring both the sides we get,

$$(h + 3)^2 + (k - 1)^2 = h^2 + (k + 2)^2$$
$$\therefore \ h^2 + 6h + 9 + k^2 - 2k + 1 = h^2 + k^2 + 4k + 4$$
$$\therefore \ 6h - 2k = 4 - 9 - 1$$
$$\therefore \ 6h - 6k = -6$$
$$\therefore \ h - k = -1 \quad \text{...(ii)}$$

...(Dividing both sides by 6)

PB = PC ...[From (i)]

Using distance formula,

$$\sqrt{(h-0)^2 + [k-(-2)]^2} = \sqrt{(h-1)^2 + (k-3)^2}$$
$$\therefore \ \sqrt{h^2 + (k+2)^2} = \sqrt{(h-1)^2 + (k-3)^2}$$

Squaring both the sides we get,

$$\therefore \ h^2 + (k + 2)^2 = (h - 1)^2 + (k - 3)^2$$
$$\therefore \ h^2 + k^2 + 4k + 4 = h^2 - 2h + 1 + k^2 - 6k + 9$$
$$\therefore \ h^2 + k^2 + 4k - h^2 + 2h - k^2 + 6k = 10 - 4$$
$$\therefore \ 2h + 10k = 6$$

$\therefore \quad h + 5k = 3$ ...(iii)

(Dividing both sides by 2)

Subtracting (ii) from (iii),

$$\begin{array}{rcl} h + 5k &=& 3 \\ h - k &=& -1 \\ (-) \quad (+) & & (+) \\ \hline 6k &=& 4 \end{array}$$

$\therefore \quad k = \frac{4}{6}$

$\therefore \quad k = \frac{2}{3}$

Substituting $k = \frac{2}{3}$ in equation (i)

$h - \frac{2}{3} = -1$

$\therefore \quad h = -1 + \frac{2}{3}$

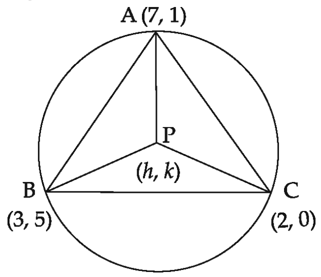
$\therefore \quad h = \frac{-3+2}{3}$

$\therefore \quad h = \frac{-1}{3}$

$\therefore$ **$P\left(\frac{-1}{3}, \frac{2}{3}\right)$ is the circumcentre of ΔABC**

(16) Find the co-ordinates of circumcentre and radius of a circumcircle of ΔABC, if A(7, 1), B(3, 5) and C(2, 0) are given. *(4 marks)*

Solution :



Let P (h, k) be the circumcentre of ABC.

PA = PB = PC ...(i) (Radii of same circle)

$\therefore$ PA = PB ...[From (i)]

Using distance formula,

$$\sqrt{(h-7)^2+(k-1)^2} = \sqrt{(h-3)^2+(k-5)^2}$$

Squaring both the sides we get,

$\therefore \quad (h-7)^2 + (k-1)^2 = (h-3)^2 + (k-5)^2$

$\therefore \quad h^2 - 14h + 49 + k^2 - 2k + 1 = h^2 - 6h + 9 + k^2 - 10k + 25$

$\therefore \quad h^2 - 14h + k^2 - 2k - h^2 + 6h - k^2 + 10k = 25 + 9 - 49 - 1$

$\therefore \quad -8h + 8k = -16$

$\therefore \quad h - k = 2$ ...(ii)

...(Dividing throughtout by –8)

PB = PC ...[From (i)]

Using distance formula,

$$\sqrt{(h-3)^2+(k-5)^2} = \sqrt{(h-2)^2+(k-0)^2}$$

Squaring both the sides we get,

$(h-3)^2 + (k-5)^2 = (h-2)^2 + (k)^2$

$\therefore \quad h^2 - 6h + 9 + k^2 - 10k + 25 = h^2 - 4h + 4 + k^2$

$\therefore \quad h^2 - 6h + k^2 - 10k - h^2 + 4h - k^2 = 4 - 9 - 25$

$\therefore \quad -2h - 10k = -30$

$\therefore \quad h + 5k = 15$ ...(iii)

(dividing both sides by – 2)

Subtracting (iii) from (ii),

$$\begin{array}{rcl} h - k &=& 2 \\ h + 5k &=& 15 \\ (-) \quad (-) & & (-) \\ \hline -6k &=& -13 \end{array}$$

$\therefore \quad k = \frac{-13}{-6}$

$\therefore \quad k = \frac{13}{6}$

Substituting $k = \frac{13}{6}$ in (ii),

$h - k = 2$

$\therefore \quad h - \frac{13}{6} = 2$

$\therefore \quad h = 2 + \frac{13}{6}$

$\therefore \quad h = \frac{25}{6}$

$\therefore$ **$P\left(\frac{25}{6}, \frac{13}{6}\right)$ are the co-ordinates of circumcentre.**

Using distance formula,

$$\text{Radius} = d(P, A) = \sqrt{\left(7-\frac{25}{6}\right)^2+\left(1-\frac{13}{6}\right)^2}$$

$$= \sqrt{\left(\frac{42-25}{6}\right)^2+\left(\frac{6-13}{6}\right)^2}$$

$$= \sqrt{\left(\frac{17}{6}\right)^2+\left(\frac{-7}{6}\right)^2}$$

$$= \sqrt{\frac{289}{36}+\frac{49}{36}}$$

$$= \sqrt{\frac{338}{36}}$$

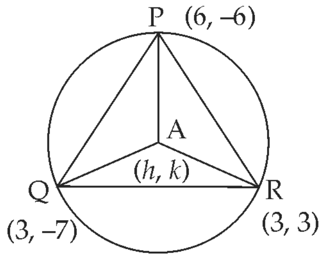
$$= \sqrt{\frac{169 \times 2}{36}}$$

$$= \frac{13}{6}\sqrt{2}$$

$\therefore$ **Radius of circumcircle = $\frac{13}{6}\sqrt{2}$ units**

(20) Find the co-ordinates of the centre of the circle passing through the point. P(6, –6), Q(3, –7) and R(3, 3) *(4 marks)*

Solution :



Let A(h, k) be the centre of the circle.

PA = QA = RA ...(i) (Radii of same circle)

i.e. PA = QA ...[From (i)]

Using distance formula,

$$\sqrt{(h-6)^2+[k-(-6)]^2} = \sqrt{(h-3)^2+[k-(-7)]^2}$$

$$\therefore \sqrt{(h-6)^2+(k+6)^2} = \sqrt{(h-3)^2+(k+7)^2}$$

Squaring both the sides,

$$h^2 - 12h + 36 + k^2 + 12k + 36 = h^2 - 6h + 9 + k^2 + 14k + 49$$

$$\therefore h^2 - 12h + k^2 + 12k - h^2 + 6h - k^2 - 14k = 9 + 49 - 36 - 36$$

$$\therefore -6h - 2k = -14$$

$$\therefore 3h + k = 7 \quad ...(ii)$$

...(Dividing both sides by –2)

QA = RA ...[From (i)]

Using distance formula,

$$\therefore \sqrt{(h-3)^2+[k-(-7)]^2} = \sqrt{(h-3)^2+(k-3)^2}$$

$$\therefore \sqrt{(h-3)^2+(k+7)^2} = \sqrt{(h-3)^2+(k-3)^2}$$

Squaring both the sides,

$$(h-3)^2 + (k+7)^2 = (h-3)^2 + (k-3)^2$$

$$k^2 + 14k + 49 = k^2 - 6k + 9$$

$$\therefore k^2 + 14k - k^2 + 6k = 9 - 49$$

$$\therefore 20k = -40$$

$$\therefore k = \frac{-40}{20}$$

$$\therefore k = -2$$

Substituting $k = -2$ in equation ....(ii)

$$3h - 2 = 7$$

$$\therefore 3h = 7 + 2$$

$$\therefore h = \frac{9}{3}$$

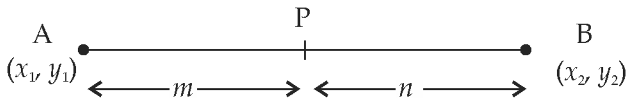
$$\therefore h = 3$$

$\therefore$ **A(3, –2) is the centre of the circle**

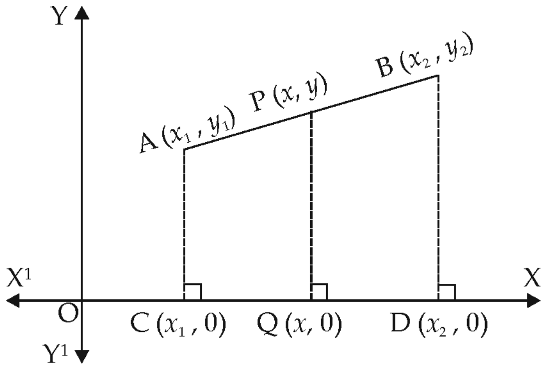
Points to Remember:

Section formula for division of a line segment :

If P(x, y) divides segment joining A(x_1, y_1) and B(x_2, y_2) in the ratio $m : n$, then



$$x = \frac{mx_2 + nx_1}{m+n}; \quad y = \frac{my_2 + ny_1}{m+n}$$



In the above figure, in XY plane point P on the seg AB, divides seg AB in the ratio $m : n$.

Let A (x_1, y_1) B (x_2, y_2) and P (x, y)

seg AC, seg PQ and seg BD are prependicular to X - axis

$\therefore$ Let C (x_1, 0), Q (x, 0) and D (x_2, 0).

$$\left.\begin{aligned}\therefore\ CQ &= x - x_1\\ \text{and } QD &= x_2 - x\end{aligned}\right\} \quad ....(i)$$

seg AC || seg PQ || seg BD.

$\therefore$ By the property of intercepts of three parallel lines, $\frac{AP}{PB} = \frac{CQ}{QD} = \frac{m}{n}$

From the figure CQ = $x - x_1$ and QD = $x_2 - x$ ...[From (i)]

$$\therefore \frac{x - x_1}{x_2 - x} = \frac{m}{n}$$

$$\therefore n(x - x_1) = m(x_2 - x)$$

$$\therefore nx - nx_1 = mx_2 - mx$$

$$\therefore mx + nx = mx_2 + nx_1$$

$$\therefore x(m + n) = mx_2 + nx_1$$

$$\therefore x = \frac{mx_2 + nx_1}{m+n}$$

Similarly drawing perpendiculars from points A, P and B to Y - axis, we get $y = \frac{my_2 + ny_1}{m+n}$

$\therefore$ The co-ordinates of point, which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2) in the ratio $m : n$ are given by $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Practice Set - 5.2 *(Textbook Page No.)*

(1) Find the co-ordinates of point P if P divides the line segment joining the points A(–1, 7) and B(4, –3) in the ratio 2 : 3. *(2 marks)*

Solution :

P(x, y) divides seg AB in the ratio 2 : 3.

A$(-1, 7) = (x_1, y_1)$

B$(4, -3) = (x_2, y_2)$

$m : n = 2 : 3$

By Section formula,

$$x = \frac{mx_2 + nx_1}{m+n}; \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{2 \times 4 + 3 \times (-1)}{2+3} \quad \text{and} \quad = \frac{2 \times (-3) + 3 \times (7)}{2+3}$$

$$= \frac{8-3}{5} \quad \text{and} \quad = \frac{-6+21}{5}$$

$$= \frac{5}{5} \quad \text{and} \quad = \frac{15}{5}$$

$$x = 1 \quad \text{and} \quad y = 3$$

∴ **The coordinates of point P are (1, 3).**

(2) In each of the following examples find the co-ordinates of point A which divides segment PQ in the ratio $a : b$. *(2 marks each)*

(i) P(–3, 7), Q(1, –4), $a : b = 2 : 1$

Solution :

A(x, y) divides seg PQ in the ratio 2 : 1.

P$(-3, 7) = (x_1, y_1)$

Q$(1, -4) = (x_2, y_2)$

$a : b = 2 : 1 = m : n$

By Section formula,

$$x = \frac{mx_2 + nx_1}{m+n}; \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{2 \times 1 + 1 \times (-3)}{2+1} \quad \text{and} \quad = \frac{2 \times (-4) + 1 \times (7)}{2+1}$$

$$= \frac{2-3}{3} \quad \text{and} \quad = \frac{-8+7}{3}$$

$$x = \frac{-1}{3} \quad \text{and} \quad y = \frac{-1}{3}$$

∴ $\boxed{\mathbf{A \equiv \left(-\frac{1}{3}, -\frac{1}{3}\right)}}$

(ii) P(–2, –5), Q(4, 3), $a : b = 3 : 4$

Solution :

A(x, y) divides seg PQ in the ratio 3 : 4.

P$(-2, -5) = (x_1, y_1)$

Q$(4, 3) = (x_2, y_2)$

$a : b = 3 : 4 = m : n$

By Section formula,

$$x = \frac{mx_2 + nx_1}{m+n}; \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{3 \times 4 + 4 \times (-2)}{3+4} \quad \text{and} \quad = \frac{3 \times 3 + 4 \times (-5)}{3+4}$$

$$= \frac{12-8}{7} \quad \text{and} \quad = \frac{9-20}{7}$$

$$x = \frac{4}{7} \quad \text{and} \quad y = \frac{-11}{7}$$

∴ $\boxed{\mathbf{A\left(\frac{4}{7}, \frac{-11}{7}\right)}}$

(iii) P(2, 6), Q(–4, 1), $a : b = 1 : 2$

Solution :

A(x, y) divides seg PQ in the ratio 1 : 2.

P$(2, 6) = (x_1, y_1)$

Q$(-4, 1) = (x_2, y_2)$

$a : b = 1 : 2 = m : n$

By Section formula,

$$x = \frac{mx_2 + nx_1}{m+n}; \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{1 \times (-4) + 2 \times 2}{1+2} \quad \text{and} \quad = \frac{1 \times 1 + 2 \times 6}{1+2}$$

$$= \frac{-4+4}{3} \quad \text{and} \quad = \frac{1+12}{3}$$

$$x = \frac{0}{3} \quad \text{and} \quad y = \frac{13}{3}$$

∴ $\boxed{\mathbf{x = \left(0, \frac{13}{3}\right)}}$

(3) Find the ratio in which point T(–1, 6) divides the line segment joining the points P(–3, 10) and Q(6, –8). *(2 marks)*

Solution :

Let point T divides seg PQ in the ratio $m : n$.

T$(-1, 6) = (x, y)$

P$(-3, 10) = (x_1, y_1)$

Q$(6, -8) = (x_2, y_2)$

By Section formula,

$$x = \frac{mx_2 + nx_1}{m+n};$$

$$-1 = \frac{m \times 6 + n(-3)}{m+n}$$

$\therefore$ $-1\,(m + n) = 6m - 3n$

$\therefore$ $-m - n = 6m - 3n$

$\therefore$ $-m - 6m = -3n + n$

$\therefore$ $-7m = -2n$

$\therefore$ $7m = 2n$

$\therefore$ $\frac{m}{n} = \frac{2}{7}$

i.e. $m : n = 2 : 7$

$\therefore$ **Point T divides seg PQ in the ratio 2 : 7.**

(5) Find the ratio in which point P(k, 7) divides the segment joining A(8, 9) and B(1, 2). Also find *k*. *(3 marks)*

Solution :

A(8, 9) $= (x_1, y_1)$

B(1, 2) $= (x_2, y_2)$

P(k, 7) $= (x, y)$

Let point P divide seg AB in the ratio $m : n$.

By Section formula,

$$y = \frac{my_2 + ny_1}{m+n}$$

$$7 = \frac{m \times 2 + n \times 9}{m+n}$$

$\therefore$ $7\,(m + n) = 2m + 9n$

$\therefore$ $7m + 7n = 2m + 9n$

$\therefore$ $7m - 2m = 9n - 7n$

$\therefore$ $5m = 2n$

$\therefore$ $\frac{m}{n} = \frac{2}{5}$

$\therefore$ $\boxed{m : n = 2 : 5}$

$$x = \frac{mx_2 + nx_1}{m+n}$$

$\therefore$ $k = \frac{2 \times 1 + 5 \times 8}{2+5}$

$\therefore$ $k = \frac{2+40}{7}$

$\therefore$ $k = \frac{42}{7}$

$\therefore$ $\boxed{k = 6}$

Problem Set - 5 *(Textbook Pg No. 122)*

(4) Find the ratio in which the line segment joining the points A(3, 8) and B(–9, 3) is divided by the Y-axis. *(2 marks)*

Solution :

A(3, 8) $= (x_1, y_1)$

B(–9, 3) $= (x_2, y_2)$

Let point P(0, a) be a point on Y-axis which divides seg AB in the ratio $m : n$.

P(0, a) $= (x, y)$

By Section formula,

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$0 = \frac{m \times (-9) + n(3)}{m+n}$$

$\therefore$ $0 \times (m + n) = -9m + 3n$

$\therefore$ $0 = -9 + 3n$

$\therefore$ $9m = 3n$

$\therefore$ $\frac{m}{n} = \frac{3}{9}$

$\therefore$ $\frac{m}{n} = \frac{1}{3}$

$\therefore$ $m : n = 1 : 3$

$\therefore$ **Y-axis divides segment joining points A and B in the ratios 1 : 3**

(17) Given A(4, –3), B(8, 5). Find the co-ordinates of the point that divides segment AB in the ratio 3 : 1. *(2 marks)*

Solution :

Let P(x, y) be the point which divides seg AB in the ratio 3 : 1.

A(4, –3) $= (x_1, y_1)$

B(8, 5) $= (x_2, y_2)$

P(x, y)

$m : n = 3 : 1$

By Section formula,

$x = \frac{mx_2 + nx_1}{m+n};$ and $y = \frac{my_2 + ny_1}{m+n}$

$= \frac{3 \times 8 + 1 \times 4}{3+1}$ and $= \frac{3 \times 5 + 1 \times (-3)}{3+1}$

$= \frac{24+4}{4}$ and $= \frac{15-3}{4}$

$= \frac{28}{4}$ and $= \frac{12}{4}$

$x = 7$ and $y = 3$

$\therefore$ **P(7, 3) divides seg AB in the ratio 3 : 1**

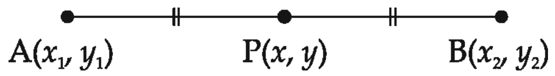
Points to Remember:

- **(Mid-point formula)**

  If M(x, y) is the midpoint of segment joining A(x_1, y_1) and B(x_2, y_2), then

$$x = \frac{x_1 + x_2}{2} \text{ and } y = \frac{y_1 + y_2}{2}$$

If point P is the midpoint of segment AB and P(x, y), A(x_1, y_1), B(x_2, y_2) then $m = n$ and values of x and y can be written as



$$x = \frac{mx_2 + nx_1}{m+n} \qquad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{mx_2 + mx_1}{m+m} (\because m = n) \qquad = \frac{my_2 + my_1}{m+m} (\because m=n)$$

$$= \frac{m(x_2 + x_1)}{2m} \qquad = \frac{m(y_1 + y_2)}{2m}$$

$$= \frac{x_1 + x_2}{2} \qquad = \frac{y_1 + y_2}{2}$$

$\therefore$ Co-ordinates of midpoint P are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

This is called as midpoint formula.

In the previous standard we have shown that $\frac{a+b}{2}$ is the midpoint of two rational numbers a and b which are on the number line. That conclusion is the special case of the above midpoint formula.

Practice Set - 5.2 *(Textbook Page No. 115)*

(6) Find the coordinates of the midpoint of the segment joining the points (22, 20) and (0, 16) *(2 marks)*

Solution :

Let A(22, 20) = (x_1, y_1) and

B (0, 16) = (x_2, y_2)

Let M (x, y) be the midpoint of seg AB.

By midpoint formula,

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$x = \frac{22 + 0}{2} \quad \text{and} \quad y = \frac{20 + 16}{2}$$

$$x = \frac{22}{2} \quad \text{and} \quad y = \frac{36}{2}$$

$$x = 11 \quad \text{and} \quad y = 18$$

$\therefore$ $\boxed{\textbf{M(11, 18)}}$

(4) Point P is the centre of the circle and AB is a diameter. Find the co-ordinates of point B if co-ordinates of point A and P are (2, –3) and (–2, 0) respectively. *(2 marks)*

Solution :

P(–2, 0) = (x, y)

A(2, –3) = (x_1, y_1)

B(x_2, y_2)

P is the centre of the circle ...(Given)

Point P is the midpoint of diameter AB.

By midpoint formula,

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$-2 = \frac{2 + x_2}{2} \quad \text{and} \quad 0 = \frac{-3 + y_2}{2}$$

$\therefore$ $-2 \times 2 = 2 + x_2$ and $0 \times 2 = -3 + y_2$

$\therefore$ $-4 - 2 = x_2$ and $0 + 3 = y_2$

$\therefore$ $x_2 = -6$ and $y_2 = 3$

$\therefore$ $\boxed{\textbf{B(–6, 3)}}$

Problem Set - 5 *(Textbook Pg No. 122)*

(3) Find the coordinates of the midpoint of the line segment joining P(0, 6) and Q(12, 20) *(2 marks)*

Solution :

P(0, 6) = (x_1, y_1) and

Q(12, 20) = (x_2, y_2)

Let M (x, y) be the midpoint of seg PQ.

By midpoint formula,

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

$$x = \frac{0 + 12}{2} \quad \text{and} \quad y = \frac{6 + 20}{2}$$

$$x = \frac{12}{2} \quad \text{and} \quad y = \frac{26}{2}$$

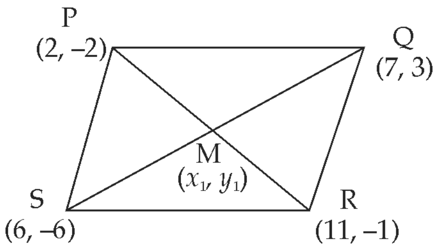
$$x = 6 \quad \text{and} \quad y = 13$$

$\therefore$ **M(6, 13) is the midpoint of segment joining P (0, 6) and Q (12, 20)**

Practice Set - 5.1 *(Textbook Page No. 108)*

(5) P(2, –2) Q(7, 3), R(11, –1) and S(6, –6) are the vertices of a parallelogram. *(4 marks)*

Solution :



Let M (x_1, y_1) be the midpoint of diagonal PR.

By midpoint formula,

$$x_1 = \frac{2+11}{2} \quad ; \quad y_1 = \frac{-2-1}{2}$$

$$x_1 = \frac{13}{2} \quad ; \quad y_1 = \frac{-3}{2}$$

$M\left(\frac{13}{2}, \frac{-3}{2}\right)$ is the midpoint of diagonal PR ...(i)

Let N(x_2, y_2) be the midpoint of diagonal QS.

By midpoint formula,

$$x_2 = \frac{7+6}{2} \quad ; \quad y_2 = \frac{3+(-6)}{2}$$

$$x_1 = \frac{13}{2} \quad ; \quad y_1 = \frac{-3}{2}$$

$N\left(\frac{13}{2}, \frac{-3}{2}\right)$ is the midpoint of diagonal QS ...(ii)

[From (i) and (ii)]

Midpoint of diagonal PR and diagonal QS is the same.

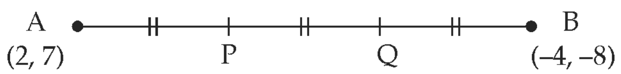
i.e. Diagonals PR and QS bisect each other.

∴ □PQRS is a parallelogram ...(A quadrilateral is a parallelogram if its diagonals bisect each other.)

Practice Set - 5.2 *(Textbook Page No. 116)*

(10) Find the coordinates of points of trisection of the line segment AB with A(2, 7) and B(–4, –8) *(4 marks)*

Solution :



Let point P and Q be two points which divide seg AB in three equal parts.

Point P divides seg AB in the ratio 1 : 2

By Section formula,

$$P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$P\left(\frac{1\times(-4)+2\times 2}{1+2}, \frac{1\times(-8)+2\times 7}{1+2}\right)$$

$$\therefore \quad P\left(\frac{-4+4}{3}, \frac{-8+14}{3}\right)$$

$$\therefore \quad P\left(\frac{0}{3}, \frac{6}{3}\right)$$

∴ P (0, 2)

Also, PQ = QB

∴ Point Q is midpoint of seg PB.

By midpoint formula,

$$\therefore \quad Q\left(\frac{0+(-4)}{2}; \frac{2+(-8)}{2}\right)$$

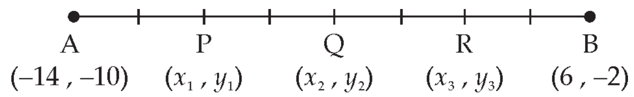
$$\therefore \quad Q\left(\frac{-4}{2}, \frac{-6}{2}\right)$$

∴ Q (–2, –3)

∴ $\boxed{\text{P(0, 2) and Q(–2, –3) are points which trisects seg AB}}$

(11) If A (–14, –10), B(6, –2) is given, find the co-ordinates of the points which divide segment AB into four equal parts. *(4 marks)*

Solution :



Let point P(x_1, y_1), Q (x_2, y_2) and R(x_3, y_3) be the three points which divides seg AB in four equal parts.

Point Q is the midpoint of seg AB.

By midpoint formula,

$$x_2 = \frac{-14+6}{2} \quad \text{and} \quad y_2 = \frac{-10+(-2)}{2}$$

$$x_2 = \frac{-8}{2} \quad \text{and} \quad y_2 = \frac{-12}{2}$$

$$x_2 = -4 \quad \text{and} \quad y_2 = -6$$

∴ Q (–4, –6)

AP = PQ ...[From (i)]

∴ P is the midpoint of seg AQ.

By midpoint formula,

$$x_1 = \frac{-14+(-4)}{2} \quad \text{and} \quad y_1 = \frac{-10+(-6)}{2}$$

$x_1 = \frac{-18}{2}$ and $y_1 = \frac{-16}{2}$

$x_1 = -9$ and $y_1 = -8$

$\therefore$ P (–9, –8)

$\therefore$ Q R = BR ...[From (i)]

R is the midpoint of seg BQ.

By midpoint formula,

$x_3 = \frac{-4+6}{2}$ and $y_3 = \frac{-6+(-2)}{2}$

$\therefore$ $x_3 = \frac{2}{2}$ and $y_3 = \frac{-8}{2}$

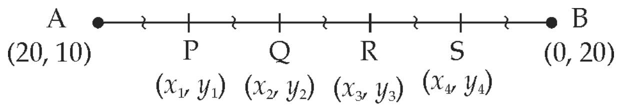
$x_3 = 1$ and $y_3 = -4$

$\therefore$ R(1, –4)

$\therefore$ **P(–9, –8), Q(–4, –6) and R(1, –4) divides seg AB in four equal parts.**

(12) If A(20, 10), B(0, 20) are given, find the co-ordinates of the points which is divide segment AB into five congruent parts. *(4 marks)*

Solution :



Let point P(x_1, y_1), Q (x_2, y_2), R(x_3, y_3) and S(x_4, y_4) be four points which divides seg AB into five congruent parts.

Point P divides seg AB in the ratio 1 : 4.

By section formula,

$x_1 = \frac{1\times 0 + 4\times 20}{1+4}$ and $y_1 = \frac{1\times 20 + 4\times 10}{1+4}$

$\therefore$ $x_1 = \frac{80}{5}$ and $y_1 = \frac{20+40}{5} = \frac{60}{5}$

$\therefore$ $x_1 = 16$ and $y_1 = 12$

$\therefore$ P(16, 12)

AP = PQ ...[From (i)]

$\therefore$ P is the midpoint of seg AQ.

By midpoint formula,

$16 = \frac{20+x_2}{2}$ and $12 = \frac{10+y_2}{2}$

$\therefore$ $16\times 2 = 20 + x_2$ and $12\times 2 = 10 + y_2$

$\therefore$ $32 - 20 = x_2$ and $24 - 10 = y_2$

$\therefore$ $x_2 = 12$ and $y_2 = 14$

$\therefore$ Q(12, 14)

PQ = QR ...[From (i)]

$\therefore$ Q is the midpoint of seg PR.

By midpoint formula,

$12 = \frac{16+x_3}{2}$ and $14 = \frac{12+y_3}{2}$

$\therefore$ $24 = 16 + x_3$ and $28 = 12 + y_3$

$\therefore$ $x_3 = 24 - 16$ and $y_3 = 28 - 12$

$\therefore$ $x_3 = 8$ and $y_3 = 16$

$\therefore$ R(8, 16)

RS = BS ...[From (i)]

$\therefore$ S is the midpoint of seg RB.

By midpoint formula,

$x_4 = \frac{0+8}{2}$ and $y_4 = \frac{16+20}{2}$

$x_4 = \frac{8}{2}$ and $y_4 = \frac{36}{2}$

$\therefore$ $x_4 = 4$ and $y_4 = 18$

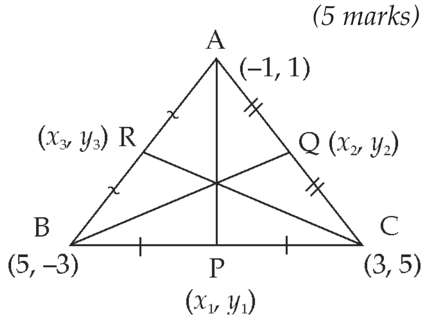
$\therefore$ S(4, 18)

$\therefore$ **P (16, 12), Q (12, 14), R (8, 16) and S (4, 18) divides seg AB in five equal parts.**

Problem Set - 5 *(Textbook Pg No. 123)*

(13) Find the lengths of the medians of a triangle whose vertices are A(–1, 1), B(5, –3) and C(3, 5). *(5 marks)*

Solution :



Let A(–1, 1), B(5, –3) and C(3, 5) be the vertices of triangle.

Let points P, Q and R be the midpoint of side BC, side AC and side AB respectively.

P is the midpoint of seg BC

$\therefore$ By midpoint formula,

$x_1 = \frac{5+3}{2}$ and $y_1 = \frac{-3+5}{2}$

$\therefore$ $x_1 = \frac{8}{2}$ and $y_1 = \frac{2}{2}$

$\therefore$ $x_1 = 4$ and $y_1 = 1$

$\therefore$ P(4, 1)

Q is the midpoint of seg AC.

$\therefore$ By midpoint formula,

$x_2 = \frac{-1+3}{2}$ and $y_2 = \frac{1+5}{2}$

$x_2 = \frac{2}{2}$ and $y_2 = \frac{6}{2}$

$x_2 = 1$ and $y_2 = 3$

$\therefore$ Q(1, 3)

R is the midpoint of seg AB.

$\therefore$ By midpoint formula,

$x_3 = \frac{-1+5}{2}$ and $y_3 = \frac{1-3}{2}$

$x_3 = \frac{4}{2}$ and $y_3 = \frac{-2}{2}$

$x_3 = 2$ and $y_3 = -1$

$\therefore$ R (2, –1)

A(–1, 1), P(4, 1)

Using distance formula,

$$d\,(A, P) = \sqrt{(4-(-1))^2 + (1-1)^2}$$

$$= \sqrt{(4+1)^2 + 0^2}$$

$$= \sqrt{5^2}$$

$\therefore$ d (A, P) = 5 units

$\therefore$ d (AP) = 5 units

B (5, –3), Q(1, 3)

Using distance formula,

$$d\,(B, Q) = \sqrt{(5-1)^2 + (-3-3)^2}$$

$$= \sqrt{4^2 + (-6)^2}$$

$$= \sqrt{16+36}$$

$$= \sqrt{52}$$

$$= \sqrt{13 \times 4}$$

$\therefore$ d(B, Q) = $2\sqrt{13}$ units

C(3, 5), R(2, –1)

Using distance formula,

$$\therefore\ d\,(C, R) = \sqrt{(3-2)^2 + [5-(-1)]^2}$$

$$= \sqrt{(1)^2 + (6)^2}$$

$$= \sqrt{1+36}$$

$$= \sqrt{37}$$

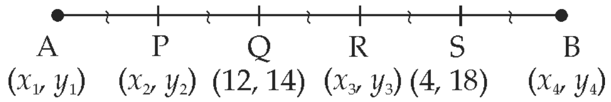
$\therefore$ d (C, R) = $\sqrt{37}$ units

$\therefore$ d (CR) = $\sqrt{37}$ units

$\therefore$ **Length of three medians are 5 units, $2\sqrt{13}$ units, $\sqrt{37}$ units.**

***(19)** **The line segment AB is divided into five congruent parts at P, Q, R and S such that A–P–Q–R–S–B. If point (12, 14) and S (4, 18) are given find the co-ordiates of A, P, R and B.** *(5 marks)*

Solution :



Points P, Q, R and S divides seg AB into five equal parts.

$\therefore$ AP = PQ = QR = RS = SB ...(i)

Let A(x_1, y_1), P(x_2, y_2), R(x_3, y_3) and B(x_4, y_4)

QR = RS ...[From (i)]

$\therefore$ Point R is the midpoint of seg QS.

$\therefore$ By midpoint formula,

$x_3 = \frac{12+4}{2}$ and $y_3 = \frac{14+18}{2}$

$\therefore$ $x_3 = \frac{16}{2}$ and $y_3 = \frac{32}{2}$

$\therefore$ $x_3 = 8$ and $y_3 = 16$

$\therefore$ R(8, 16)

RS = SB ...[From (i)]

Point S is the midpoint of seg RB.

$\therefore$ By midpoint formula,

$4 = \frac{8+x_4}{2}$ and $18 = \frac{16+y_4}{2}$

$\therefore$ $8 = 8 + x_4$ and $36 = 16 + y_4$

$\therefore$ $x_4 = 8 - 8$ and $36 - 16 = y_4$

$\therefore$ $x_4 = 0$ and $y_4 = 20$

$\therefore$ B(0, 20)

PQ = QR ...[From (i)]

$\therefore$ Q is the midpoint of seg PR

$\therefore$ By midpoint formula,

$12 = \frac{x_2+8}{2}$ and $14 = \frac{y_2+16}{2}$

$\therefore$ $12 \times 2 = x_2 + 8$ and $14 \times 2 = y_2 + 16$

$\therefore$ $24 - 8 = x_2$ and $28 - 16 = y_2$

$\therefore$ $x_2 = 16$ and $y_2 = 12$

$\therefore$ P (16, 12)

AP = PQ ...[From (i)]

$\therefore$ P is the midpoint of seg AQ

$\therefore$ By midpoint formula,

$16 = \frac{x_1+12}{2}$ and $12 = \frac{y_1+14}{2}$

$\therefore$ $32 = x_1 + 12$ and $24 = y_1 + 14$

$\therefore$ $32 - 12 = x_1$ and $24 - 14 = y_1$

$\therefore$ $x_1 = 20$ and $y_1 = 10$

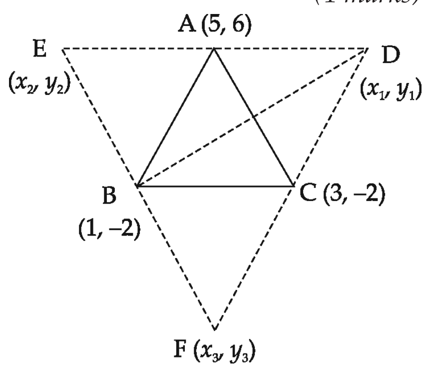
$\therefore$ A(20, 10)

$\therefore$ $\boxed{\textbf{A(20, 10), P(16, 12), R(8, 16) and B(0, 20)}}$

***(21) Find the possible pairs of co-ordinates of the fourth vertex D of the parallelogram, if three of its vertices are A(5, 6), B(1, –2) and C(3, –2).**

(4 marks)

Solution :



Let A (5, 6), B(1, –2) and C(3, –2) be the three vertices of a parallelogram.

Fourth vertex can be point D or point E or point F as shown in the above figure.

For parallelogram ABCD, let D (x_1, y_1) be the fourth vertex. Diagonals of a parallelogram bisect each other.

$\therefore$ Diagonal AC and diagonal BD have the same midpoint.

Using midpoint formula,

$$\left(\frac{5+3}{2}, \frac{6+(-2)}{2}\right) = \left(\frac{1+x_1}{2}, \frac{-2+y_1}{2}\right)$$

$\frac{8}{2} = \frac{1+x_1}{2}$ and $\frac{6-2}{2} = \frac{-2+y_1}{2}$

$\therefore$ $x_1 = 8 - 1$ and $4 = -2 + y_1$

$\therefore$ $x_1 = 7$ and $y_1 = 6$

$\therefore$ $\boxed{\textbf{D(7, 6)}}$

For parallelogram ACBE, let E(x_2, y_2) be the fourth vertex Diagonals of a parallelogram bisect each other.

$\therefore$ Diagonal AB and diagonal CE have the same midpoint.

By midpoint formula,

$$\left(\frac{3+x_2}{2}, \frac{-2+y_2}{2}\right) = \left(\frac{5+1}{2}, \frac{6+(-2)}{2}\right)$$

$\therefore$ $\frac{3+x_2}{2} = \frac{6}{2}$ and $\frac{-2+y_2}{2} = \frac{6-2}{2}$

$\therefore$ $x_2 = 6 - 3$ and $y_2 = 4 + 2$

$\therefore$ $x_2 = 3$ and $y_2 = 6$

$\therefore$ $\boxed{\textbf{E(3, 6)}}$

For parallelogram ABFC, let F(x_3, y_3) be the fourth vertex. Diagonals of a parallelogram bisect each other.

$\therefore$ Diagonal AF and diagonal BC have the same midpoint

Using midpoint formula,

$$\left(\frac{5+x_3}{2}, \frac{6+y_3}{2}\right) = \left(\frac{1+3}{2}, \frac{-2+(-2)}{2}\right)$$

$\therefore$ $\frac{x_3+5}{2} = \frac{1+3}{2}$ and $\frac{y_3+6}{2} = \frac{-2-2}{2}$

$\therefore$ $x_3 = 4 - 5$ and $y_3 = -4 - 6$

$\therefore$ $x_3 = -1$ and $y_3 = -10$

$\therefore$ $\boxed{\textbf{F(–1, –10)}}$

Points to Remember:

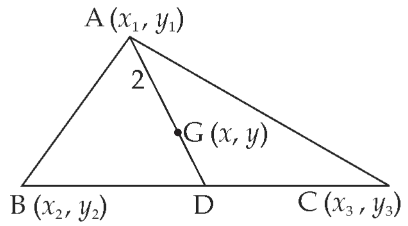
- **Centroid formula**

Now we will see by using section formula how the co-ordinates of centroid are found if the co-ordinates of vertices of a triangle are given

In ABC, point G is a centroid

A(x_1, y_1) B(x_2, y_2) and C(x_3, y_3).

$$x = \frac{x_1 + x_2 + x_3}{3} \text{ and } y = \frac{y_1 + y_2 + y_3}{3}$$



If A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are vertices of ABC and seg AD is median of ABC, G (x, y) is the centroid of this triangle.

D is the mid point of the line segment BC.

$\therefore$ Co-ordinates of point D are

$x = \frac{x_2 + x_3}{2}$, $y = \frac{y_2 + y_3}{2}$ ...(By midpoint theorem)

Point G (x, y) is centroid of triangle ABC.

$\therefore$ AG : GD = 2 : 1

$\therefore$ According to section formula,

$$x = \frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2+1} = \frac{x_2 + x_3 + x_1}{3} = \frac{x_1 + x_2 + x_3}{3}$$

$$y = \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2+1} = \frac{y_2 + y_3 + y_1}{3} = \frac{y_1 + y_2 + y_3}{3}$$

co-ordinates of centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

This is called as centroid formula.

Practice Set - 5.2 *(Textbook Page No. 115)*

(7) In each of the following vertices of a triangles are given. Find the coordinates of centroid of each triangle. *(2 marks each)*

(i) (–7, 6), (2, –2), (8, 5)

Solution :

Let A(–7, 6) = (x_1, y_1) be the vertices of ABC

B(2, –2) = (x_2, y_2)

C(8, 5) = (x_3, y_3)

Let G(x, y) be the centroid of ABC .

By centroid formula,

$x = \frac{x_1 + x_2 + x_3}{3}$ and $y = \frac{y_1 + y_2 + y_3}{3}$

$= \frac{-7 + 2 + 8}{3}$ and $= \frac{6 - 2 + 5}{3}$

$= \frac{3}{3}$ and $= \frac{9}{3}$

$x = 1$ and $y = 3$

$\therefore$ **G(1, 3)**

(ii) (3, –5), (4, 3), (11, –4)

Solution :

Let A(3, –5) = (x_1, y_1)

B(4, 3) = (x_2, y_2)

C(11, –4) = (x_3, y_3) be the vertices of ABC

Let G(x, y) be the centroid of ABC .

By centroid formula,

$x = \frac{x_1 + x_2 + x_3}{3}$ and $y = \frac{y_1 + y_2 + y_3}{3}$

$= \frac{3 + 4 + 11}{3}$ and $= \frac{-5 + 3 - 4}{3}$

$= \frac{18}{3}$ and $= \frac{-6}{3}$

$x = 6$ and $y = -2$

$\therefore$ **G(6, –2)**

(iii) (4, 7), (8, 4), (7, 11)

Solution :

Let A(4, 7) = (x_1, y_1)

B(8, 4) = (x_2, y_2)

C(7, 11) = (x_3, y_3) be the vertices of ABC

Let G(x, y) be the centroid of ABC .

By centroid formula,

$x = \frac{x_1 + x_2 + x_3}{3}$ and $y = \frac{y_1 + y_2 + y_3}{3}$

$= \frac{4 + 8 + 7}{3}$ and $= \frac{7 + 4 + 11}{3}$

$x = \frac{19}{3}$ and $y = \frac{22}{3}$

$\therefore$ $\boxed{G\left(\frac{19}{3}, \frac{22}{3}\right)}$

(8) In ABC, G(–4, –7) is the centroid. If A(–14, –19) and B(3, 5) then find co-ordinates of C. *(2 marks)*

Solution :

A(–14, –19) = (x_1, y_1)

B(3, 5) = (x_2, y_2)

Let C (x_3, y_3)

G(–4, –7) = (x, y)

Point G is the centroid of ABC .

By centroid formula,

$x = \frac{x_1 + x_2 + x_3}{3}$ and $y = \frac{y_1 + y_2 + y_3}{3}$

$-4 = \frac{-14 + 3 + x_3}{3}$ and $-7 = \frac{-19 + 5 + y_3}{3}$

$\therefore$ $-4 \times 3 = -11 + x_3$ and $-7 \times 3 = -14 + y_3$

$\therefore$ $-12 + 11 = x_3$ and $-21 + 14 = y_3$

$\therefore$ $x_3 = -1$ and $y_3 = -7$

$\therefore$ **C(–1, –7)**

(9) A(h, –6), B(2, 3) and C(–6, k) are the co-ordinates of vertices of a triangle whose centroid is G(1, 5). find h and k. *(2 marks)*

Solution :

Let A(h, –6) = (x_1, y_1)

B(2, 3) = (x_2, y_2)

and C(–6, k) = (x_3, y_3)

G(1, 5) = (x, y)

Point G is the centroid of ABC .

By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3} \quad \text{and} \quad y = \frac{y_1 + y_2 + y_3}{3}$$

$$1 = \frac{h + 2 - 6}{3} \quad \text{and} \quad 5 = \frac{-6 + 3 + k}{3}$$

$\therefore \quad 1 \times 3 = h - 4 \quad$ and $\quad 5 \times 3 = -3 + k$

$\therefore \quad 3 + 4 = h \quad$ and $\quad 15 + 3 = k$

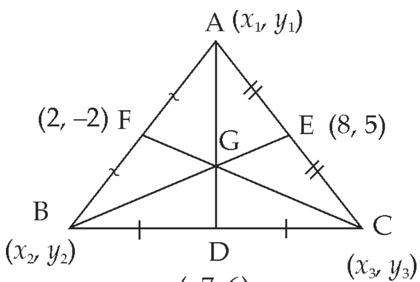
$\therefore \quad h = 7 \quad$ and $\quad k = 18$

$\therefore \quad \boxed{h = 7 \text{ and } k = 18}$

Problem Set - 5 *(Textbook Pg No. 123)*

***(14) Find the co-ordinates of centroid of the triangles if points D (–7, 6), E(8, 5) and F(2, –2) are the mid points of the sides of that triangle.** *(4 marks)*

Solution :



In ABC, seg AD, seg BE and seg CE are the medians.

Point G is the centroid.

D(–7, 6), E(8, 5), F(2, –2)

Let G(x, y), A(x_1, y_1), B (x_2, y_2) and C(x_3, y_3)

D is the midpoint of seg BC.

By midpoint formula,

$$-7 = \frac{x_2 + x_3}{2} \quad \text{and} \quad 6 = \frac{y_2 + y_3}{2}$$

$\therefore \quad -14 = x_2 + x_3 \quad$ ...(i) and $\quad 12 = y_2 + y_3 \quad$ ...(ii)

E is the midpoint of seg AC.

By midpoint formula,

$$8 = \frac{x_1 + x_3}{2} \quad \text{and} \quad 5 = \frac{y_1 + y_3}{2}$$

$\therefore \quad 8 \times 2 = x_1 + x_3 \quad$ and $\quad 5 \times 2 = y_1 + y_3$

$\therefore \quad 16 = x_1 + x_3 \quad$ ...(iii) and $\quad 10 = y_1 + y_3 \quad$ ...(iv)

F is the midpoint of side AB.

By midpoint formula,

$$2 = \frac{x_1 + x_2}{2} \quad \text{and} \quad -2 = \frac{y_1 + y_2}{2}$$

$\therefore \quad 4 = x_1 + x_2 \quad$ ...(v) $\quad$ and $\quad -4 = y_1 + y_2 \quad$ ...(vi)

$\therefore \quad$ Adding (i), (iii) and (v) we get,

$2x_1 + 2x_2 + 2x_3 = 6$

$\therefore \quad x_1 + x_2 + x_3 = 3 \quad$ ...(vii)

$\therefore \quad$ Adding (ii), (iv) and (vi) we get,

$2y_1 + 2y_2 + 2y_3 = 18$

$\therefore \quad y_1 + y_2 + y_3 = 9 \quad$ ...(viii)

G is the centroid of ABC .

By centroid formula,

$$x = \frac{x_1 + x_2 + x_3}{3} \text{ and } y = \frac{y_1 + y_2 + y_3}{3}$$

$\therefore \quad x = \frac{3}{3} \quad$ and $\quad y = \frac{9}{3} \quad$ ...[From (vii) and (viii)]

$\therefore \quad x = 1 \quad$ and $\quad y = 3$

$\therefore \quad$ **G(1, 3) is the centroid of ΔABC.**

[**Note** : G (1, 3) is also the centroid of Δ DEF]

Points to Remember:

Solpe of Line

(A) Using inclination:

Inclination of a line

Angle formed by a line with positive X-axis is called inclination of a line.

It is represented by 'θ'

Slope of a line = tan θ.

Solpe of Line - Using ratio in triganometry

In the adjoining figure, point P (x_1, y_1) and Q (x_2, y_2) are two points on line l.

Line l intersects X axis in point T.

seg QS $\perp$ X - axis, seg PR $\perp$ seg QS

$\therefore \quad$ seg PR || seg TS $\quad$ ...(Correspondence angle test)

$\therefore \quad$ QR $= y_2 - y_1$ and PR $= x_2 - x_1$

$\therefore \quad \dfrac{QR}{PR} = \dfrac{y_2 - y_1}{x_2 - x_1} \quad$ ...(i)

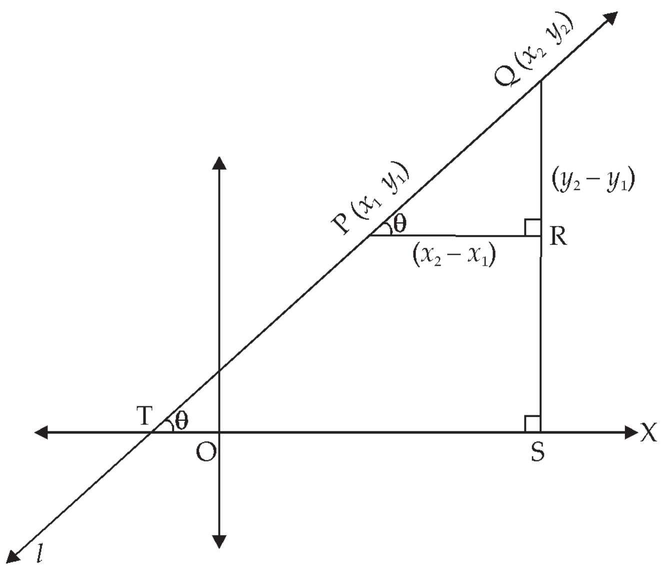
Line TQ makes an angle θ with the X - axis and seg PR || line TS.

$\therefore \quad \dfrac{QR}{PR} = \tan\theta \quad$ ...(ii)

$\therefore \quad$ From (i) and (ii), $\dfrac{y_2 - y_1}{x_2 - x_1} = \tan\theta$

$\therefore \quad$ m = tanθ

$\therefore \quad \angle$QPR = $\angle$QTS...(Correspondence angle theorm)



From this we can define slope as this way. Then the ratio of the angle made by the line with the positive direction of X - axis is called as slope of that line.

When any two lines have same slope, these lines make equal angles with the positive direction of X - axis.

$\therefore$ These two lines are parrallel.

Practice Set - 5.3 *(Textbook Page No.)*

(1) Angles made by the line with the positive direction of X-axis are given. Find the slope of these lines. *(2 marks)*

(i) 45° (ii) 60° (iii) 90°

Solution :

(i) Inclination of the line (θ) = 45°

Slope = $\tan \theta$

= $\tan 45°$

$\boxed{\textbf{Slope} = 1}$

(ii) Inclination of the line (θ) = 60°

Slope = $\tan \theta$

= $\tan 60°$

$\boxed{\textbf{Slope} = \sqrt{3}}$

(iii) Inclination of the line (θ) = 90°

Slope = $\tan \theta$

= $\tan 90°$

$\boxed{\textbf{Slope = Not defined}}$

Points to Remember:

(B) Slope of line;

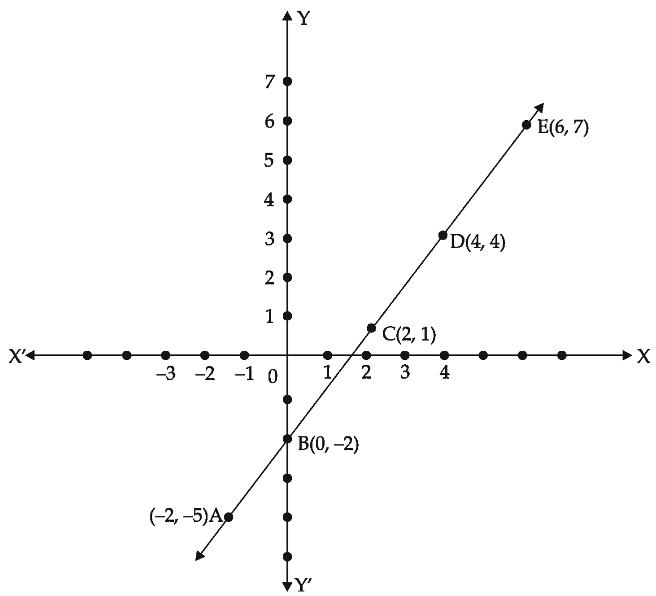
If A(x_1, y_1) B(x_2, y_2) are two points, then slope of line passing through points A and B can be given by,

Slope of line AB = $\dfrac{y_2 - y_1}{x_2 - x_1}$ or

Slope of line AB = $\dfrac{y_1 - y_2}{x_1 - x_2}$

Activity:

As in the figure below, points A(–2, –5) B(0, –2), C(2, 1), D(4, 4) and E(6, 7) lie on line l. Using these coordinates, complete the following table.



Sr. No.	First Point	Second Point	Coordinates of first point	Coordinates of second point	$\dfrac{y_2 - y_1}{x_2 - x_1}$
1	C	E	(2, 1)	(6, 7)	$\dfrac{7-1}{6-2} = \dfrac{6}{4} = \dfrac{3}{2}$
2	A	D	(–2, –5)	(4, 4)	$\dfrac{4-(-5)}{4-(-2)} = \dfrac{9}{6} = \dfrac{3}{2}$

3	D	A	(4, 4)	(–2, –5)	$\frac{-5-4}{-2-4} = \frac{-9}{-6} = \frac{3}{2}$
4	B	C	$\boxed{(0, -2)}$	$\boxed{(2, 1)}$	$\frac{1-(-2)}{2-0} = \frac{3}{2}$
5	C	A	$\boxed{(2, 1)}$	$\boxed{(-2, -5)}$	$\frac{-5-1}{-2-2} = \frac{-6}{-4} = \frac{3}{2}$
6	A	C	$\boxed{(-2, -5)}$	$\boxed{(2, 1)}$	$\frac{1-(-5)}{2-(-2)} = \frac{6}{4} = \frac{3}{2}$

From the above table we can say that for any two points on line l, $\frac{y_2 - y_1}{x_2 - x_1}$ remains the same.

$\frac{y_2 - y_1}{x_2 - x_1}$ is called the slope of line l and it is denoted by letter m

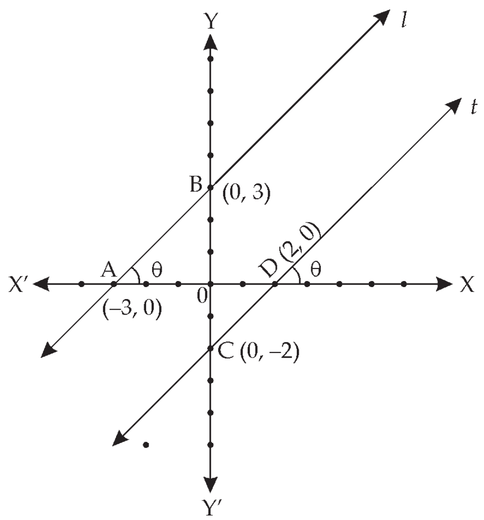
$\therefore \quad m = \frac{y_2 - y_1}{x_2 - x_1}$

Note: (1) Slope of X-axis is 0.

(2) Slope of Y-axis cannot be determined.

Slope of Parallel lines

As given in the figure, line l and line t makes an angle θ with the positive x axis.



line l || line t ....(alternate angles test)

Point A(–3, 0) and Point B(0, 3) lie on line l

$\therefore$ Slope of line l = $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{\boxed{3} - \boxed{0}}{\boxed{0} - \boxed{-3}} = \frac{\boxed{3}}{\boxed{3}} = 1$$

Similarly, take any two points on line t and find the slope of line t.

Let the point be C(0, –2) and D(2, 0).

$\therefore$ Slope of CD = $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2-0}{0-2} = \frac{-2}{-2} = 1$$

$\therefore$ Slope of line AB = Slope of line CD

$\therefore$ Slopes of parallel lines are equal hence proved.

Pracitce Set - 5.3 *(Textbook Page No. 121)*

(2) Find the slopes of lines passing through the given point. *(2 marks each)*

(i) A(2, 3) and B(4, 7)

Solution :

A(2, 3) $= (x_1, y_1)$

B(4, 7) $= (x_2, y_2)$

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-3}{4-2}$

$= \frac{4}{2}$

$\therefore$ $\boxed{\textbf{Slope of line AB} = 2}$

(ii) P(–3, 1) and Q(5, –2)

Solution :

P(–3, 1) $= (x_1, y_1)$

Q(5, –2) $= (x_2, y_2)$

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-1}{5-(-3)}$

$= \frac{-3}{5+3} = \frac{-3}{8}$

$\therefore$ $\boxed{\textbf{Slope of line PQ} = \frac{-3}{8}}$

(iii) C(5, –2) and D(7, 3)

Solution :

C(5, –2) $= (x_1, y_1)$

D(7, 3) $= (x_2, y_2)$

Slope of line CD $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3-(-2)}{7-5}$

$= \dfrac{3+2}{2} = \dfrac{5}{2}$

$\therefore$ $\boxed{\textbf{Slope of line CD} = \dfrac{5}{2}}$

(iv) L(–2, –3) and M(–6, –8)

Solution :

L(–2, –3) $= (x_1, y_1)$

M(–6, –8) $= (x_2, y_2)$

Slope of line LM $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-8-(-3)}{-6-(-2)}$

$= \dfrac{-8+3}{-6+2}$

$= \dfrac{-5}{-4} = \dfrac{5}{4}$

$\therefore$ $\boxed{\textbf{Slope of line LM} = \dfrac{5}{4}}$

(v) E(–4, –2) and F(6, 3)

Solution :

E(–4, –2) $= (x_1, y_1)$

F(6, 3) $= (x_2, y_2)$

Slope of line EF $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3-(-2)}{6-(-4)}$

$= \dfrac{3+2}{6+4}$

$= \dfrac{5}{10}$

$\therefore$ $\boxed{\textbf{Slope of line EF} = \dfrac{1}{2}}$

(vi) T(0, –3) and S(0, 4)

Solution :

T(0, –3) $= (x_1, y_1)$

S(0, 4) $= (x_2, y_2)$

Slope of line TS $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{4-(-3)}{0-0}$

$= \dfrac{4+3}{0} = \dfrac{7}{0}$

$\therefore$ $\boxed{\textbf{Slope of line TS = Not defined}}$

Practice Set - 5.1 *(Textbook Page No. 107)*

(2) Determine whether the points are collinear.

(2 marks each)

(i) A(1, –3), B(2, –5) and C(–4, 7)

Solution :

A(1, –3) $= (x_1, y_1)$

B(2, –5) $= (x_2, y_2)$

C(–4, 7) $= (x_3, y_3)$

Slope of line AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-5-(-3)}{2-1}$

$= \dfrac{-5+3}{1}$

$= -2$

$\therefore$ Slope of line AB $= -2$ ...(i)

Slope of line BC $= \dfrac{y_3 - y_2}{x_3 - x_2}$

$= \dfrac{7-(-5)}{-4-2}$

$= \dfrac{7+5}{-6}$

$\therefore$ Slope of line BC $= \dfrac{12}{-6} = -2$ ...(ii)

$\therefore$ Slope of line AB = Slope of line BC

...[From (i) and (ii)]

Line AB and line BC have equal slopes and have a common point B.

$\therefore$ $\boxed{\textbf{Points A, B and C are collinear.}}$

(ii) L(–2, 3), M(1, –3), N(5, 4)

Solution :

L(–2, 3) $= (x_1, y_1)$

M(1, –3) $= (x_2, y_2)$

N(5, 4) $= (x_3, y_3)$

Slope of line LM $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-3-3}{1-(-2)}$

$= \dfrac{-6}{1+2}$

$= \dfrac{-6}{3}$

$\therefore$ Slope of line LM $= -2$ ...(i)

Slope of line MN $= \dfrac{y_3 - y_2}{x_3 - x_2}$

$= \dfrac{4-(-3)}{5-1}$

$= \dfrac{4+3}{4}$

$\therefore$ Slope of line MN $= \dfrac{7}{4}$ ...(ii)

$\therefore$ Slope of line LM Slope of line MN

...[From (i) and (ii)]

$\therefore$ **Points L, M and N are not collinear.**

(iii) R(0, 3), D(2, 1) and S(3, –1)

Solution :

R(0, 3) $= (x_1, y_1)$

D(2, 1) $= (x_2, y_2)$

S(3, –1) $= (x_3, y_3)$

Slope of line RD $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1-3}{2-0}$

$= \dfrac{-2}{2}$

$\therefore$ Slope of line RD $= -1$ ...(i)

Slope of line DS $= \dfrac{y_3 - y_2}{x_3 - x_2}$

$= \dfrac{-1-1}{3-2}$

$= \dfrac{-2}{1}$

$\therefore$ Slope of line DS $= -2$ ...(ii)

$\therefore$ Slope of line RD slope of line DS

...[From (i) and (ii)]

$\therefore$ **Points R, D and S are not collinear**

(iv) P(–2, 3), Q(1, 2), R(4, 1)

Solution :

P(–2, 3) $= (x_1, y_1)$

Q(1, 2) $= (x_2, y_2)$

R(4, 1) $= (x_3, y_3)$

Slope of line PQ $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{2-3}{1-(-2)}$

$= \dfrac{-1}{1+2}$

$\therefore$ Slope of line PQ $= \dfrac{-1}{3}$ ...(i)

Slope of line QR $= \dfrac{y_3 - y_2}{x_3 - x_2}$

$= \dfrac{1-2}{4-1}$

$\therefore$ Slope of line QR $= \dfrac{-1}{3}$ ...(ii)

$\therefore$ Slope of line PQ = slope of line QR

...[From (i) and (ii)]

Line PQ and line QR have equal slopes and have a common point Q.

$\therefore$ **Points P, Q and R are collinear**

Practice Set - 5.3 *(Textbook Page No. 121)*

(3) Determine whether following points are collinear. *(3 marks)*

(i) A(–1, –1), B(0, 1), C(1, 3)

Solution :

A(–1, –1) $= (x_1, y_1)$

B(0, 1) $= (x_2, y_2)$

C(1, 3) $= (x_3, y_3)$

Slope of line AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1-(-1)}{0-(-1)}$

$= \dfrac{1+1}{0+1}$

$\therefore$ Slope of line AB $= \dfrac{2}{1}$ ...(i)

Slope of line BC $= \dfrac{y_3 - y_2}{x_3 - x_2}$

$= \dfrac{3-1}{1-0}$

$\therefore$ Slope of line BC $= \dfrac{2}{1}$ ...(ii)

$\therefore$ Slope of line AB = Slope of line BC

...[From (i) and (ii)]

Also, both lines have a common point B.

$\therefore$ **Points A, B and C are collinear points.**

(ii) D(–2, –3), E(1, 0), F(2, 1)

D(–2, –3) = (x_1, y_1)

E(1, 0) = (x_2, y_2)

F(2, 1) = (x_3, y_3)

Solution :

$$\text{Slope of line DE} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-3)}{1-(-2)} = \frac{0+3}{1+2} = \frac{3}{3} = 1$$

$\therefore$ Slope of line DE = 1 ...(i)

$$\text{Slope of line EF} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{1-0}{2-1} = \frac{1}{1}$$

$\therefore$ Slope of line EF = 1 ...(ii)

$\therefore$ Slope of line DE = Slope of line EF ...[From (i) and (ii)]

Also, both lines have a common point E.

$\therefore$ **Points D, E and F are collinear points.**

(iii) L(2, 5), M(3, 3), N(5, 1)

Solution :

L(2, 5) = (x_1, y_1)

M(3, 3) = (x_2, y_2)

N(5, 1) = (x_3, y_3)

$$\text{Slope of line LM} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{3-2} = \frac{-2}{1}$$

$\therefore$ Slope of line LM = –2 ...(i)

$$\text{Slope of line MN} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{1-3}{5-3} = \frac{-2}{2}$$

$\therefore$ Slope of line MN = –1 ...(ii)

$\therefore$ Slope of line LM $\neq$ Slope of line MN ...[From (i) and (ii)]

$\therefore$ **Points L, M and N are non-collinear points.**

(iv) P(2, –5), Q(1, –3), R(–2, 3)

Solution :

P(2, –5) = (x_1, y_1)

Q(1, –3) = (x_2, y_2)

R(–2, 3) = (x_3, y_3)

$$\text{Slope of line PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-5)}{1-2} = \frac{-3+5}{-1} = \frac{2}{-1}$$

$\therefore$ Slope of line PQ = –2 ...(i)

$$\text{Slope of line QR} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{3-(-3)}{-2-1} = \frac{3+3}{-3} = \frac{6}{-3}$$

$\therefore$ Slope of line QR = –2 ...(ii)

$\therefore$ Slope of line PQ = Slope of line QR

...[From (i) and (ii)]

Also, both lines have a common point Q.

$\therefore$ **Points P, Q and R are collinear points.**

(v) R(1, –4), S(–2, 2), T(–3, 4)

Solution :

R(1, –4) = (x_1, y_1)

S(–2, 2) = (x_2, y_2)

T(–3, 4) = (x_3, y_3)

$$\text{Slope of line RS} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{-2-1} = \frac{2+4}{-3} = \frac{6}{-3}$$

$\therefore$ Slope of line RS = –2 ...(i)

$$\text{Slope of line ST} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{4-2}{-3-(-2)}$$

$$= \frac{2}{-3+2}$$

$$= \frac{2}{-1}$$

$\therefore$ Slope of line ST = –2 ...(ii)

$\therefore$ Slope of line RS = Slope of line ST ...[From (i) and (ii)]

Also, they have a common point S.

$\therefore$ **Points R, S and T are collinear points.**

(vi) A(–4, 4), K(–2, $\frac{5}{2}$), N(4, –2)

Solution :

A(–4, 4) = (x_1, y_1)

K(–2, $\frac{5}{2}$) = (x_2, y_2)

N(4, –2) = (x_3, y_3)

Slope of line AK $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{\frac{5}{2} - 4}{-2-(-4)}$$

$$= \left(\frac{5-8}{2}\right) \div (-2+4)$$

$$= \frac{-3}{2} \div 2$$

$$= \frac{-3}{2} \times \frac{1}{2}$$

$\therefore$ Slope of line AK $= \frac{-3}{4}$ ...(i)

Slope of line AN $= \frac{y_3 - y_1}{x_3 - x_1}$

$$= \frac{-2-4}{4-(-4)}$$

$$= \frac{-6}{4+4}$$

$$= \frac{-6}{8}$$

$\therefore$ Slope of line AN $= \frac{-3}{4}$ ...(ii)

$\therefore$ Slope of line AK = Slope of line AN ...[From (i) and (ii)]

Also, they have a common point A.

$\therefore$ **Points A, K and N are collinear points.**

Problem Set - 5 *(Textbook Pg No. 122)*

(2) Determine whether the given points are collinear.

(2 marks each)

(i) A(0, 2), B(1, –0.5), C(2, –3)

Solution :

Slope of line AB $= \frac{-0.5-2}{1-0}$

$$= \frac{-2.5}{1}$$

$\therefore$ Slope of line AB = –2.5 ...(i)

Slope of line AC $= \frac{-3-2}{2-0}$

$$= \frac{-5}{2}$$

$\therefore$ Slope of line AC = –2.5 ...(ii)

$\therefore$ Slope of line AB = Slope of line AC ...[From (i) and (ii)]

Also, they have a common point A.

$\therefore$ **Points A, B and C are collinear points.**

(ii) P(1, 2), Q (2, $\frac{8}{5}$), R (3, $\frac{6}{5}$)

Solution :

Slope of line PQ $= \frac{\frac{8}{5} - 2}{2-1}$

$$= \frac{\frac{8-10}{5}}{1}$$

$\therefore$ Slope of line PQ $= \frac{-2}{5}$ ...(i)

Slope of line QR $= \frac{\frac{6}{5} - \frac{8}{5}}{3-2}$

$$= \frac{\frac{6-8}{5}}{1}$$

$\therefore$ Slope of line QR $= \frac{-2}{5}$ ...(ii)

$\therefore$ Slope of line PQ = Slope of line QR ...[From (i) and (ii)]

Also, they have a common point Q.

$\therefore$ **Points P, Q and R are collinear points.**

(iii) L(1, 2) , M(5, 3) , N(8, 6)

Solution :

Slope of line LM $= \frac{3-2}{5-1}$

$\therefore$ Slope of line LM $= \frac{1}{4}$ ...(i)

Slope of line MN $= \frac{6-3}{8-5}$

$= \frac{3}{3}$

$\therefore$ Slope of line MN = 1 ...(ii)

$\therefore$ Slope of line LM $\neq$ Slope of line MN ...[From (i) and (ii)]

$\therefore$ **Points L, M and N are not collinear points**

Pracitce Set - 5.3 *(Textbook Page No. 121)*

(4) If A(1, –1), B(0, 4), C(–5, 3) are vertices of a triangle, then find the slope of each side. *(3 marks)*

Solution :

A(1, –1), B(0, 4), C(–5, 3)

By using slope formula,

Slope of AB $= \frac{4-(-1)}{0-1}$

$= \frac{4+1}{-1}$

$= \frac{5}{-1}$

$\boxed{\textbf{Slope of AB} = -5}$

Slope of BC $= \frac{3-4}{-5-0}$

$= \frac{-1}{-5}$

$\boxed{\textbf{Slope of BC} = \frac{1}{5}}$

Slope of AC $= \frac{3-(-1)}{-5-1}$

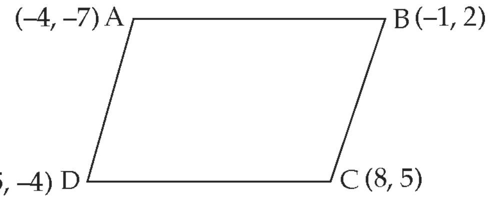
$= \frac{3+1}{-6}$

$= \frac{4}{-6}$

$\boxed{\textbf{Slope of AC} = \frac{-2}{3}}$

(5) Show that A(–4, –7), B(–1, 2), C(8, 5) and D(5, –4) are the vertices of a parallelogram. *(4 marks)*

Solution :



Slope of AB $= \frac{2-(-7)}{-1-(-4)}$

$= \frac{2+7}{-1+4}$

$= \frac{9}{3}$

Slope of AB $= 3$ ...(i)

Slope of BC $= \frac{5-2}{8-(-1)}$

$= \frac{3}{8+1}$

$= \frac{3}{9}$

Slope of BC $= \frac{1}{3}$ ...(ii)

Slope of AD $= \frac{-4-(-7)}{5-(-4)}$

$= \frac{-4+7}{5+4}$

$= \frac{3}{9}$

Slope of AD $= \frac{1}{3}$ ...(iii)

Slope of CD $= \frac{-4-5}{5-8}$

$= \frac{-9}{-3}$

Slope of CD $= 3$ ...(iv)

Slope of line AB = Slope of line CD ...[From (i) and (iv)]

$\therefore$ Line AB || Line CD ...(v)

$\therefore$ Slope of line BC = Slope of line AD ...[From (ii) and (iii)]

$\therefore$ Line BC || Line AD ...(vi)

In □ABCD, AB || CD ...[From (v)]

BC || AD ...[From (vi)]

$\therefore$ **□ABCD is a parallelogram.** ...(Definition)

(6) Find k, if R(1, –1), S(–2, k) and slope of line RS is –2. *(2 marks)*

Solution :

R(1, –1) $= (x_1, y_1)$

S(–2, k) $= (x_2, y_2)$

Slope of line RS $= \frac{y_2 - y_1}{x_2 - x_1}$

$-2 = \frac{k-(-1)}{-2-1}$

$\therefore -2 = \frac{k+1}{-3}$

$\therefore (-2) \times (-3) = k+1$

$\therefore 6 = k+1$

$\therefore \quad k = 6 - 1$

$\therefore \quad \boxed{k = 5}$

(7) Find k, if B(k, –5), C(1, 2) and slope of the line is 7. *(2 marks)*

Solution :

B(k, –5) $= (x_1, y_1)$

C(1, 2) $= (x_2, y_2)$

$$\text{Slope of line BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \quad 7 = \frac{2-(-5)}{1-k}$$

$\therefore \quad 7(1-k) = 2 + 5$

$\therefore \quad 7(1-k) = 7$

$$\therefore \quad 1 - k = \frac{7}{7}$$

$\therefore \quad 1 - k = 1$

$\therefore \quad 1 - 1 = k$

$\therefore \quad \boxed{k = 0}$

(8) Find k, if PQ || RS and P(2, 4), Q(3, 6), R(3, 1) and S(5, k). *(2 marks)*

Solution :

Line PQ || Line RS ...(Given)

$\therefore$ Slope of line PQ = Slope of line RS

$$\frac{6-4}{3-2} = \frac{k-1}{5-3}$$

$$\therefore \quad \frac{2}{1} = \frac{k-1}{2}$$

$\therefore \quad 2 \times 2 = k - 1$

$\therefore \quad 4 + 1 = k$

$\therefore \quad \boxed{k = 5}$

Problem Set - 5 *(Textbook Pg No. 123)*

9) Find k if the line passing through points P(–12, –3) and Q(4, k) has slope $\frac{1}{2}$. *(2 marks)*

Solution :

P(–12, –3), Q(4, k) ...(Given)

$$\text{Slope of PQ} = \frac{1}{2} \quad \text{...(Given)}$$

$$\text{Slope of PQ} = \frac{k-(-3)}{4-(-12)} \quad \text{...(Given)}$$

$$\therefore \quad \frac{1}{2} = \frac{k+3}{4+12}$$

$$\therefore \quad \frac{16}{2} = k + 3$$

$\therefore \quad k + 3 = 8$

$\therefore \quad k = 8 - 3$

$\therefore \quad \boxed{k = 5}$

(10) Show that the line joining the points A(4, 8) and B(5, 5) is parallel to the line joining the points C(2, 4) and D(1, 7) *(2 marks)*

Solution :

A(4, 8)

B (5, 5)

$$\text{Slope of line AB} = \frac{8-5}{4-5} = \frac{3}{-1}$$

$\therefore$ Slope of line AB $= -3$ ...(i)

C (2, 4)

D (1, 7)

$$\text{Slope of line CD} = \frac{7-4}{1-2} = \frac{3}{-1}$$

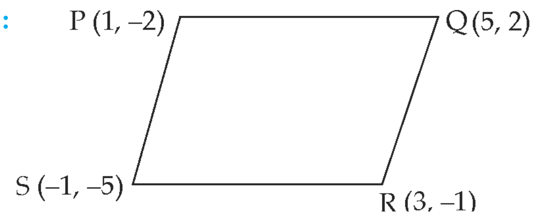
$\therefore$ Slope of line CD $= -3$ ...(ii)

$\therefore$ Slope of line AB = Slope of line CD ...[From (i) and (ii)]

$\therefore$ **Line AB || Line CD**

(11) Show that points P (1, –2), Q (5, 2), R (3, –1), S (–1, –5) are the vertices of a parallelogram. *(4 marks)*

Solution :



P (1, –2), Q (5, 2), R (3, –1), S (–1, –5)

$$\text{Slope of line PQ} = \frac{2-(-2)}{5-1} = \frac{2+2}{4} = \frac{4}{4} = 1 \quad \text{...(i)}$$

$$\text{Slope of line QR} = \frac{2-(-1)}{5-3} = \frac{2+1}{2} = \frac{3}{2} \quad \text{...(ii)}$$

$$\text{Slope of line RS} = \frac{-1-(-5)}{3-(-1)} = \frac{-1+5}{3+1} = \frac{4}{4} = 1 \quad \text{...(iii)}$$

$$\text{Slope of line PS} = \frac{-2-(-5)}{1-(-1)} = \frac{-2+5}{1+1} = \frac{3}{2} \quad \text{...(iv)}$$

Slope of line PQ = Slope of line RS ...[From (i) and (iii)]

$\therefore$ Line PQ || Line RS ...(v)

Slope of line QR = Slope of line PS ...[From (ii) and (iv)]

$\therefore$ Line QR || Line PS ...[From (vi)]

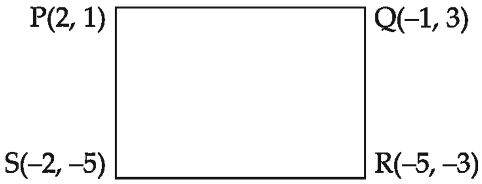
In □PQRS, PQ || RS ...[From (v)]

QR || PS ...[From (vi)]

$\therefore$ **□PQRS is a parallelogram** ...(Definition)

(12) Show that the □PQRS formed by P(2, 1), Q(–1, 3), R(–5, –3), S(–2, –5) is a rectangle. *(4 marks)*

Solution :



P (2, 1), Q (–1, 3), R (–5, –3), S (–2, –5)

Slope of line PQ $= \dfrac{3-1}{-1-2} = \dfrac{2}{-3} = \dfrac{-2}{3}$ ...(i)

Slope of line QR $= \dfrac{3-(-3)}{-1-(-5)} = \dfrac{3+3}{-1+5} = \dfrac{6}{4} = \dfrac{3}{2}$ ...(ii)

Slope of line RS $= \dfrac{-5-(-3)}{-2-(-5)} = \dfrac{-5+3}{-2+5} = \dfrac{-2}{3}$ ...(iii)

Slope of line PS $= \dfrac{1-(-5)}{2-(-2)} = \dfrac{1+5}{2+2} = \dfrac{6}{4} = \dfrac{3}{2}$ ...(iv)

Slope of line PQ = Slope of line RS ...[From (i) and (iii)]

$\therefore$ Line PQ || Line RS ...(v)

$\therefore$ Slope of line QR = Slope of line PS ...[From (ii) and (iv)]

$\therefore$ Line QR || Line PS ...(vi)

In □PQRS, side PQ || side RS ...[From (v)]

$\therefore$ side RS || side PR ...[From (vi)]

□PQRS is a parallelogram ...(vii)(Definition)

Using distance formula,

$$d(P, R) = \sqrt{[2-(-5)]^2 + [1-(-3)]^2}$$

$$= \sqrt{(2+5)^2 + (1+3)^2}$$

$$= \sqrt{7^2 + 4^2}$$

$$= \sqrt{49+16}$$

$\therefore$ $d(P, R) = \sqrt{65}$ units ...(viii)

$$d(Q, S) = \sqrt{[-1-(-2)]^2 + [3-(-5)]^2}$$

$$= \sqrt{(-1+2)^2 + (3+5)^2}$$

$$= \sqrt{1^2 + 8^2}$$

$$= \sqrt{1+64}$$

$\therefore$ $d(Q, S) = \sqrt{65}$ units ...(ix)

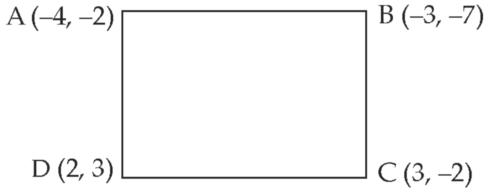
In parallelogram PQRS,

diagonal PR ≅ diagonal QS ...[From (viii) and (ix)]

$\therefore$ **□PQRS is a rectangle** ...(A parallelogram is a rectangle if its diagonals are congruent)

***(18) Find the type of the quadrilateral if points A(–4, –2), B(–3, –7) C(3, –2) and D(2, 3) are joined serially.** *(4 marks)*

Solution :



A(–4, –2), B(–3, –7) C(3, –2), D(2, 3)

Slope of line AB $= \dfrac{-7-(-2)}{-3-(-4)}$

$= \dfrac{-7+2}{-3+4}$

$= \dfrac{-5}{1}$

$\therefore$ Slope of line AB $= -5$ ...(i)

Slope of line BC $= \dfrac{-2-(-7)}{3-(-3)}$

$= \dfrac{-2+7}{3+3}$

$= \dfrac{5}{6}$

$\therefore$ Slope of line BC $= \dfrac{5}{6}$ ...(ii)

Slope of line CD $= \dfrac{-2-3}{3-2}$

Slope of line CD $= \dfrac{-5}{1} = -5$ ...(iii)

Slope of line AD $= \dfrac{-2-3}{-4-2}$

$= \dfrac{-5}{-6}$

Slope of line AD $= \dfrac{5}{6}$ ...(iv)

Slope of line AB = Slope of line CD ...[From (i) and (iii)]

$\therefore$ Line AB || Line CD ...(v)

Slope of line BC = Slope of line AD

...[From (ii) and (iv)]

$\therefore$ Line BC || Line AD ...(vi)

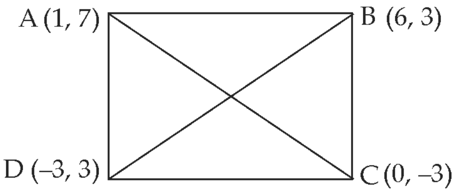
In □ ABCD, AB || CD ...[From (v)]

BC || AD ...[From (vi)]

$\therefore$ **□ ABCD is a parallelogram.**

(22) Find the slope of the diagonals of a quadrilateral with vertices A (1, 7), B(6, 3) C(0, –3) and D(–3, 3). *(3 marks)*

Solution :



A(1, 7), B(6, 3) C(0, –3) and D(–3, 3)

□ ABCD has two diagonals seg AC and seg BD.

$$\text{Slope of AC} = \frac{7-(-3)}{1-0} = \frac{7+3}{1} = \frac{10}{1}$$

$\therefore$ $\boxed{\textbf{Slope of AC = 10}}$

$$\text{Slope of BD} = \frac{3-3}{6-(-3)} = \frac{0}{6+3}$$

Slope of BD = 0

$\therefore$ $\boxed{\textbf{Slope of BD = 0}}$

Problem Set - 5 *(Textbook Pg No. 122)*

MCQ's

(1) Fill in the blanks using correct alternatives *(1 mark each)*

(1) Seg AB is parallel to Y-axis and co-ordinates of point A are (1, 3) then co-ordinates of point B can be ....

(A) (3, 1) (B) (5, 3) (C) (3, 0) (D) (1, –3)

(2) Out of the following, point .... lies to the right of the origin on X-axis.

(a) (–2, 0) (b) (0, 2) (c) (2, 3) (d) (2, 0)

(3) Distance of point (–3, 4) from the origin is ....

(A) 7 (B) 1 (C) 5 (D) –5

(4) A line makes an angle of 30° with the positive direction of X-axis. So the slope of the line is ....

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

Additional MCQ's

(5) What is the slope of line with indination 60 ?

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) 0

(6) Find the inclination of a line with slope 1.

(A) 60° (B) 45° (C) 90° (D) Can't say

(7) Line l is parallel to line m. It slopes of line l is $\frac{1}{2}$ then slope of line m is ....

(A) –2 (B) 0 (C) $\frac{1}{2}$ (D) Can't say

(8) What is slope of line passing through points (4, 6) and (1, –2).

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{8}{5}$ (D) $\frac{8}{3}$

(9) Slope of X - axis is ....

(A) 0 (B) 1 (C) –1 (D) Not defined

(10) Slope of Y - axis is ....

(A) 0 (B) 1 (C) –1 (D) Not defined

(11) Distance of point A (7, 24) from the origin is ....

(A) 17 (B) –17

(C) 25 (D) Can not be found

(12) Find the co-ordinates of the point P which bisects seg having co-ordinates (3, 2) and (5, –2) ....

(A) (–3, 5) (B) (0, 4) (C) (4, 0) (D) (5, –3)

(13) Find the co-ordinates of the point which divides line seg QR in the ratio 1 : 2 where Q (1, 1) and R(1, –2).

(A) (–5, 3) (B) (1, 0) (C) (–3, 2) (D) (4, 0)

(14) In what ratio does the point (1, 6) divide the line segment joining the points (3, 6) and (–5, 6).

(A) 1 : 3 (B) 2 : 3 (C) 3 : 1 (D) 3 : 2

ANSWERS

(1) (D) (1, –3) (2) (D) (2, 0) (3) (C) 5

(4) (C) $\frac{1}{\sqrt{3}}$ (5) (A) $\sqrt{3}$ (6) (B) 45°

(7) (C) $\frac{1}{2}$ **(8)** (D) $\frac{8}{3}$ **(9)** (A) 0

(10) (D) Not defined **(11)** (C) 25 **(12)** (C) (4, 0)

(13) (B) (1, 0) **(14)** (A) 1 : 3

PROBLEMS FOR PRACTICE

Based on Practice Set 5.1

(1) Find the distance between the given points. *(1 mark each)*

(i) A (3, –4), B (–5, 6)

(ii) P (10, –8), Q (–3, –2)

(iii) K (0, –5), L (–5, 0)

(iv) I (3.5, 6.8), J (1.5, 2.8)

(2) Show that the point (5, 11) is equidistant from the points (–5, 13) and (3, 1). *(2 marks)*

(3) Check whether points (3, 3), (–4, –1) and (3, –5) are the vertices of an isosceles triangle. *(2 marks)*

(4) Find the relation between x and y, where point (x, y) is equidistant from (2, –4) and (–2, 6). *(3 marks)*

(5) Show that the point (0, 9) is equidistant from the point (–4, 1) and (4, 1) *(2 marks)*

(6) Find the coordinates of the point on Y-axis which is equidistant from the points M(6, 5) and point N(–4, 3). *(3 marks)*

(7) Using distance formula, check whether following points are collinear or not. *(2 marks each)*

(i) L(4, –1) M(1, –3), N(–2, –5)

(ii) A(–5, 4), B(–2, –2), C(3, –12)

(8) Find the distance of point Z(–2.4, –1), from the origin. *(2 marks)*

(9) Show that the points A(4, 7) B(8, 4) and C(7, 11) are the vertices of a right angled triangle. *(3 marks)*

(10) Show that A(4, –1), B(6, 0), C(7, –2) and D(5, –3) are the vertices of a square. *(4 marks)*

(11) Find the coordinates of the circumcentre of PQR if P(2, 7), Q(–5, 8) and R(–6, 1).

(12) Show that the points (2, 4), (2, 6) and $(2+\sqrt{3}, 5)$ are the vertices of an equilateral triangle. *(3 marks)*

(13) Find the coordinates of the circumcentre of ABC, if A(2, 3), B(4, –1) and C(5, 2). Also, find circumradius. *(3 marks)*

Based on Practice Set 5.2

(14) Show that points A(1, –5), B(–4, –8), C(–1, –13) and D(4, –10) are the vertices of a rhombus. *(4 marks)*

(15) Find the coordinates of the point P which divides line segment QR in the ratio m : n in the following examples. *(2 marks each)*

(i)	Q(–5, 8),	R(4, –4)	$m : n = 2 : 1$
(ii)	Q(–2, 7),	R(–2, –5)	$m : n = 1 : 3$
(iii)	Q(1, 7),	R(–3, 1)	$m : n = 1 : 2$
(iv)	Q(6, –5),	R(–10, 2)	$m : n = 3 : 4$
(v)	Q(5, 8),	R(–7, –8)	$m : n = 4 : 1$

(16) Find the coordinates of the midpoint of segment QR, if Q(2.5, –4.3) and R(–1.5, 2.7) *(2 marks)*

(17) Find the coordinates of the midpoint P of seg AB, if A(3.5, 9.5) and B(–1.5, 0.5) *(2 marks)*

(18) In what ratio does the point (1, 3) divide line segment joining the points (3, 6) and (–5, –6)? *(3 marks)*

(19) Find the lengths of the median of ABC whose vertices are A(7, –3), B(5, 3), C(3, –1). *(4 marks)*

(20) Show that the line segment joining the points (5, 7), (3, 9) and (8, 6), (0, 10) bisect each other. *(4 marks)*

(21) Segments AB and CD bisects each other at point M. If A(4, 3), B(–2, 5), C(–3, 5), then find coordinates of D. *(4 marks)*

(22) Find the ratio in which the line segment joining the points (6, 4) and (1, –7) is divided by X-axis. *(3 marks)*

(23) Find the coordinates of the points which divide the line segment joining the points (–2, 2) and (6, –6) in four equal parts. *(3 marks)*

(24) Find the coordinates of the points which divide segment AB into four equal parts, if A(5, 7) and B(–3, –1) *(4 marks)*

(25) If A-P-Q-B, point P and Q trisects seg AB and A(3, 1), Q(–1, 3), then find coordinates of points B and P. *(4 marks)*

(26) Find the coordinates of centroid G of ABC, if *(3 marks each)*

(i) A(8, 9), B(4, 5), C(6, 2)

(ii) A(11, 8), B(–6, 5), C(1, –28)

(27) The origin 'O' is the centroid of ABC in which A(–4, 3) B(3, k) and C(h, 5). Find h and k. *(4 marks)*

(28) Find the coordinates of the points dividing the segment joining A(–5, 7) and B(11, –1) into four equal parts. *(4 marks)*

Based on Practice Set 5.3

(29) Find the slope of a line which makes an angle with the positive X-axis. *(1 mark each)*

(i) 0° **(ii)** 30° **(iii)** 45° **(iv)** 60° **(v)** 90°

(30) Find the slope of the line passing through the points. *(2 marks each)*

(i) (–1, 4) (3, –7) **(ii)** (5, 5), (1, 6)

(iii) (1, 7) (4, 8) **(iv)** (4, 8), (5, 5)

(v) (4, 1) (2, –3) **(vi)** (4, 4), (3, 5)

(31) Using slope concept, check whether the following points are collinear. *(2 marks each)*

(i) (–2, –1) (4, 0) (3, 3)

(ii) (–2, –3), $\left(\frac{33}{8}, 4\right)$ (5, 5)

(iii) (4, 4) (3, 5) (–1, –1)

(iv) (2, 10), (0, 4) (3, 13)

(v) (5, 0) (10, –3) (–5, 6)

(vi) (2, 5), (5, 7) (8, 9)

(32) Find the value of k, if (5, k), (–3, 1) and (–7, –2) are collinear. *(3 marks)*

(33) Find the value of k, if (2, 1) (4, 3) and (0, k) are collinear 1. *(3 marks)*

(34) Find the value of k, if the slope of the line passing through (2, 5) and (k, 3) is 2. *(2 marks)*

(35) P(3, 4), Q(7, 2) and R(–2, –1) are the vertices of PQR. Write down the slope of each side of the triangle. *(4 marks)*

(36) Show that line joining (4, –1) and (6, 0) is parallel to line joining (7, –2) and (5, –3). *(4 marks)*

(37) Show that □ ABCD is a parallelogram, if A(–1, 2), B(–5, –6) C(3, –2) and D(7, 6)

(38) Show that P(3, 4), Q(7, –2), R(1, 1) and S(–3, 7) are the vertices of a parallelogram.

ANSWERS

(1) **(i)** $2\sqrt{41}$ **(ii)** $\sqrt{205}$ **(iii)** $5\sqrt{2}$ **(iv)** $2\sqrt{5}$

(4) $5y = x + 5$ **(6)** (0, 9)

(7) **(i)** collinear **(ii)** non-collinear

(8) 2.6 units **(11)** (–2, 4)

(13) (3, 1) circumradius = $\sqrt{5}$ units

(15) **(i)** (1, 0) **(ii)** (–2, 4) **(iii)** $\left(-\frac{1}{3}, 5\right)$ **(iv)** $\left(-\frac{6}{7}, -2\right)$ **(v)** $\left(\frac{23}{5}, -\frac{24}{5}\right)$

(16) (0.5, –0.8) **(17)** (1, 5) **(18)** 1 : 3

(19) 5, 5, $\sqrt{10}$ **(21)** (5, 3) **(22)** 4 : 7

(23) (0, 0) (2, –2) (4, –4) **(24)** (3, 5) (1, 3) (–1, 1)

(25) (–3, 4) (1, 2) **(26)** (i) (6, 5.33) (ii) (2, –5)

(27) $h = 1, k = -8$ **(28)** (–1, 5) (3, 3) (7, 1)

(29) **(i)** 0 **(ii)** $\frac{1}{\sqrt{3}}$ **(iii)** 1 **(iv)** $\sqrt{3}$ **(v)** not defined

(30) **(i)** $-1\frac{1}{4}$ **(ii)** $-\frac{1}{4}$ **(iii)** $\frac{1}{3}$ **(iv)** –3 **(v)** 2 (vi) –1

(31) **(ii), (iv), (v), (vi)** are collinear.

(32) 7 **(33)** –1 **(34)** 1 **(35)** $-\frac{1}{2}, \frac{1}{3}, 1$

ASSIGNMENT – 5

Time : 1 Hr. **Marks : 20**

Q.1. A. Choose the proper alternative answer for the questions given below: **(2)**

(1) Distance of point (–3, 4) from the origin is ....

(A) 7 (B) 1 (C) 5 (D) –5

(2) Line l is parallel to line m. If slope of line l is $\frac{1}{2}$ then slope of line m is ....

(A) –2 (B) 0 (C) $\frac{1}{2}$ (D) Can't say

Q.1. B. Solve the following questions: **(2)**

(1) Slope of a line is $\sqrt{3}$. Find its inclination.

(2) Find the distance between (2, 3) and (4, 1).

Q.2. Perform any one of the following activities: **(2)**

(1) Seg AB is a dimeter of a circle with centre P (1, 2). If A (–4, 2), then find the co-ordinates of point B.

(2) If P – T – Q and P (–3, 10), Q (6, –8) and T (–1, 6), then find the ratio in which point T divides seg PQ (Complete the following activity)

Let point T divides seg PQ in the ratio $m : n$

P (–3, 10) = (x_1, y_1) Q (6, –8) = (x_2, y_2) T (–3, 10) = (x, y)

By section formula,

$$X = \frac{\square \times \square + \square \times \square}{\square + \square}$$

$$\therefore \quad -3\left(\square + \square\right) = \square \times \square + \square \times \square$$

$$\therefore \quad -3\square + -3\,\square = \square \times \square + \square \times \square$$

$$\therefore \quad -3\square - \square \times \square = \square \times \square + 3\,\square$$

$$\therefore \quad \square\, m = \square\, n$$

$$\therefore \quad m : n = \square : \square$$

(3) A (–7, 6), B (2 –2) and B (8, 5) are co-ordinates of vertices of ΔABC. Find the co-ordinates of centroid of ΔABC.

Q.3. Solve any two of the folllowing questions: **(6)**

(1) Decide (2, 10), (0, 4) and (3, 13) are collinear or not.

(2) Line PQ || Line RS. P (2, 4), Q (3, 6) R (3, 1) and S(5, K).

(3) Prove that $\left(\sqrt{2}, \sqrt{2}\right)$, $\left(-\sqrt{2}, -\sqrt{2}\right)$ and $\left(-\sqrt{6}, \sqrt{6}\right)$ are the vertices of an equilateral triangle.

Q.4. Attempt the following: **(8)**

(1) Find the co-ordinates of circumcentre of ΔABC if A (7, 1), B (3, 5) and C (2, 0)

(2) Find the possible co-ordinates of the fourth vertex of the parallelogram, if three of its vertices are (5, 6), (1, –2) and (–3, 2).

(3) Find the o-ordinates of the points which divide the line segment joining the points (–2, 2) and (6, – 6) into four equal parts.

❖ ❖ ❖

6 Trigonometry

... INDEX ...

Points to Remember:

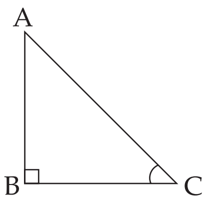
- **Introduction:**

  The word 'trigonometry' is derived from Greek words **Tri** meaning **three, gona** meaning **sides** and **metron** meaning **measure.**

  Thus, trigonometry deals with measurements of sides and angles of a right angled triangle.

  In ΔABC,

  $m\angle ABC = 90$



(1) seg AC is the hypotenuse.

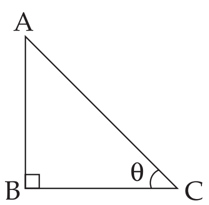
(2) For $\angle ACB$, seg AB is the opposite side.

(3) For $\angle ACB$, seg BC is the adjacent side.

- **Trigonometric ratios of an acute angle in a right angled triangle:**

  For any acute angle in a right angled triangle, the three above mentioned sides, can be arranged two at a time, in six different ratios. These ratios are called **Trigonometric ratios.**

  In ΔABC, $m\angle ABC = 90$, $m\angle ACB = \theta$



Sine ratio of $\theta = \sin\theta = \dfrac{\text{Opposite side}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

Cosine ratio of $\theta = \cos\theta = \dfrac{\text{Adjacent side}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

Tangent ratio of $\theta = \tan\theta = \dfrac{\text{Opposite side}}{\text{Adjacent side}} = \dfrac{AB}{BC}$

Cosecant ratio of $\theta = \operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{Opposite side}} = \dfrac{AC}{AB}$

Secant ratio of $\theta = \sec\theta = \dfrac{\text{Hypotenuse}}{\text{Adjacent side}} = \dfrac{AC}{BC}$

Cotangent ratio $\theta = \cot\theta = \dfrac{\text{Adjacent side}}{\text{Opposite side}} = \dfrac{BC}{AB}$

- **Relations between Trigonometric Ratios:**

(1) $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ (2) $\sec\theta = \dfrac{1}{\cos\theta}$

(3) $\cot\theta = \dfrac{1}{\tan\theta}$ (4) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(5) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

- **Trigonometric Identities:**

(1) $\sin^2\theta + \cos^2\theta = 1$ (2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Table of Trigonometric Ratios for Angles 0 , 30 , 45 , 60 and 90

Trigonometric Ratios	Ange θ				
	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\text{cosec}\ \theta = \frac{1}{\sin\theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta = \frac{1}{\cos\theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta = \frac{1}{\tan\theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

MASTER KEY QUESTION SET - 6

Practice Set - 6.1 *(Textbook Page No. 131)*

(1) **If $\sin\theta = \frac{7}{25}$ then find $\cos\theta$ and $\tan\theta$.** *(2 marks)*

Solution :

$\sin\theta = \frac{7}{25}$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \left(\frac{7}{25}\right)^2 + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \frac{49}{625}$

$\therefore \cos^2\theta = \frac{625 - 49}{625}$

$\therefore \cos^2\theta = \frac{576}{625}$

$\therefore \boxed{\cos\theta = \frac{24}{25}}$ ...(Taking square roots)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\therefore \tan\theta = \frac{7}{25} \div \frac{24}{25}$

$\therefore \tan\theta = \frac{7}{25} \times \frac{25}{24}$

$\therefore \boxed{\tan\theta = \frac{7}{24}}$

Problem Set - 6 *(Textbook Page No. 138)*

(2) **If $\sin\theta = \frac{11}{61}$ find the values of cosq using trigonometric identity.** *(2 marks)*

Solution :

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \frac{121}{3721}$

$\cos^2\theta = \frac{3721 - 121}{3721}$

$\therefore \cos^2\theta = \frac{3600}{3721}$

$\therefore \boxed{\cos\theta = \frac{60}{61}}$ ...(Taking square roots)

Practice Set - 6.1 *(Textbook Page No. 131)*

(2) **If $\tan\theta = \frac{3}{4}$ then find the value of $\sec\theta$ and $\cos\theta$.** *(2 marks)*

Solution :

$\tan\theta = \frac{3}{4}$

$1 + \tan^2\theta = \sec^2\theta$

$\therefore 1 + \left(\frac{3}{4}\right)^2 = \sec^2\theta$

$\therefore 1 + \frac{9}{16} = \sec^2\theta$

$\therefore \frac{16+9}{16} = \sec^2\theta$

$\therefore \sec^2\theta = \frac{25}{16}$

$\therefore \boxed{\sec\theta = \frac{5}{4}}$ ...(Taking square roots)

$\cos\theta = \frac{1}{\sec\theta}$

$\therefore \cos\theta = 1 \div \frac{5}{4}$

$\therefore \cos\theta = 1 \times \frac{4}{5}$

$\therefore \boxed{\cos\theta = \frac{4}{5}}$

Problem Set - 6 *(Textbook Page No. 138)*

(3) If $\tan\theta = 2$ then find values of other trigonometric ratios. *(3 marks)*

Solution :

$\tan\theta = 2$

$1 + \tan^2\theta = \sec^2\theta$

$\therefore\ 1 + 2^2 = \sec^2\theta$

$\therefore\ \sec^2\theta = 1 + 4$

$\therefore\ \sec^2\theta = 5$

$\therefore\ \boxed{\sec\theta = \sqrt{5}}$ ...(Taking square roots)

$\cos\theta = \dfrac{1}{\sec\theta}$

$\therefore\ \boxed{\cos\theta = \dfrac{1}{\sqrt{5}}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\therefore\ \sin\theta = \tan\theta \times \cos\theta$

$\therefore\ \sin\theta = 2 \times \dfrac{1}{\sqrt{5}}$

$\therefore\ \boxed{\sin\theta = \dfrac{2}{\sqrt{5}}}$

$\mathrm{cosec}\,\theta = \dfrac{1}{\sin\theta}$

$\therefore\ \boxed{\mathrm{cosec}\,\theta = \dfrac{\sqrt{5}}{2}}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\therefore\ \boxed{\cot\theta = \dfrac{1}{2}}$

Practice Set - 6.1 *(Textbook Page No. 131)*

(3) If $\cot\theta = \dfrac{40}{9}$, find the value of cosec θ and $\sin\theta$. *(2 marks)*

Solution :

$\cot\theta = \dfrac{40}{9}$

$\mathrm{cosec}^2\theta = 1 + \cot^2\theta$

$= 1 + \left(\dfrac{40}{9}\right)^2$

$= 1 + \dfrac{1600}{81}$

$= \dfrac{81 + 1600}{81}$

$\therefore\ \mathrm{cosec}^2\theta = \dfrac{1681}{81}$

$\therefore\ \boxed{\mathrm{cosec}\,\theta = \dfrac{41}{9}}$ ...(Taking square roots)

$\sin\theta = \dfrac{1}{\mathrm{cosec}\,\theta}$

$\therefore\ \sin\theta = 1\ \dfrac{41}{9}$

$\therefore\ \sin\theta = 1 \times \dfrac{9}{41}$

$\therefore\ \boxed{\sin\theta = \dfrac{9}{41}}$

Problem Set - 6 *(Textbook Page No. 138)*

(4) If $\sec\theta = \dfrac{13}{12}$, find values of other trigonometric ratios. *(3 marks)*

Solution :

$\sec\theta = \dfrac{13}{12}$

$\cos\theta = \dfrac{1}{\sec\theta}$

$\therefore\ \boxed{\cos\theta = \dfrac{12}{13}}$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore\ \sin^2\theta + \left(\dfrac{12}{13}\right)^2 = 1$

$\therefore\ \sin^2\theta = 1 - \dfrac{144}{169}$

$\therefore\ \sin^2\theta = \dfrac{169 - 144}{169}$

$\therefore\ \sin^2\theta = \dfrac{25}{169}$

$\therefore\ \boxed{\sin\theta = \dfrac{5}{13}}$ ...(Taking square roots)

$\mathrm{cosec}\,\theta = \dfrac{1}{\sin\theta}$

$\therefore\ \boxed{\mathrm{cosec}\,\theta = \dfrac{13}{5}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{5}{13}\ \dfrac{12}{13}$

$$= \frac{5}{13} \times \frac{13}{12}$$

$$\therefore \boxed{\tan\theta = \frac{5}{12}}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\therefore \boxed{\cot\theta = \frac{12}{5}}$$

Practice Set - 6.1 *(Textbook Page No. 131)*

(4) **If $5\sec\theta - 12\operatorname{cosec}\theta = 0$, find the values of $\sec\theta$, $\cos\theta$ and $\sin\theta$.** *(3 marks)*

Solution :

$$5\sec\theta - 12\operatorname{cosec}\theta = 0$$

$$\therefore \quad 5\sec\theta = 12\operatorname{cosec}\theta$$

$$\therefore \quad \frac{5}{\cos\theta} = \frac{12}{\sin\theta}$$

$$\therefore \quad \frac{\sin\theta}{\cos\theta} = \frac{12}{5}$$

$$\therefore \boxed{\tan\theta = \frac{12}{5}}$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$= 1 + \left(\frac{12}{5}\right)^2$$

$$= 1 + \frac{144}{25}$$

$$= \frac{25 + 144}{25}$$

$$\sec^2\theta = \frac{169}{25}$$

$$\therefore \boxed{\sec\theta = \frac{13}{5}} \qquad \text{...(Taking square roots)}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\therefore \quad \cos\theta = 1 \quad \frac{13}{5}$$

$$\therefore \quad \cos\theta = 1 \times \frac{5}{13}$$

$$\therefore \boxed{\cos\theta = \frac{5}{13}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \quad \sin\theta = \tan\theta \times \cos\theta$$

$$\therefore \quad \sin\theta = \frac{12}{5} \times \frac{5}{13}$$

$$\therefore \boxed{\sin\theta = \frac{12}{13}}$$

(5) **If $\tan\theta = 1$ then find the value of $\dfrac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta}$** *(4 marks)*

Solution :

$$\tan\theta = 1$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$= 1 + (1)^2$$

$$= 1 + 1$$

$$\therefore \quad \sec^2\theta = 2$$

$$\therefore \quad \sec\theta = \sqrt{2} \qquad \text{...(Taking square roots)}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\therefore \quad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\tan\theta = 1$$

$$\therefore \quad \frac{\sin\theta}{\cos\theta} = 1$$

$$\therefore \quad \sin\theta = \cos\theta$$

$$\therefore \quad \sin\theta = \frac{1}{\sqrt{2}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\therefore \quad \operatorname{cosec}\theta = \sqrt{2}$$

$$\frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta} = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \div \left(\sqrt{2} + \sqrt{2}\right)$$

$$= \frac{2}{\sqrt{2}} \quad 2\sqrt{2}$$

$$\therefore \quad \frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta} = \frac{2}{\sqrt{2}} \times \frac{1}{2\sqrt{2}}$$

$$\therefore \boxed{\frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta} = \frac{1}{2}}$$

(6) **Prove that:** *(2 marks)*

(i) $\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

Solution :

Proof: LHS $= \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \sec\theta$

$=$ R.H.S.

$\therefore \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

(ii) $\cos^2\theta\,(1 + \tan^2\theta) = 1$ *(2 marks)*

Solution :

Proof: LHS $= \cos^2\theta\,(1 + \tan^2\theta)$

$= \cos^2\theta \times \sec^2\theta$ ...$(\because 1 + \tan^2\theta = \sec^2\theta)$

$= \cos^2\theta \times \dfrac{1}{\cos^2\theta}$

$= 1$

$=$ R.H.S.

$\therefore \cos^2\theta\,(1 + \tan^2\theta) = 1$

(iii) $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$ *(3 marks)*

Solution :

Proof: LHS $= \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\dfrac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$

$= \sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= \sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\dfrac{(1-\sin\theta)^2}{\cos^2\theta}}$ $(\because \sin^2\theta + \cos^2\theta = 1$
$\therefore 1 - \sin^2\theta = \cos^2\theta)$

$= \dfrac{1-\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \sec\theta - \tan\theta$

$=$ R.H.S.

$\therefore \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

(iv) $(\sec\theta - \cos\theta)(\cot\theta + \tan\theta) = \tan\theta \,.\, \sec\theta$ *(3 marks)*

Solution :

Proof: LHS $= (\sec\theta - \cos\theta)(\cot\theta + \tan\theta)$

$= \left(\dfrac{1}{\cos\theta} - \cos\theta\right)\left(\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}\right)$

$= \left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)\left(\dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta.\sin\theta}\right)$

$= \dfrac{\sin^2\theta}{\cos\theta} \times \dfrac{1}{\cos\theta.\sin\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1$
$\therefore 1 - \cos^2\theta = \sin^2\theta)$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\cos\theta}$

$= \tan\theta \times \sec\theta$

$=$ R.H.S.

$\therefore (\sec\theta - \cos\theta)(\cot\theta + \tan\theta) = \tan\theta \,.\, \sec\theta$

(v) $\cot\theta + \tan\theta = \operatorname{cosec}\theta \,.\, \sec\theta$ *(2 marks)*

Solution :

LHS $= \cot\theta + \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cdot\cos\theta}$

$= \dfrac{1}{\sin\theta\cdot\cos\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \operatorname{cosec}\theta \,.\, \sec\theta$

$=$ R.H.S.

$\therefore \cot\theta + \tan\theta = \operatorname{cosec}\theta \,.\, \sec\theta$

(vi) $\dfrac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$ *(2 marks)*

Solution :

LHS $= \dfrac{1}{\sec\theta - \tan\theta}$

$= \dfrac{1 \times (\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$

$= \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$

$= \dfrac{\sec\theta + \tan\theta}{1}$ $\left(\begin{array}{l}\sec^2\theta = 1 + \tan^2\theta \\ \therefore \sec^2\theta - \tan^2\theta = 1\end{array}\right)$

$= \text{R.H.S.}$

$\therefore \quad \mathbf{\dfrac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta}$

(vii) $\mathbf{\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta}$ *(3 marks)*

Solution :

Proof: LHS $= \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta)^2 - (\cos^2\theta)^2$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= 1 \quad (\sin^2\theta - \cos^2\theta) \quad \left(\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ \therefore \sin^2\theta = 1 - \cos^2\theta\end{array}\right)$

$= 1 - \cos^2\theta - \cos^2\theta$

$= 1 - 2\cos^2\theta$

$= \text{R.H.S.}$

$\therefore \quad \mathbf{\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta}$

(viii) $\mathbf{\sec\theta + \tan\theta = \dfrac{\cos\theta}{1 - \sin\theta}}$ *(3 marks)*

Solution :

Proof: LHS $= \sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1 + \sin\theta}{\cos\theta}$

$= \dfrac{(1 + \sin\theta)(1 - \sin\theta)}{\cos\theta\,(1 - \sin\theta)}$

$= \dfrac{1 - \sin^2\theta}{\cos\theta\,(1 - \sin\theta)}$

$= \dfrac{\cos^2\theta}{\cos\theta\,(1 - \sin\theta)} \quad \left(\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ \therefore \cos^2\theta = 1 - \sin^2\theta\end{array}\right)$

$= \dfrac{\cos\theta}{1 - \sin\theta}$

$= \text{R.H.S.}$

$\therefore \quad \mathbf{\sec\theta + \tan\theta = \dfrac{\cos\theta}{1 - \sin\theta}}$

(ix) **If** $\mathbf{\tan\theta + \dfrac{1}{\tan\theta} = 2}$ **then show that prove** $\mathbf{\tan^2\theta + \dfrac{1}{\tan^2\theta} = 2}$ *(2 marks)*

Solution :

Proof : $\tan\theta + \dfrac{1}{\tan\theta} = 2$

squaring both sides,

$\left(\tan\theta + \dfrac{1}{\tan\theta}\right)^2 = 4$

$\therefore \quad \tan^2\theta + 2\tan\theta\,\dfrac{1}{\tan\theta} + \dfrac{1}{\tan^2\theta} = 4$

$\therefore \quad \tan^2\theta + 2 + \dfrac{1}{\tan^2\theta} = 4$

$\therefore \quad \tan^2\theta + \dfrac{1}{\tan^2\theta} = 4 - 2$

$\therefore \quad \mathbf{\tan^2\theta + \dfrac{1}{\tan^2\theta} = 2}$

(x) $\mathbf{\dfrac{\tan A}{(1 + \tan^2 A)^2} + \dfrac{\cot A}{(1 + \cot^2 A)^2} = \sin A \cos A}$. *(3 marks)*

Solution :

Proof: LHS $= \dfrac{\tan A}{(1 + \tan^2 A)^2} + \dfrac{\cot A}{(1 + \cot^2 A)^2}$

$= \dfrac{\tan A}{(\sec^2 A)^2} + \dfrac{\cot A}{(\operatorname{cosec}^2 A)^2} \quad \left(\begin{array}{l}\because 1 + \tan^2 A = \sec^2 A \\ \text{and } 1 + \cot^2 A = \operatorname{cosec}^2 A\end{array}\right)$

$= \dfrac{\tan A}{\sec^4 A} + \dfrac{\cot A}{\operatorname{cosec}^4 A}$

$= \tan A \,.\, \cos^4 A + \cot A \,.\, \sin^4 A$

$= \dfrac{\sin A}{\cos A}\,\cos^4 A + \dfrac{\cos A}{\sin A}\cdot \sin^4 A$

$= \sin A \,.\, \cos^3 A + \cos A \,.\, \sin^3 A$

$= \sin A \,.\, \cos A\,(\cos^2 A + \sin^2 A)$

$= \sin A \,.\, \cos A \quad 1 \qquad [\because \sin^2 A + \cos^2 A = 1]$

$= \sin A \,.\, \cos A$

$= \text{R.H.S.}$

$\therefore \quad \mathbf{\dfrac{\tan A}{(1 + \tan^2 A)^2} + \dfrac{\cot A}{(1 + \cot^2 A)^2} = \sin A \cos A}$

(xi) $\mathbf{\sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A = 1}$. *(3 marks)*

Solution :

Proof: LHS $= \sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A$

$= \sec^4 A\,(1 + \sin^2 A)(1 - \sin^2 A) - 2\tan^2 A$

$= \sec^4 A \quad (1 + \sin^2 A) \quad \cos^2 A - 2\tan^2 A$

$\left(\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ \therefore 1 - \sin^2\theta = \cos^2\theta\end{array}\right)$

$= \dfrac{1}{\cos^4 A} \quad (1 + \sin^2 A) \quad \cos^2 A - \dfrac{2\sin^2 A}{\cos^2 A}$

$= \dfrac{1 + \sin^2 A}{\cos^2 A} - \dfrac{2\sin^2 A}{\cos^2 A}$

$= \dfrac{1 + \sin^2 A - 2\sin^2 A}{\cos^2 A}$

$= \dfrac{1 - \sin^2 A}{\cos^2 A} \quad \left(\begin{array}{l}\sin^2 A + \cos^2 A = 1 \\ \therefore 1 - \sin^2 A = \cos^2 A\end{array}\right)$

$= \dfrac{\cos^2 A}{\cos^2 A}$

$= 1$

$= \text{R.H.S.}$

$\therefore \quad \mathbf{\sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A = 1}$

(xii) $\dfrac{\tan\theta}{\sec\theta - 1} = \dfrac{\tan\theta + \sec\theta + 1}{\tan\theta + \sec\theta - 1}$ *(3 marks)*

Solution :

Proof: $1 + \tan^2\theta = \sec^2\theta$

$\therefore \tan^2\theta = \sec^2\theta - 1$

$\therefore \tan\theta \quad \tan\theta = (\sec\theta + 1)(\sec\theta - 1)$

$\therefore \dfrac{\tan\theta}{\sec\theta - 1} = \dfrac{\sec\theta + 1}{\tan\theta}$

$\therefore \dfrac{\tan\theta}{\sec\theta - 1} = \dfrac{\tan\theta + \sec\theta + 1}{\sec\theta - 1 + \tan\theta}$ ...... (Theorem on equal ratios)

$\therefore \mathbf{\dfrac{\tan\theta}{\sec\theta - 1} = \dfrac{\tan\theta + \sec\theta + 1}{\tan\theta + \sec\theta - 1}}$

Problem Set - 6 *(Textbook Page No. 138)*

(5) Prove the following:

(i) sec θ (1 – sin θ) (sec θ + tan θ) = 1 *(2 marks)*

Solution :

Proof: LHS = sec θ (1 – sin θ) (sin θ + tan θ)

$= \dfrac{1}{\cos\theta}(1 - \sin\theta) \times \left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)$

$= \dfrac{(1 - \sin\theta)}{\cos\theta}\ \dfrac{(1 + \sin\theta)}{\cos\theta}$

$= \dfrac{1 - \sin^2\theta}{\cos^2\theta}$ ...$[\because (a+b)(a-b) = a^2 - b^2]$

$= \dfrac{\cos^2\theta}{\cos^2\theta}$ $\left(\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1\\ \therefore 1 - \sin^2\theta = \cos^2\theta\end{array}\right)$

$= 1$

= R.H.S.

$\therefore$ **secθ (1 – sinθ) (sec θ + tan θ) = 1**

(ii) (sec θ + tan θ) (1 – sin θ) = cos θ *(2 marks)*

Solution :

Proof: LHS = (sec θ + tan θ) (1 – sin θ)

$= \left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)$

$= \dfrac{(1 + \sin\theta)}{\cos\theta}(1 - \sin\theta)$

$= \dfrac{1 - \sin^2\theta}{\cos\theta}$ ...$[\because (a+b)(a-b) = a^2 - b^2]$

$= \dfrac{\cos^2\theta}{\cos\theta}$ $\left(\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1\\ \therefore 1 - \sin^2\theta = \cos^2\theta\end{array}\right)$

= cos θ

= R.H.S.

$\therefore$ **(sec θ + tan θ) (1 – sin θ) = cos θ**

(iii) sec²θ + cosec²θ = sec²θ × cosec²θ *(2 marks)*

Solution :

Proof: LHS = $\sec^2\theta + \text{cosec}^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \times \sin^2\theta}$

$= \dfrac{1}{\cos^2\theta \times \sin^2\theta}$ ...$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \sec^2\theta \,.\, \text{cosec}^2\theta$

= R.H.S.

$\therefore$ $\mathbf{\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta \times \text{cosec}^2\theta}$

(iv) cot²θ – tan²θ = cosec²θ – sec²θ *(2 marks)*

Solution :

Proof: LHS = $\cot^2\theta - \tan^2\theta$ $(\because 1 + \cot^2\theta = \text{cosec}^2\theta$

$\therefore \cot^2\theta = \text{cosec}^2\theta - 1)$

$= (\text{cosec}^2\theta - 1) - (\sec^2\theta - 1)$ ...$\left(\begin{array}{l}\because \sec^2\theta = 1 + \tan^2\theta\\ \therefore \tan^2\theta = \sec^2\theta - 1\end{array}\right)$

$= \text{cosec}^2\theta - 1 - \sec^2\theta + 1$

$= \text{cosec}^2\theta - \sec^2\theta$

= R.H.S.

$\therefore$ $\mathbf{\cot^2\theta - \tan^2\theta = \text{cosec}^2\theta - \sec^2\theta}$

(v) tan⁴θ + tan²θ = sec⁴θ – sec²θ *(2 marks)*

Solution :

Proof: LHS = $\tan^4\theta + \tan^2\theta$

$= \tan^2\theta\ (\tan^2\theta + 1)$ $\left(\begin{array}{l}\because 1 + \tan^2\theta = \sec^2\theta\\ \therefore \tan^2\theta = \sec^2\theta - 1\end{array}\right)$

$= (\sec^2\theta - 1)(\sec^2\theta)$

$= \sec^4\theta - \sec^2\theta$

= R.H.S.

$\therefore$ $\mathbf{\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta}$

(vi) $\mathbf{\dfrac{1}{1 - \sin\theta} + \dfrac{1}{1 + \sin\theta} = 2\sec^2\theta}$ *(3 marks)*

Solution :

Proof: LHS = $\dfrac{1}{1 - \sin\theta} + \dfrac{1}{1 + \sin\theta}$

$= \dfrac{1 + \sin\theta + 1 - \sin\theta}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \dfrac{2}{1 - \sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$ $\left(\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1\\ \therefore \cos^2\theta = 1 - \sin^2\theta\end{array}\right)$

$= 2\sec^2\theta$

$=$ R.H.S.

$$\therefore \quad \frac{\mathbf{1}}{\mathbf{1-\sin\theta}} + \frac{\mathbf{1}}{\mathbf{1+\sin\theta}} = \mathbf{2\sec^2\theta}$$

(vii) $\mathbf{\sec^6 x - \tan^6 x = 1 + 3\sec^2 x \times \tan^2 x}$ *(3 marks)*

Solution :

Proof: LHS $= \sec^6 x - \tan^6 x$

$= (\sec^2 x)^3 - (\tan^2 x)^3$

$= (\sec^2 x - \tan^2 x)^3 + 3\sec^2 x \tan^2 x.(\sec^2 x - \tan^2 x)$

...$[\because a^3 - b^3 = (a-b)^3 + 3ab\,(a-b)]$

$= (1)^3 + 3\sec^2 x \,.\, \tan^2 x\ (1)$

$\left(\begin{array}{l} 1+\tan^2 x = \sec^2 x \\ \therefore \sec^2 x - \tan^2 x = 1 \end{array}\right)$

$= 1 + 3\sec^2 x \,.\, \tan^2 x$

$=$ R.H.S.

$\therefore \quad \mathbf{\sec^6 x - \tan^6 x = 1 + 3\sec^2 x \times \tan^2 x}$

(viii) $\dfrac{\mathbf{\tan\theta}}{\mathbf{\sec\theta+1}} = \dfrac{\mathbf{\sec\theta-1}}{\mathbf{\tan\theta}}$ *(4marks)*

Solution :

Proof: LHS $= \dfrac{\tan\theta}{\sec\theta+1}$

$$= \frac{\tan\theta}{\sec\theta+1}\ \frac{\sec\theta-1}{\sec\theta-1}$$

$$= \frac{\tan\theta\,(\sec\theta-1)}{\sec^2\theta-1} \qquad [\because (a+b)(a-b) = a^2-b^2]$$

$$= \frac{\tan\theta\,(\sec\theta-1)}{\tan^2\theta} \qquad \left[\because 1+\tan^2\theta = \sec^2\theta \;\; \therefore \sec^2\theta - 1 = \tan^2\theta\right]$$

$$= \frac{\sec\theta-1}{\tan\theta}$$

$=$ R.H.S.

$$\therefore \quad \frac{\mathbf{\tan\theta}}{\mathbf{\sec\theta+1}} = \frac{\mathbf{\sec\theta-1}}{\mathbf{\tan\theta}}$$

(ix) $\dfrac{\mathbf{\tan^3\theta-1}}{\mathbf{\tan\theta-1}} = \mathbf{\sec^2\theta + \tan\theta}$ *(3 marks)*

Solution :

Proof: LHS $= \dfrac{\tan^3\theta-1}{\tan\theta-1}$

$$= \frac{\tan^3\theta-1^3}{\tan\theta-1}$$

$$= \frac{(\tan\theta-1)\,(\tan^2\theta+\tan\theta+1)}{(\tan\theta-1)}$$

$= \tan^2\theta + 1 + \tan\theta \qquad (\because \tan^2\theta + 1 = \sec^2\theta)$

$= \sec^2\theta + \tan\theta$

$=$ R.H.S.

$$\therefore \quad \frac{\mathbf{\tan^3\theta-1}}{\mathbf{\tan\theta-1}} = \mathbf{\sec^2\theta + \tan\theta}$$

(x) $\dfrac{\mathbf{\sin\theta-\cos\theta+1}}{\mathbf{\sin\theta+\cos\theta-1}} = \dfrac{\mathbf{1}}{\mathbf{\sec\theta-\tan\theta}}$ *(4 marks)*

Solution :

Proof: R.H.S. $= \dfrac{1}{\sec\theta-\tan\theta}$

$$= 1\left[\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right]$$

$$= 1\left[\frac{1-\sin\theta}{\cos\theta}\right]$$

$$= \frac{\cos\theta}{1-\sin\theta} \qquad \text{...(i)}$$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \sin^2\theta$

$\therefore \cos\theta \ \ \cos\theta = (1-\sin\theta)\,(1+\sin\theta)$

$$\therefore \frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$\therefore \frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta-\cos\theta}{\cos\theta-(1-\sin\theta)} \qquad \text{...(By theorem on equal ratios)}$$

$$\therefore \frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta-\cos\theta}{\cos\theta-1+\sin\theta} \qquad \text{...(ii)}$$

From (i) and (ii)

$$\therefore \quad \frac{\mathbf{\sin\theta-\cos\theta+1}}{\mathbf{\sin\theta+\cos\theta-1}} = \frac{\mathbf{1}}{\mathbf{\sec\theta-\tan\theta}}$$

Points to Remember:

- **Application of trignomety:**

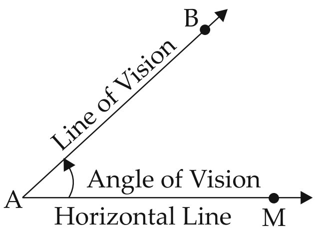
  Height and Distances:

  Many times, we require to find the height of a tower, building, tree or distance of a ship from the lighthouse or width of the river etc. We cannot measure them actually, we can find the heights and distances with the help of trigonometric ratios.

(i) **Line of vision:** The line connecting the eye of the observer and the objects is called the **Line of vision.**

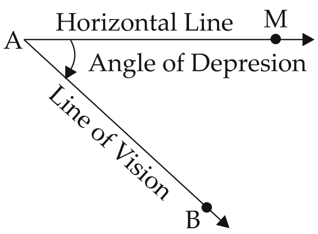
(ii) **Angle of Elevation:**

If A, B are two points such that B is at higher level than A and AM is horizontal line through A, then



$\angle$MAB is the **angle of elevation** of B with respect to A.

(iii) **Angle of Depression:** If A, B are two points such that B is at lower level than A and AM is the horizontal line through A, then $\angle$MAB is the **angle of depression** of B with respect to A.

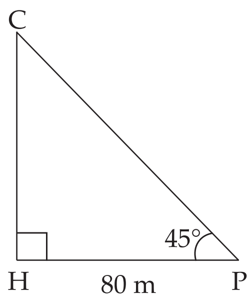


Practice Set - 6.2 *(Textbook Page No. 137)*

(1) **A person is standing at a distance of 80m from a church looking at its top. The angle of elevation is of 45°. Find the height of the church.** *(3 marks)*

Solution :

CH represents the height of the church and C represents its top. P is the position of the person at a distance of 80 m from the church.



$\therefore$ PH = 80 m

$\angle$CPH is the angle of the elevation

$\therefore$ $\angle$CPH = 45°

In ΔPHC, $\angle$CHP = 90

$\therefore\ \tan \angle CPH = \frac{CH}{PH}$ ...(By definition)

$\therefore\ \tan 45 = \frac{CH}{80}$

$\therefore\ 1 = \frac{CH}{80}$

$\therefore$ CH = 80 m

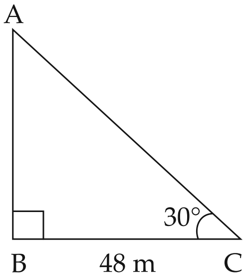
$\therefore$ **Height of the church is 80 m**

Problem Set - 6 *(Textbook Page No. 139)*

(6) **A boy standing at a distance of 48 meters from a building observes the top of the building and makes an angle of elevation of 30°.Find the height of the building.** *(3 marks)*

Solution :

AB represents the height of the building. C represents the position of the boy at a distance of 48 m from the building.



$\angle$ACB is the angle of elevation

$\therefore$ $\angle$ACB = 30°,

In ΔABC, $\angle$ABC = 90

$\therefore\ \tan \angle ACB = \frac{AB}{BC}$ ...(By definition)

$\therefore\ \tan 30 = \frac{AB}{48}$

$\therefore\ \frac{1}{\sqrt{3}} = \frac{AB}{48}$

$\therefore\ AB = \frac{48}{\sqrt{3}}$

$\therefore\ AB = \frac{48 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$\therefore\ AB = \frac{48\sqrt{3}}{3}$

$\therefore\ AB = 16\sqrt{3}$

$\therefore$ AB = 16 1.73

$\therefore$ AB = 27.68 m

$\therefore$ **The height of the building is 27.68 m.**

Practice Set - 6.2 *(Textbook Page No. 137)*

(2) **From the top of a lighthouse, an observer looking at a ship makes angle of depression of 60°. If the height of the lighthouse is 90 metre, then find how far the ship is from the lighthouse.** $\left(\sqrt{3} = 1.73\right)$ *(3 marks)*

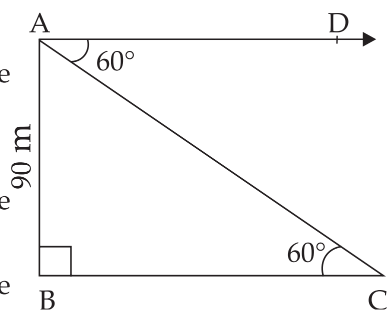
Solution :

AB represents the height of the lighthouse.

AB = 90 m

C represents the position of ship.

$\angle$DAC is the angle of depression



$\therefore$ $\angle$DAC = 60°,

$\therefore$ $\angle$DAC = $\angle$ACB = 60° ...(Alternate angle theorem)

In $\triangle ABC$, $\angle ABC = 90$

$\therefore$ $\tan 60 = \frac{AB}{BC}$ ...(By definition)

$\therefore$ $\sqrt{3} = \frac{90}{BC}$

$\therefore$ $BC = \frac{90}{\sqrt{3}}$

$\therefore$ $BC = \frac{90\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$\therefore$ $BC = \frac{90\sqrt{3}}{3}$

$\therefore$ $BC = 30\sqrt{3}$

$\therefore$ $= 30\ (1.73)$

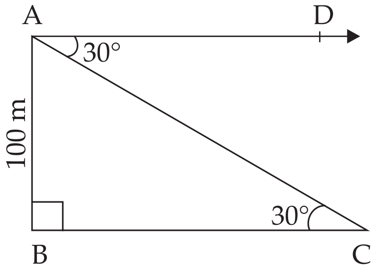
$\therefore$ $BC = 51.9$ m

$\therefore$ **The distance of the ship from the lighthouse is 51.90 m.**

Problem Set - 6 *(Textbook Page No. 139)*

(7) **From the top of a lighthouse, an observer looks at a ship and finds the angle of depression to be 30°. If the height of the lighthouse is 100 m, then find how far is that ship from the lighthouse.** *(3 marks)*

Solution :



AB represents the height of the lighthouse.

$\therefore$ AB = 100 m

C represents the position of ship.

$\angle DAC$ is the angle of depression.

$\therefore$ $\angle DAC = \angle ACB = 30°$ ...(BA alternate angle theorem

In $\triangle ABC$, $\angle ABC = 90$

$\therefore$ $\tan \angle ACB = \frac{AB}{BC}$ ...(By definition)

$\therefore$ $\tan 30 = \frac{100}{BC}$

$\therefore$ $\frac{1}{\sqrt{3}} = \frac{100}{BC}$

$\therefore$ $BC = 100\sqrt{3}$

$\therefore$ $BC = 100 \quad 1.73$

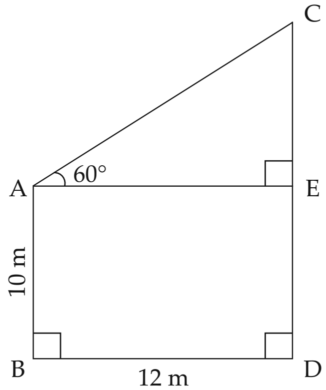
$\therefore$ $BC = 173$ m

$\therefore$ **The distance of the ship from the lighthouse is 173 m.**

Practice Set - 6.2 *(Textbook Page No. 137)*

(3) **Two buildings are facing each other on either side of a road of width 12 m. From the top of the first building, which is 10 m. high, the angle of elevation of the top of the second is 60°. What is the height of the second building?** *(4 marks)*

Solution :



AB and CD represents the height of the two buildings, on either side of a road.

AB = 10 m, BD = 12 m

$\angle CAE$ is the angle of the elevation.

$\therefore$ $\angle CAE = 60°$,

$\square$ABDE is a rectangle ...(By definition)

AB = DE = 10 m (Opposite sides of a rectangle)

AE = BD = 12 m

In $\triangle AEC$, $\angle AEC = 90$

$\therefore$ $\tan \angle CAE = \frac{CE}{AE}$ ...(By definition)

$\therefore$ $\tan 60 = \frac{CE}{12}$

$\therefore$ $\sqrt{3} = \frac{CE}{12}$

$\therefore$ $CE = 12 \quad \sqrt{3}$

$\therefore$ $CE = 12 \quad 1.73$

$\therefore$ $CE = 20.76$ m

$CD = CE + DE$ ...(C – E – D)

$= 20.76 + 10$

$CD = 30.76$ m

$\therefore$ **Height of the second building is 30.76 m.**

Problem Set - 6 *(Textbook Page No. 139)*

(8) **Two buildings are in front of each other on a road of width 15 meters. From the top of the first building, having a height of 12 meter, the angle of elevation of the top of the second building is 30°.What is the height of the second building?** *(4 marks)*

Solution :

AB and CD represents the height of two buildings at distance of 15 m, i.e. BC = 15 m, AB = 12 m

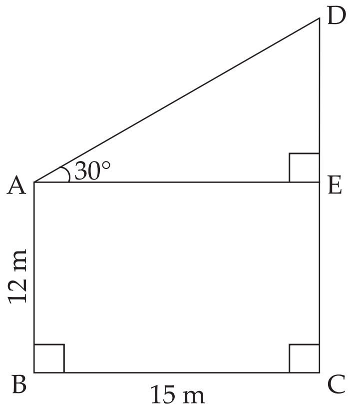
$\angle DAE$ is the angle of elevation.

$\therefore$ $\angle DAE = 30°$

$\square ABCE$ is a rectangle. ...(By definition)

$\therefore$ AB = EC = 12 m

BC = AE = 15 m ...(Opposite sides of rectangle)



In $\triangle AED$, $\angle AED = 90$

$\therefore$ $\tan \angle DAE = \dfrac{DE}{AE}$ ...(By definition)

$\therefore$ $\tan 30 = \dfrac{DE}{15}$

$\therefore$ $\dfrac{1}{\sqrt{3}} = \dfrac{DE}{15}$

$\therefore$ $DE = \dfrac{15}{\sqrt{3}}$

$= \dfrac{15 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$= \dfrac{15\sqrt{3}}{3}$

$= 5\sqrt{3}$

$= 5 \quad 1.73$

$\therefore$ DE = 8.65 m

CD = CE + DE ...(C – E – D)

= 12 + 8.65

CD = 20.65 m

$\therefore$ **Height of the second building is 20.65 m.**

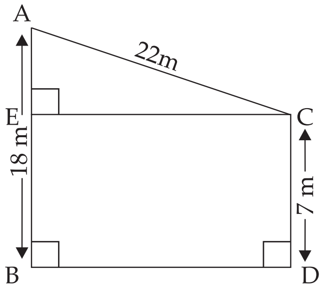
Practice Set - 6.2 *(Textbook Page No. 137)*

(4) Two poles of heights 18 metre and 7 metre are erected on a ground. The length of the wire fastened at their tops in 22 metre. Find the angle made by the wire with the horizontal. *(4 marks)*

Solution :

AB and CD represents the height of the two poles.

AC is the length of the wire of length 22 m joining top A and top C of two poles.



AC = 22 m

$\angle ACE$ is the angle made by the wire with the horizontal.

$\square EBDC$ is a rectangle ...(By definition)

BE = CD = 7 m ...(Opposite sides of rectangle)

AB = AE + BE ...(A – E – B)

$\therefore$ 18 = AE + 7

$\therefore$ 18 – 7 = AE

$\therefore$ AE = 11 m

In $\triangle AEC$, $\angle AEC = 90$

$\therefore$ $\sin \angle ACE = \dfrac{AE}{AC}$ ...(By definition)

$\therefore$ $\sin \angle ACE = \dfrac{11}{22}$

$\therefore$ $\sin \angle ACE = \dfrac{1}{2}$

But, $\sin 30 = \dfrac{1}{2}$

$\therefore$ $\sin \angle ACE = \sin 30$

$\therefore$ $\angle ACE = 30$

$\therefore$ **The angle made by the wire with the horizontal is 30°.**

(5) A storm broke a tree and the treetop rested 20 m from the base of the tree, making an angle of 60° with the horizontal. Find the height of the tree. *(4 marks)*

Solution :

AB represents the height of the tree.

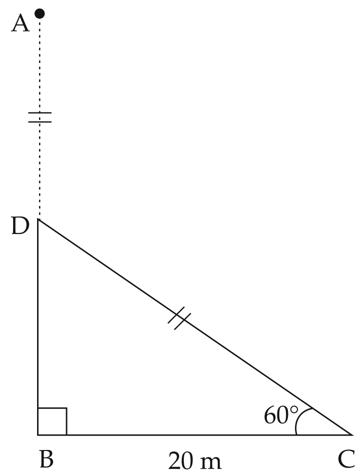
Tree breaks at D.

AD represents the broken part of the tree which takes the position DC.

$\therefore$ AD = DC

$\angle DCB = 60°$

BC = 20 m



In $\triangle DBC$, $\angle DBC = 90$

$\therefore$ $\tan 60 = \dfrac{DB}{BC}$ ...(By definition)

$\therefore$ $\sqrt{3} = \dfrac{DB}{20}$

$\therefore$ $DB = 20\sqrt{3}$

$= 20\,(1.73)$

$\therefore$ DB = 34.60 m

$$\cos 60 = \frac{BC}{DC} \quad \text{...(By definition)}$$

$$\therefore \frac{1}{2} = \frac{20}{DC}$$

$\therefore$ DC = 40 m

$\therefore$ AD = DC = 40 m

$\therefore$ AB = AD + DB ...(A – D – B)

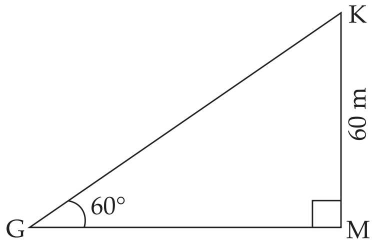
$\therefore$ AB = 40 + 34.60

$\therefore$ AB = 74.60 m

$\therefore$ **The height of the tree is 74.60 m.**

(6) A kite is flying at a height of 60 m above the ground. The string attached to the kite is tied at the ground. It makes an angle of 60° with the ground. Assuming that the string is straight, find the length of the string. $\left(\sqrt{3} = 1.73\right)$ *(3 marks)*

Solution :



'K' is the position of kite in the sky, 60 m above the ground level,

KG represents the length of the string.

$\angle$KGM is the angle between string and the ground $\angle$KGM = 60°

In ΔKMG, $\angle$KMG = 90

$$\therefore \sin \angle KGM = \frac{KM}{GK} \quad \text{...(By definition)}$$

$$\therefore \sin 60 = \frac{60}{GK}$$

$$\therefore \frac{\sqrt{3}}{2} = \frac{60}{GK}$$

$$\therefore GK = \frac{60 \times 2}{\sqrt{3}}$$

$$\therefore GK = \frac{120}{\sqrt{3}}$$

$$\therefore GK = \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\therefore GK = \frac{120\sqrt{3}}{3}$$

$\therefore$ GK = $40\sqrt{3}$

$\therefore$ GK = 40 1.73

$\therefore$ GK = 69.20 m

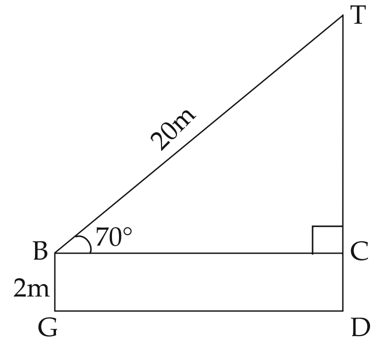
$\therefore$ **Length of the string is 69.20 m.**

Problem Set - 6 *(Textbook Page No. 139)*

(9) A ladder on the platform of a fire brigade van can be elevated at an angle of 70° to the maximum. The length of the ladder can be extended upto 20 m. If the platform is 2 m above the ground, find the maximum height from the ground upto which the ladder can reach. (sin = 70° = 0.94)

(5 marks)

Solution :



GD is ground level. BC is base of the ladder of the fire brigads van at a height of 2 m from ground level.

'T' is top of ladder of the fire brigads van at the maximum height

$\angle$TBC = 70° ...(Angle of elevation)

BT is the length of the ladder

BT = 20 m, BG = 2 m

□BGDC is a rectangle ...(By definition)

BG = CD = 2 m ...(Opposite sides of a rectangle)

In ΔBCT, $\angle$BCT = 90

$$\therefore \sin \angle TBC = \frac{TC}{TB} \quad \text{...(By definition)}$$

$$\therefore \sin 70 = \frac{TC}{20}$$

$$\therefore 0.94 = \frac{TC}{20}$$

$\therefore$ TC = 0.94 20

$\therefore$ TC = 18.80 m

TD = TC + CD ...(T – C – D)

$\therefore$ TD = 18.80 + 2

$\therefore$ TD = 20.80 m

$\therefore$ **Other end of the ladder can reach 20.80 m above the ground ladder.**

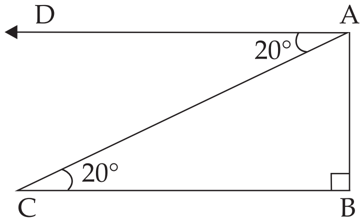
*** (10) While landing at an airport, a pilot made an angle of depression of 20°. Average speed of the plane was 200 km/hr. The plane reached the ground after 54 seconds. Find the height at which the plane was when it started landing.**
(sin 20° = 0.342) *(5 marks)*

Solution :

A represents the position of the plane above the ground.

'C' is the landing point of the plane on the ground

AB represents the height of the plane from the ground.



$\angle DAC$ is the angle of depression

$\angle DAC = \angle ACB = 20$

Distance (AC) = speed × time

= 200 km/ hr × 54 sec

$= 200 \text{ km/ hr} \times \frac{54}{3600}$ hr

($\because$ 1 hr= 3600 sec)

$= 200 \times \frac{54}{3600}$

= 3km

$\therefore$ AC = 3000 m

In $\triangle ABC$, $\angle ABC = 90$

$\therefore \sin \angle ACB = \frac{AB}{AC}$ ...(By definition)

$\therefore \sin 20 = \frac{AB}{3000}$

$\therefore 0.342 = \frac{AB}{3000}$

$\therefore$ AB = 0.342 3000

$\therefore$ AB = 1026 km.

$\therefore$ **Plane was at a height of 1026 km, when it started landing.**

Problem Set - 6 *(Textbook Page No. 138)*

MCQ's

Choose the correct alternative answer for the following questions.

(1) **$\sin \theta . \operatorname{cosec} \theta$ = ...**

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$

(2) **cosec45° =?**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$

(3) **$1 + \tan^2 \theta$ = ?**

(A) $\cot^2 \theta$ (B) $\operatorname{cosec}^2 \theta$ (C) $\sec^2 \theta$ (D) $\tan^2 \theta$

(4) **When we see at a higher level from the horizontal line, angle formed is ....**

(A) Angle of Elevation (B) Angle of Depression

(C) 0 (D) Straight angle

Additional MCQ's

(5) **If $\sin \theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$, then $\tan \theta$ =**

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$

(C) $\frac{12}{25}$ (D) can not be calculated

(6) **If $\operatorname{cosec} \theta = \frac{61}{60}$, $\sec \theta = \frac{61}{11}$, then $\cot \theta$ = ...**

(A) $\frac{61^2}{660}$ (B) $\frac{60}{11}$

(C) $\frac{11}{60}$ (D)can not be calculated

(7) **If $\sin \theta = \frac{24}{25}$, then $\cos \theta$ = ...**

(A) $\frac{\sqrt{24}}{5}$ (B) $\frac{25}{24}$ (C) $\frac{25}{7}$ (D) $\frac{7}{25}$

(8) **If $\tan \theta = 1$, then $\sec \theta$ = ...**

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) 0

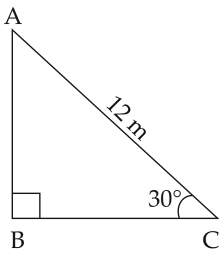
(9) **If $\cot \theta = \frac{3}{4}$, then $\tan \theta$ = ...**

(A) $\frac{4}{3}$ (B) $\frac{9}{16}$ (C) $\frac{16}{9}$ (D) $\frac{5}{4}$

(10) **If $\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$, then θ = ...**

(A) 0 (B) 45 (C) 30 (D) 60

(11) **In the adjoining figure, if $\angle B = 90°$, $\angle C = 30°$, AC = 12 m. then AB = ...**



(A) $12\sqrt{3}$ m (B) $6\sqrt{3}$ m (C) 12 m (D) 6 m

(12) **If $(\sec \theta - 1)(\sec \theta + 1) = \frac{1}{3}$, then $\cos \theta$ = ...**

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{3}$

(13) **If $\sin \theta + \cos \theta = a$, and $\sin \theta - \cos \theta = b$, then = ...**

(A) $a^2 + b^2 = 1$ (B) $a^2 - b^2 = 1$

(C) $a^2 + b^2 = 2$ (D) $a^2 - b^2 = 2$

(14) **If sin θ = 1, then find cot θ = ...**

(A) 0 (B) 1 (C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

ANSWERS

(1) (A) 1 (2) (B) $\sqrt{2}$ (3) (C) $\sec^2\theta$

(4) (A) Angle of Elevation (5) (A) $\frac{4}{3}$ (6) (C) $\frac{11}{60}$

(7) (D) $\frac{7}{25}$ (8) (B) $\sqrt{2}$ (9) (A) $\frac{4}{3}$ (10) (D) 60

(11) (D) 6 cm (12) (C) $\frac{\sqrt{3}}{2}$ (13) (C) $a^2 + b^2 = 2$

(14) (A) 0

PROBLEMS FOR PRACTICE

Based on Practice Set 6.1

(1) If tan θ = 2, find the values of other trigonometric ratios using the identities. *(3 marks)*

(2) If $\cot\theta = \frac{7}{24}$, find the values of other trigonometric ratios using the identity. *(3 marks)*

(3) 3 sin θ – 4 cos θ = 0, then find the values of all trigonometric ratios. *(3 marks)*

(4) If $\sqrt{3}$ tan θ = 3 sin θ, find the value of $\sin^2\theta - \cos^2\theta$. *(3 marks)*

(5) Simplify : sin θ (cosec θ – sin θ). *(2 marks)*

(6) Prove: *(3 marks each)*

(i) $\cos^2\theta + \frac{1}{1+\cot^2\theta} = 1$

(ii) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

(iii) $(1 + \tan^2\theta)\ (1 + \sin\theta)\ (1 - \sin\theta) = 1$

(iv) $(1 + \cot^2\theta)\ (1 + \cos\theta)\ (1 - \cos\theta) = 1$

(v) $\cot^2\theta - \frac{1}{\sin^2\theta} = -1$

(vi) $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

(vii) $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

(viii) $\frac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

(ix) $\frac{\tan^3 A - 1}{\tan A - 1} = \sec^2 A + \tan A$

(x) $\frac{\sin\theta + \tan\theta}{\cos\theta} = \tan\theta\ (1 + \sec\theta)$

(xi) $\text{cosec}^2 A - \cos^2 A = \frac{\sec^2 A - \sin^2 A}{\tan^2 A}$

(xii) $\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right) = (\sec\theta - \tan\theta) = 1$

(xiii) $\frac{\cos^2 A + \tan^2 A - 1}{\sin^2 A} = \tan^2 A$

(xiv) $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1+\sin A}{\cos A}$

(xv) $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = = 1 + \sin\theta$

Based on Practice Set 6.2

(7) For a person standing at a distance of 80 m from a temple, the angle of elevation of its top is 45 . Find the height of the church. *(3 marks)*

(8) From the top of a lighthouse, an observer looks at a ship and finds the angle of depression to be 60. If the lighthouse is 90 m, then find how far is that ship from the lighthouse? $\left(\sqrt{3} = 1.73\right)$ *(4 marks)*

(9) A building is 200 $\sqrt{3}$ metres high. Find the angle of elevation if its top is 200 m away from its foot. *(2 marks)*

(10) A straight road leads to the foot of a tower of height 50 m. From the top of the tower, the angle of depression of two cars standing on the road are 30 and 60 . What is the distance between the two cars? *(4 marks)*

(11) A ship of height 24 m is sighted from a lighthouse. From the top of the lighthouse, the angle of depression to the top of the mast and base of the ship is 30 and 45 respectively. How far is the ship from the lighthouse? $\left(\sqrt{3} = 1.73\right)$ *(4 marks)*

(12) From a point on the roof of a house, 11 m high, it is observed that the angles of depression of the top and foot of a lamp post are 30 and 60 respectively. What is the height of the lamp post? *(4 marks)*

ANSWERS

	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
(1)	$\frac{2}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	2	$\frac{1}{2}$	$\sqrt{5}$	$\frac{\sqrt{5}}{2}$
(2)	$\frac{24}{25}$	$\frac{7}{25}$	$\frac{24}{7}$	$\frac{7}{24}$	$\frac{25}{7}$	$\frac{25}{25}$
(3)	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{5}{3}$	$\frac{5}{4}$

(4) $\sin^2\theta - \cos^2\theta = \frac{1}{2}$ (5) $\cos^2\theta$

(7) 80 m (8) 51.9 m (9) 60

(10) $\frac{100}{\sqrt{3}}$ (11) 56.76 m (12) 7.33 m

ASSIGNMENT – 6

Time : 1 Hr. **Marks : 20**

Q.1. (A) Choose the correct alternative answer for the following: **(2)**

(1) **sin θ. cosec θ =** ....

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$

(2) **If sin θ =** $\frac{4}{5}$ **and cosθ =** $\frac{3}{5}$**, then tan θ =** ....

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{12}{25}$ (D) can not be calculated

Q.1. (B) Solve the following: **(2)**

(1) If $\sin\theta = \frac{\sqrt{3}}{2}$, then find θ .

(2) Find the value of tan 40° × tan 50°.

Q.2. Perform any one of the following activities **(2)**

(1) If tan θ = 1, then complete the following activity to find cos θ.

Sol. $1 + \tan^2\theta = \boxed{}$ ...(Identity)

$\therefore$ $1 + (1)^2 = \boxed{}$

$\therefore$ $2 = \boxed{}$

Taking square roots

$\therefore$ $\boxed{} = \sqrt{2}$

$\therefore$ $\cos\theta = \frac{1}{\boxed{}}$

$\therefore$ $\cos\theta = \frac{1}{\sqrt{2}}$

(2) A boy is at a distance of 60 m from a tree, makes an angle of elevation of 60 with the top of the tree. What is the height of the tree?

(3) Prove that $\frac{\sin^2 A}{\cos A} + \cos A = \sec A$.

Q.3. Solve the following: **(6)**

(1) If $x = r\cos\theta$ and $y = r\sin\theta$, then proof $x^2 + y^2 = r^2$

(2) Using Pythagoras theorem, prove that $1 + \cot^2\theta = \operatorname{cosec}^2\theta$,

(3) Two poles of height 18 m and 7 m are erected on the gound. A wire of length 22 m tied to the top of the poles. Find the angle made by the wire with the horizontal.

Q.4. Solve any two of the following questions: **(8)**

(1) Prove that $\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$

(2) In a right angled ΔABC, ∠A = 90 and $\frac{5\sin^2 B + 7\cos^2 C + 4}{3 + 8\tan^2 60^\circ} = \frac{7}{27}$. If AC = 3, find the perimeters of ΔABC.

(3) Prove : $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$

❖ ❖ ❖

7 Mensuration

... INDEX ...

Points to Remember:

- **Introduction :**

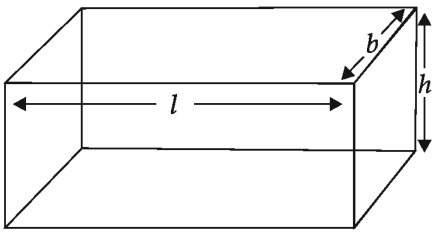
  Mensuration is a special branch of mathematics that deals with the measurement of geometric figures.

  In previous classes we have studied certain concepts related to areas of plane figures (shapes) such as triangles, quadrilaterals, polygons and circles.

- **Cuboid [Rectangular Parallelopiped]**

  A cuboid is a solid figure bounded by six rectangular faces, where the opposite faces are equal.

  A cuboid has a **length**, **breadth** and **height** denoted as '*l*', '*b*' and '*h*' respectively as shown in the figure,



  In our day to day life we come across cuboids such as rectangular room, rectangular box, brick, rectangular fish tank, etc.

FORMULAE

(1) Total surface area of a cuboid $= 2\,(lb + bh + lh)$

(2) Lateral surface area of a cuboid $= 2\,(l + b) \times h$

(3) Volume of a cuboid $= l \times b \times h$

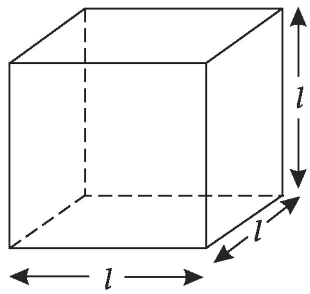
(4) Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$

- **Cube**

  A cube is a cuboid bounded by six equal squares faces.

  Hence its length, breadth and height are equal.

  $\therefore$ The edge of the cube = length = breadth = height



  The edge of the cube is denoted as '*l*'.

  A dice is an example of cube.

FORMULAE

(1) Total surface area of a cube $= 6l^2$

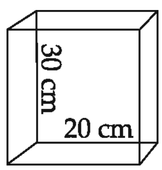
(2) Lateral surface area of a cube $= 4l^2$

(3) Volume of cube $= l^3$

(4) Diagonal of the cube $= \sqrt{3}l$

Solve the following example

(1) The length, breadth and height of an oil can are 20 cm, 20 cm and 30 cm respectively as shown in the adjacent figure. How much oil will it contain? (1 litre = 1000 cm^3)



Solution :

For oil can, length (l) = 20 cm, breadth (b) = 20 cm, height (h) = 30 cm.

Volume of oil can $= l \times b \times h$

$= 20 \times 20 \times 30$

$= 12000 \text{ cm}^3$

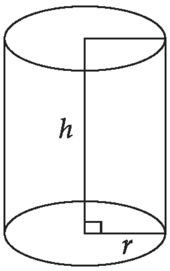
Capacity of oil it contains = Volume of oil can

$= \dfrac{12000}{1000}$ [$\because$ 1 ltr = 1000cm^3]

= 12 litres

$\therefore$ **The oil can will contain 12 litres of oil**

- **Right Circular Cylinder**



A right circular cylinder (Cylinder) is a solid figure bounded by two flat circular surfaces and a curved surface.

The perpendicular distance between the two base faces is called height of the cylinder and is denoted by '*h*'. The radius of the base of the cylinder is denoted by '*r*'.

The cylinders which we see regularly are drum, pipe, road roller, coins, test tube, refill of a ball pen, syringe etc.

FORMULAE

(1) Curved surface area of a right circular cylinder $= 2\pi rh$

(2) Total surface area of a right circular cylinder $= 2\pi r\,(r + h)$

(3) Volume of a right circular cylinder $= \pi r^2 h$

Practice Set - 7.1 *(Textbook Page No. 145)*

(3) **Find the total surface area of a cylinder if the radius of its base is 5 cm and height is 40 cm.** *(1 mark)*

Solution :

For a cylinder,

$r = 5$ cm ...(Given)

$h = 40$ cm ...(Given)

Total surface area $= 2\pi r\,(r + h)$

$= 2 \times 3.14 \times 5 \times (5 + 40)$

$= 31.4 \times 45$

$\therefore$ **Total surface area = 1,413 cm²**

(9) **In the adjoining figure a cylindrical wrapper of flat tablets is shown. The radius of a tablet is 7 mm and its thickness is 5 mm. How many such tablets are wrapped in the wrapper?** *(3marks)*

Solution :

For cylindrical wrapper,

Diameter = 14 mm

Radius (R) $\dfrac{14}{2}$ mm = 7 mm

Height (H) = 10 cm

i.e. H = 100 mm

For cylindrical tablet,

Radius (r) = 7 mm, Height (h) = 5 mm

Let 'N' number of tablets can be wrapped in the given wrapper.

$\therefore$ N × Volume of tablet = Volume of wrapper.

$\therefore\ N \times \pi r^2 h = \pi R^2 H$

$\therefore\ N \times \pi \times 7 \times 7 \times 5 = \pi \times 7 \times 7 \times 100$

$\therefore\ N = \dfrac{\pi \times 7 \times 7 \times 100}{\pi \times 7 \times 7 \times 5}$

$\therefore\ N = 20$

$\therefore$ **20 tablets can be packed in the given wrapper.**

Problem Set - 7 *(Textbook Pg No. 161)*

***(3)** **Some plastic balls of radius 1 cm were melted and cast into a tube. The thickness, length and outer radius of the tube were 2 cm, 90 cm and 30 cm respectively. How many balls were melted to make the tube?** *(4 marks)*

Solution :

For spherical solid ball, $r = 1$ cm,

For cylindrical pipe, Outer radius (r_1) = 30 cm

Thickness (t) = 2 cm

Height (h) = 90 cm

Inner radius (r_2) = $r_1 - t$

= 30 – 2

r_2 = 28 cm

Volume of cylindrical pipe = Number of spherical balls required (N) × Volume of spherical ball

$\therefore\ \pi \times h\,(r_1^2 - r_2^2) = N \times \dfrac{4}{3}\pi r^3$

$\therefore\ \pi \times 90\,(30^2 - 28^2) = N \times \dfrac{4}{3}\pi \times (1)^3$

$\therefore\ \dfrac{\pi \times 90 \times (30+28)(30-28) \times 3}{4 \times \pi} = N$

$\therefore\ \dfrac{90 \times 58 \times 2 \times 3}{4} = N$

$\therefore\ N = 7{,}830$

$\therefore$ **Number of spherical balls required is 7,830**

(4) A metal parallelopiped of measures 16 cm × 11 cm × 10 cm was melted to make coins. How many coins were made if the thickness and diameter of each coin was 2 mm and 2 cm respectively? *(2 marks)*

Solution :

For the metallic cuboid,

$l = 16$ cm, $b = 11$ cm, $h = 10$ cm

For the cylindrical coin,

Diameter = 2 cm, Thickness(h_1) = 2 mm = 0.2 cm

i.e. Radius (r_1) = 1 cm

Let number of coins made be N.

$\therefore$ N × Volume of coin = Volume of cuboid

$\therefore$ $N \times \pi r_1^2 h_1 = l \times b \times h$

$\therefore$ $N \times \frac{22}{7} \times 1 \times 1 \times \frac{2}{10} = 16 \times 11 \times 10$

$\therefore$ $N = \frac{16 \times 11 \times 10 \times 10 \times 7}{22 \times 2}$

$\therefore$ N = 2800

$\therefore$ **Number of coins made are 2800**

(5) The diameter and length of a roller is 120 cm and 84 cm respectively. To level the ground, 200 rotations of the roller are required. Find the expenditure to level the ground at the rate of ₹ 10 per sq. m. *(3 marks)*

Solution :

For circular roller,

Diameter = 120 cm, $\therefore$ radius $(r) = \frac{120}{2} = 60$ cm

length (h) = 84 cm

Number of rotations required to level the ground (N) = 200

Rate of levelling (R) = ₹ 10 per sq. metre

Area levelled in 1 rotation = curved surface area of the roller.

$\therefore$ Area levelled in 200 rotations (A) = $200 \times 2\pi rh$

$= 200 \times 2 \times \frac{22}{7} \times 60 \times 84$

$= 6336000 \text{ cm}^2$

$= \frac{6336000}{100 \times 100} m^2$

$\therefore$ A = 633.6 m^2

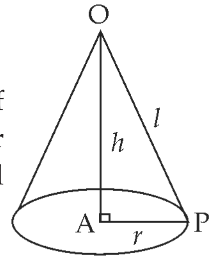
Cost of levelling = A × R

= 633.6 × 10 = ₹ 6336

$\therefore$ **Cost of levelling the ground is ₹ 6336.**

Points to Remember:

- **Right Circular Cone**



An ice-cream cone, a clown's hat, a funnel are examples of cones. A cone has one circular flat surface and one curved surface.

In the diagram alongside,

seg OA is the height of the cone denoted by *'h'*.

seg AP is the radius of the base denoted by *'r'*.

seg OP is the slant height of the cone denoted by *'l'*.

FORMULAE

(1) The h, r and l of a cone represents the sides of a right angled triangle where l is the hypotenuse.

$\therefore$ $l^2 = r^2 + h^2$

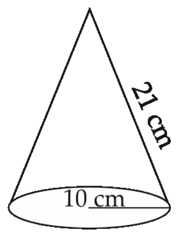
(2) Curved surface area of a right circular cone = πrl

(3) Total surface area of a right circular cone $= \pi r(r + l)$

(4) Volume of a right circular cone = $\frac{1}{3} \times \pi r^2 h$

Solve the following example (Texbook pg no. 141)

(1) The adjoining figure shows the measures of a joker's cap. How much cloth is needed to make such a cap?



Solution :

For the jokers cap,

radius (r) = 10 cm, slant height (l) = 21 cm

cloth required to make such cap

= curved surface area of cone

$= \pi rl$

$= \frac{22}{7} \times 10 \times 21$

= 660 cm [$\because$ 1 m = 100 cm]

= 6.6 m

$\therefore$ **6.6 m cloth is needed to make such a cap.**

Pracitce Set - 7.1 *(Textbook Page No. 145)*

(1) Find the volume of cone if the radius of its base is 1.5 cm and its perpendicular height is 5 cm. *(1 mark)*

Solution :

For the cone.

radius (r) = 1.5 cm, height (h) = 5 cm

$$\text{Volume (V)} = \frac{1}{3}\pi r^2 h$$

$$\therefore \quad = \frac{1}{3} \times 3.14 \times 1.5 \times 1.5 \times 5$$

$\therefore$ **V = 11.775 cm³**

$\therefore$ **Volume of the cone is 11.775 cm³**

(5) The dimensions of a cuboid are 44 cm, 21 cm, 12 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base. *(3 marks)*

Solution :

For the solid cuboid,

$l = 44$ cm, $b = 21$ cm, $h = 12$ cm

For the solid cone.

$h_1 = 24$ cm

$r = ?$

Cone is made by melting the cuboid.

$\therefore$ Volume of cone = volume of cuboid.

$$\frac{1}{3}\pi r^2 h_1 = l \times b \times h$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 44 \times 21 \times 12$$

$$\therefore \quad r^2 = \frac{44 \times 21 \times 12 \times 3 \times 7}{22 \times 24}$$

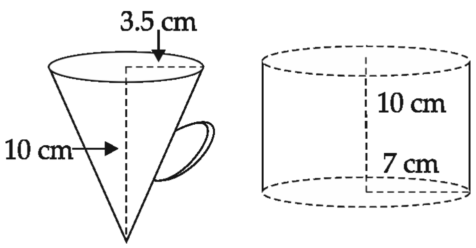
$$\therefore \quad r^2 = 21 \times 3 \times 7$$

$$\therefore \quad r^2 = 21 \times 21$$

$\therefore \quad r = 21$ cm ...(Taking square roots)

$\therefore$ **Radius of the cone is 21 cm**

(6) Observe the measures of pots in below figure How many jugs of water can the cylindrical pot hold. *(3 marks)*



Solution :

For the conical watering, $r = 3.5$ cm, $h = 10$ cm

For the cylindrical water pot, $r_1 = 7$ cm, $h_1 = 10$ cm

Let 'N' be the number of jugs required to fill the cylindrical pot completely.

$\therefore$ N × Volume of cone = Volume of cylinder

$$\therefore \quad N \times \frac{1}{3}\pi r^2 h = \pi r_1^2 h_1$$

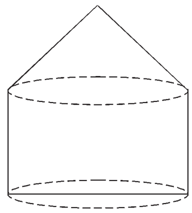
$$\therefore \quad N \times \frac{1}{3}\pi \times 3.5 \times 3.5 \times 10 = \pi \times 7 \times 7 \times 10$$

$$\therefore \quad N = \frac{\pi \times 7 \times 7 \times 10 \times 3}{\pi \times 3.5 \times 3.5 \times 10}$$

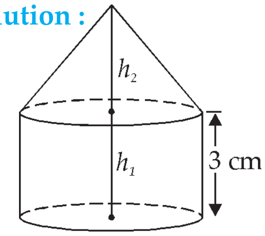
$\therefore$ N = 12

$\therefore$ **Number of conical jugs required to fill up cylindrical pot completely is 12.**

(7) A cylinder and a cone have equal bases. The height of the cylinder is 3 cm and the area of its base is 100cm². The cone is placed upon the cylinder. Volume of the solid figure so formed is 500 cm². Find the total height of figure. *(3 marks)*



Solution :



Let the radius of base of each part be r.

Height of the cylinder (h_1) = 3 cm.

Let the height of the cone be h_2

Area of the base = 100 cm²

$\therefore \quad \pi r^2 = 100 \text{ cm}^2$ ...(i)

Volume of the solid figure formed =

Volume of the cylinder + Volume of the cone

$$\therefore \quad 500 = \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$$

$$\therefore \quad 500 = \pi r^2 (h_1 + \frac{1}{3} h_2) \qquad \text{...[From (i)]}$$

$$\therefore \quad 500 = 100 \left(3 + \frac{h_2}{3}\right)$$

$$\therefore \quad \frac{500}{100} = 3 + \frac{h_2}{3}$$

$$\therefore \quad 5 = 3 + \frac{h_2}{3}$$

$$\therefore \quad 5 - 3 = \frac{h_2}{3}$$

$$\therefore \quad \frac{h_2}{3} = 2$$

$$\therefore \quad h_2 = 6$$

Total height = $h_1 + h_2$

= 3 + 6

Total height of the figure is 9 cm

Problem Set - 7 *(Textbook Pg No. 161)*

(7) A cylinder bucket of diameter 28 cm and height 20 cm was full of sand. When the sand in the bucket was poured on the ground, the sand got converted into a shape of a cone. If the height of

the cone was 14 cm, what was the base area of the cone? *(3 marks)*

Solution :

For the cylindrical bucket,

diameter = 28 cm

$\therefore$ Radius (r) = 14 cm.

height (h) = 20 cm

Volume of sand in the bucket =

Volume of the bucket $= \pi r^2 h$

For conical shape sand

height (h_1) = 14 cm

Let the radius be r_1

Sand from the bucket is emptied to form a cone.

$\therefore$ Volume of sand in the conical shape = Volume of the sand in the bucket

$$\therefore \quad \frac{1}{3}\pi r_1^2 h_1 = \pi r^2 h$$

$$\therefore \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{22}{7} \times 14 \times 14 \times 20$$

$$\therefore \quad \frac{1}{3} \times \pi r_1^2 \times 14 = \frac{22}{7} \times 14 \times 14 \times 20$$

$$\therefore \quad \pi r_1^2 = \frac{22}{7} \times 14 \times 14 \times 20 \times \frac{3}{14}$$

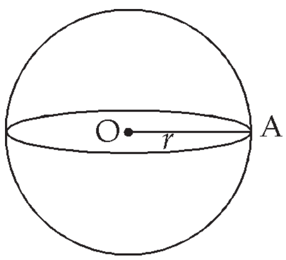
$$\pi r_1^2 = 2{,}640 \text{ sq.cm}$$

$\therefore$ **Area of base of the cone is 2640 cm^2**

Points to Remember:

- **Sphere**

The set of all points in the space which are at a fixed distance from a fixed point is called a sphere.



The fixed point is called the **centre** and the fixed distance is called the **Radius of the sphere.**

In the adjoining figure, point O is the centre of the sphere and seg OA is the radius of the sphere which is denoted as 'r'.

Since the entire surface of the sphere is curved, its area is called as curved surface area or simply surface area of the sphere.

Some common examples of a sphere are cricket ball, football, globe, spherical soap bubble etc.

FORMULAE

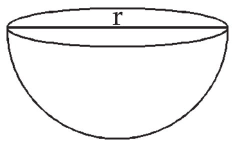
(1) Surface area (curved surface area) of a sphere $= 4\pi r^2$

(2) Volume of a sphere $= \frac{4}{3} \times \pi r^3$

- **Hemisphere**

Half of a sphere is called as hemisphere.

Any hemisphere is made up of a curved surface and a plane circular surface.



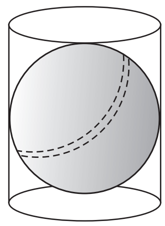
FORMULAE

(1) Curved surface area of a hemisphere $= 2\pi r^2$

(2) Total surface area of a solid hemisphere $= 3\pi r^2$

(3) Volume of a hemisphere $= \frac{2}{3} \times \pi r^3$

As shown in the adjacent figure, a sphere is placed in a cylinder. It touches the top, bottom and the curved surface of the cylinder. If radius of the base of the cylinder is 'r',



(1) What is the ratio of the radii of the sphere and the cylinder?

(2) What is the ratio of the curved surface area of the cylinder and the surface area of the sphere?

(3) What is the ratio of the volumes of the cylinder and the sphere?

Solution :

radius of the base of the cylinder $= r$

$\because$ ball touches the top, bottom and curved surface of the cylinder,

$$\left.\begin{array}{l}\text{radius of the sphere} = r \\ \text{height of cylinder} = \text{diameter of sphere} = 2r\end{array}\right\} \quad \text{...(i)}$$

(i) $\dfrac{\text{Radius of sphere}}{\text{Radius of cylinder}} = \dfrac{r}{r} = 1$ ...[From (i)]

(ii) Curved surface area of the cylinder

$= 2\,\pi r h$

$= 2\,\pi r\,(2r)$ ...[From (i)]

$= 4\,\pi r^2$

Also surface area of sphere $= 4\pi r^2$

$$\therefore \quad \frac{\text{Curved surface area of cylinder}}{\text{Surface area of sphere}} = \frac{4\pi r^2}{4\pi r^2} = 1$$

(iii) Volume of cylinder $= \pi r^2 h$

$= \pi r^2\,(2r)$ ...[From (i)]

$$= 2\,\pi r^3$$

Volume of sphere $= \dfrac{4}{3}\pi r^3$

$\therefore \dfrac{\text{Volume of cylinder}}{\text{Volume of sphere}} = \dfrac{2\pi r^3}{\frac{4}{3}\pi r^3} = \dfrac{3}{2}$

Practice Set - 7.1 *(Textbook Page No. 145)*

(2) Find the volume of a sphere with diameter 6 cm. *(1 mark)*

Solution :

For the sphere,

Diameter = 6 cm

$\therefore$ Radius (r) $= \dfrac{6}{2}$ cm = 3 cm

Volume of the sphere $= \dfrac{4}{3} \times \pi r^3$

$= \dfrac{4}{3} \times 3.14 \times 3 \times 3 \times 3$

$= 113.04 \text{ cm}^2$

$\therefore$ **Volume of the sphere 113. 04 cm³**

(4) Find the surface area of sphere of radius 7 cm. *(1 mark)*

Solution :

For the sphere,

Radius (r) = 7 cm

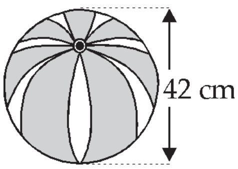
Cruved surface area of the sphere $= 4\pi r^2$

$= 4 \times \dfrac{22}{7} \times 7 \times 7$

$= 616 \text{ cm}^2$

$\therefore$ **Curved surface area of the sphere is 616 cm²**

(11) Find the surface area and the volume of a beach ball shown in the figure. *(3 marks)*



Solution :

For a spherical beach ball.

Diameter = 42 cm ...(Given)

$\therefore$ Radius (r) $= \dfrac{42}{2} = 21$ cm

Volume of the spherical beach ball $= \dfrac{4}{3} \times \pi r^3$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times 21 \times 21 \times 21$

$= 38{,}808 \text{ cm}^3$

$\therefore$ **Volume of the spherical beach ball is 38,808 cm³**

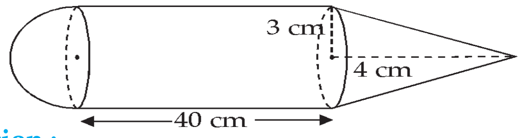
Surface area of the spherical beach ball $= 4\pi r^2$

$= 4 \times \dfrac{22}{7} \times 21 \times 21$

$= 5{,}544 \text{ cm}^3$

$\therefore$ **Surface area of the spherical beach ball is 5,544 cm²**

(8) In below figure, a toy made from a hemisphere, a cylinder and a cone is shown. Find the total area of the toy. *(4 marks)*



Solution :

Toy is made up of a cone, cylinder and hemisphere of equal radii.

For the conical part	For the cylindrical part	For hemisphere
Radius (r) = 3 cm	Radius (r) = 3 cm	Radius (r) = 3cm
Height (h) = 4 cm	height (h_1) = 40 cm	

Let the slant height of conical part be l.

$l^2 = r^2 + h^2$

$\therefore \quad = 3^2 + 4^2$

$= 9 + 16$

$\therefore l^2 = 25$

$\therefore l = 5$ cm (Taking square roots)

Total surface area of the toy =

Curved surface area of the hemisphere

+ Curved surface area of the cylinder

+ Curved surface area of the cone

$= 2\pi r^2 + 2\pi\, r\, h_1 + \pi\, r\, l$

$= \pi\, r(2r + 2h_1 + l)$

$= \dfrac{22}{7} \times 3 \times (3 \times 2 + 2 \times 40 + 5)$

$= \dfrac{22}{7} \times 3 \times (6 + 80 + 5)$

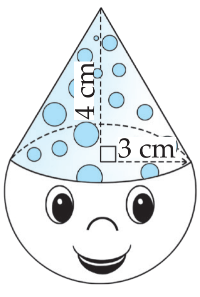
$= \dfrac{22}{7} \times 3 \times 91$

$= 22 \times 3 \times 13$

$= 858 \text{ cm}^2$

$\therefore$ **Total Surface area of the toy is 858 sq.cm**

(10)



The adjoining figure shows a toy. Its lower part is a hemisphere and the upper part is a cone. Find the volume and the surface area of the toy from the measures shown in the figure. (π = 3.14) *(4 marks)*

Solution :

For the conical part

Radius (r) = 3 cm

Height (h) = 4 cm

Let l be the slant height of conical part

$l^2 = r^2 + h^2$

$\therefore \quad = 3^2 + 4^2$

$= 9 + 16$

$\therefore \quad l^2 = 25$

$\therefore \quad l = 5$ cm (Taking square roots)

For the hemisphere

Radius (r) = 3 cm

Volume of the toy = Volume of the Cone + volume of the hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} \times 3.14 \times 3 \times 3\ (4 + 2 \times 3)$

$= 3.14 \times 3\ (4 + 6)$

$= 3.14 \times 3 \times 10$

$= 3.14 \times 30$

$= 94.2 \text{ cm}^3$

$\therefore$ **Volume of the toy is 94.2cm³**

Surface area of toy = Curved surface area of the cone + Curved surface area of the hemisphere

$= \pi r l + 2\pi r^2$

$= \pi r (l + 2r)$

$= 3.14 \times 3\ (5 + 2 \times 3)$

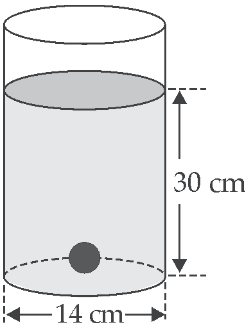
$= 3.14 \times 3\ (5 + 6)$

$= 3.14 \times 3 \times 11$

$= 3.14 \times 33$

$= 103.62 \text{ cm}^2$

$\therefore$ **Surface area of toy is 103.62 cm²**

(12) 

As shown in the figure, a cylindrical glass contains water. A metal sphere of diameter 2 cm is immersed in it water. Find the volume of water. *(3 marks)*

Solution :

For the cylindrical vessel

Diameter = 14 cm

Radius (r_1) = 7cm

Height of the water level (h_1) = 30 cm

(When sphere is immersed)

For the Sphere

Diameter = 2 cm

Radius(r_2) = 1 cm

Apparent volume of water when sphere is immersed in water (V_1) $= \pi r_1^2 h_1$

$= \frac{22}{7} \times 7 \times 7 \times 30$

$= 4620 \text{ cm}^3$

Volume of the Sphere (V_2) $= \frac{4}{3} \times \pi r_2^3$

$= \frac{4}{3} \times 3.14 \times 1 \times 1 \times 1$

$= 4.19 \text{ cm}^3$

Actual volume of water $= V_1 - V_2$

$= 4620 - 4.19$

$= 4615.81 \text{ cm}^3$

$\therefore$ **Actual volume of Water is 4615.81 cm³**

Problem Set - 7 *(Textbook Pg No. 161)*

(6) The diameter and thickness of a hollow metallic sphere are 12 cm and 0.01 m respectively. The density of the metal is 8.88 gm per cm³. Find the outer surface area and mass of the sphere. *(4 marks)*

Solution :

For the metallic hollow sphere,

Outer diameter $= 12$ cm

$\therefore$ Outer radius (r_1) $= \frac{12}{2} = 6$ cm

Thickness $= 0.01$ m

$= 0.01 \times 100$ ...[$\because$ 1 m = 100 cm]

$= 1$ cm

Inner radius (r_2) $= r_1 -$ thickness

$= 6 - 1$

$= 5$ cm

Outer surface area of the hollow sphere

$= 4\pi r_1^2$

$= 4 \times 3.14 \times 6 \times 6$

$= 452.16 \text{ cm}^2$

$\therefore$ **Outer surface area is 452.16 sq. cm.**

Volume of metal in the hollow metalic sphere

= Volume of the outer sphere − Volume of the inner sphere

$$= \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_2^3$$

$$= \frac{4}{3}\pi (r_1^3 - r_2^3)$$

$$= \frac{4}{3} \times \frac{22}{7} \times (6^3 - 5^3)$$

$$= \frac{4}{3} \times \frac{22}{7} \times (216 - 125)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 91$$

$$= \frac{4}{3} \times 22 \times 13$$

$$V = \frac{1144}{3} \text{cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \qquad \text{...(Formula)}$$

$\therefore$ Mass = Density × Volume

Mass of the hollow sphere =

Volume of the hollow sphere × Density of Sphere

$$V = \frac{1144}{3} \times 8.88$$

$$= 1144 \times 2.96$$

$$= 3386.24 \text{ gm}$$

$\therefore$ **Mass of the hollow sphere is 3386.24 gm**

(8) The radius of a metallic sphere is 9 cm. It was melted to make a wire of diameter 4 mm. Find the length of the wire. *(3 marks)*

Solution :

For the sphere, $r = 9$ cm

For the wire, Thickness (diameter) = 4 mm

$\therefore$ Radius $(r_1) = \frac{4}{2}$ mm = 2 mm = $\frac{2}{10}$ cm ...[1 cm = 10 mm]

Let the length of wire be h_1

Wire is made by melting the sphere,

Volume of the wire = Volume of the sphere

$$\therefore \pi r_1^2 h_1 = \frac{4}{3} \times \pi r^3$$

$$\therefore \pi \times \frac{2}{10} \times \frac{2}{10} \times h_1 = \frac{4}{3} \pi \times 9 \times 9 \times 9$$

$$\therefore h_1 = \frac{4 \times \pi \times 9 \times 9 \times 9 \times 10 \times 10}{3 \times \pi \times 2 \times 2}$$

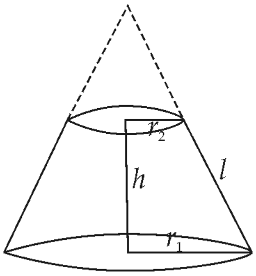
$$\therefore h_1 = 24{,}300 \text{ cm}$$

$$\therefore h_1 = 243 \text{ m} \qquad \text{...}[\because 1 \text{ m} = 100 \text{ cm}]$$

$\therefore$ **Length of the wire formed is 243 m.**

Points to Remember:

- **Frustum Of The Cone :**



If the cone is cut off by a plane parallel to the base not passing through the vertex, two parts are formed as

(i) cone (a part towards the vertex)

(ii) frustum of cone (the part left over on the other side i.e. towards base of the original cone)

FORMULAE

(1) Slant height (l) of the frustum = $\sqrt{h^2 + (r_1 - r_2)^2}$

(2) Curved surface area = $\pi (r_1 + r_2) l$

(3) Total surface area of the frustum = $\pi (r_1 + r_2) l + \pi r_1^2 + \pi r_2^2$

(4) Volume of the frustum = $\frac{1}{3} \pi (r_1^2 + r_2^2 + r_1 \times r_2) h$

Practice Set - 7.2 *(Textbook Page No. 148)*

(1) The radii of two circular ends of frustum shape bucket are 14 cm and 7 cm. Height of the bucket is 30 cm. How many litres of water it can hold? (1 litre = 1000 cm³) *(3 marks)*

Solution :

For a frustum shaped bucket,

radius of bigger circle (r_1) = 14 cm

radius of smaller circle (r_2) = 7 cm

height (h) = 30 cm

Capacity of the bucket = Volume of the bucket

$$= \frac{1}{3} \pi h \ (r_1^2 + r_2^2 + r_1 \times r_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 30\ (14^2 + 7^2 + 14 \times 7)$$

$$= \frac{22}{7} \times 10 \times (196 + 49 + 98)$$

$$= \frac{22}{7} \times 10 \times 343$$

$$= 22 \times 10 \times 49$$

$$= 10{,}780 \text{ cm}^3$$

$$= \frac{10780}{1000} \qquad \text{...}(\because 1 \text{ litre} = 1000 \text{ cm}^3)$$

$$= 10.78 \text{ litres}$$

$\therefore$ **Capacity of the bucket is 10.780 litres**

(2) The radii of ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its (i) curved surface area (ii) Total surface area (iii) Volume ($\pi = 3.14$). *(4 marks)*

Solution :

For a frustum

radius of bigger circle (r_1) = 14 cm

radius of smaller circle (r_2) = 6 cm

height (h) = 6 cm

Slant height of the frustum (l) = $\sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{6^2 + (14-6)^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

= 10 cm ...(Taking square roots)

(i) Curved surface area of the frustum = $\pi (r_1 + r_2)\, l$

$= 3.14 \times (14 + 6) \times 10$

$= 3.14 \times 20 \times 10$

$= 628 \text{ cm}^2$

Curved surface area of the frustum is 628 cm²

(ii) Total surface area of the frustum

$= \pi (r_1 + r_2)\, l + \pi r_1^2 + \pi r_2^2$

$= 628 + \pi (14^2 + 6^2)$

$= 628 + \pi (196 + 36)$

$= 628 + 3.14 \times 232$

$= 628 + 728.48$

$= 1,356.48 \text{ cm}^2$

Total surface area of the frustum is 1356.48 cm²

(iii) Volume of the frustum = $\frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 \times r_2)$

$= \frac{1}{3} \times 3.14 \times 6 \,(14^2 + 6^2 + 14 \times 6)$

$= 3.14 \times 2\, (196 + 36 + 84)$

$= 3.14 \times 2 \times 316$

$= 1,984.48 \text{ cm}^3$

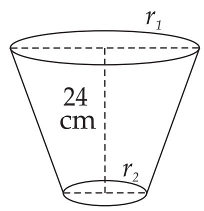
$\therefore$ **Volume of the frustum is 1,984.48 cm³**

(3) The circumferences of circular faces of a frustum are 132 cm and 88 cm and its height is 24 cm. To find curved surface area of frustum complete the following activity. ($\pi = \frac{22}{7}$) *(3 marks)*

Solution :

For the frustum

Circumference$_1$ = $2 \pi r_1 = 132$



$\therefore$ $r_1 = \frac{132}{2\pi} = \boxed{21 \text{ cm}}$

Circumference$_2$ = $2 \pi r_2 = 88$

$\therefore$ $r_2 = \frac{88}{2\pi} = \boxed{14 \text{ cm}}$

Slant height of the frustum = (l) = $\sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(24)^2 + (21-14)^2}$

$= \sqrt{(24)^2 + (7)^2}$

$l = 25$ cm

Curved surface area of frustum = $\pi (r_1 + r_2)\, l$

$= \pi \times \boxed{35} \times \boxed{25}$

$= \boxed{2,750}$ sq. cm.

Problem Set - 7 *(Textbook Pg No. 161)*

(2) A washing tub in the shape of a frustum of a cone has height 21 cm. The radii of the circular top and bottom are 20 cm and 15 cm respectively. What is the capacity of the tub? ($\pi = \frac{22}{7}$) *(3 marks)*

Solution :

For frustum shaped tub,

$r_1 = 20$ cm, $r_2 = 15$ cm, $h = 21$ cm

Quantity of water that can be contained in the tub

= Inner volume of tub

$= \frac{1}{3} \pi \times h (r_1^2 + r_2^2 + r_1 \times r_2)$

$= \frac{1}{3} \times \frac{22}{7} \times 21\, (20^2 + 15^2 + 20 \times 15)$

$= 22\, (400 + 225 + 300)$

$= 22 \times 925$

$= 20,350 \text{ cm}^3$

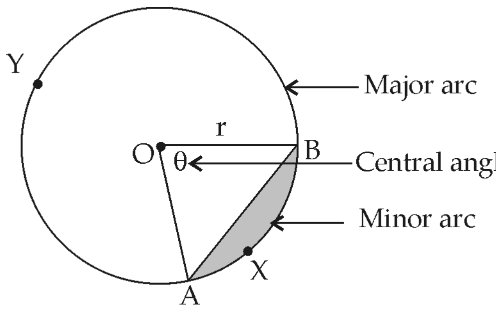
$= \frac{20350}{1000}$ litres [$\because$ 1 litres = 1000 cm]

= 20.35 litres

$\therefore$ **Quantity of water that can be contained in the tub is 20.35 litres**

Points to Remember:

Now we will study how to find some measurements related to circle



- **Circle : Arc, Sector, Segment**

Area of sector :

Sector of a circle is the part of the circle enclosed by two radii of the circle and their intercepted arc. (i.e. arc between the two ends of radii)

Area of the sector (A) = $\dfrac{\theta}{360} \times \pi r^2$

Length of an arc :

Length of an arc of a circle (arc length) is the distance along the curved line making up the arc.

Length of the arc (l) = $\dfrac{\theta}{360} \times 2\pi r$

Relation between the area of the sector and the length of an arc :

Activity I : (Textbook page no. 154)

Area of a sector, (A) $= \dfrac{\theta}{360} \times \pi r^2$ ...(i)

Length of the arc, (l) $= \dfrac{\theta}{360} \times 2\pi r$

$\therefore \quad = \dfrac{\theta}{360} = \dfrac{l}{2\pi r}$ ...(ii)

$\therefore$ A $= \dfrac{l}{2\pi r} \times \pi r^2$ ...[From (i) and (ii)]

$\therefore$ A $= \dfrac{l}{2} r = \dfrac{lr}{2}$

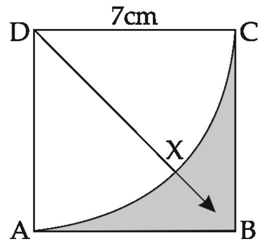
$\therefore$ Area of the sector = $\dfrac{\text{Length of the arc} \times \text{Radius}}{2}$

Similarity $= \dfrac{A}{\pi r^2} = \dfrac{l}{2\pi r} = \dfrac{\theta}{360}$

Area of circle = A (minor sector O – AXB)
+ A (major sector O – AYB)

Activity II : (Textbook page no. 154)

In the figure given, side of square ABCD is 7 cm with centre D and radius DA. Sector D - AXC is drawn. Fill in the following boxes propertly and find out the area of the shaded region.



Solution :

Area of a square $= \boxed{(\text{side})^2}$

$= \boxed{7^2}$

$= 49 \text{ cm}^2$

Area of sector (D – AXC) $= \boxed{\dfrac{\theta}{360} \times \pi r^2}$

$= \dfrac{\boxed{90}}{360} \times \dfrac{22}{7} \times 7^2$

$= 38.5 \text{ cm}^2$

A (shaded region) = A $\boxed{\square \text{ ABCD}}$ – A $\boxed{\text{sector (D - AXC)}}$

$= \boxed{49} \text{ cm}^2 - \boxed{38.5} \text{ cm}^2$

$= \boxed{10.5} \text{ cm}^2$

Practice Set - 7.3 *(Textbook Page No. 154)*

(1) Radius of a circle is 10 cm. Measure of an arc of the circles 54°. Find the area of the sector associated with the arc. ($\pi = 3.14$) *(2 marks)*

Solution :

For the sector, $r = 10$ cm, $\theta = 54°$

Area of the sector $= \dfrac{\theta}{360} \times \pi r^2$

$= \dfrac{54}{360} \times 3.14 \times 10 \times 10$

$= \dfrac{3}{20} \times 314$

$= \dfrac{942}{20}$

$= 47.1 \text{ cm}^2$

$\therefore$ **Area of the sector is 47.1 cm²**

(2) Measure of an arc of a circle is 80 cm and its radius is 18 cm. Find the length of the arc ($\pi = 3.14$) *(2 marks)*

Solution :

For a sector, r = 18 cm, $\theta = 80$

Length of an arc $= \dfrac{\theta}{360} \times 2\pi r$

$= \dfrac{80}{360} \times 2 \times 3.14 \times 18$

$= 3.14 \times 8$

$= 25.12$ cm

$\therefore$ **Length of the arc is 25.12 cm**

(3) Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector. *(1 mark)*

Solution :

For the sector, $r = 3.5$ cm, length of arc $(l) = 2.2$ cm

Area of the sector $= l \times \frac{r}{2}$

$= 2.2 \times \frac{3.5}{2}$

$= 3.85 \text{ cm}^2$

$\therefore$ **Area of the sector is 3.85 cm²**

(4) Radius of a circle is 10 cm. Area of a sector of the circle is 100 cm². Find the area its corresponding major sector. ($\pi = 3.14$) *(2 marks)*

Solution :

For the circle, $r = 10$ cm

Area of minor sector = 100 cm²

Area of the circle = πr^2

$= 3.14 \times 10 \times 10$

$= 314 \text{ cm}^2$

Area of a major circle =

Area of the circle – Area of corresponding minor sector

$= 314 - 100$

$= 214 \text{ cm}^2$

$\therefore$ **Area of the major sector is = 214 cm²**

(5) Area of a sector of a circle of radius 15 cm is 30 cm². Find the length of the arc of the sector *(2 marks)*

Solution :

For the circle, $r = 15$ cm

Area of the sector = 30 cm²

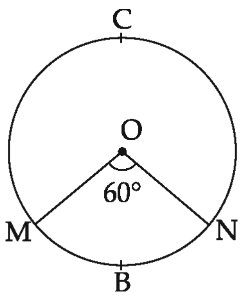
Area of the sector = Length of arc $\times \frac{r}{2}$

$\therefore$ 30 = Length of arc $\times \frac{15}{2}$

$\therefore$ Length of arc $= \frac{30 \times 2}{15}$

$\therefore$ Length of arc = 4 cm

$\therefore$ **Length of the arc is 4 cm**

(6) 

In the adjoining figure, the radius of the circle is 7 cm and m (arc MBN) = 60. Find (i) Area of the circle (ii) A (O – MBN) (iii) A (O – MCN) *(3 marks)*

Solution :

For the circle, r = 7 cm

m(arc MBN) = $\theta = 60°$

(i) Area of the circle $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7$

$= 154 \text{ cm}^2$

Area of the circle is 154 cm²

(ii) A (sector O – MBN) $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times 154$

$= \frac{1}{6} \times 154$

$= 25.67 \text{ cm}^2$

A (sector O – MBN) is 25.67 cm²

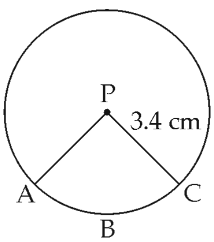
(iii) A (sector O – MCN) =

Area of the circle – A (sector O – MBN)

$= 154 - 25.67$

$= 128.33 \text{ cm}^2$

A (sector O – MCN) is 128.33 cm²

(7) 

In the adjoining figure, radius of circle is 3.4 cm and perimeter of sector P–ABC is 12.8 cm. Find A(P–ABC). *(2 marks)*

Solution :

For the circle, $r = 3.4$ cm

perimeter of sector P–ABC = 12.8 cm

P(P–ABC) = Length of arc $(l) + r + r$

$\therefore$ 12.8 $= l + 3.4 + 3.4$

$\therefore$ 12.8 – 6.8 $= l$

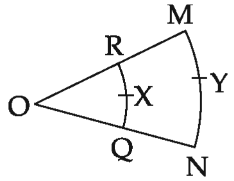
$\therefore$ l = 6 cm

Area of the sector $= l \times \frac{r}{2}$

$= 6 \times \frac{3.4}{2} \times 7 \times 7$

Area of the sector = 10.2 cm²

Area of the sector is 10.2 cm²

(8) 

In the adjoining figure, 'O' is centre of arcs. ∠ROQ = ∠MON = 60°, OR = 7 cm, OM = 21 cm. Find the lengths of arc RXQ and arc MYN. $\left(\pi = \frac{22}{7}\right)$ *(3 marks)*

Solution :

(i) For arc RXQ, θ = ∠ROQ = 60°

OR (r) = 7 cm

Length of arc RXQ $= \frac{\theta}{360} \times 2\pi r$

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 7$

$= 7.33$ cm

Length of arc RXQ is 7.33 cm

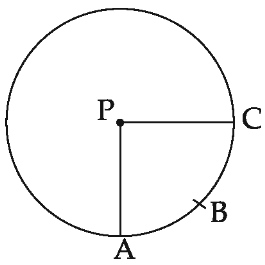
(ii) For arc MYN, OM (r) = 21 cm, $\theta = \angle MON = 60°$

Length of arc MYN $= \frac{\theta}{360} \times 2\pi r$

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$

$= 22$ cm

Length of arc (MYN) is 22 cm

(9) 

In the adjoining figure A (P – ABC) = 154 cm² and radius of the circle is 14 cm. Find (i) ∠APC (ii) *l* (arc ABC). *(3 marks)*

Solution :

Region P - ABC is a sector

A (P - ABC) $= \frac{\theta}{360} \times \pi r^2$

$\therefore 154 = \frac{\theta}{360} \times \frac{22}{7} \times 14 \times 14$

$\therefore \frac{154 \times 360 \times 7}{22 \times 14 \times 14} = \theta$

$\therefore \theta = 90°$

$\therefore$ **∠APC = 90°**

length of arc ABC $= \frac{\theta}{360} \times 2\pi r$

$= \frac{90}{360} \times 2 \times \frac{22}{7} \times 14$

length of arc ABC = 22 cm

$\therefore$ ***l* (arc ABC) is 22 cm**

(10) Radius of a sector of a circle is 7 cm. If measure of arc of the sector is (1) 30° (2) 210° (3) three right angles; find the area of the sector in each case *(3 marks)*

Solution :

For the circle, r = 7 cm

(i) For the sector, θ = 30°

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

$= 12.83$

$\therefore$ **Area of the sector is 12.83 cm²**

(ii) For the sector, θ = 210°

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{210}{360} \times \frac{22}{7} \times 7 \times 7$

$= 89.83$

$\therefore$ **Area of sector is 89.83 cm²**

(iii) For the sector, θ = 3 right angles = 3 × 90° = 270°

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{270}{360} \times \frac{22}{7} \times 7 \times 7$

$= 115.5$

$\therefore$ **Area of the sector is 115.50 cm²**

(11) The area of a minor sector of a circle is 3.85 cm² and the measure of its central angle is 36°. Find the radius of the circle. *(2 marks)*

Solution :

For the sector, Area = 3.85 cm², θ = 36°

$A = \frac{\theta}{360} \times \pi r^2$

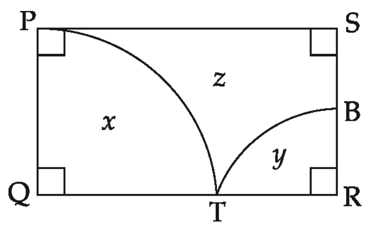
$\therefore 3.85 = \frac{36}{360} \times \frac{22}{7} \times r^2$

$\therefore \frac{3.85 \times 360 \times 7}{36 \times 22} = r^2$

$\therefore r^2 = 12.25$

$\therefore r = 3.5$ cm

$\therefore$ **Radius of the circle is 3.5 cm**

(12) 

In the adjoining figure, □PQRS is a rectangle. PQ = 14 cm, QR = 21cm, find the areas of the parts *x*, *y* and *z*. *(4 marks)*

Solution :

□PQRS is a rectangle with l = 21cm and b = 14cm

A (□PQRS) $= l \times b$

$= 21 \times 14$

A (□PQRS) = 294 cm² ...(i)

For region x ie. for sector Q - PT,

r = 14 cm, θ = 90°

A (region x) = A (sector Q - PT)

$= \frac{\theta}{360} \times \pi r^2$

$$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14$$

$$= 154 \text{ cm}^2$$

A (region x) is 154 cm² ...(ii)

QP = QT ...(Radii of the same circle)

$\therefore$ QT = 14 cm ...(iii)

QR = QT + RT ...(Q – T – R)

$\therefore$ 21 = 14 + RT ...(From (iii) and given)

$\therefore$ RT = 21 – 14 = 7 cm ...(iv)

For region y i.e. for (sector R – B T)

$r_1 = 7$cm, $\theta = 90°$

A (region y) = A (sector R – B T)

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= 38.5 \text{ cm}^2$$

A (region y) is 38.5 cm²

A (rectangle PQRS) =

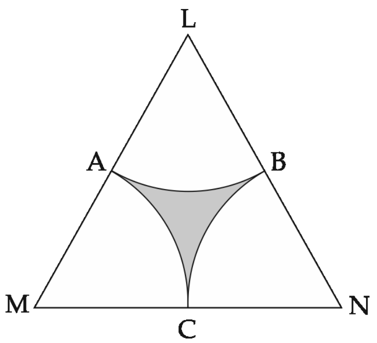
A (region x) + A (region y) + A (region z) [Area addition property]

294 = 154 + A (region z) + 38.5

A (region z) = 294 – 154 – 38.5 [From (i), (ii) and (iv)]

$= 101.5 \text{ cm}^2$

A (region z) is 101.5 cm²

(13) 

ΔLMN is an equilateral triangle. LM = 14 cm. As shown in the figure, three sectors are drawn with vertices as centre and radius 7 cm. Find,

(i) A (ΔLMN) (ii) Area of any one of the sectors. (iii) Total area of all the three sectors (iv) Area of the shaded region. *(4 marks)*

Solution :

(i) ΔLMN is an equilateral triangle with side 14 cm

$$\text{A } (\Delta\text{LMN}) = \frac{\sqrt{3}}{4} \times \text{side}^2$$

$$= \frac{1.73}{4} \times 14 \times 14$$

$$= 87.77 \text{ cm}^2$$

A (ΔLMN) is 84.77 cm² ...(i)

(ii) For sector L – AB, r = 7 cm

θ = 60° (Angle of an equilateral triangle)

$$\text{A (sector L – AB)} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{77}{3}$$

$\therefore$ A (sector L – AB) = 25.67 cm²

$\therefore$ **Area of a sector is 25.67 cm²**

(iii) For all three sectors, radii and central angles are equal

$\therefore$ Area of all sectors are equal.

$\therefore$ Total of areas of all three sectors =

3 × A (sector A – LB)

$$= 3 \times \frac{77}{3}$$

$$= 77 \text{ cm}^2$$

$\therefore$ Total areas of all three sectors = 77 cm²

(iv) Area of the shaded region =

A (ΔLMN) – Area of three sectors

= 84.77 – 77 ...[From (i) and (ii)]

= 7.77 cm²

$\therefore$ **Area of the shaded region is 7.77 cm²**

Problem Set - 7 *(Textbook Pg No. 161)*

(9) The area of a sector of a circle of 6 cm radius is 15π sq. cm. Find the measure of the arc and length of the arc corresponding to the sector. *(3 marks)*

Solution :

For the sector,

Radius (r) = 6 cm

Area of the sector = 15π cm²

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$

$$\therefore \quad 15\pi = \frac{\theta}{360} \times \pi \times 6 \times 6$$

$$\therefore \quad \frac{15\pi \times 360}{\pi \times 6 \times 6} = \theta$$

$$\therefore \quad \theta = 150$$

$\therefore$ Measure of the arc is 150°

Area of the sector =

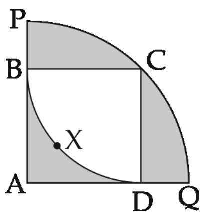
Length of corresponding arc (l) $\times \frac{r}{2}$

$\therefore \quad 15\pi = l \times \frac{6}{2}$

$\therefore \quad \frac{15\pi \times 2}{6} = l$

$\therefore \quad 5\pi = l$

Length of the corresponding arc is 5π cm

(11) 

In the adjoining figure, square ABCD is inscribed in the sector A-PCQ. The radius of sector C-BXD is 20 cm. Complete the following activity to find the area of shaded region. *(4 marks)*

Solution :

Side of square ABCD = Radius of sector

C - BXD = $\boxed{20}$ cm

Area of square = (side)^2 = $\boxed{20}^2 = \boxed{400 \text{ sq cm}}$ ...(i)

Area of shaded region inside the square

$= $ A (square ABCD) – A (sector C – BXD)

$= \boxed{400} - \frac{\theta}{360} \times \pi r^2$

$= \boxed{400} - \frac{90}{360} \times \frac{3.14}{1} \times \frac{400}{1}$

$= \boxed{400} - 314$

$= \boxed{86}$ sq cm

Radius of bigger sector = Length of diagonal of square ABCD

$= 20\sqrt{2}$ cm

Area of shaded portion outside square within bigger sector

$=$ A (A – PCQ) – A ($\square$ABCD)

$= \frac{\theta}{360} \times \pi r^2 - \boxed{\text{side}}^2$

$= \frac{90}{360} \times 3.14 \times (20\sqrt{2})^2 - (20)^2$

$= \boxed{628} - \boxed{400}$

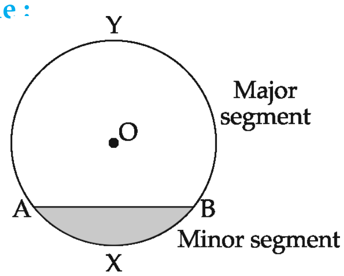
$= 228$

$\therefore$ **Total Area of the shaded region = 86 + 228 = 314 sq cm**

Points to Remember:

- **Segment of a circle :**

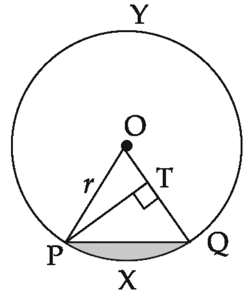
A segment of a circle is the region bounded by a chord and an arc.



Minor segment: The area enclosed by a chord and its corresponding minor arc is called a minor segment. In the figure, segment AXB is a minor segment.

Major segment: The area enclosed by a chord and its corresponding minor arc is called a major segment. In the figure, segment AYB is a major segment.

- **Area of a segment :**



A (segment PXQ) = A (O–PXQ) – A(ΔOPQ)

$= \frac{\theta}{360} \times \pi r^2 - A(\Delta OPQ)$ ...(i)

Seg PT $\perp$ radius OQ,

In ΔOTP, $\sin\theta = \frac{PT}{OP}$

$\therefore \quad PT = OP \sin\theta$

$\therefore \quad PT = r \times \sin\theta$ ($\because OP = r$)

$A(\Delta OPQ) = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times OQ \times PT$

$= \frac{1}{2} \times r \times r \sin\theta$

$= \frac{1}{2} \times r^2 \sin\theta$ ...(ii)

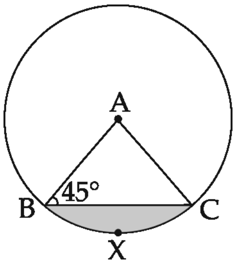
$\therefore$ A(ssegment PXQ) $= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$...[From (i) and (ii)]

$= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$

Also,

Area of circle = A (minor seg PXQ) + A (major seg PRQ)

Practice Set - 7.4 *(Textbook Page No. 159)*

(1) 

In the adjoining figure, is the centre of the circle. ∠ABC 45° and AC = 7$\sqrt{2}$ cm. Find the area of segment BXC. (π = 3.14, $\sqrt{2}$ = 1.41) *(3 marks)*

Solution :

In ΔABC, AB = AC ...(Radii of same circle)

∴ ∠ABC = ∠ACB ...(Isosceles triangle theorem)

But ∠ABC = 45° ...(given)

∴ ∠ACB = 45° ...(i)

In ΔABC, ∠ACB = ∠ABC = 45° ...[From (i) and given]

∴ ∠BAC = 90° ...(Remaining angle of ΔABC)

For segment BXC, θ = 90°, $r = 7\sqrt{2}$ cm

Arc of segment BXC) = $r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$

$$= \left(7\sqrt{2}\right)^2\left[\frac{3.14\times 90}{360} - \frac{\sin\theta}{2}\right]$$

$$= \left(7\sqrt{2}\right)^2\left[\frac{3.14\times 90}{360} - \frac{\sin 90}{2}\right]$$

$$= 98 \times \left[\frac{1.57}{2} - \frac{1}{2}\right]$$

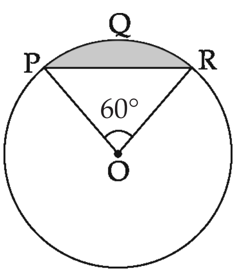
$$= 98 \times \left[\frac{1.57-1}{2}\right]$$

$$= 98 \times \frac{0.57}{2}$$

$$= 49 \times 0.57$$

$$= 27.93$$

∴ **Arc of segment BXC is 27.93 sq cm**

(2) 

In the adjoining figure, point 'O' is the centre of the circle, m(acr PQR) = 60°, OP = 10cm. Find the area of the shaded portion. (π = 3.14, $\sqrt{3}$ = 1.73) *(3 marks)*

Solution :

m (arc PQR) = m ∠POR ...(Definition of measure of minor arc)

m∠PQR = 60° ...(i)

For segment PQR, r = OP = 10 cm

θ = m ∠POR = 60° [From (i)]

Area of shaded portion = A (segment PQR)

$$= r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$$

$$= 10 \times 10\left[\frac{3.14\times 60}{360} - \frac{\sin 60}{2}\right]$$

$$= 100\left[\frac{3.14}{6} - \frac{\sqrt{3}}{2\times 2}\right]$$

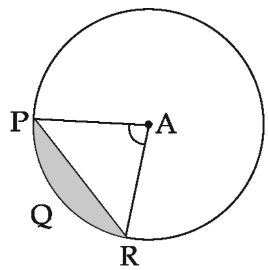
$$= 100\left[\frac{3.14\times 2}{6\times 2} - \frac{1.73\times 3}{4\times 3}\right]$$

$$= 100\left[\frac{6.28-5.19}{12}\right]$$

$$= 100 \times \frac{1.09}{12}$$

$$= 9.08 \text{ cm}^2$$

∴ **Area of shaded potion = 9.08 cm²**

(3) 

In the adjoining figure, if A is the centre of the circle. ∠PAR = 30° AP = 7.5, find the area of segment PQR. (π = 3.14) *(3 marks)*

Solution :

For segment PQR, r = AP = 7.5 units

θ = ∠PAR = 30°

A (segment PQR) = $r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$

$$= (7.5)^2\left[\frac{3.14\times 30}{360} - \frac{\sin 30}{2}\right]$$

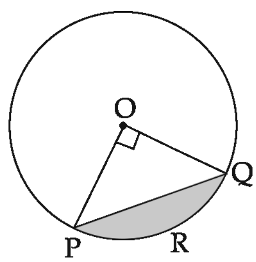
$$= 56.25\left[\frac{3.14}{12} - \frac{1}{2\times 2}\right]$$

$$= 56.25\left[\frac{3.14-3}{12}\right]$$

$$= 56.25 \times \frac{0.14}{12}$$

$$= 0.66$$

∴ **A (segment PQR) is 0.66 sq. units**

(4) 

In the adjoining figure, if O is the centre of the circle, PQ is a chord. ∠POQ = 90°, area of shaded region is 114 cm², find the radius of the circle (π = 3.14) *(3 marks)*

Solution :

For segment PRQ, $\theta = \angle POQ = 90°$

A (segment PRQ) $= 114 \text{ cm}^2$

A (segment PQR) $= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$

$\therefore \quad 114 = r^2 \left[\frac{3.14 \times 90}{360} - \frac{\sin 90}{2}\right]$

$\therefore \quad 114 = r^2 \left[\frac{3.14}{4} - \frac{1}{2}\right]$

$\therefore \quad 114 = r^2 \left[\frac{1.57 - 1}{2}\right]$

$\therefore \quad 114 = r^2 \left[\frac{0.57}{2}\right]$

$\therefore \quad \frac{114 \times 2}{0.57} = r^2$

$\therefore \quad r^2 = \frac{114 \times 2 \times 100}{57}$

$\therefore \quad r^2 = 2 \times 2 \times 10 \times 10$

$\therefore \quad r = 20$ ...(Taking square roots)

$\therefore$ **Radius of the circle is 20 cm**

(5) A chord PQ of a circle with radius 15 cm subtends an angle of 60° with the centre of the circle. Find the area of the minor as well as the major segment. ($\pi = 3.14$, $\sqrt{3} = 1.73$) *(4 marks)*

Solution :

For minor segment, $r = 15$ cm and $\theta = 60°$

Area of minor segment $= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$

$= 15 \times 15 \left[\frac{3.14 \times 60}{360} - \frac{\sin 60}{2}\right]$

$= 225 \times \left[\frac{3.14}{6} - \frac{\sqrt{3}}{2 \times 2}\right]$

$= 225 \times \left[\frac{3.14 \times 2}{6 \times 2} - \frac{1.73 \times 3}{4 \times 3}\right]$

$= 225 \times \left[\frac{6.28 - 5.19}{12}\right]$

$= 225 \times \frac{1.09}{12}$

$= 20.44$

$\therefore$ **Area of minor segment is 20.44 cm²**

Area of circle $= \pi r^2$

$= 3.14 \times 15 \times 15$

$= 706.5 \text{ cm}^2$

$\therefore$ **Area of the circle is 706.5 cm²**

Area of major segment =

Area of circle – Area of minor segment

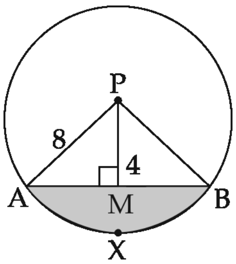
$= 706.5 - 20.44$

$= 686.06 \text{ cm}^2$

$\therefore$ **Area of the major segment is 686.06 cm²**

Problem Set - 7 *(Textbook Pg No. 161)*

(10)



In the adjoining figure, seg AB is a chord of a circle with centre P. If PA = 8 cm and distance of chord AB from the centre P is 4 cm, find the area of the shaded portion. ($\pi = 3.14$, $\sqrt{3} = 1.73$) *(4marks)*

Solution :

Radius of the circle is = 8 cm ...(i) (Given)

i.e. PA = 8 cm

seg PM ⊥ chord AB, A – M – B ...(ii)

Distance of the chord AB from the centre P is 4 cm

i.e. PM = 4 cm ...(iii)

In ΔPMA, $\angle PMA = 90°$ ...[From (ii)]

$PM = \frac{1}{2} PA$ ...[From (i) and (iii)]

$\therefore \angle PAM = 30°$ ...(iv) (Converse of 30° – 60° – 90° theorem)

In ΔPMA, $\angle PMA = 90°$ ...[From (ii)]

$\angle PAM = 30°$ ...[From (iv)]

$\angle APM = 60°$ ...(v) (Remaining angle of triangle)

Similarly, we can prove. $\angle BPM = 60°$ ...(vi)

$\angle APB = \angle APM + \angle BPM$ ...(Angle addition property)

$\therefore \angle APB = 60° + 60°$ ...[From (v) and (vi)]

$\therefore \angle APB = 120°$ ...(vii)

For sector (P –AXB)

$\theta = \angle APB = 120°$

r = 8 cm

Area (sector P – AXB) $= \frac{\theta}{360} \times \pi r^2$

$\therefore$ Area (sector P – AXB) $= \frac{120}{360} \times 3.14 \times 8 \times 8$

Area (sector P – AXB) = 66.99 sq cm ...(viii)

ΔPMA is 30° – 60° – 90° triangle ...[From (ii), (iv) and (v)]

$AM = \frac{\sqrt{3}}{2} \times PA$ ...(Side opposite to 30°)

$AM = \frac{\sqrt{3}}{2} \times 8 = 4\sqrt{3}$ cm ...(ix)

seg PM $\perp$ chord AB [From (i)]

$\therefore$ AB = 2AM (Perpendicular drawn from the centre to the chord bisects the chord)

$AB = 2 \times 4\sqrt{3}$ cm ...[From (ix)]

$AB = 8\sqrt{3}$ cm ...(x)

$A(\Delta PAB) = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times AB \times PM$

$= \frac{1}{2} \times 8\sqrt{3} \times 4$

$= 16\sqrt{3}$

$= 16 \times 1.73$

$A(\Delta PAB) = 27.68$ sq cm ...(xi)

A (sector P – AXB) =

A (ΔPAB) + A (segment AXB) ...(Area addition property)

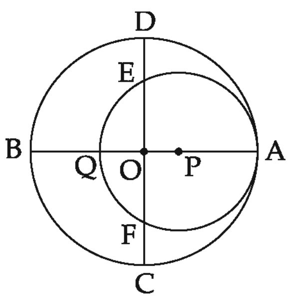
$\therefore$ 66.99 = 27.68 + A (segment AXB) ...[From (viii) and (xi)]

$\therefore$ A (segment AXB) = 66.99 – 27.68

A (segment AXB) = 39.28 sq. cm

Area of shaded portion = A (seg PXB) = 39.28 sq. cm

$\therefore$ **Area of the shaded portion is 39.28 sq cm.**

(12) 

In the adjoining figure, two circles with centres O and P are touching internally at point A. If BQ = 9, DE = 5, complete the following activity to find the radii of the circles. *(4 marks)*

Solution :

Let the radius of the bigger circle be R and the radius of the smaller circle be r

OA, OB, OC and OD are radii of the bigger circle.

$\therefore$ OA = OB = OC = OD = R

PQ = PA = r

OQ = OB – BQ = $\boxed{R - 9}$

OE = OD – DE = $\boxed{R - 5}$

As the chords QA and EF of the circle with centre P intersect in the interior of the circle, so by the property of internal division of two chords of a circle,

OQ × OA = OE × OF ...($\because$ OE = OF)

$\boxed{R - 9} \times R = \boxed{R - 5} \times \boxed{R - 5}$

$R^2 - 9R = R^2 - 10R + 25$

$-9R = -10R + 25$

$-9R + 10R = 25$

$\therefore$ $\boxed{R = 25 \text{ units}}$

AQ = 2*r* = AB – BQ

$\therefore$ 2*r* = 50 – 9 = 41

$\therefore$ $r = \frac{41}{2} = \boxed{r = 20.5}$

Problem Set - 7 *(Textbook Pg No. 161)*

MCQ's

Choose the correct alternative answer for each of the following questions. *(1 mark each)*

(1) **The ratio of circumference and area of a circle is 2 : 7. Find its circumference.**

(A) 14π (B) $\frac{7}{\pi}$ (C) 7π (D) $\frac{14}{\pi}$

(2) **If measure of an arc of circle is 160° and its length is 44 cm, find the circumference of the circle.**

(A) 66 cm (B) 44 cm (C) 160 cm (D) 99 cm

(3) **Find the perimeter of a sector of a circle if its measure is 90° and radius is 7 cm.**

(A) 44 cm (B) 25 cm (C) 36 cm (D) 56 cm

(4) **Find the curved surface area of a cone of radius 7 cm and height 24 cm.**

(A) 440 cm^2 (B) 550 cm^2 (C) 330 cm^2 (D) 110 cm^2

(5) **The curved surface area of a cylinder is 440cm^2 and its radius is 5 cm. Find its height.**

(A) $\frac{44}{\pi}$ cm (B) 22π cm (C) 44π cm (D) $\frac{44}{\pi}$ cm

(6) **A cone was melted and cast into a cylinder of the same radius as that of the base of the cone. If the height of the cylinder is 5 cm, find the height of the cone.**

(A) 15 cm (B) 10 cm (C) 18 cm (D) 5 cm

(7) **Find the volume of a cube of side 0.01 cm.**

(A) 1 cm^3 (B) 0.001 cm^3

(C) 0.0001 cm^3 (D) 0.000001 cm^3

(8) **Find the side of a cube of volume 1 m^3.**

(A) 1 cm (B) 10 cm (C) 100 cm (D) 1000 cm

Additional MCQ's

In each of the following, choose the correct alternative.

(9) **Vertical surface area of a cuboid is ....**
(A) $2(l \times b) + h$ (B) $2(l \times b) \times h$
(C) $2(l + b) + h$ (D) $2(l + b) \times h$

(10) **Total surface area of a cube is 216 cm^2. Find its volume.**
(A) 36 cm^3 (B) 100 cm^3
(C) 216 cm^3 (D) 400 cm^3

(11) **The length, breadth , height of cuboid are in the ratio 1:1:2, its total surface area is 1000cm^2. Therefore $\therefore$ Its length is ....**
(A) 10 cm (B) 15 cm (C) 20 cm (D) 12 cm

(12) **A tent is made up of cylinder and mounted by a conical top. In order to calculate its total surface area, find sum of their.**
(A) Volumes (B) Total surface area
(C) Curved surface area (D) Base areas

(13) **If diameter of a semicircle is 35 cm. Find its length.**
(A) 110 cm (B) 55 cm
(C) 90 cm (D) 70 cm

(14) **If $r = 7$ cm and $\theta = 180°$. Length of arc is ....**
(A) 44 cm (B) 22 cm (C) 10 cm (D) 18 cm

(15) **If $r = 7$ cm and $\theta = 36°$ then area of sector is ....**
(A) 15.4 cm^2 (B) 20.36 cm^2
(C) 10.46 cm^2 (D) 18.2 cm^2

(16) **Bricks of dimensions 15 cm × 8 cm × 5 cm are used to build a wall of dimensions 120 cm × 16 cm × 200 cm. How many bricks are used?**
(A) 1280 (B) 640 (C) 160 (D) 320

(17) **If the volume of cylinder is 12436 cm^3 and radius and height of cylinder are in the ratio 2:3, find its height.**
(A) 21 cm (B) 7 cm (C) 14 cm (D) 18 cm

(18) **Find the volume of a right circular cone if $r = 14$ cm and $h = 9$ cm.**
(A) 161 cm^3 (B) 2438 cm^3
(C) 1848 cm^3 (D) 1488 cm^3

(19) **The volume of two spheres are in the ration 8:27, find the ratio of their radii.**
(A) 2:3 (B) 2:9 (C) 1:3 (D) 4:9

ANSWERS

(1) (A) 14π **(2)** (D) 99 cm **(3)** (B) 25 cm **(4)** (B) 550 cm^2
(5) (A) $\frac{44}{\neq}$ **(6)** (A) 15 cm **(7)** (D) 0.000001 cm^3
(8) (C) 100 cm **(9)** (D) $2(l + b) \times h$ **(10)** (C) 216 cm^3
(11) (A) 10 cm **(12)** (C) Curved surface area
(13) (B) 55 cm **(14)** (B) 22 cm **(15)** (A) 15.4 cm^2
(16) (B) 640 **(17)** (A) 21 cm **(18)** (C) 1848 cm^3
(19) (A) 2:3

PROBLEMS FOR PRACTICE

Based on Practice Set 7.1

(1) Two cubes each with 12 cm edge, are joined end to end. Find the surface area of the resulting cuboid. *(2 marks)*

(2) A solid cube with edge 'l' was divided exactly into two equal halves. Find the ratio of the total surface area of the given cube and that of the cuboid formed. *(3 marks)*

(3) A beam 4 m long, 50 m wide and 20 m deep is made of wood, which weighs 25 kg per m^3. Find the weight of the beam. *(3 marks)*

(4) A fish tank is in the form of a cuboid, external measures of that cuboid are 80 cm × 40 cm × 30 cm. The base, side faces and back face are to be covered with a coloured paper. Find the area of the paper needed. *(4 marks)*

(5) The base radii of two right circular cones of the same height are in the ratio 2 : 3. Find ratio of their volumes. *(3 marks)*

(6) If the radius of a sphere is doubled, what will be the ratio of its surface area and volume as to that of the first? *(4 marks)*

(7) The dimensions of a metallic cuboid are 44 cm × 42 cm × 21 cm. It is molten and recast into a sphere. Find the surface area of the sphere. *(4 marks)*

Based on Practice Set 7.2

(8) If the radii of the conical frustum bucket are 14 cm and 7 cm. If its height is 30 cm, then find (i) Its total surface area (ii) capacity of the bucket. *(4 marks)*

(9) The slant height of the frustum of the cone is 6.3 cm and the perimeters of its circular bases are 18 cm and 6 cm respectively. Find the curved surface area of the frustum. *(4 marks)*

(10) The radii of the circular ends of a frustum of a cone are 14 cm and 8 cm. If the height of the frustum is 8 cm. Find (i) Curved surface area of the frustum (ii) Total surface area of the frustum (iii) Volume of the frustum. *(4 marks)*

(11) The curved surface area of the frustum of a cone is 180 sq cm and the circumfernce of its circular bases are 18 cm and 6 cm respectively. Find the slant height of the frustum of a cone. *(4 marks)*

Based on Practice Set 7.3

(12) A sector of a circle with radius 10 cm has central angle 72°. Find the area of the sector ($\pi = 3.14$) *(3 marks)*

(13) If the area of a sector is $\frac{1}{12}$ th of the area of the circle, then what is the measure of the corresponding central angle. *(3 marks)*

(14) In a clock, the minute hand is of length 14 cm. Find the area covered by the minute hand in 5 minutes. *(3 marks)*

(15) The radius of the circle is 3.5 cm and the area of sector is 3.85 sq cm. Find the measure of the arc of the circle. *(3 marks)*

(16) Find the area of the sector of a circle of radius 6 cm and arc with length 15 cm.

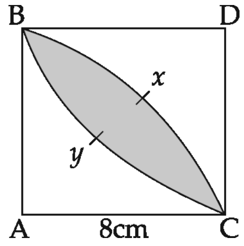
(17) Find the length of the arc of the circle of diametr 8.4 cm with area of the sector 18.48 cm^2. Also find measure of the arc. *(3 marks)*

Based on Practice Set 7.4

(18) Find the area of minor segment of a circle of radius 6 cm when its chord subtends an angle of 60° at its centre. ($\sqrt{3} = 1.73$) *(3 marks)*

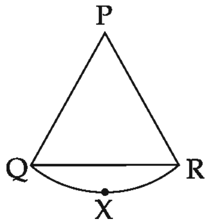
(19) Area of segment PRQ is 114 sq cm. Chord PQ subtends centre angle ∠POQ measuring 90°. Find the radius of the circle. ($\pi = 3.14$) *(3 marks)*

(20)



In the adjoining figures, arc AXC and arc AYC are drawn with radius 8 cm and centres as point B and point D respectively. Find the area of shaded region if □ABCD is a square with side 8 cm. *(4 marks)*

(21)



In the adjoining figure, P is the centre of the circle with radius 18 cm. If the area of the ΔPQR is 100 cm^2. Find the central angle QPR. *(4 marks)*

ANSWERS

(1) 1440 sq cm **(2)** 3 : 2 **(3)** 10 kg **(4)** 8000 cm^3

(5) 4 : 9 **(6)** 4 : 1, 8 : 1 **(7)** 5544 cm^2

(8) (i) $\left(770 + 66\sqrt{449}\right)$ sq cm (ii) 10,780 cm^3

(9) 75.6 cm^2 **(10)** (i) 690.8 cm^2 (ii) 157.2 cm^2 (iii) 3114.88 cm^3 **(11)** 15 cm **(12)** 62.8 cm^2 **(13)** 30°

(14) 51.33 cm^2 **(15)** 36° **(16)** 45 cm^2 **(17)** 8.8 cm, 120°

(18) 3.29 cm^2 **(19)** 20 cm **(20)** 36.48 cm^2 **(21)** 40°

❖ ❖ ❖

ASSIGNMENT – 7

Time : 1 Hr. **Marks : 20**

Q.1. Attempt the following: **(2)**

(1) Find the area of a circle with radius 7 cm.

(2) Length of arc of a circle, with radius 5 cm, is 10 cm. Find the area of corresponding sector.

Q.2. Attempt the following: **(4)**

(1) Find the volume of a sphere of diameter 6 cm. ($\pi = 3.14$)

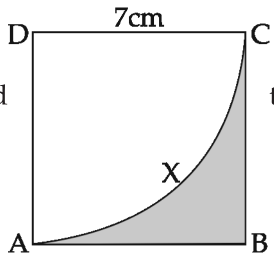
(2) Radii of the top and the base of a frustum are 14 cm, 8 cm respectively. Its height is 8 cm. Find its slant height and curved surface area.

Q.3. Attempt any two of the following: **(6)**

(1) The radius of a circle with centre P is 10 cm. If chord AB of the circle substends a right angle at P, find the areas of minor segment and the major segment ($\pi = 3.14$)

(2) The diameter and length of a roller is 120 cm and 84 cm respectively. To level the ground, 200 rotations of the roller are required. Find the expenditure to level the ground at the rate of ₹ 10 per sq. m.

(3) Complete the following activity to solve the following question
In the adjoining figure, $\square$ABCD is a square with side 7 cm.
With centre D and radius DA, sector D – AXC is drawn. Find the area of shaded portion.



Sol. Area of square $= \boxed{\quad}$ (Formula)

$= \boxed{\quad}$

$= 49 \text{ cm}^2$

Area of sector Δ-AXC $= \boxed{\quad}$ (Formula)

$$= \frac{\boxed{\quad}}{\boxed{\quad}} \times \frac{22}{7} \times \boxed{\quad}$$

$= 38.5 \text{ cm}^2$

Area of shaded region $= \text{A}\boxed{\quad} - \text{A}\boxed{\quad}$

$= \boxed{\quad}\text{ cm}^2 - \boxed{\quad}\text{ cm}^2$

$= \boxed{\quad}\text{ cm}^2$

Q.4. Attempt any two of the following: **(8)**

(1) A regular hexagon is inscribed in a circle of radius 14 cm. Find the area of the region between the circle and the hexagon.

(2) The radius of a metallic sphere is 9 cm. It was melted to make a wire of diameter 4 mm. Find the length of the wire.

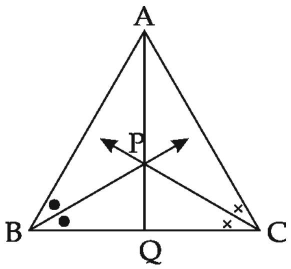
(3) The radius and height of a cylindrical water reservoir is 2.8 m and 3.5 m respectively. How much maximum water can the tank hold? A person needs 70 litres of water per day. For how many persons is the water sufficient for a day? ($\pi = \frac{22}{7}$).

❖ ❖ ❖

Challenging Questions

1. Similarity

(1) Bisectors of $\angle B$ and $\angle C$ in ΔABC meet each other at P. Line AP cuts the side BC at Q. Then prove that : $\dfrac{AP}{PQ} = \dfrac{AB + AC}{BC}$ *(4 marks)*



Proof :

In ΔABQ, ray BP bisects $\angle ABQ$. [Given]

$\therefore \dfrac{AP}{PQ} = \dfrac{AB}{BQ}$ ... (i) [By property of an angle bisector of a triangle]

In ΔACQ, ray CP bisects $\angle ACQ$. [Given]

$\therefore \dfrac{AP}{PQ} = \dfrac{AC}{CQ}$ ... (ii) [By property of an angle bisector of a triangle]

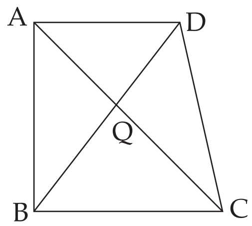
$\therefore \dfrac{AP}{PQ} = \dfrac{AB}{BQ} = \dfrac{AC}{CQ}$ ... [From (i) and (ii)]

$\therefore \dfrac{AP}{PQ} = \dfrac{AB + AC}{BQ + CQ}$ [By theorem on equal ratios]

$\therefore \dfrac{AP}{PQ} = \dfrac{AB + AC}{BC}$

$\therefore \boxed{\dfrac{AP}{PQ} = \dfrac{AB + AC}{BC}}$

(2) In $\square$ABCD, side BC $\parallel$ side AD. Diagonal AC and diagonal BD intersects in point Q. If $AQ = \dfrac{1}{3}AC$, then show that $DQ = \dfrac{1}{2}BQ$. *(4 marks)*



Proof :

Side AD $\parallel$ side BC on transversal BD.

$\angle ADB \cong \angle CBD$ ... (i) [Alternate angles]

In ΔAQD and ΔCQB,

$\angle ADQ \cong \angle CBQ$ ... [From (i), B-Q-D]

$\angle AQD \cong \angle CQB$ ... [Vertically opposite angles]

$\therefore \Delta AQD \sim \Delta CQB$

... [By AA test of similarity]

$\therefore \dfrac{AQ}{CQ} = \dfrac{DQ}{BQ}$ ... (ii) [c.s.s.t.]

Now, $AQ = \dfrac{1}{3}AC$ ... [Given]

$\therefore 3AQ = AC$

$\therefore 3AQ = AQ + CQ$

$\therefore 3AQ - AQ = CQ$

$\therefore 2AQ = CQ$

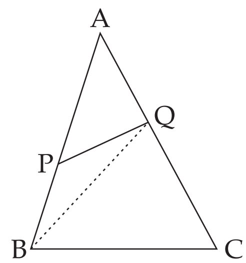
$\therefore \dfrac{AQ}{CQ} = \dfrac{1}{2}$ ... (iii)

$\therefore \dfrac{1}{2} = \dfrac{DQ}{BQ}$ ... [From (ii) and (iii)]

$\therefore \boxed{DQ = \dfrac{1}{2}BQ}$

(3) A line cuts two sides AB and side AC of ΔABC in points P and Q respectively.

Show that $\dfrac{A(\Delta ABC)}{A(\Delta APQ)} = \dfrac{AP \times AQ}{AB \times AC}$ *(4 marks)*



Construction : Join seg BQ

Proof :

Considering ΔAPQ and ΔABQ,

$\dfrac{A(\Delta APQ)}{A(\Delta ABQ)} = \dfrac{AP}{AB}$ ... (i)

[Ratio of areas of two triangles having equal height is equal to the ratio of their corresponding bases]

Considering ΔABQ and ΔABC

$\dfrac{A(\Delta ABQ)}{A(\Delta ABC)} = \dfrac{AQ}{AC}$ ... (ii)

[Ratio of areas of two triangles having equal height is equal to the ratio of their corresponding bases]

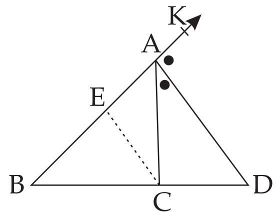
$\dfrac{A(\Delta APQ)}{A(\Delta ABQ)} \times \dfrac{A(\Delta APQ)}{A(\Delta ABQ)} = \dfrac{AP}{AB} \times \dfrac{AQ}{AC}$

... [Multiplying (i) and (ii)]

$\therefore \boxed{\dfrac{A(\Delta ABC)}{A(\Delta APQ)} = \dfrac{AP \times AQ}{AB \times AC}}$

(4) In the adjoining figure, AD is the bisector of the exterior ∠A of ΔABC. Seg AD intersects the side BC produced in D.

Prove that : $\dfrac{BD}{CD} = \dfrac{AB}{AC}$ *(4 marks)*



Construction : Draw seg CE ∥ seg DA meeting BA at E

Proof :

In ΔABD,

seg CE ∥ seg DA ... (Given)

$\therefore \dfrac{BC}{CD} = \dfrac{BE}{EA}$ ... [By B.P.T.]

$\therefore \dfrac{BC + CD}{CD} = \dfrac{BE + EA}{EA}$ ... [By componendo]

$\therefore \dfrac{BD}{CD} = \dfrac{AB}{EA}$ ... (i) [By B-E-A, B-C-D]

seg CE ∥ seg DA, on transversal BK

∠KAD ≅ ∠AEC ... (ii) [Corresponding angles theorem]

seg CE ∥ seg DA, on transveral AC

∠CAD ≅ ∠ACE ... (iii) [Alternate angles theorem]

Also, ∠KAD ≅ ∠CAD ... (iv) [Ray AD bisects ∠KAC]

∴ ∠AEC ≅ ∠ACE ... (v) [From (ii), (iii) and (iv)]

In ΔAEC,

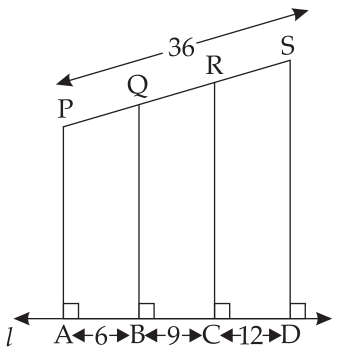
∠AEC ≅ ∠ACE ... [From (v)]

∴ EA = AC ... (vi)[By converse of isosceles triangle theorem]

$\therefore \boxed{\dfrac{BD}{CD} = \dfrac{AB}{AC}}$ ... [From (i) and (vi)]

(5) In the adjoining figure, each of the segments PA, QB, RC and SD is perpendicular to line *l*. If AB = 6, BC = 9, CD = 12, PS = 36, then determine PQ, QR and RS. *(4 marks)*

Proof :



Seg PA ⊥ line *l*
Seg QB ⊥ line *l*
Seg RC ⊥ line *l*
Seg SD ⊥ line *l* ... [Given]

∴ seg PA ∥ seg QB ∥ seg RC ∥ seg SD ... (i) [∵ lines perpendicular to same lines are parallel]

Now, seg PA ∥ seg QB ∥ seg RC

$\therefore \dfrac{PQ}{QR} = \dfrac{AB}{BC}$ ... [Property of three parallel lines and their transversals]

$\therefore \dfrac{PQ}{QR} = \dfrac{6}{9}$ ... [Given AB = 6, BC = 9]

$\therefore \dfrac{PQ}{QR} = \dfrac{2}{3}$ ... (ii)

Also, seg QB ∥ seg RC ∥ seg SD ... [From (i)]

$\therefore \dfrac{QR}{RS} = \dfrac{BC}{CD}$ ... [Property of three parallel lines and their transversals]

$\therefore \dfrac{QR}{RS} = \dfrac{9}{12}$ ... [Given, BC = 9, CD = 12]

$\therefore \dfrac{QR}{RS} = \dfrac{3}{4}$ ... (iii)

∴ PQ : QR : RS = 2 : 3 : 4 ... [From (ii) and (iii)]

Let the common multiple be x.

∴ PQ = $2x$, QR = $3x$, RS = $4x$

Now, PQ + QR + RS = PS ... [P-Q-R, Q-R-S]

∴ $2x + 3x + 4x = 36$

∴ $9x = 36$ ∴ $x = \dfrac{36}{9} = 4$

∴ PQ = $2x$ = 2 × 4 = 8 units

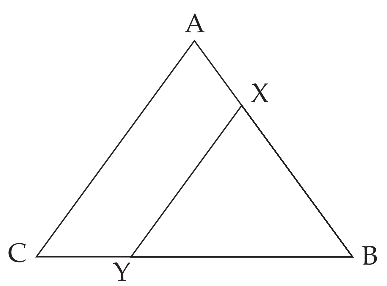
∴ QR = $3x$ = 3 × 4 = 12 units

∴ RS = $4x$ = 4 × 4 = 16 units

∴ $\boxed{\text{PQ = 8 units, QR = 12 units, RS = 16 units}}$

(6) In the adjoining figure, XY ∥ AC and XY divides the triangular region ABC into two equal areas. Determine AX : AB. *(4 marks)*

Proof :



seg XY ∥ side AC on transversal BC.

∠XYB ≅ ∠ACB ... (i) [Corresponding angles]

In ΔXYB and ΔACB,

∠XYB ≅ ∠ACB ... [From (i)]

$\angle ABC \cong \angle XBY$ ... [Common angle]

$\therefore$ $\Delta XYB \sim \Delta ACB$ ... [BY AA test of similarity]

$\dfrac{A(\Delta XYB)}{A(\Delta ACB)} = \dfrac{XB^2}{AB^2}$ ... (ii) [By theorem on areas of similar triangles]

Now, $A(\Delta XYB) = \dfrac{1}{2} A(\Delta ACB)$ ... [$\because$ seg XY divides the triangular region ABC into two equal areas]

$\therefore$ $\dfrac{A(\varnothing XYB)}{A(\varnothing ACB)} = \dfrac{1}{2}$ ... (iii)

$\therefore$ $\dfrac{XB^2}{AB^2} = \dfrac{1}{2}$ ... [From (ii) and (iii)]

$\therefore$ $\dfrac{XB}{AB} = \dfrac{1}{\sqrt{2}}$ ... [Taking square root on both sides]

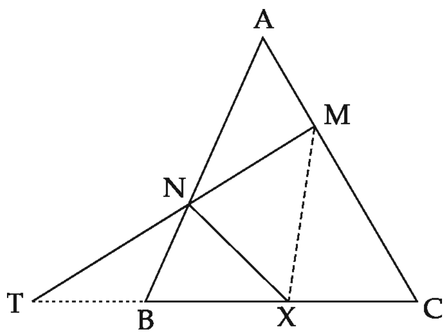
$\therefore$ $1 - \dfrac{XB}{AB} = 1 - \dfrac{1}{\sqrt{2}}$ ... [Subtracting both sides from 1]

$\therefore$ $\dfrac{AB - XB}{AB} = \dfrac{\sqrt{2}-1}{\sqrt{2}}$ $\therefore$ $\dfrac{AX}{AB} = \dfrac{\sqrt{2}-1}{\sqrt{2}}$ ...[A-X-B]

$\therefore$ $AX : AB = (\sqrt{2} - 1) : \sqrt{2}$

$\therefore$ $\boxed{\mathbf{AX : AB = (\sqrt{2} - 1) : \sqrt{2}}}$

(7) Let X be any point on side BC of ΔABC, XM and XN are drawn parallel to BA and CA. MN meets in T. Prove that TX² = TB. TC. *(4 marks)*



Proof :

In ΔTXM,

seg BN || seg XM ... [Given]

$\therefore$ $\dfrac{TN}{NM} = \dfrac{TB}{BX}$ ... (i) [By B.P.T.]

In ΔTMC,

seg XN || seg CM ... [Given]

$\therefore$ $\dfrac{TN}{NM} = \dfrac{TX}{CX}$ ... (ii) [By B.P.T.]

$\therefore$ $\dfrac{TB}{BX} = \dfrac{TX}{CX}$ ... [From (i) and (ii)]

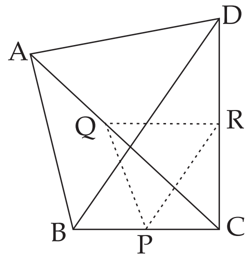
$\therefore$ $\dfrac{BX}{TB} = \dfrac{CX}{TX}$ ... [By invertendo]

$\therefore$ $\dfrac{BX + TB}{TB} = \dfrac{CX + TX}{TX}$ ... [By componendo]

$\therefore$ $\dfrac{TX}{TB} = \dfrac{TC}{TX}$ ... [T-B-X, T-X-C]

$\therefore$ $\boxed{\mathbf{TX^2 = TB \cdot TC}}$

(8) Two triangles, ΔABC and ΔDBC, lie on the same side of the base BC. From a point P on BC, PQ || AB and PR || BD are drawn. They intersect AC at Q and DC at R. Prove that QR || AD. *(4 marks)*



Proof :

In ΔCAB,

seg PQ || seg AB ... [Given]

$\therefore$ $\dfrac{CP}{PB} = \dfrac{CQ}{AQ}$ ... (i) [By B.P.T.]

In ΔBCD,

seg PR || seg BD ... [Given]

$\therefore$ $\dfrac{CP}{PB} = \dfrac{CR}{RD}$ ... (ii) [By B.P.T.]

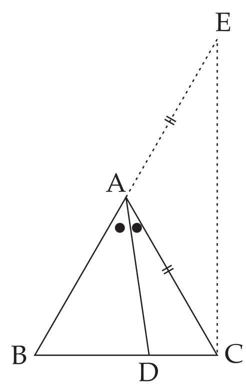
In ΔACD,

$\therefore$ $\dfrac{CQ}{AQ} = \dfrac{CR}{RD}$ ... [From (i) and (ii)]

$\therefore$ $\boxed{\textbf{seg QR || seg AD}}$ ... [By converse of B.P.T.]

(9) In ΔABC, D is a point on BC such that $\dfrac{BD}{DC} = \dfrac{AB}{AC}$. Prove that AD is the bisector of ∠A. (Hint : Produce BA to E such that AE = AC. Join EC) *(4 marks)*

Proof :



Proof : Seg BA is produced to point E such that AE = AC and seg EC is drawn.

$\therefore$ $\dfrac{BD}{DC} = \dfrac{AB}{AC}$ ... (i) [Given]

AC = AE ... (ii) [By construction]

$\therefore$ $\frac{BD}{DC} = \frac{AB}{AE}$ ... (iii) [Substituting (ii) in (i)]

$\therefore$ seg AD $\parallel$ seg EC ... [By converse of B.P.T.]

On transversal BE,

$\angle BAD \cong \angle BEC$ ... [Corresponding angles theorem]

$\therefore$ $\angle BAD \cong \angle AEC$ ... (iv) [$\because$ B-A-E]

On transversal AC,

$\angle CAD \cong \angle ACE$ ... (v) [Alternate angles theorem]

In ΔACE,

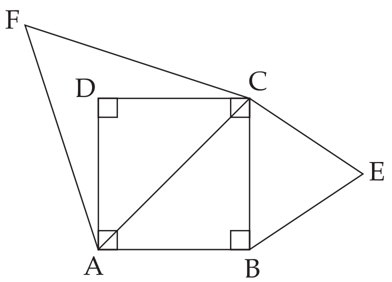
seg AC $\cong$ seg AE ... [By construction]

$\angle AEC \cong \angle ACE$ ... (vi) [By isosceles triangle theorem]

$\therefore$ $\angle BAD \cong \angle CAD$ ... [From (iv), (v) and (vi)]

$\therefore$ **Ray AD is the bisector of $\angle BAC$.**

(10) In the adjoining figure, □ABCD is a square. ΔBCE on side BC and ΔACF on the diagonal AC are similar to each other. Then, show that $A(\Delta BCE) = \frac{1}{2} A(\Delta ACF)$ *(4 marks)*



Proof :

□ABCD is a square. ... [Given]

$\therefore$ $AC = \sqrt{2}\, BC$ ... (i) [$\because$ Diagonal of a square $= \sqrt{2} \times$ side of square]

$\Delta BCE \sim \Delta ACF$ ... [Given]

$\therefore$ $\frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{(BC)^2}{(AC)^2}$ ... (ii)[By theorem on areas of similar triangles]

$\therefore$ $\frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{(BC)^2}{(\sqrt{2}.BC)^2}$ ... [From (i) and (ii)]

$\therefore$ $\frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{BC^2}{2BC^2}$

$\therefore$ $\frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{1}{2}$

$\therefore$ $\boxed{A(\Delta BCE) = \frac{1}{2} A(\Delta ACF)}$

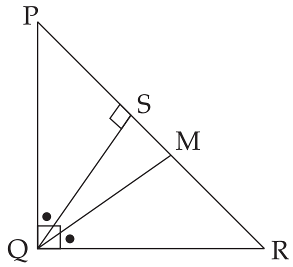
❄❄❄

2. Theorem of Pythagoras

(1) In ΔPQR, ∠PQR = 90°, as shown in figure, seg QS ⊥ side PR, seg QM is angle bisector of ∠PQR.

Prove that : $\frac{PM^2}{MR^2} = \frac{PS}{SR}$ *(4 marks)*

Proof :



In ΔPQR,

Seg QM bisects $\angle PQR$ ... [Given]

$\therefore$ $\frac{PM}{MR} = \frac{PQ}{QR}$ [Property of an angle bisector of a triangle]

$\therefore$ $\frac{PM^2}{MR^2} = \frac{PQ^2}{QR^2}$ ... (i) [Squaring both sides]

In ΔPQR,

$m\angle PQR = 90^\circ$ ... [Given]

Seg QS $\perp$ hypotenuse PR ... [Given]

$\therefore$ $\Delta PQR \sim \Delta PSQ \sim \Delta QSR$ ... (ii) [Theorem on similarity of right angled triangles]

$\Delta PSQ \sim \Delta PQR$ ... [From (ii)]

$\therefore$ $\frac{PQ}{PR} = \frac{PS}{PQ}$ ... [c.s.s.t.]

$\therefore$ $PQ^2 = PR \times PS$ ... (iii)

Also, $\Delta QSR \sim \Delta PQR$ ... [From (ii)]

$\therefore$ $\frac{QR}{PR} = \frac{SR}{QR}$ ... [c.s.s.t.]

$\therefore$ $QR^2 = PR \times SR$ ... (iv)

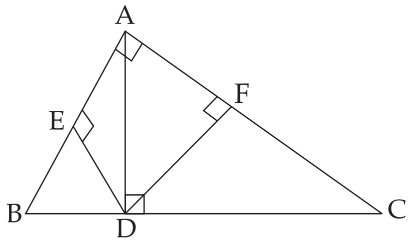
$\therefore$ $\frac{PM^2}{MR^2} = \frac{PR \times PS}{PR \times SR}$ ... [From (i), (iii) and (iv)]

$\therefore$ $\boxed{\frac{PM^2}{MR^2} = \frac{PS}{SR}}$

(2) In ΔABC, m∠BAC = 90°, seg DE ⊥ side AB, seg DF ⊥ side AC, seg AD ⊥ side BC.

Prove : A(□AEDF) = $\sqrt{AE \times EB \times AF \times FC}$) *(4 marks)*

Proof :



In ΔADB,

$m\angle ADB = 90^\circ$ ... [Given]

seg DE $\perp$ side AB ... [Given]

$\therefore$ $DE^2 = AE \times EB$ ... (i) [By property of geometric mean]

In ΔADC,

$m\angle ADC = 90^\circ$ ... [Given]

Seg DF $\perp$ side AC ... [Given]

$\therefore$ $DF^2 = AF \times FC$ ... (ii) [By property of geometric mean]

Multiplying (i) and (ii), we get

$\therefore$ $DE^2 \times DF^2 = AE \times EB \times AF \times FC$

$\therefore$ $DE \times DF = \sqrt{AE \times EB \times AF \times FC}$ ... (iii)

In $\square$AEDF,

$\therefore$ $m\angle EAF = m\angle AED = m\angle AFD = 90^\circ$ ... [Given]

$\therefore$ $m\angle EDF = 90^\circ$ ... [Remaining angle]

$\therefore$ $\square$AEDF is a rectangle ... [By definition]

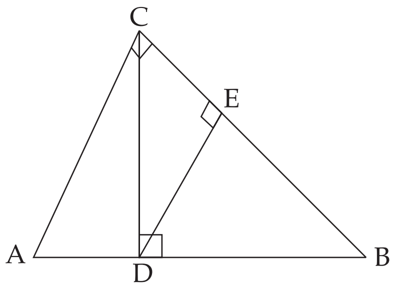
$\therefore$ $A(\square AEDF) = DE \times DF$ ... (iv) [Area of rectangle = $l \times b$]

From (iii) and (iv), we get

$\therefore$ $\boxed{A(\square AEDF) = \sqrt{AE \times EB \times AF \times FC}}$

(3) In ΔABC, $\angle ACB = 90^\circ$,
seg CD $\perp$ side AB,
seg DE $\perp$ seg CB.
Show that : $CD^2 \times AC = AD \times AB \times DE$ *(4 marks)*

Proof :



In ΔACB,

$\angle ACB = 90^\circ$, ... [Given]

seg CD $\perp$ hypotenuse AB,

$\therefore$ $CD^2 = AD \times DB$ ... (i) [By property of geometric mean]

In ΔDEB and ΔACB,

$\angle DEB \cong \angle ACB$ ... [Each is 90°]

$\angle DBE \sim \angle ABC$ ... [Common angle]

$\therefore$ $\Delta DEB \cong \Delta ACB$ ... [By AA test of similarity]

$\therefore$ $\dfrac{DE}{AC} = \dfrac{DB}{AB}$ ... [c.s.s.t.]

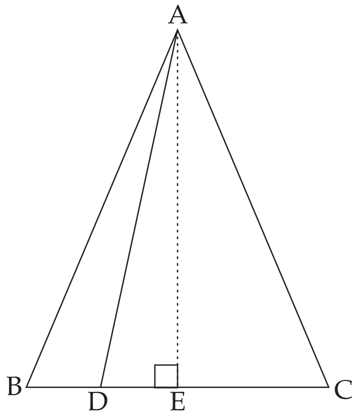
$\therefore$ $AC = \dfrac{DE \times AB}{DB}$ ... (ii)

$\therefore$ $CD^2 \times AC = AD \times DB \times \dfrac{DE \times AB}{DB}$ ... [Multiplying (i) and (ii)]

$\therefore$ $\boxed{CD^2 \times AC = AD \times AB \times DE}$

(4) In an equilateral ΔABC, the side BC is trisected at D. Prove that $9AD^2 = 7AB^2$. (Hint : AE $\perp$ BC) *(4 marks)*

Proof :



Construction : AE $\perp$ BC is drawn

In ΔAED,

$m\angle AED = 90^\circ$ ... [By construction]

$\therefore$ $AD^2 = AE^2 + DE^2$ ... (i) [By Pythagoras theorem]

$\therefore$ $\angle ABC = 60^\circ$ ... (ii) [Angle of an equilateral triangle]

In ΔAEB,

$m\angle AEB = 90^\circ$ ... [By construction]

$\therefore$ $\angle ABE = 60^\circ$ ... [B-E-C]

$\therefore$ $\angle BAE = 30^\circ$ ... [Remaining angle of a triangle]

$\therefore$ ΔAEB is a 30° - 60° - 90° triangle

$\therefore$ $AE = \dfrac{\sqrt{3}}{2}(AB)$ ... (iii) [Side opposite to 60°]

$\therefore$ $BE = \dfrac{1}{2}(AB)$ ... (iv) [Side opposite to 30°]

$\therefore$ $BD = \dfrac{1}{3}(BC)$ ... [Given]

$\therefore$ $BD = \dfrac{1}{3}(AB)$ ... (v) [Since, BC = AB, sides of an equilateral triangle]

$\therefore$ $DE + BD = BE$ ... [B-D-E]

$\therefore$ DE = BE – BD

$\therefore$ DE = $\frac{1}{2}$ AB – $\frac{1}{3}$ AB ... [From (iv) and (v)]

$\therefore$ DE = $\frac{3AB - 2AB}{6}$

$\therefore$ DE = $\frac{1}{6}$ (AB) ... (vi)

$\therefore$ $AD^2 = \left[\frac{\sqrt{3}}{2}AB\right]^2 + \left[\frac{1}{6}AB\right]^2$ ... [Substituting (iii), (vi) in (i)]

$\therefore$ $AD^2 = \frac{3}{4}AB^2 + \frac{1}{36}AB^2$

$\therefore$ $AD^2 = \frac{27AB^2 + AB^2}{36}$

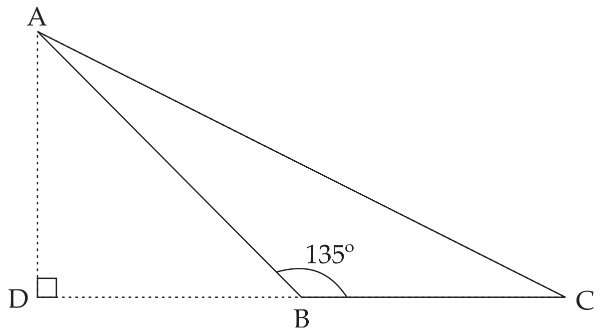
$\therefore$ $AD^2 = \frac{28AB^2}{36}$

$\therefore$ $AD^2 = \frac{7}{9}AB^2$

$\therefore$ $\boxed{9AD^2 = 7AB^2}$

(5) In ΔABC, ∠ABC = 135°.
Prove that : $AC^2 = AB^2 + BC^2 + 4A(\Delta ABC)$.
Construction : Draw seg AD ⊥ side BC, such that D-B-C. *(4 marks)*

Proof :



m ∠ABC + m ∠ABD = 180° ...(Angles forming linear pair)

$\therefore$ 135° + m ∠ABD = 180°

$\therefore$ m ∠ABD = 180° – 135°

$\therefore$ m ∠ABD = 45° ...(i)

In ΔADB,

m ∠ADB = 90° ...(Given)

m ∠ABD = 45° ...[From (i)]

$\therefore$ m ∠BAD = 45° ...(ii) (Remaining angle)

In ΔABD,

∠ABD ≅ ∠BAD ...[From (i) and (ii)]

$\therefore$ seg AD ≅ seg DB ...(iii) (Converse of isosceles triangle theorem)\

In ΔADB,

m ∠ADB = 90° (Construction)

$\therefore$ $AB^2 = AD^2 + DB^2$ ...(iv) (By Pythagoras theorem)

In ΔADC,

m ∠ADC = 90° ...(Construction)

$\therefore$ $AC^2 = AD^2 + DC^2$ ...(By Pythagoras theorem)

$\therefore$ $AC^2 = AD^2 + (DB + BC)^2$ ...(∵ D - B - C)

$\therefore$ $AC^2 = AD^2 + DB^2 + 2 \times DB \times BC + BC^2$

$\therefore$ $AC^2 = AB^2 + BC^2 + 2 \times DB \times BC$ ...[From (iv)]

$\therefore$ $AC^2 = AB^2 + BC^2 + 2 \times AD \times BC$ ...(v) [From (iii)]

Area of triangle = $\frac{1}{2}$ × base × height

$\therefore$ A (ΔABC) = $\frac{1}{2}$ × BC × AD

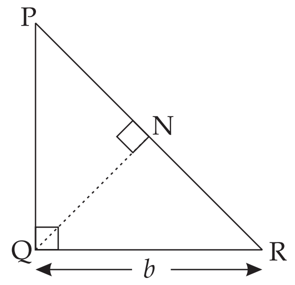
$\therefore$ 4A (ΔABC) = 4 × $\frac{1}{2}$ × BC × AD ...(Multiplying throughout by 4)

$\therefore$ 4A (ΔABC) = 2 × AD × BC ...(vi)

$\therefore$ $\boxed{AC^2 = AB^2 + BC^2 + 4A(\Delta ABC)}$

(6) In ΔPQR is a right angled triangle, right angled at Q such that QR = b and A(ΔPQR) = a.
If QN ⊥ PR, then show that QN = $\frac{2a.b}{\sqrt{b^2 + 4a^2}}$. *(4 marks)*

Proof :



Area of triangle = $\frac{1}{2}$ × base × height

$\therefore$ A (ΔPQR) = $\frac{1}{2}$ × QR × PQ

$\therefore$ $a = \frac{1}{2} \times b \times PQ$ ...(Given)

$\therefore$ $\frac{2a}{b} = PQ$ ...(i)

Also,

A (ΔPQR) = $\frac{1}{2}$ × PR × QN

$\therefore$ $a = \frac{1}{2}$ × PR × QN ...(Given)

$\therefore$ QN = $\frac{2a}{PR}$ ...(ii)

In ΔPQR,

m ∠PQR = 90° ...(Given)

$\therefore$ $PR^2 = PQ^2 + QR^2$ ...(By Pythagoras theorem)

$\therefore$ $PR = \sqrt{PQ^2 + QR^2}$ ...(Taking square roots)

$\therefore$ PR $= \sqrt{\left(\frac{2a}{b}\right)^2 + b^2}$ ...[From (i) and given]

$\therefore$ PR $= \sqrt{\left(\frac{4a^2}{b^2}\right) + b^2}$

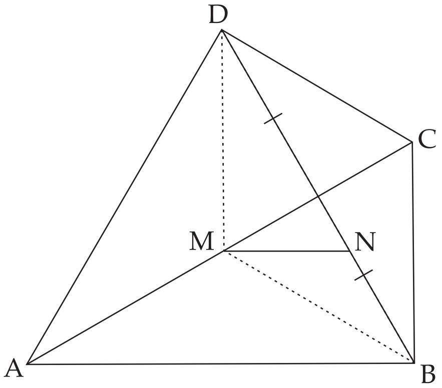
$\therefore$ PR $= \frac{\sqrt{b^4 + 4a^2}}{b}$ ...(iii)

$\therefore$ QN $= \frac{2a}{\frac{\sqrt{b^4+4a^2}}{b}}$ ...[From (ii) and (iii)]

$\therefore$

$\therefore$ $\boxed{\mathbf{QN = \frac{2ab}{\sqrt{b^4 + 4a^2}}}}$

(7) In □ABCD is a quadrilateral. M is the midpoint of diagonal AC and N is the midpoint of diagonal BD. Prove that : $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4MN^2$. *(4 marks)*



Given : □ABCD is a quadrilateral.

M and N are the midpoints of diagonal AC and BD respectively.

To prove : $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4MN^2$

Construction : Join seg DM and seg BM.

Proof :

In ΔADC,

seg DM is the median.

$\therefore$ $AD^2 + CD^2 = 2DM^2 + 2CM^2$ ... [Apollonius theorem]

$\therefore$ $AD^2 + CD^2 = 2DM^2 + 2[\frac{1}{2}AC]^2$ ... [M is the midpoint of AC]

$\therefore$ $AD^2 + CD^2 = 2DM^2 + 2 \times \frac{1}{4}AC^2$

$\therefore$ $AD^2 + CD^2 = 2DM^2 + \frac{1}{2}AC^2$ ... (i)

Similarly, in ΔABC seg BM is the median.

$AB^2 + BC^2 = 2BM^2 + \frac{1}{2}AC^2$ ... (ii)

$\therefore$ $AD^2 + CD^2 + AB^2 + BC^2 = 2DM^2 + \frac{1}{2}AC^2 + 2BM^2 + \frac{1}{2}AC^2$ ... [Adding (i) and (ii)]

$\therefore$ $AB^2 + BC^2 + CD^2 + DA^2 = 2BM^2 + 2DM^2 + AC^2$ ... (iii)

In ΔDMB,

seg MN is the median

$\therefore$ $BM^2 + DM^2 = 2MN^2 + 2BN^2$ ... [Apollonius theorem]

$\therefore$ $BM^2 + DM^2 = 2MN^2 + 2[\frac{1}{2}BD]^2$ ... [N is the midpoint of BD]

$\therefore$ $BM^2 + DM^2 = 2MN^2 + 2 \times \frac{1}{4}BD^2$

$\therefore$ $BM^2 + DM^2 = 2MN^2 + \frac{1}{2}BD^2$

$\therefore$ $2BM^2 + 2DM^2 = 4MN^2 + 2 \times \frac{1}{2}BD^2$ ... [Multiplying by 2]

$\therefore$ $2BM^2 + 2DM^2 = 4MN^2 + BD^2$ ... (iv)

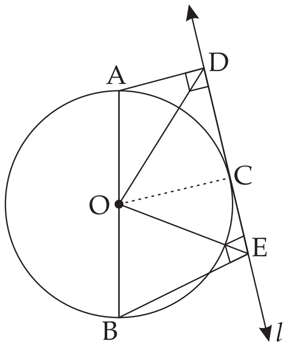
$\therefore$ $AB^2 + BC^2 + CD^2 + DA^2 = (4MN^2 + BD^2) + AC^2$ ... [Substituting (iv) in (iii)]

$\therefore$ $\boxed{\mathbf{AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4MN^2}}$

❁ ❁ ❁

3. Circle

(1) From the end points of a diameter of circle perpendiculars are drawn to a tangent of the same circle. Show that their feet on the tangent are equidistant from the centre of the circle. *(4 marks)*



Given : (i) A circle with centre O.

(ii) Seg AB is the diameter of the circle.

(iii) Line l is tangent to the circle at point C.

(iv) Seg AD ⊥ line l

(v) Seg BE ⊥ line l

To Prove : OD = OE

Construction : Draw seg OC

Proof :

Seg AD $\perp$ line l ... [Given]

Seg OC $\perp$ line l ... [Radius is perpendicular to the tangent]

Seg BE $\perp$ line l ... [Given]

$\therefore$ Seg AD $\parallel$ seg OC $\parallel$ seg BE ... [Perpendiculars drawn to the same line are parallel to each other]

$\therefore$ On transversal AB and DE,

$\frac{AO}{OB} = \frac{DC}{CE}$ ... (i) [Property of three parallel lines and their transversals]

But, AO = OB ... [Radii of the same circle]

$\therefore$ $\frac{AO}{OB} = 1$ ... (ii)

$\therefore$ $\frac{DC}{CE} = 1$ ... [From (i) and (ii)]

$\therefore$ DC = CE ... (iii)

In ΔOCD and ΔOCE ,

seg OC $\cong$ seg OC ... [Common side]

$\angle$OCD $\cong$ $\angle$OCE ... [Each is a right angle]

seg DC $\cong$ seg CE ... [From (iii)]

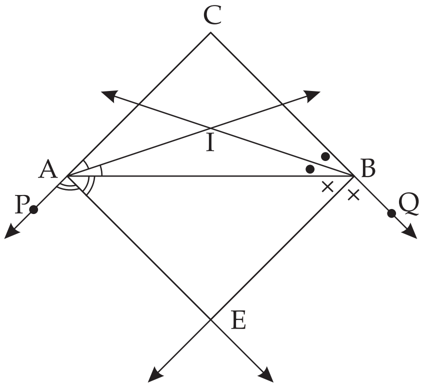
$\therefore$ ΔOCD $\cong$ ΔOCE ... [By SAS test of congruence]

$\therefore$ seg OD $\cong$ seg OE ... (c.s.c.t)

$\therefore$ $\boxed{\text{OD = OE}}$

(2) The bisectors of the angles A, B of ΔABC intersects in I, the bisectors of the corresponding exterior angles intersect in E. Prove that $\square$AIBE is cyclic. *(4 marks)*

Solution :



Proof : Take point P and Q as show in fig.

m$\angle$CAB + m$\angle$BAP = 180° ... [Linear pair axiom]

$\therefore$ $\frac{1}{2}$m$\angle$CAB + $\frac{1}{2}$m$\angle$BAP = $\frac{1}{2}$ × 180° ... [Multiplying throughout by $\frac{1}{2}$]

$\therefore$ m$\angle$IAB + m$\angle$BAE = 90° ... [$\because$ Ray AI and ray AE bisects $\angle$CAB and $\angle$BAP respectively]

$\therefore$ m$\angle$IAE = 90° ... (i) [Angle addition property]

Similarly,

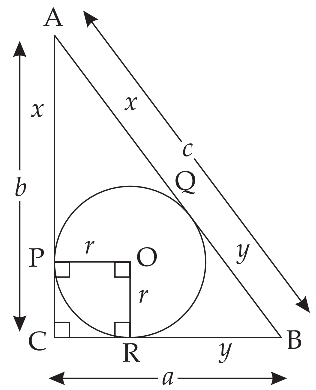
$\therefore$ m$\angle$IBE = 90° ... (ii)

$\therefore$ m$\angle$IAE + m$\angle$IBE = 90° + 90° ... [Adding (i) and (ii)]

$\therefore$ m$\angle$IAE + m$\angle$IBE = 180°

$\therefore$ $\boxed{\square\text{AIBE is cyclic}}$ ... [If opposite angles of a quadrilateral are supplementary then quadrilateral is cyclic]

(3) In a right angled ΔABC, $\angle$ACB = 90°. A circle is inscribed in the triangle with radius *r*. *a*, *b*, *c* are the lengths of the sides BC, AC and AB respectively. Prove that 2*r* = *a* + *b* – *c*. *(4 marks)*



Proof :

Let the centre of the inscribed circle be 'O'

Let AP = AQ = x ...(i)
CP = CR = y ...(ii)
BR = BQ = z ...(iii)
(The lengths of the two tangent segments to a circle drawn from an external point are equal)

$a + b - c$ = BC + AC – AB

$\therefore$ $a + b - c$ = CR + RB + AP + PC – (AQ + QB) (B – R – C, A – P – C , A – Q – B)

$\therefore$ $a + b - c$ = $y + z + x + y - (x + z)$ [From (i), (ii) and (iii)]

$\therefore$ $a + b - c$ = $y + z + x + y - x - z$

$\therefore$ $a + b - c$ = $2y$

$\therefore$ $a + b - c$ = 2CP ...(iv) [From (ii)]

In $\square$ PCRO

m $\angle$OPC = m $\angle$ORC = 90° ...(Radius is perpendicular to tangent)

m $\angle$PCR = 90° ...(Given)

$\therefore$ m $\angle$POR = 90° ...(Remaining angle)

$\therefore$ □PCRO is a rectangle ...(By definition)

$\therefore$ CP = OR ...(v)

(Opposite sides of a rectangle)

$\therefore$ $a + b - c = 2$ OR ...[From (iv) and (v)]

$\therefore$ $\boxed{a + b - c = 2r}$

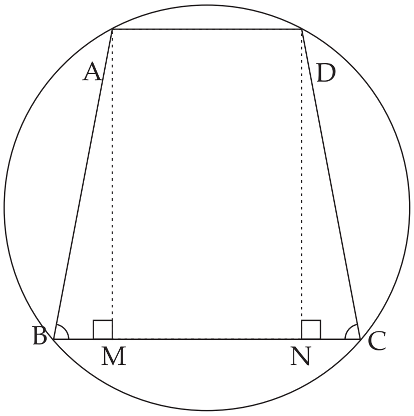
(4) If two consecutive angles of cyclic quadrilateral are congruent, then prove that one pair of opposite sides is congruent and other is parallel. *(4 marks)*

Given : □ABCD is a cyclic quadrilateral

$\angle ABC \cong \angle BCD$

To Prove : side DC ≅ side AB, AD || BC

Construction : Draw seg AM and seg DN both perpendicular to side BC.



Proof :

I $\angle ABC \cong \angle BCD$ ... (i) [Given]

$\angle ABC + \angle ADC = 180^\circ$... (ii) [Opposite angles of a cyclic quadrilateral are supplementary]

$\angle BCD + \angle ADC = 180^\circ$ ... [From (i) and (ii)]

$\therefore$ Side AD || side BC ... [Interior angles test]

In ΔDNC and ΔAMB,

seg DN ≅ seg AM ... [Perpendicular distance between two parallel lines]

$\angle DNC \cong \angle AMB$ ... [Each is 90°]

$\angle DCN \cong \angle ABM$ ... [Given B-M-N-C]

$\therefore$ $\Delta DNC \cong \Delta AMB$ ... [By SAA test of congruence]

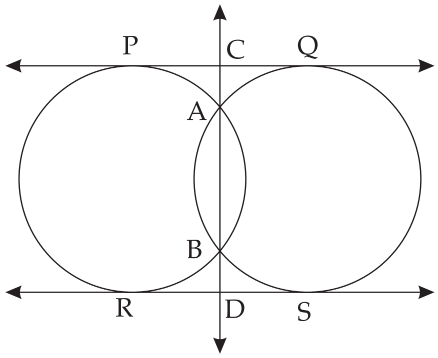
$\therefore$ $\boxed{\text{side DC} \cong \text{side AB}}$ ... [c.s.c.t.]

(5) As shown in the adjoining figure, two circles intersect each other in points A and B. Two tangents touch these circles in points P, Q and R, S as shown. Line AB intersects seg PQ in C and seg RS in D. Show that C and D are midpoints of seg PQ and seg RS respectively. *(3 marks)*

Given : Two circles intersect each other in points A and B.

Line PQ and RS are the common tangents and line CD is a common secant.

To Prove : C and D are midpoints of seg PQ and seg RS.



Proof :

Line CP is a tangent and line CD is a secant.

$\therefore$ $CP^2 = CA \times CB$ ... (i) [Tangent secant segment theorem]

Similarly, $CQ^2 = CA \times CB$ ... (ii)

$\therefore$ $CP^2 = CQ^2$ ... [From (i) and (ii)]

$\therefore$ CP = CQ ... [Taking square root on both sides]

$\therefore$ **C is the midpoint of seg PQ** ... [P - C - Q]

Line RD is a tangent and line CD is a secant

$\therefore$ $RD^2 = DB \times DA$ ... (iii) [Tangent secant

Similarly, $SD^2 = DB \times DA$ ... (iv) segment theorem]

$\therefore$ $RD^2 = SD^2$ ... [From (iii) and (iv)]

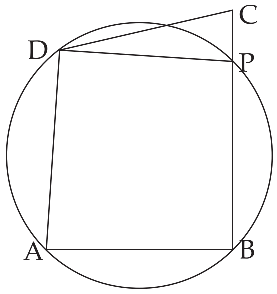
$\therefore$ RD = SD ... [Taking square root on both sides]

$\therefore$ **D is the midpoint of seg RS** ... [R - D - S]

(6) □ABCD is a parallelogram. A circle passing through D, A, B cuts BC in P. Prove that DC = DP. *(3 marks)*

Given : □ABCD is a parallelogram .

Prove : DC = DP



Proof :

□ABPD is a cyclic quadrilateral. [By definition]

$\therefore$ $\angle BAD = \angle DPC$ ...(i) [Exterior angle of a cyclic quadrilateral equals to the interior opposite angle]

□ABCD is a parallelogram ... [Given]

$\therefore$ $\angle BAD = \angle DCB$ ... [Opposite angles of a parallelogram are equal]

$\therefore$ $\angle BAD = \angle DCP$ ... (ii) [C - P - B]

In ΔDCP,

$\angle DPC = \angle DCP$ ... [From (i) and (ii)]

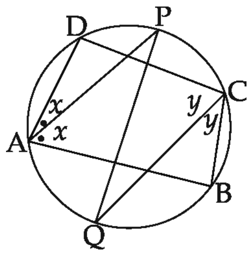
$\therefore$ DC = DP ... [Converse of isosceles triangle theorem]

(7) In a cyclic quadrilateral ABCD, the bisectors of opposite angles A and C meet the circle at P and Q respectively. Prove that PQ is a diameter of the circle. *(4 marks)*

Given : □ABCD is a cyclic quadrilateral.

Ray AP and ray CQ bisect $\angle BAD$ and $\angle BCD$

To Prove : seg PQ is a diameter of the circle.



Proof :

$\angle DAP \cong \angle BAP$ ... ($\because$ ray AP bisects $\angle DAB$)

Let m $\angle DAP$ = m $\angle BAP = x°$ ...(i)

$\angle DCQ \cong \angle BCQ$ ... ($\because$ ray CQ bisects $\angle DCB$)

Let, m $\angle DCQ$ = m $\angle BCQ = y°$ ...(ii)

□ABCD is cyclic ...(Given)

$\therefore$ m $\angle DAB$ + m $\angle DCB = 180°$

(Opposite angles of a cyclic quadrilateral are supplementary)

$\therefore$ m$\angle DAP$ + m $\angle BAP$ + $\angle DCQ$ + m $\angle BCQ = 180°$

...(Angle addition property)

$\therefore$ $x + x + y + y = 180°$ ...[From (i) and (ii)]

$\therefore$ $2x + 2y = 180°$ ...(iii)

$\therefore$ m$\angle DAP = \frac{1}{2}$ m (arc DP)

...(Inscribed angle theorem)

$\therefore$ $x = \frac{1}{2}$ m (arc DP) ...[From (i)]

$\therefore$ m (arc DP) $= 2x°$ ...(iv)

m $\angle DCQ = \frac{1}{2}$ m (arc DQ)

...(Inscribed angle theorem)

$\therefore$ $y = \frac{1}{2}$ m (arc DQ) ...[From (ii)]

$\therefore$ m (arc DQ) $= 2y°$ ...(v)

m (arc DP) + m (arc DQ) $= 2x + 2y$

...[Adding (iv) and (v)]

$\therefore$ m (arc PDQ) $= 180°$

...[Arc addition property and from (iii)]

$\therefore$ Arc PDQ is a semicircle

$\therefore$ $\boxed{\text{seg PQ is a diameter of the circle.}}$

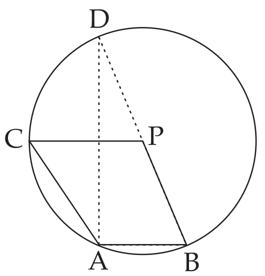
(8) In ΔABC, $\angle A$ is an obtuse angle, P is the cirumcentre of ΔABC.

Prove that $\angle PBC = \angle A - 90°$ *(4 marks)*

Given : P is centre of the circle.

To Prove : $\angle PBC = \angle A - 90°$

Construction : Extend seg BP to intersect the circle at point D, B - P - D. Join seg AD.



Proof :

Seg BD is the diameter

$\therefore$ $\angle BAD = 90°$ ... (i) [Angles inscribed in a semicircle]

$\therefore$ $\angle DBC \cong \angle DAC$ ... [Angles inscribed in a same arc are congruent]

$\therefore$ $\angle PBC = \angle DAC$ ... (ii) [B - P - D]

Now, $\angle BAC = \angle BAD + \angle DAC$ ... [Angle addition property]

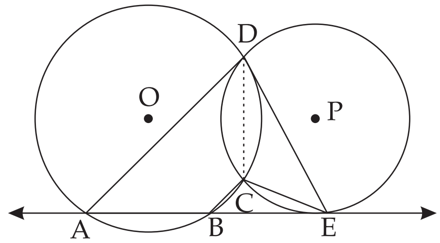
$\therefore$ $\angle A = 90° + \angle PBC$ ... [From (i) and (ii)]

$\therefore$ $\angle A - 90° = \angle PBC$

$\therefore$ $\boxed{\angle PBC = \angle A - 90°}$

(10) Two circles with centre O and P intersect each other in point C and D. Chord AB of the circle with centre O touches the circle with centre P in point E. Prove that $\angle ADE + \angle BCE = 180°$ *(4 marks)*

Construction : Draw seg CD.



Proof :

In ΔABCE,

$\therefore$ $\angle$CBE + $\angle$CEB + $\angle$BCE = 180° ... (i) [Sum of measures of angles of a triangle is 180°]

□ABCD is a cyclic quadrilateral, $\angle$CBE is its exterior angle.

$\therefore$ $\angle$CBE = $\angle$ADC ... (ii) [Exterior angle of a cyclic quadrilaterals equals to interior opposite angle]

$\angle$CED = $\angle$EDC ... (iii) [Angles in alternate segments]

$\therefore$ $\angle$ADC + $\angle$EDC + $\angle$BCE = 180° ... [Substituting (ii) and (iii) in (i)]

$\therefore$ $\boxed{\angle ADE + \angle BCE = 180^\circ}$ ... [Angle addition property]

❁ ❁ ❁

4. Geometric Constructions

(1) **Draw a ΔABC with side BC = 6 cm, $\angle$B = 45° and $\angle$A = 100°, then construct a triangle whose sides are $\frac{4}{7}$ times the corresponding sides of ΔABC.** *(4 marks)*

Solution :

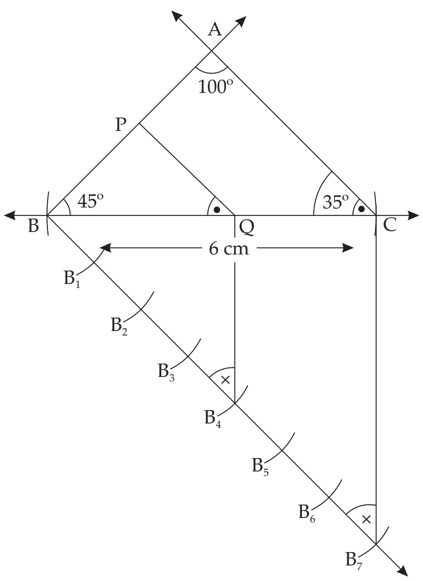
Analysis :

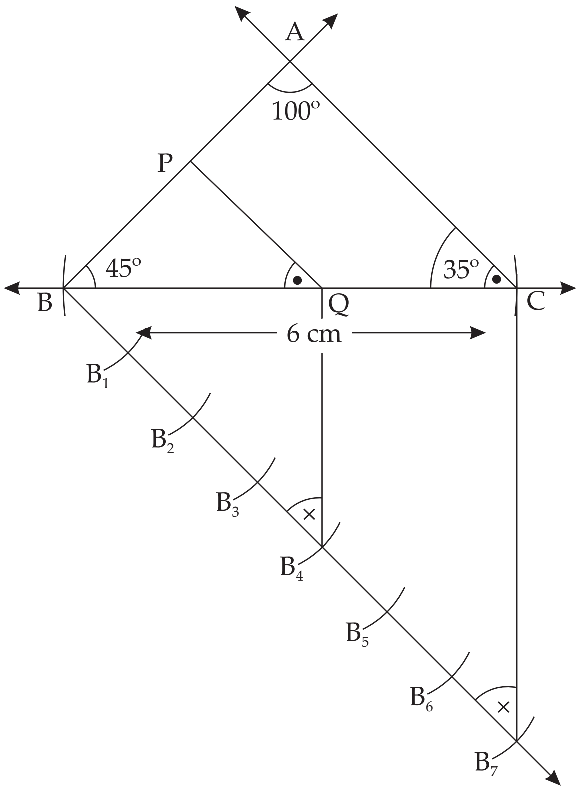
In ΔABC,

m$\angle$A = 100°
m$\angle$B = 45° } ... [Given]

$\therefore$ m$\angle$C = 35° (Remaining angles of a triangle)

(Analytical Figures)



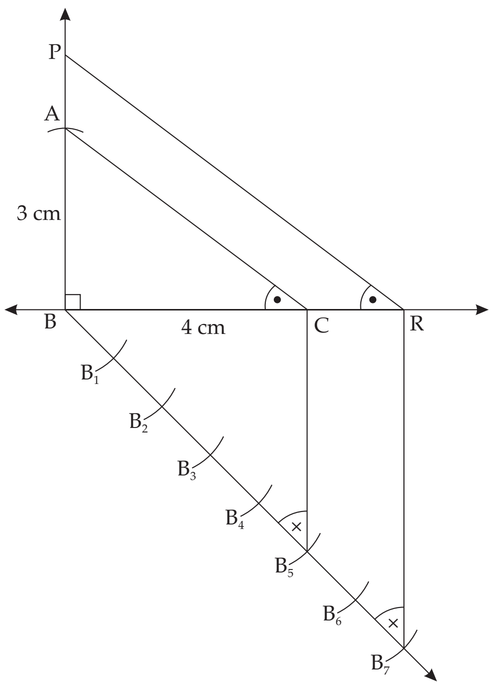


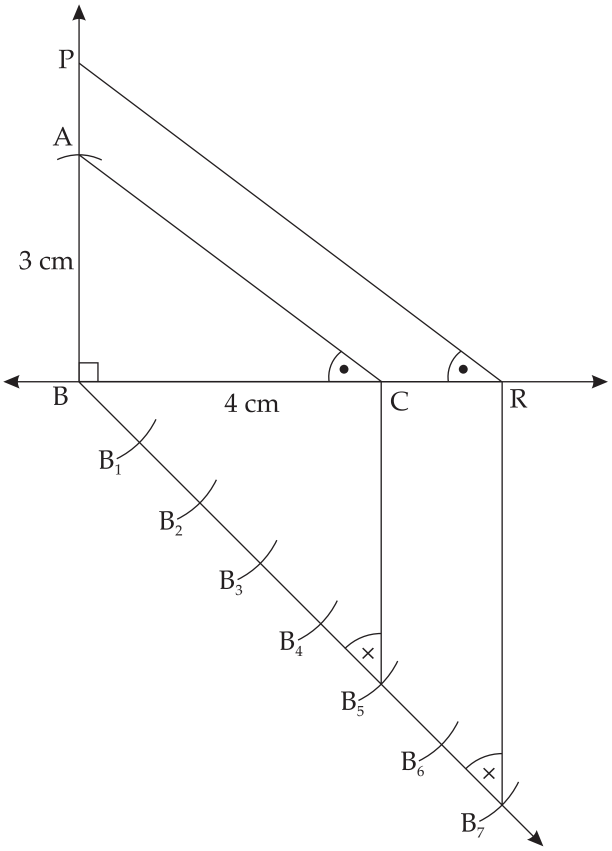
ΔPBQ is the required triangle similar to the given ΔABC.

(2) **Draw a ΔABC, right angled at B such that AB = 3 cm and BC = 4 cm. Now, construct a triangle similar to ΔABC, each of whose sides is $\frac{7}{5}$ times the corresponding sides of ΔABC.** *(4 marks)*

Solution :

(Analytical Figure)

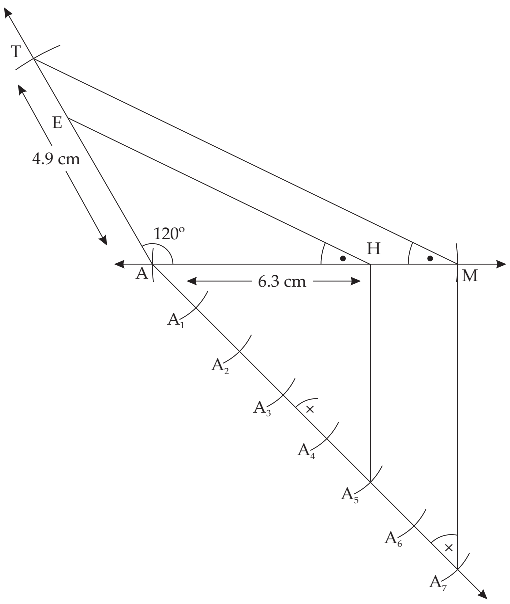


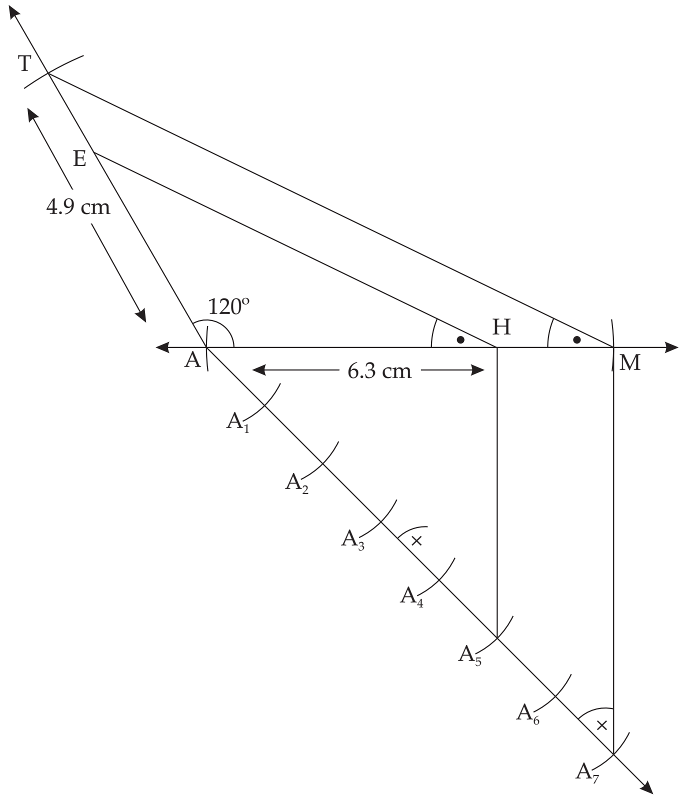


ΔPBR is the required triangle similar to the given ΔABC.

(3) **$\Delta AMT \sim \Delta AHE$, In ΔAMT, MA = 6.3 cm, $\angle MAT = 120^\circ$, AT = 4.9 cm and $\frac{MA}{HA} = \frac{7}{5}$, construct ΔAHE.** *(4 marks)*

Solution :





(4) **$\Delta LTR \sim \Delta HYD$. Construct ΔHYD, where HY = 7.2 cm, YD = 6 cm, $\angle Y = 40^\circ$ and $\frac{LR}{HD} = \frac{5}{6}$ and construct ΔLTR.** *(4 marks)*

Solution :

$\Delta LTR \sim \Delta HYD$

$\therefore \frac{LT}{HY} = \frac{TR}{YD} = \frac{LR}{HD} = \frac{5}{6}$ ... (i) (c.s.s.t.)

$\therefore \angle T = \angle Y = 40^\circ$ (c.a.s.t.)

$\frac{LT}{HY} = \frac{5}{6}$ [From (i)]

$\therefore \frac{LT}{7.2} = \frac{5}{6}$

$\therefore LT = \frac{7.2 \times 5}{6} = \frac{36}{6}$

$\therefore$ **LT = 6 cm**

$\frac{TR}{YD} = \frac{5}{6}$ [From (i)]

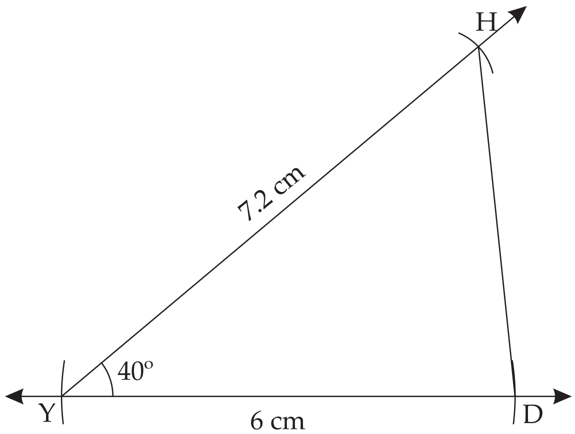
$\therefore \frac{TR}{6} = \frac{5}{6}$

$\therefore TR = \frac{5 \times 6}{6} = \frac{30}{6}$

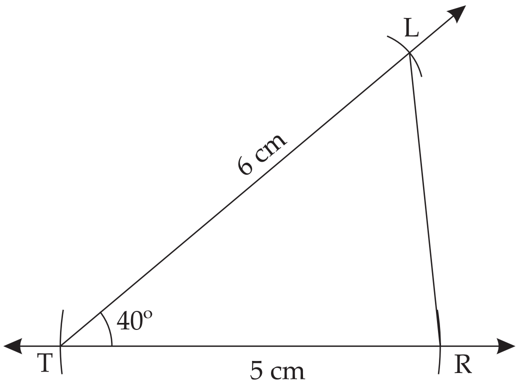
$\therefore$ **TR = 5 cm**

Information for constructing ΔLTR is complete.

(Given triangle)



(Required triangle)



(5) **Draw a circle with centre O and radius 3.5 cm. Draw tangents PA and PB to the circle, from a point P outside the circle, at points A and B respectively. ∠APB = 80°.** *(4 marks)*

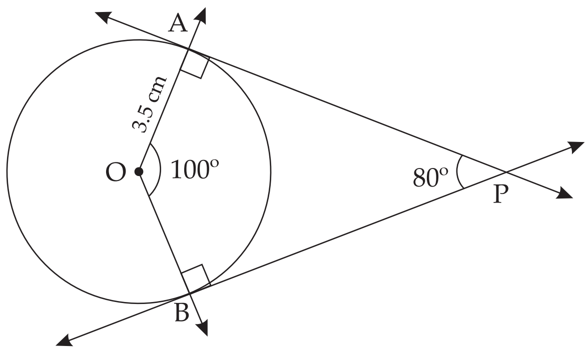
Solution :

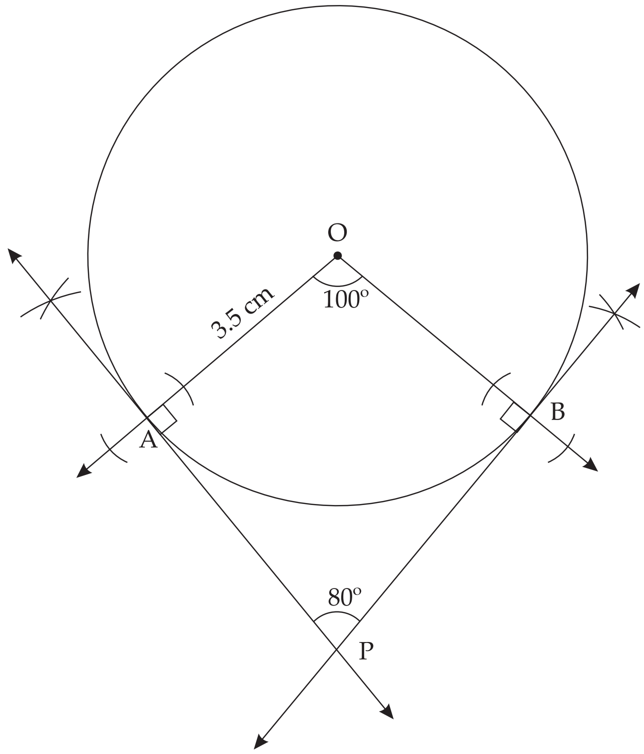
In □OAPB, ∠P = 80° ... [Given]

∠OAP = ∠OBP = 90° [Tangent perpendicularity theorem]

∴ ∠AOB = 100 [Remaining angle of □]

Analytical figure :

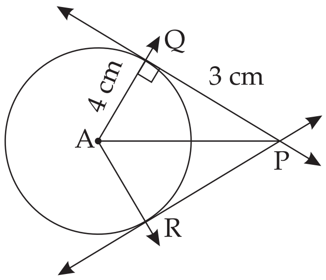




(6) **Draw a circle with centre A and radius 4 cm. Draw tangent segments PQ and PR from an external point P such that PQ = PR = 3 cm.** *(4 marks)*

Solution :

In ΔAPQ, ∠AQP = 90° [Tangent theorem]



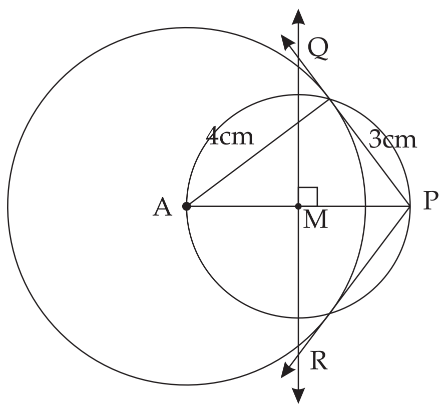
∴ $AP^2 = AQ^2 + PQ^2$ [Pythagoras theorem]

∴ $AP^2 = 4^2 + 3^2$

$= 16 + 9$

∴ $AP^2 = 25$

∴ AP = 5cm



5. Co-ordinate Geometry

(1) If A(–14, 10) and B (6, –2) find the coordinates of the points which divides seg AB into four equal parts. *(4 marks)*

Solution :

A	P	Q	R	B
(–14, 10)	(x_1, y_1)	(x_2, y_2)	(x_3, y_3)	(6, –2)

Let points $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ be three points which divide seg AB into four equal parts.

$\therefore$ AP = PQ = QR = RB ... (i)

AQ = AP + PQ = AP + AP = 2AP ... (ii)
[A - P - Q and From (i)]

BQ = BR + RQ = AP + AP = 2 AP ... (iii)
[B - R - Q and From (i)]

$\therefore$ AQ = BQ [From (ii) and (iii)]

$\therefore$ Q is midpoint of seg AB

$\therefore$ $x_2 = \dfrac{-14+6}{2}$; $y_2 = \dfrac{10+(-2)}{2}$ [Midpoint formula]

$\therefore$ $x_2 = \dfrac{-8}{2}$; $y_2 = \dfrac{8}{2}$

$\therefore$ $x_2 = -4$; $y_2 = 4$

$\therefore$ $\boxed{\mathbf{Q = (-4, 4)}}$

P is midpoint of seg AQ. [From (i)]

$\therefore$ $x_1 = \dfrac{-14-4}{2}$; $y_1 = \dfrac{10+4}{2}$ [Midpoint formula]

$\therefore$ $x_1 = \dfrac{-18}{2}$; $y_1 = \dfrac{14}{2}$

$\therefore$ $x_1 = -9$; $y_1 = 7$

$\therefore$ $\boxed{\mathbf{P = (-9, 7)}}$

R is midpoint of seg BQ. [From (i)]

$\therefore$ $x_3 = \dfrac{-4+6}{2}$; $y_3 = \dfrac{4+(-2)}{2}$ [Midpoint formula]

$\therefore$ $x_3 = \dfrac{2}{2}$; $y_3 = \dfrac{2}{2}$

$\therefore$ $x_3 = 1$; $y_3 = 1$

$\therefore$ $\boxed{\mathbf{R = (1, 1)}}$

(2) If A(20, 10) and B (0, 20) are end points of a seg AB then find the coordinates of points which divide seg AB into 5 congruent parts. *(4 marks)*

Solution :

B	P	Q	R	S	A
(0, 20)	(x_1, y_1)	(x_2, y_2)	(x_3, y_3)	(x_4, y_4)	(20, 10)

Let points $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ and $S(x_4, y_4)$ be four points which divide seg AB into five equal parts.

$\therefore$ BP = PQ = QR = RS = AS ... (i)

BS = BP + PQ + QR + RS [B - P - Q - R - S]

$\therefore$ BS = BP + BP +BP + BP ... [From (i)]

$\therefore$ BS = 4BP

$\therefore$ BS = 4AS [From (i)]

$\therefore$ $\dfrac{BS}{AS} = \dfrac{4}{1}$

$\therefore$ Point S divides seg BA in the ratio 4 : 1.

i.e. $m : n = 4 : 1$

By section formula,

$\therefore$ $x_4 = \dfrac{4\times 20+1\times 0}{4+1}$; $y_4 = \dfrac{4\times 10+1\times 20}{4+1}$

$\therefore$ $x_4 = \dfrac{80}{5}$; $y_4 = \dfrac{40+20}{5} = \dfrac{60}{5}$

$\therefore$ $x_4 = 16$; $y_4 = 12$

$\therefore$ $\boxed{\mathbf{S(16, 12)}}$

Q is midpoint of seg BS.

$\therefore$ $x_2 = \dfrac{0+16}{2}$; $y_2 = \dfrac{20+12}{2}$ [Midpoint formula]

$\therefore$ $x_2 = \dfrac{16}{2}$; $y_2 = \dfrac{32}{2}$

$\therefore$ $x_2 = 8$; $y_2 = 16$

$\therefore$ $\boxed{\mathbf{Q(8, 16)}}$

P is midpoint of seg BQ.

$\therefore$ $x_1 = \dfrac{0+8}{2}$; $y_1 = \dfrac{20+16}{2}$

$\therefore$ $x_1 = \dfrac{8}{2}$; $y_1 = \dfrac{36}{2}$

$\therefore$ $x_1 = 4$; $y_1 = 18$

$\therefore$ $\boxed{\mathbf{P(4, 18)}}$

R is midpoint of seg QS.

$\therefore$ $x_3 = \dfrac{8+16}{2}$; $y_3 = \dfrac{16+12}{2}$

$\therefore$ $x_3 = \dfrac{24}{2}$; $y_3 = \dfrac{28}{2}$

$\therefore$ $x_3 = 12$; $y_3 = 14$

$\therefore$ $\boxed{\mathbf{R(12, 14)}}$

$\therefore$ **Point P(4, 18), Q(8, 16), R(12, 14) and S(16, 12) divides seg AB into five equal parts.**

(3) Find the coordinates of the circumcentre and the radius of the circumcircle of ΔABC if A(2, 3), B(4, –1) and C(5, 2). *(4 marks)*

Solution :

Let point P(h, k) be the circumcentre of ΔABC.

$\therefore$ PA = PB = PC ... (i) [Radii of a circle]

$\therefore$ PA = PB ... [From (i)]

Using distance formula,

$$\sqrt{(h-2)^2+(k-3)^2} = \sqrt{(h-4)^2+(k+1)^2}$$

Squaring both sides

$\therefore$ $(h-2)^2 + (k-3)^2 = (h-4)^2 + (k+1)^2$

$\therefore$ $h^2 - 4h + 4 + k^2 - 6k + 9 = h^2 - 8h + 16 + k^2 + 2k + 1$

$\therefore$ $-4h - 6k + 13 = -8h + 2k + 17$

$\therefore$ $-4h + 8h - 6k - 2k = 17 - 13$

$\therefore$ $4h - 8k = 4$

$\therefore$ $h - 2k = 1$ ... (ii)

PA = PC [From (i)]

Using distance formula,

$(h-2)^2 + (k-3)^2 = (h-5)^2 + (k-2)^2$

$\therefore$ $h^2 - 4h + 4 + k^2 - 6k + 9 = h^2 - 10h + 25 + k^2 - 4k + 4$

$\therefore$ $-4h - 6k + 13 = -10h - 4k + 29$

$\therefore$ $-4h + 10h - 6k + 4k = 29 - 13$

$\therefore$ $6h - 2k = 16$ ... (iii)

Subtracting equation (ii) from equation (iii)

$$\begin{array}{rrcr} & 6h - 2k & = & 16 \\ - & h - 2k & = & 1 \\ & (-)\quad (+) & & (-) \\ \hline & 5h & = & 15 \end{array}$$

$\therefore$ $h = 3$

Substituting $h = 3$ in equation (ii)

$3 - 2k = 1$

$\therefore$ $-2k = 1 - 3$

$\therefore$ $-2k = -2$

$\therefore$ $k = 1$

$\therefore$ P(3, 1) is the centre of the circle.

Radius PA $= \sqrt{(2-3)^2+(3-1)^2}$ [Distance formula]

$= \sqrt{(-1)^2+(2)^2}$

$= \sqrt{1+4}$

$\therefore$ $\boxed{\textbf{Radius PA} = \sqrt{5}\ \textbf{unit}}$

(4) Point M(–3, 7) and N(–1, 6) divides segment AB into three equal parts. Find the coordinates of point A and point B. *(4 marks)*

Solution :

A	M	N	B
(x_1, y_1)	(–3, 7)	(–1, 6)	(x_2, y_2)

Let A(x_1, y_1), and B(x_2, y_2)

Points M and N divides seg AB into three equal parts.

$\therefore$ AM = MN = NB ... (i)

$\therefore$ AM = MN

$\therefore$ Point M is midpoint of seg AN.

$\therefore$ $-3 = \dfrac{x_1 + (-1)}{2}$ and $7 = \dfrac{y_1 + 6}{2}$ [Midpoint formula]

$\therefore$ $-6 = x_1 - 1$ and $14 = y_1 + 6$

$\therefore$ $x_1 = -6 + 1$ and $y_1 = 14 - 6$

$\therefore$ $x_1 = -5,\ y_1 = 8$

$\therefore$ $\boxed{\textbf{A(–5, 8)}}$

MN = NB ... [From (i)]

$\therefore$ Point N is midpoint of seg MB.

$\therefore$ $-1 = \dfrac{-3 + x_2}{2}$; $6 = \dfrac{7 + y_2}{2}$ [Midpoint formula]

$\therefore$ $-2 = -3 + x_2$; $12 = 7 + y_2$

$\therefore$ $-2 + 3 = x_2$; $12 - 7 = y_2$

$\therefore$ $x_2 = 1$; $y_2 = 5$

$\therefore$ $\boxed{\textbf{B(1, 5)}}$

(5) Segment AB is divided into four equal parts by points P, Q and R such that A - P - Q - R - B. If P(12, 9) and R(0, 11), then find the coordinates of point A, Q and B. *(4 marks)*

Solution :

A	P	Q	R	B
(x_1, y_1)	(12, 9)	(x_2, y_2)	(0, 11)	(x_3, y_3)

AP = PQ = QR = RB ... (i)

Let A(x_1, y_1), Q(x_2, y_2) and B(x_3, y_3)

Q is midpoint of seg PR. [From (i)]

$\therefore$ $x_2 = \dfrac{12 + 0}{2}$; $y_2 = \dfrac{9 + 11}{2}$ [Midpoint formula]

$\therefore$ $x_2 = \dfrac{12}{2}$; $y_2 = \dfrac{20}{2}$

$\therefore$ $x_2 = 6$; $y_2 = 10$

$\therefore$ $\boxed{\textbf{Q(6, 10)}}$

P is midpoint of seg AQ. [From (i)]

$\therefore$ $\dfrac{x_1 + x_2}{2} = 12$ and $\dfrac{y_1 + y_2}{2} = 9$

$\therefore$ $6 + x_1 = 24$ and $y_1 + 10 = 18$

$\therefore$ $x_1 = 24 - 6$ and $y_1 = 18 - 10$

$\therefore$ $x_1 = 18$ and $y_1 = 8$

$\therefore$ $\boxed{\textbf{A(18, 8)}}$

R is midpoint of seg BQ. [From (i)]

$\therefore \quad \dfrac{x_2 + x_3}{2} = 0$ and $\dfrac{y_2 + y_3}{2} = 11$

$\therefore \quad x_3 + 6 = 0 \times 2$ and $y_3 + 10 = 11 \times 2$

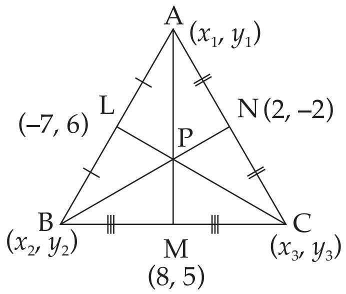
$\therefore \quad x_3 = 0 - 6$ and $y_3 = 22 - 10$

$\therefore \quad x_3 = -6$ and $y_3 = 12$

$\therefore \quad \boxed{\mathbf{B(-6, 12)}}$

(6) If (–7, 6), (8, 5) and (2, –2) are the midpoints of the sides of a triangle. Find the coordinates of its centroid. *(4 marks)*

Solution :



Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be three vertices of ΔABC.

L (–7, 6) is midpoint of seg AB.

M (8, 5) is midpoint of seg BC.

N (2, –2) is midpoint of seg AC.

Let $G(x, y)$ be centroid of ΔABC.

L is midpoint of seg AB.

$\therefore \quad \dfrac{x_1 + x_2}{2} = -7$ and $\dfrac{y_1 + y_2}{2} = 6$ [Midpoint formula]

$\therefore \quad x_1 + x_2 = -14$ and $y_1 + y_2 = 12$ ... (i)

M is midpoint of seg BC.

$\therefore \quad \dfrac{x_2 + x_3}{2} = 8$ and $\dfrac{y_2 + y_3}{2} = 5$ [Midpoint formula]

$\therefore \quad x_2 + x_3 = 16$ and $y_2 + y_3 = 10$ ... (ii)

N is midpoint of seg AC.

$\therefore \quad \dfrac{x_1 + x_3}{2} = 2$ and $\dfrac{y_1 + y_3}{2} = -2$ [Midpoint formula]

$\therefore \quad x_1 + x_3 = 4$ and $y_1 + y_3 = -4$ ... (iii)

Adding (i), (ii) and (iii)

$2x_1 + 2x_2 + 2x_3 = 6$ and $2y_1 + 2y_2 + 2y_3 = 18$

$\therefore \quad x_1 + x_2 + x_3 = 3$ and $y_1 + y_2 + y_3 = 9$ ... (iv)

$\therefore \quad$ G is centroid of ΔABC

$\therefore \quad x = \dfrac{x_1 + x_2 + x_3}{3}$ and $y = \dfrac{y_1 + y_2 + y_3}{3}$ [Centroid formula]

$\therefore \quad x = \dfrac{3}{3}$ and $y = \dfrac{9}{3}$

$\therefore \quad x = 1$ and $y = 3$

$\therefore \quad \boxed{\mathbf{G = (1, 3)}}$

❁ ❁ ❁

6. Trigonometry

(1) If $\sqrt{1 + x^2} \sin\theta = x$, prove that $\tan^2\theta + \cot^2\theta = x^2 + \dfrac{1}{x^2}$ *(3 marks)*

Proof :

$\sqrt{1 + x^2}\ \sin\theta = x$ ... [Given]

$\therefore \quad \sin\theta = \dfrac{x}{\sqrt{1 + x^2}}$ ... (i)

$\therefore \quad \sin^2\theta + \cos^2\theta = 1$ ... (Identity)

$\therefore \quad \dfrac{x^2}{1 + x^2} + \cos^2\theta = 1$ ... [From (i)]

$\therefore \quad \cos^2\theta = 1 - \dfrac{x^2}{1 + x^2}$

$\therefore \quad \cos^2\theta = \dfrac{1 + x^2 - x^2}{1 + x^2}$

$\therefore \quad \cos^2\theta = \dfrac{1}{1 + x^2}$ ... (ii)

$\tan^2\theta = \sin^2\theta \div \cos^2\theta$

$= \dfrac{x^2}{(1 + x^2)} \div \dfrac{1}{(1 + x^2)}$ [From (i) and (ii)]

$= \dfrac{x^2}{(1 + x^2)} \times (1 + x^2)$

$\tan^2\theta = x^2$ ... (iii)

$\therefore \quad \cot^2\theta = \dfrac{1}{\tan^2\theta}$

$\therefore \quad \cot^2\theta = \dfrac{1}{x^2}$ ... (iv) [From (iii)]

$\therefore \quad \tan^2\theta + \cot^2\theta = x^2 + \dfrac{1}{x^2}$ [Adding (iii) and (iv)]

$\therefore \quad \boxed{\mathbf{\tan^2\theta + \cot^2\theta = x^2 + \dfrac{1}{x^2}}}$

(2) Prove : $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta$ *(4 marks)*

Proof :

L.H.S. $= \dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta}$

$= \left[\dfrac{\sin\theta}{\cos\theta} \div \left(1 - \dfrac{\cos\theta}{\sin\theta}\right)\right] + \left[\dfrac{\cos\theta}{\sin\theta} \div \left(1 - \dfrac{\sin\theta}{\cos\theta}\right)\right]$

$= \left[\dfrac{\sin\theta}{\cos\theta} \div \left(\dfrac{\sin\theta - \cos\theta}{\sin\theta}\right)\right]$

$+ \left[\dfrac{\cos\theta}{\sin\theta} \div \left(\dfrac{\cos\theta - \sin\theta}{\cos\theta}\right)\right]$

$$= \left[\frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{(\sin\theta-\cos\theta)}\right] + \left[\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{(\cos\theta-\sin\theta)}\right]$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta\times(-1)(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta.\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{(\sin\theta-\cos\theta)\sin\theta.\cos\theta}$$

$$= \frac{\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta}{\sin\theta.\cos\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta.\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\sin\theta.\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}+1+\frac{\cos\theta}{\sin\theta}$$

L.H.S. $= \tan\theta + 1 + \cot\theta$

R.H.S. $= 1 + \tan\theta + \cot\theta$

$\therefore$ **L.H.S. = R.H.S.**

(3) Prove :

$\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

(3 marks)

Proof :

L.H.S. $= \sin^8\theta - \cos^8\theta$

$= (\sin^4\theta)^2 - (\cos^4\theta)^2$

$= (\sin^4\theta - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$

$= [(\sin^2\theta)^2 - (\cos^2\theta)^2][\sin^4\theta + \cos^4\theta]$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)[\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta - 2\sin^2\theta\cos^2\theta]$

$= 1(\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$[\because \sin^2A + \cos^2A = 1]$

$= (\sin^2\theta - \cos^2\theta)(1^2 - 2\sin^2\theta\cos^2\theta)$

L.H.S. $= (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

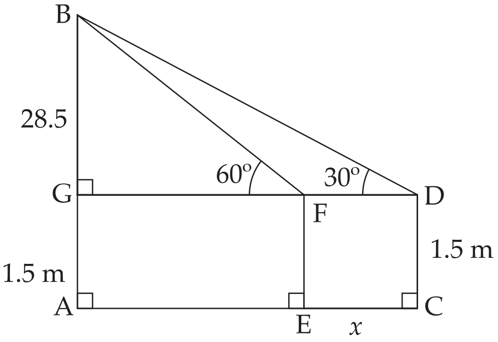
R.H.S. $= (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

$\therefore$ **L.H.S. = R.H.S.**

(4) A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

(4 marks)

Solution :



AB ⇒ Building of height 30 m

CD ⇒ First position of boy of height 1.5m

EF ⇒ Second position of boy,

Let the distance travelled be x m.

CD = EF = AG

$\therefore$ EF = AG = 1.5 m

BG = AB – AG ... [A - G - B]

$\therefore$ BG = 30 – 1.5 m = 28.5 m

In ΔBGF, $\angle$BGF = 90°

$\therefore \tan 60° = \frac{BG}{GF}$ [Definition]

$\therefore \sqrt{3} = \frac{28.5}{GF}$

$\therefore GF = \frac{28.5}{\sqrt{3}} = \frac{28.5\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$\therefore GF = 9.5\sqrt{3}$ m ... (i)

In ΔBGD, $\angle$BGD = 90°

$\therefore \tan 30° = \frac{BG}{GD}$

$\therefore \frac{1}{\sqrt{3}} = \frac{28.5}{GD}$

$\therefore GD = 28.5\sqrt{3}$ m ... (ii)

AC = GD, GF = AE ... (iii)

$\therefore AC = 28.5\sqrt{3}$, $AE = 9.5\sqrt{3}$

[From (i), (ii) and (iii)]

AC = AE + CE [A - E - C]

$\therefore 28.5\sqrt{3} = x + 9.5\sqrt{3}$

$\therefore x = 28.5\sqrt{3} - 9.5\sqrt{3}$

$\therefore x = 19\sqrt{3}$ m

$\therefore$ **Distance he walked towards the building is $19\sqrt{3}$ m.**

(4) Prove :

$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

(4 marks)

Proof :

L.H.S. $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + 2\sin A \,.\, \text{cosec } A + \text{cosec}^2 A$
$\quad + \cos^2 A + 2\cos A \sec A + \sec^2 A$

$= \sin^2 A + \cos^2 A + 2\sin A \dfrac{1}{\sin A} + 2\cos A \dfrac{1}{\cos A} + \text{cosec}^2 A + \sec^2 A$

$= 1 + 2 + 2 + (1 + \cot^2 A) + (1 + \tan^2 A)$

[$\because \sin^2\theta + \cos^2\theta = 1$,
$\text{cosec}^2\theta = 1 + \cot^2\theta$,
$\sec^2\theta = 1 + \tan^2\theta$]

$= 5 + 1 + \cot^2 A + 1 + \tan^2 A$

L.H.S. $= 7 + \cot^2 A + \tan^2 A$

R.H.S. $= 7 + \tan^2 A + \cot^2 A$

L.H.S. = R.H.S.

$\therefore$ $\boxed{(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A.}$

(5) Prove that :

$$\frac{1 - \sin\theta\cos\theta}{\cos\theta(\sec\theta - \text{cosec}\theta)} \times \frac{\sin^2\theta - \cos^2\theta}{\sin^3\theta + \cos^3\theta} = \sin\theta$$

(4 marks)

Proof :

L.H.S. $= \dfrac{1 - \sin\theta\cos\theta}{\cos\theta(\sec\theta - \text{cosec}\theta)} \times \dfrac{\sin^2\theta - \cos^2\theta}{\sin^3\theta + \cos^3\theta}$

$= \dfrac{1 - \sin\theta\cos\theta}{\cos\theta\left(\dfrac{1}{\cos\theta} - \dfrac{1}{\sin\theta}\right)} \times \dfrac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}$

$= \dfrac{(1 - \sin\theta\cos\theta)}{\cos\theta\left(\dfrac{\sin\theta - \cos\theta}{\sin\theta.\cos\theta}\right)} \times \dfrac{(\sin\theta - \cos\theta)}{(1 - \sin\theta.\cos\theta)}$

[$\because \sin^2 A + \cos^2 A = 1$]

$= \dfrac{1}{\dfrac{(\sin\theta - \cos\theta)}{\sin\theta}} \times (\sin\theta - \cos\theta)$

L.H.S. $= \sin\theta$

R.H.S. $= \sin\theta$

$\therefore$ $\boxed{\text{L.H.S.} = \text{R.H.S.}}$

(6) Prove that :

$$\frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = 2\text{cosec}A$$

(4 marks)

Proof :

L.H.S. $= \dfrac{\tan A}{\sec A - 1} + \dfrac{\tan A}{\sec A + 1}$

$= \tan A \left[\dfrac{1}{\sec A - 1} + \dfrac{1}{\sec A + 1}\right]$

$= \tan A \left[\dfrac{\sec A + 1 + \sec A - 1}{(\sec A - 1)(\sec A + 1)}\right]$

$= \tan A \times \dfrac{2\sec A}{(\sec^2 A - 1)}$

$= \tan A \times \dfrac{2\sec A}{\tan^2 A}$ [$\because 1 + \tan^2\theta = \sec^2\theta$, $\therefore \tan^2\theta = \sec^2\theta - 1$]

$= 2\sec A \div \tan A$

$= \dfrac{2}{\cos A} \div \dfrac{\sin A}{\cos A}$

$= \dfrac{2}{\cos A} \times \dfrac{\cos A}{\sin A}$

$= \dfrac{2}{\sin A}$

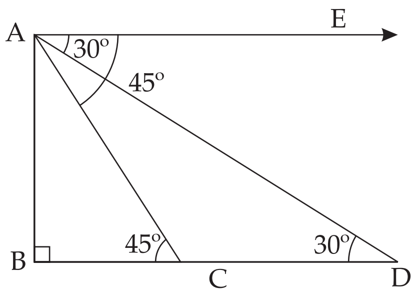
L.H.S. $= 2\text{cosec } A$

R.H.S. $= 2\text{cosec } A$

$\therefore$ $\boxed{\text{L.H.S.} = \text{R.H.S.}}$

(7) From the top of a light house, 80 metres high, two ships on the same side of light house are observed. The angles of depression of the ships as seen from the light house are found to be of 45° and 30°. Find the distance between the two ships. (Assume that the two ships and the bottom of the lighthouse are in a line) *(4 marks)*

Solution :



In the above figure, AB represents lighthouse of height 80 m. D and C are positions of two ships.

$\angle EAD$ and $\angle EAC$ are angles of depression.

$\angle EAD = 30^\circ$ and $\angle EAC = 45^\circ$ [Given]

$\therefore$ $\angle ADB = 30^\circ$ and $\angle ACB = 45^\circ$ ... (i)

[Alternate angles]

In ΔABD, $\angle B = 90^\circ$

$\therefore$ $\tan \angle ADB = \dfrac{AB}{BD}$ [Definition]

$\therefore$ $\tan 30^\circ = \dfrac{80}{BD}$ [From (i)]

$\therefore$ $\dfrac{1}{\sqrt{3}} = \dfrac{80}{BD}$

$\therefore$ $BD = 80\sqrt{3}$ m ... (ii)

In ΔABC, $\angle ABC = 90^\circ$

$\therefore$ $\tan \angle ACB = \dfrac{AB}{BC}$ [Definition]

$\therefore$ $\tan 45^\circ = \dfrac{80}{BC}$

$\therefore$ $1 = \dfrac{80}{BC}$

$\therefore$ BC = 80 m ... (iii)

$\therefore$ BD = BC + CD [B - C - D]

$\therefore$ $80\sqrt{3} = 80 + CD$

$\therefore$ $CD = 80\sqrt{3} - 80$

$\therefore$ $CD = 80(\sqrt{3} - 1)$ m

$\therefore$ **The distance between the two ships is $80(\sqrt{3} - 1)$ m.**

(8) If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$ *(4 marks)*

Proof :

$m = a\cos\theta + b\sin\theta$ ... [Given]

$\therefore$ $m^2 = (a\cos\theta + b\sin\theta)^2$

$\therefore$ $m^2 = a^2\cos^2\theta + 2ab\cos\theta\sin\theta + b^2\sin^2\theta$

$\therefore$ $m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta$ ... (i)

$n = a\sin\theta - b\sin\theta$

$\therefore$ $n^2 = (a\sin\theta - b\cos\theta)^2$

$\therefore$ $n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta$ ... (ii)

$\therefore$ $m^2 + n^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta + a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta$

$m^2 + n^2 = a^2\sin^2\theta + a^2\cos^2\theta + b^2\sin^2\theta + b^2\cos^2\theta$

$\therefore$ $m^2 + n^2 = a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$\therefore$ $\boxed{m^2 + n^2 = a^2 + b^2}$ $(\because \sin^2A + \cos^2A = 1]$

(9) If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$, where $\theta \neq 0$. *(4 marks)*

Solution :

$\sqrt{3}\tan\theta = 3\sin\theta$

$\therefore$ $\dfrac{\tan\theta}{\sin\theta} = \dfrac{3}{\sqrt{3}}$

$\therefore$ $\dfrac{\sin\theta}{\cos\theta} \div \sin\theta = \dfrac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

$\therefore$ $\dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\sin\theta} = \sqrt{3}$

$\therefore$ $\dfrac{1}{\cos\theta} = \sqrt{3}$

$\therefore$ $\cos\theta = \dfrac{1}{\sqrt{3}}$

$\therefore$ $\cos^2\theta = \dfrac{1}{3}$ ... (i)

$\sin^2\theta + \cos^2\theta = 1$ ... [From (ii)]

$\therefore$ $\sin^2\theta + \dfrac{1}{3} = 1$

$\therefore$ $\sin^2\theta = 1 - \dfrac{1}{3}$

$\therefore$ $\sin^2\theta = \dfrac{3-1}{3} = \dfrac{2}{3}$ ... (ii)

$\therefore$ $\sin^2\theta - \cos^2\theta = \dfrac{2}{3} - \dfrac{1}{3}$ [From (i) and (ii)]

$\therefore$ $\boxed{\sin^2\theta - \cos^2\theta = \dfrac{1}{3}}$

(10) Prove that :

$$\left(1+\frac{1}{\tan^2A}\right)\left(1+\frac{1}{\cot^2A}\right) = \frac{1}{\sin^2A - \sin^4A}$$

(4 marks)

Proof : :

$$\text{L.H.S.} = \left(1+\frac{1}{\tan^2A}\right)\left(1+\frac{1}{\cot^2A}\right)$$

$= (1 + \cot^2A)(1 + \tan^2A)$

$= \operatorname{cosec}^2A \,.\, \sec^2A$

$= \dfrac{1}{\sin^2A.\cos^2A}$

$= \dfrac{1}{\sin^2A(1-\sin^2A)}$ $[\because \sin^2A + \cos^2A = 1$ $\therefore \cos^2A = 1 - \sin^2A]$

$\text{L.H.S.} = \dfrac{1}{\sin^2A - \sin^4A}$

$\text{R.H.S.} = \dfrac{1}{\sin^2A - \sin^4A}$

$\therefore$ $\boxed{\textbf{L.H.S. = R.H.S.}}$

❁ ❁ ❁

7. Mensuration

(1) A tin maker converts a cubical metallic box into 10 cylindrical tins. Side of the cube is 50 cm and radius of the cylinder is 7 cm. Find the height of each cylinder so made if the wastage incurred was 12%. ($\pi = \dfrac{22}{7}$) *(4 marks)*

Solution :

Total surface area of cube $= 6l^2$ (Formula)

$= 6 \times 50 \times 50$

$= 15000 \text{ cm}^2$

Wastage incurred $= 12\%$ of 15000

$= \dfrac{12}{100} \times 15000$

$= 1800 \text{ cm}^2$

$\therefore$ Area of metal sheet used to make 10 cylindrical tins $= 15000 - 1800$

$= 13,200 \text{ cm}^2$

$\therefore$ Area of metal sheet used to make 1 cylindrical tin $= \dfrac{13200}{10} = 1{,}320 \text{ cm}^2$

$\therefore$ Area of metal sheet required to make 1 cylindrical tin $=$ Total surface area of a cylinder

$\therefore$ Total surface area of a cylinder $= 1{,}320$

$\therefore$ $2\pi r\,(r + h) = 1{,}320$

$\therefore$ $2 \times \dfrac{22}{7} \times 7 \times (7 + h) = 1{,}320$

$\therefore$ $7 + h = \dfrac{1320}{44}$

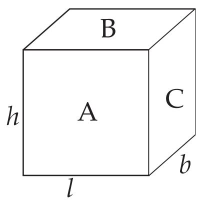
$\therefore$ $h = 30 - 7$

$\therefore$ $h = 23$ cm

$\therefore$ **Height of each cylinder = 23 cm**

(2) The three faces, A, B, C having a common vertex of a cuboid have areas 450 cm², 600 cm² and 300 cm² respectively. Find the volume of the cuboid. *(4 marks)*

Solution :



Area of surface A = 450 cm²

$\therefore$ $l \times h = 450 \text{ cm}^2$ ... (1)

Area of surface B = 600 cm²

$\therefore$ $l \times b = 600 \text{ cm}^2$ ... (2)

Area of surface C = 300 cm²

$\therefore$ $b \times h = 300 \text{ cm}^2$ ... (3)

Multiplying (1), (2) and (3),

$l^2 \times b^2 \times h^2 = 600 \times 450 \times 300$

$\therefore$ $l^2 \times b^2 \times h^2 = 300 \times 2 \times 450 \times 300$

$= 300 \times 900 \times 300$

$\therefore$ $l^2 \times b^2 \times h^2 = 300 \times 300 \times 30 \times 30$

$\therefore$ $l \times b \times h = 300 \times 30$ [Taking square roots]

$\therefore$ $l \times b \times h = 9000 \text{ cm}^3$ ... (4)

Volume of cuboid $= l \times b \times h$

$\therefore$ **Volume of cuboid = 9000 cm³**

(3) Oil tins of cuboidal shape are made from a metallic sheet with length 8 m and breadth 4 m. Each tin has dimensions 60 × 40 × 20 in cm and is open from the top. Find the number of such tins that can be made? *(4 marks)*

Solution :

Area of metallic sheet = 8 m × 4 m

$\therefore$ Area of metallic sheet = 800 cm × 400 cm ...(1)

Total surface area of a tin $= 2(l + b) \times h + l \times b$

$\therefore$ Total surface area of a tin

$= 2(60 + 40) \times 20 + 60 \times 40$

$= 2 \times 100 \times 20 + 2400$

$= 4000 + 2400$

Total surface area of a tin = 6400 cm² ... (2)

Number of tins can be made

$= \dfrac{\text{Area of metallic sheet}}{\text{Total surface area of open tin}}$

$= \dfrac{800 \times 400}{6400}$

$\therefore$ **Number of tins can be made = 50**

(4) Plastic drum of cylindrical shape is made by melting spherical solid plastic balls of radius 1 cm. Find the number of balls required to make a drum of thickness 2 cm, height 90 cm and outer radius 30 cm. *(4 marks)*

Solution :

For drum, thickness = 2 cm

Inner radius (r_i) = outer radius $(r_o) - 2$

$\therefore$ $r_i = 30 - 2 = 28$ cm

Inner height (h_i) = Outer height $(h_o) - 2$

$\therefore$ $h_i = 90 - 2 = 88$ cm

Volume of plastic (V_1) required for cylindrical drum = Volume of outer cylinder − Volume of inner cylinder

$= \pi r_0^2 h_0 - \pi r_i^2 h_i$

$= \pi[30^2 \times 90 - 28^2 \times 88]$

$= \pi[900 \times 90 - 784 \times 88]$

$= \pi[81{,}000 - 68{,}992]$

$\therefore$ $V_1 = 12008\pi \text{ cm}^3$ ... (1)

Volume of one plastic ball $(V_2) = \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times \pi \times 1^3$

$(V_2) = \dfrac{4}{3}\pi$ ... (2)

Number of plastic balls required to make the drum $= \dfrac{V_1}{V_2}$

$= \dfrac{12008\pi}{\frac{4\pi}{3}}$ [From (1) and (2)]

$= 12008 \times \dfrac{3}{4}$

$= 3002 \times 3$

$= 9006$

$\therefore$ **Number of plastic balls required to make the cylindrical drum is 9006.**

(5) Water drips from a tap at the rate of 4 drops in every 3 seconds. Volume of one drop of 0.4 cm³. If dripped water is collected in a cylinder vessel of height 7 cm and diameter is 8 cm. In what time vessel be completely filled? What is the volume of water collected? How many such vessels will be completely filled in 3 hours in 40 minutes? *(4 marks)*

Solution :

Volume of water collected

$= $ Volume of cylindrical vessel

$= \pi r^2 h$

$= \frac{22}{7} \times 4 \times 4 \times 7$

Volume of water collected $(V_1) = 352 \text{ cm}^3$

Volume of 1 drop of water $= 0.4 \text{ cm}^3$

Volume of 4 drops of water $= 4 \times 0.4 = 1.6 \text{ cm}^3$

4 drops drips in 3 seconds

$\therefore$ Volume of water dripped in 3 seconds $= 1.6 \text{ cm}^3$

$\therefore$ Volume of water dripped in 1 seconds (V_2)

$= \frac{1.6}{3} \text{ cm}^3$

$\therefore$ Volume of time required to fill the cylindrical vessel $= \frac{V_1}{V_2}$

$= 352 \div \frac{1.6}{3}$

$= 352 \times \frac{3}{16} \times 10$

$= 660$ seconds

$= 11$ minutes

3 hours and 40 minutes $= (3 \times 60 + 40)$ min

$= 220$ min.

Number of vessels that can be completely filled $= \frac{220}{11}$

$= 20$

$\therefore$ **20 vessels can be filled in 3 hours and 40 minutes.**

(6) A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights. *(3 marks)*

Solution :

As cone and hemisphere have equal bases and they have equal radii.

Let the radius of each be r

For cone height $= h_1$

For hemisphere height $(h_2) = r$ ... (1)

Volume of cone = Volume of hemisphere (Given)

$\therefore \frac{1}{3} \pi r^2 h_1 = \frac{2}{3} \pi r^3$

$\therefore \frac{1}{3} \pi r^2 h_1 = \frac{2}{3} \pi r^2 \times r$

$\therefore \frac{1}{3} \pi r^2 \times h_1 = \frac{2}{3} \pi r^2 \times h_2$ [From (2)]

$\therefore \frac{h_1}{h_2} = \frac{2}{3} \pi r^2 \times \frac{3}{1} \times \frac{1}{\pi r^2}$

$\therefore \frac{h_1}{h_2} = \frac{2}{1}$

$\therefore$ **Ratio of heights of cone and hemisphere = 2 : 1.**

(7) A sphere and a cube have the same surface area. Show that the ratio of the volume of the sphere to that of cube is $\sqrt{6} : \sqrt{\pi}$. *(4 marks)*

Solution :

Surface area of sphere $= 4\pi r^2$ ... (1)

Surface area of cube $= 6\, l^2$ ... (2)

Surface area of sphere = Surface area of cube (Given)

$\therefore 4\pi r^2 = 6l^2$ [From (1) and (2)]

$\therefore \frac{r^2}{l^2} = \frac{6}{4\pi}$

$\therefore \frac{r}{l} = \frac{\sqrt{6}}{2\sqrt{\pi}}$ ... [Taking square roots]

$\therefore \frac{r^3}{l^3} = \frac{\sqrt{6} \times \sqrt{6} \times \sqrt{6}}{2 \times 2 \times 2 \times \sqrt{\pi} \times \sqrt{\pi} \times \sqrt{\pi}}$

$\therefore \frac{r^3}{l^3} = \frac{6\sqrt{6}}{8\pi\sqrt{\pi}}$ ... (3)

Volume of sphere $= \frac{4}{3} \pi r^3$

Volume of cube $= l^3$

$\frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{4\pi r^3}{3 \times l^3}$

$= \frac{4}{3} \times \pi \times \frac{3\sqrt{6}}{4\pi\sqrt{\pi}}$ [From (3)]

$= \frac{\sqrt{6}}{\sqrt{\pi}}$

$\therefore$ **Ratio of volume of sphere and cube is $\sqrt{6} : \sqrt{\pi}$.**

(8) ₹ 5 coins were made by melting a solid cuboidal block of metal with dimensions 16 × 11 × 10 in cm. How many coins of thickness 2 mm and diameter 2 cm can be made. ($\pi = \frac{22}{7}$) *(3 marks)*

Solution :

For cylindrical coin,

height (h) = 2 mm = $\frac{2}{10}$ cm

Diameter = 2 cm $\therefore$ Radius (r) = 1 cm

For cuboidal block,

$l_1 = 16$ cm, $b_1 = 11$ cm and $h_1 = 10$ cm

Number of coins can be made

$$= \frac{\text{Volume of cuboid}}{\text{Volume of a coin}}$$

$$= \frac{l_1 \times b_1 \times h_1}{\pi r^2 h}$$

$$= \frac{16 \times 11 \times 10}{\frac{22}{7} \times 1^2 \times \frac{2}{10}}$$

$$= \frac{7 \times 16 \times 11 \times 10 \times 10}{22 \times 2}$$

$\therefore$ $\boxed{\textbf{Number of coins made = 2800}}$

(9) If the radius of a sphere is doubled, what will be the ratio of its surface area and volume as to that of the first sphere? *(4 marks)*

Solution :

Let r_1 be the radius of first sphere

and r_2 be the radius of second sphere.

$r_2 = 2 \times r_1$ ... (1) [Given]

Let S_1 and S_2 be the surface areas of first and second sphere.

$\therefore$ $S_1 = 4\pi r_1^2$ ... (2)

$\therefore$ $S_2 = 4\pi r_2^2$

$\therefore$ $S_2 = 4\pi \times (2r_1)^2$

$\therefore$ $S_2 = 4\pi \times 4 \times r_1^2$

$\therefore$ $S_2 = 4 \times 4\pi r_1^2$

$\therefore$ $S_2 = 4 \times S_1$ [From (2)]

$\therefore$ $\frac{S_2}{S_1} = 4$

Let V_1 and V_2 be the volumes of first and second sphere respectively.

$\therefore$ $V_1 = \frac{4}{3}\pi r_1^3$ ... (3)

$\therefore$ $V_2 = \frac{4}{3}\pi r_2^3$

$\therefore$ $V_2 = \frac{4}{3}\pi (2r_1)^3$ [From (1)]

$\therefore$ $V_2 = \frac{4}{3} \times \pi \times 8 \times r_1^3$

$\therefore$ $V_2 = 8 \times \frac{4}{3}\pi r_1^3$

$\therefore$ $V_2 = 8 \times V_1$ [From (3)]

$\therefore$ $\frac{V_2}{V_1} = 8$

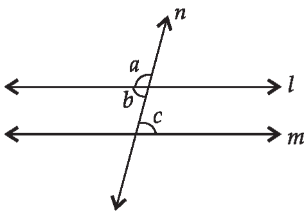
$\therefore$ **Ratio of surface area is 4 : 1 and the ratio of volume is 8 : 1.**

❁❁❁

Model Activity Sheet – 1

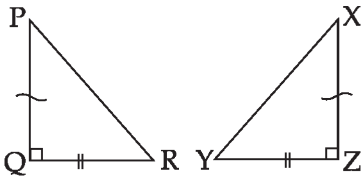
Time : 2 Hrs. **Marks : 40**

Q.1. (A) Solve the following questions. (Any 4) **(4)**

(1) 

In the adjoining figure, line l || line m and line n is the transversal. $\angle a = 100°$. Find measure of $\angle c$

(2) Write the converse of the statement. 'The diagonals of a rectangle are congruent'. Is the converse statement true?

(3) 

'Δ PQR ≅ Δ XYZ. [Hypotenuse side test]'
With respect to adjoining figure
Is the above statement true? If no, then correct it.

(4) 'Two pairs of sides of which of the following quadrilaterals are equal?
Kite, Isosceles trapezium, Rectangle.

(5) A line is parallel to X axis is at a distance of 4 units from X- axis. Write possible equations for this line.

(6) Find $\tan\theta$ if $\sin\theta = \frac{4}{5}$ and $5 \times \cos\theta = 3$.

Q.1. (B) Solve the following: (Any 2) **(4)**

(1) Total surface area of a cuboid is 400 cm. Height of the cuboid is 20 cm. Find the perimeter of the base of the cuboid.

(2) Draw an equilateral Δ ABC with side measuring 5 cm. Find its incentre.

(3) In ΔPQR, $\angle P = 40°$, $\angle R = \angle P + 10°$. State the longest side of Δ PQR giving reason.

Q.2. (A) Choose the correct alternative: **(4)**

(1) Out of the dates given below which date constitutes a pythagorean triplet?
(A) 15/08/17 (B) 16/08/17 (C) 3/5/17 (D) 4/9/15

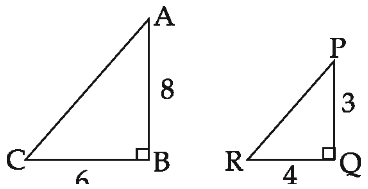
(2) sin 35 × cos 55 = ...
(A) Not possible to find (B) tan 55 (C) cot 35 (D) 1

(3) If $A = r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$ the A in the formula is ....
(A) Length of an arc (B) Area of circle (C) Area of sector (D) Area of a segment

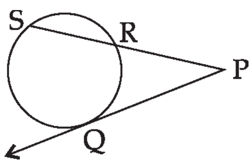
(4) Slope of a line parallel to X axis is
(A) 1 (B) 0 (C) Not defined (D) None of the above.

(B) Solve the following: (Any 2) **(4)**

(1) A circle with centre 'O' and radius 12 cm has a chord AB. $\angle AOB = 30°$. Find A(ΔAOB).

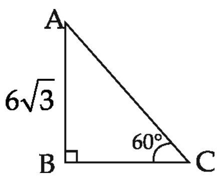
(2) 

Using the information in the adjoining figure, prove $\Delta ABC \sim \Delta PQR$.

(3) 

In the adjoining figure, ray PQ touches the circle at point Q. Line PRS is a secant, If PQ = 12, PR = 8 then find PS and RS

Q.3. (A) Complete the following activities: (Any 2) **(4)**

(1) 

Using the information in the given diagram, complete the following activity.

In ΔABC, $\angle A + \angle B + \angle C = \square$

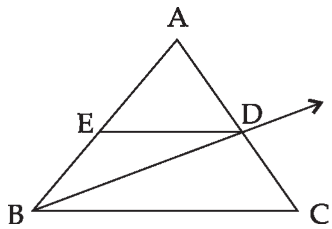
$\therefore$ $\angle A = \square - \square = \square$

$\therefore$ ΔABC is .... – .... – .... triangle.

Applying .... theorem

$AB = \dfrac{\square}{\square} \times AC$

$\therefore$ $AC = \square$ unit.

(2) 

In ΔABC, ray BD bisects $\angle ABC$, A – D – C, Side DE II Side BC.

A – E – B. then prove that

AB : BC = AE : EB

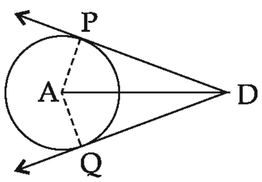
Proof: In ΔABC, ray BD bisects $\angle B$

$\therefore$ $\dfrac{AB}{\square} = \dfrac{AD}{DC}$ ..... (1) [Angle bisector property]

In ΔABC, Side DE II Side BC

$\therefore$ $\dfrac{AE}{\square} = \dfrac{AD}{DC}$ ..... (2) [By Basic Proportionality theorem]

$\therefore$ $\dfrac{AB}{\square} = \dfrac{AE}{\square}$ .... From (1) and (2)

(3) 

In the adjoining figure, A is the centre of the circle. Point D is in the exterior of the circle. Line DP and Line DQ are tangents at points P and Q respectively. Prove that DP = DQ.

Proof:

In ΔPAD and ΔQAD

Seg PA $\cong$ Seg $\square$ [Radii of the same circle]

	Seg AD ≅ Seg AD	[$\boxed{\qquad\qquad}$]
	$\angle$APD = $\angle$AQD = $\boxed{\quad}$	[Tangent Theorem]
∴	ΔPAD ≅ ΔQAD	[$\boxed{\qquad\qquad}$]
∴	Seg DP ≅ Seg DQ	[$\boxed{\qquad\qquad}$]

Q.3 (B) Solve the following questions: (Any 2) **(4)**

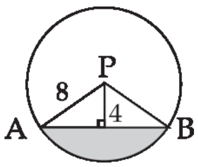
(1) Draw a circle with centre P and radius 3.5 cm. Take a point A on it. Draw a tangent at point A.

(2) Find the coordinates of centroid of a triangle whose vertices are (3, –5), (4, 3), (11, –4).

(3) If $\tan\theta + \frac{1}{\tan\theta} = 2$, then show that $\tan^2\theta + \frac{1}{\tan^2\theta} = 2$.

Q.4. Solve the following questions: (Any 3) **(9)**

(1) A person is standing at a distance of 80 m from a church looking at its top. The angle of elevation is of 45°. Find the height of the church.

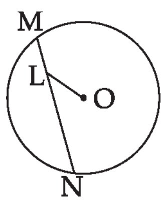
(2) 

In the adjoining figure, seg AB is a chord of a circle with centre P. If PA = 8 cm and distance of the chord from the centre P is 4 cm. Find the area of the shaded portion.

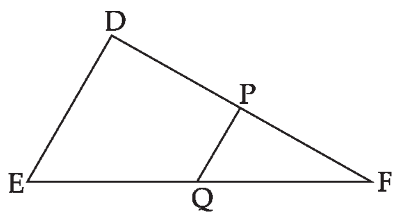
(3) ΔABC ~ ΔPQR, in ΔABC, AB = 5.4 cm, BC = 4.2 cm, AC = 6.0 cm, AB : PQ = 3 : 2. Construct ΔABC and ΔPQR

(4) Prove that, the sum of the squares of the diagonals of a rhombus is equal to the sum of the squares of the sides.

Q.5. Solve the following questions: (Any 1) **(4)**

(1) 

Seg MN is a chord of a circle with centre O. MN = 25, L is a point on chord MN such that ML = 9 and *d*(O, L) = 5. Find the radius of the circle.

(2) 

In the adjoining figure, seg PQ || seg DE, A (ΔPQF) = 20 units, PF = 2 DP, then find A(□DPQE).

Q.6. Solve the following questions: (Any 1) **(3)**

(1) If two lines are perpendicular to each other then product of its slopes is '–1'. Find the slope of a line which is perpendicular to line AB if A (3, 0) and B(0, 2).

(2) Volume of a solid with uniform cross-sectional area is given by area of base × height. Find the volume of a hexagonal prism whose base has each side measuring 6 cm. and height is $\frac{4}{\sqrt{3}}$ cm.

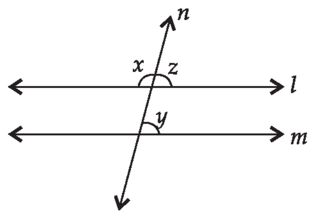
(Hint: Area of regular hexagon = $\frac{3\sqrt{3}}{2} \times \text{side}^2$)

❖ ❖ ❖

MODEL ACTIVITY SHEET – 2

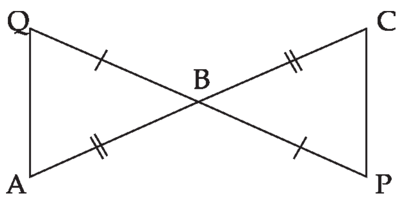
Time : 2 Hrs. **Marks : 40**

Q.1. (A) Solve the following questions. (Any 4) **(4)**

(1) 

In the adjoining figure, line n is transversal of line l and line m $\angle x = 140°$, $\angle y = 40°$. Is line $l \parallel$ line m ?

(2) Write the converse of the statement. 'Diagonals of a rhombus bisects the opposite angles'. Is the converse statement true?

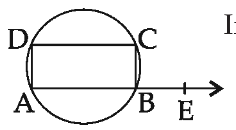
(3) 

Using the given information,

prove Δ ABQ $\cong$ Δ PBC

(4) State the equation of (a) X- axis (b) Y - axis

(5) Find the value of 2 sin 45 . cos 45

(6) 

If $\angle ADC = 80°$ then $\angle CBE$ = ?

Q.1. (B) Solve the following questions: (Any 2) **(4)**

(1) Area of a circle is 154 cm^2. Find its circumcircle.

(2) Draw a Δ ABC such that AC = 10 cm, AB = 6 cm and BC = 8 cm. Find its circumcentre.

(3) In ΔABC, points M and N are midpoints of side AB and side AC respectively, If MN = 5 cm, then find BC.

Q.2. (A) Choose the correct alternative: **(4)**

(1) In ΔABC, AB = $6\sqrt{3}$, AC = 12 cm, BC = 6 cm. Find the measure of $\angle$A

(A) 30 (B) 60 (C) 90 (D) 45

(2) If $\cot \theta = \frac{10}{24}$, then $24 \cos \theta - 10 \sin \theta$ = ?

(A) $\frac{26}{10}$ (B) $\frac{10}{26}$ (C) 0 (D) 1

(3) The perimeter of a sector of a circle with its measure 90° and radius 7 cm is ....

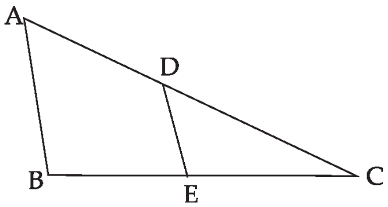
(A) 44 cm (B) 25 cm (C) 36 cm (D) 56 cm

(4) Slope of a line parallel to Y axis is

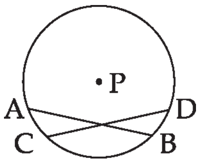
(A) zero (B) cannot be determined (C) Positive (D) Negative.

(B) Solve the following questions: (Any 2) **(4)**

(1) The area of a minor sector of a circle is 3.85 cm^2 and the measure of its central angle is 36°. Find the radius of the circle.

(2) 

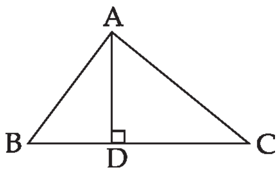
In the adjoining figure, seg DE $\parallel$ side AB. AD = 5, DC = 3 and BC = 6.4. Find BE.

(3) 

In a circle with centre P, chord AB $\cong$ chord CD.

Prove that arc AC $\cong$ arc BD

Q.3. (A) Complete the following activities: (Any 2) **(4)**

(1) 

In ΔABC, seg AD $\perp$ seg BC, Prove that $AB^2 + CD^2 = AC^2 + BD^2$

Proof: In ΔADC, $\angle ADC = 90°$

$\therefore$ $\boxed{} = \boxed{} + \boxed{}$ [Pythagoras Theorem)

$\therefore$ $AD^2 = \boxed{} - \boxed{}$ .....(1)

In ΔADB, $\angle ADB = 90°$

$\therefore$ $\boxed{} = \boxed{} + \boxed{}$ [Pythagoras Theorem)

$\therefore$ $AD^2 = \boxed{} - \boxed{}$ .....(2)

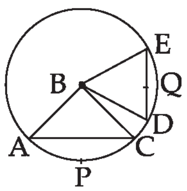
$\therefore$ $\boxed{} - \boxed{} = \boxed{} - \boxed{}$

$\therefore$ $AB^2 + CD^2 = AC^2 + BD^2$

(2) To prove that "The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines". Complete the following instructions.

(a) Draw 3 parallel lines and 2 transversals. Name these 5 lines. Also name 6 points of intersection.

(b) Write 'given' and 'to prove' from the figure drawn.

(3) 

Given : (1) A circle with centre B.

(2) Arc APC $\cong$ Arc DQE

To Prove: chord AC $\cong$ chord DE

Complete the following activity for the proof.

Proof: In ΔABC and ΔDBE

side AB $\cong$ side DB [$\boxed{}$]

side $\boxed{}$ $\cong$ side $\boxed{}$ [$\boxed{}$]

$\angle ABC \cong \angle DBE$ [Measures of congruent arcs]

$\therefore$ $\Delta ABC \cong \Delta DBE$ [$\boxed{}$ Test]

chord AC $\cong$ chord DE [$\boxed{}$]

(B) Solve the following questions: (Any 2) **(4)**

(1) Draw a circle of radius 3.6 cm. Draw a tangent to the circle at any point on it without using the centre.

(2) Find k, if R(1, –1), S(–2, k) and slope of line RS is –2.

(3) Eliminate θ if $a \sin\theta = x$ and $b \cos\theta = y$

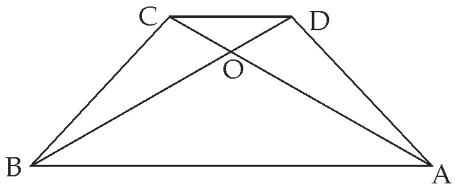
Q.4. Solve the following questions: (Any 3) **(9)**

(1) A storm broke a tree and the treetop rested 20 m from the base of the tree, making an angle of 60° with the horizontal. Find the height of the tree.

(2) The dimensions of a cuboid are 44 cm, 21 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base.

(3) $\Delta ATM \sim \Delta AHE$. In ΔAMT, AM = 6.3 cm, $\angle TAM = 50°$, and AT = 5.6 cm. $\frac{AM}{AH} = \frac{7}{5}$. Construct ΔAHE.

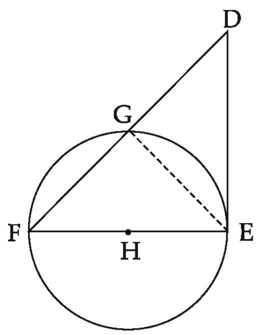
(4)



In trapezium ABCD, side AB || side DC. Diagonals AC and BD meet in O. If AB = 20, DC = 6, OB = 15. Find OD.

Q.5. Solve the following questions: (Any 1) **(4)**

(1)

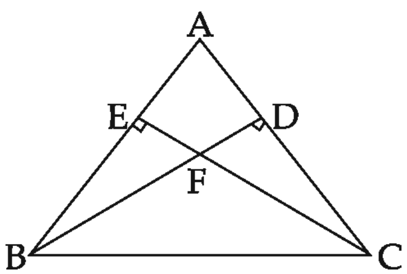


In the adjoining figure, seg EF is the diameter of the circle with centre H. Line DE is tangent at point E. If r is the radius of the circle, then prove that $DE \times GE = 4r^2$

(2) State and prove 'Pythagoras Theorem'.

Q.6. Solve the following questions: (Any 1) **(3)**

(2)

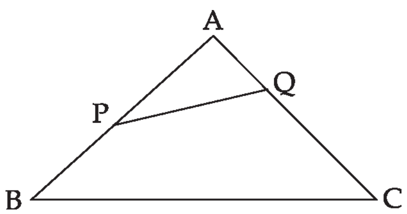


In the adjoining diagras, seg BD and seg CE are altitudes.

Prove that (i) □AEFD is cyclic quadrilateral

(ii) Points B, E, D, C are non-cyclic points.

(3)



A line cuts two sides AB and AC of ΔABC in points P and Q respectively.

Show that $\frac{A(\Delta APQ)}{A(\Delta ABC)} = \frac{AP \times AQ}{AB \times AC}$

❖ ❖ ❖