Class => XIIth O.P Malhotra Chater - 1

Relations

Exercise 1 :-

Ans 1 => a) Given S be the given relation "greater than" for the set of real numbers.

Hence $(a, b) \in S$

$\Rightarrow a > b \ \forall \ a, b \in R$

Reflexive :

$a \not> a \ \forall \ a \in R$ thus,

$(a, a) \notin S$

Hence S is not a reflexive relation.

Symmetric:

$\forall \ a, b \in R$

So that $(a, b) \in S \Rightarrow a > b$

$\Rightarrow b < a$

i.e. $b \not> a$ i.e $(b, a) \notin S$

Thus S is not a symmetric relation.

Transitive :

let $(a, b) \in S$ and $(b, c) \in S$

$\forall \ a, b, c \in R$

$\Rightarrow$ a is greater than and b is greater than c
$\Rightarrow$ $a > c$ $\Rightarrow$ $(a, c) \in S$
Thus S is transitive.

Hence, Ans(T)

(b) Given the relation S be "is the cube of" for the set of all real numbers.
Since the 1 is the cube of 1, hence $(1,1) \in S$
$2^3 = 8 \neq 2$ Hence $(2, 2) \notin S$
$\therefore$ S is not reflexive
Since $(8, 2) \in S$ as $8 = (2)^3$
but $(2, 8) \notin S$ since 2 is not equal to 8^3
Thus S is not a symmetric relation.

As $(a, b) \in S$ and $(b, c) \in S$ need not imply that $(a, c) \in S$
Since $(512, 8) \in S$ since $512 = 8^3$ and
$(8, 2) \in S$ since $8 = 2^3$
but $(512, 2) \notin S$ since $512 \neq 2^3$
Thus S is not a transitive relation.

(c) Given that $S = \{(a, b) : a$ is sister of $b\}$
Reflexive:
Since a can not be the sister of itself
Hence $(a, a) \notin S$ $\therefore$ S is not a reflexive relation.

Symmetric :-

let $(a,b) \in S$.
i.e. a is a sister of b.
Then b may not be a sister of a in case of be be a boy. Thus (b,a) may or may not be related to a i.e. $(b,a) \notin R$
Hence S is not symmetric relation.

Transitive :-

let $(a,b) \in S$
$\Rightarrow$ a be a sister of b and $(b,c) \in S$
$\Rightarrow$ b be the sister of c
$\Rightarrow$ a be a sister of c
$\Rightarrow (a,c) \in S$
Thus, the S is a transitive relation. Hence, Ans(T)

(d) $S = \{(a,b) : a$ is similar to $b\}$
Since the every triangle is similar to itself
Hence, $(a,a) \in S$
Thus S is symmetric relation
~~Ref~~ If $(a,b) \in S \Rightarrow$ a is similar to triangle b
Then triangle b is similar to triangle a

$\Rightarrow (b, a) \in R$, Thus R is symmetric.

Let $(a, b) \in R$, $(b, c) \in R$

i.e. triangle a is similar to triangle b and triangle b is similar to triangle c. Then triangle a is similar to triangle c

Hence $(a, c) \in R$

Thus R is transitive on set of triangles.

Hence, Ans R, S, T.

(e) Let L be the set of coplanar lines & relation

$R = \{(a, b) : a \perp b,\ a, b \in L\}$

Reflexive: Since a line can't be $\perp$ to itself.

Hence $(a, a) \notin R \Rightarrow R$ is not reflexive

Symmetric:

Let $(a, b) \in R \ \forall\ a, b \in L$

$\Rightarrow a \perp b \Rightarrow b \perp a \Rightarrow (b, a) \in R$

Hence, R is a symmetric relation.

Transitive:

Let $(a, b) \in R$, $(b, c) \in R \ \forall\ a, b, c \in L$

Now $(a, b) \in R \Rightarrow a \perp b$

and $(b, c) \in R \Rightarrow b \perp c$

Then a is not perpendicular to c so a is parallel to c

Thus, $(a,c) \notin R$ $\therefore$ R is not transitive relation

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Sol 2 ⇒ (a) let $R = \{(a,b) : a > b,\ a, b \in R\}$

Since $a \not> a$ Hence, $(a,a) \notin R$ or R is not reflexive

let $(a,b) \in R\ \forall\ a,b \in R$

$\Rightarrow a > b \Rightarrow b \not> a \Rightarrow (b,a) \notin R$

Hence, R is not symmetric

let $(a,b) \in R, (b,c) \in R\ \forall\ a,b,c \in R$

Since $(a,b) \in R \Rightarrow a > b$ —(1)

$(b,c) \in R \Rightarrow b > c$ —(2)

Hence $a > c$ by using (1) and (2)

$\Rightarrow (a,c) \in R$ Hence R is transitive.

(b) let $R = \{(a,b) ;$ a is perpendicular to b; $a, b \in L\}$

where L be the set of coplaner lines.

Since a line can't be $\perp$ to itself

$\therefore (a,a) \in R$ Hence R is reflexive.

Hence $(a,b) \in R\ \forall\ a,b \in L$.

$\Rightarrow a \perp b \Rightarrow b \perp a$

$\therefore (b,a) \in R$

Thus R is symmetric.

Let $(a, b) \in R, (b, c) \in R \ \forall a, b, c \in L$
Since $(a, b) \in R \Rightarrow a \perp b$
& $(b, c) \in R \Rightarrow b \perp c$

Thus a is parallel to c. Thus $(a, c) \notin R$
Thus R is symmetric only.

(c) Let $R = \{(a, b) ; a$ multiple of $b, a, b \in R\}$
Clearly a is a multiple of $I\,a$ as $a = a.I$
Hence $(a, a) \in R \Rightarrow R$ is reflexive.
Let $(a, b) \in R \Rightarrow a$ is multiple b
$\Rightarrow a = \lambda b$ where $\lambda \in I$
Clearly b is a factor of $I\,a$ i.e. not a multiple of a. Hence $(b, a) \notin R$
Thus R is not symmetric.
Let $(a, b) \in R \Rightarrow (b, c) \in R$.
Since $(a, b) \in R \Rightarrow a$ is a multiple of b
$\Rightarrow a = \lambda b$ and $(b, c) \in R$
$\Rightarrow b$ is a multiple of c where $\lambda \in I$
$\Rightarrow b = \mu c$ where $\mu \in I$
$\Rightarrow a = \lambda \mu c \Rightarrow a = \lambda' c$ where $\lambda' = \lambda \mu \in I$
Thus a be a multiple of $c \Rightarrow (a, c) \in R$
Therefore R is transitive relation.

(d) $R = \{(a,b);$ a is a friend of b$\}$

Since a must be a friend of itself

Hence $(a,a) \in R$

Hence, R is reflexive

let $(a,b) \in R \Rightarrow$ a is a friend of b

$\Rightarrow$ b is a friend of a $\Rightarrow (b,a) \in R$

Thus R is symmetric

let $(a,b) \in R \Rightarrow$ a is a friend of b

$\Rightarrow$ b is a friend of a $\Rightarrow (b,a) \in R$

Thus R is symmetric relation.

let $(a,b) \in R \Rightarrow$ a is a friend of b

$(b,c) \in R \Rightarrow$ b is a friend of c

Then a may not be a friend of c

$\Rightarrow (a,c) \notin R$ (always)

Hence, R is not transitive.

Aliter: let $A = \{1, 2, 3\}$ and relation R on a given by $\{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3)(3,1)\}$

Clearly $(1,1),(2,2),(3,3) \in R$

$\Rightarrow$ R is reflexive

$(1,3),(3,1),(2,1),(1,2) \in R$

$\Rightarrow$ R is symmetric.

i.e if $(a,b) \in R$ then $(b,a) \in R$

Now $(2,1),(1,3) \in R$ but $(2,3) \notin R$

i.e $(a,b), (b,c) \in R$ but $(a,c) \notin R$

$\Rightarrow R$ is not transitive relation.

Sol 3 ⇒ Given that if A is the set of members of a family & R means "is brother of"

$\forall a \in A$

i.e. a can't be a brother of itself

$\Rightarrow (a,a) \notin R \Rightarrow R$ is not reflexive.

$\Rightarrow \forall a, b \in A$ s.t $(a,b) \in R \Rightarrow$ a is a brother of b. Then b may not be a brother of a (in case when b is female).

$\Rightarrow (b,a) \notin R$

$\Rightarrow R$ is not symmetric on A.

$\forall a, b, c \in A$ show that $(a,b) \in A$, $(b,c) \in A$

Now $(a,b) \in A \Rightarrow$ a is brother of b

$(b,c) \in A \Rightarrow$ b is a brother of c

Thus a is a brother of c $\Rightarrow (a,c) \in R$

Therefore, R is a transitive on A.

Sol 4 ⇒ Let $A = \{1, 2, 3\}$ and relation R on A = $\{(1,2), (2,1)\}$

Clearly $(1,1), (2,2), (3,3) \notin R$

Hence, R is not reflexive relation

Now $(1,2) \in R$ ($\forall a, b \in A$) and $(2,1) \in R$.

i.e, $(a, b) \in R \;\forall\; a, b \in A$

Then $(b, a) \in R$ ∴ R is symmetric on A.

Clearly $(1, 2), (2, 1) \in R$ but $(1,1) \notin R$

∴ R is transitive on A.

Sol 5 ⇒ Let $A = \{1, 2, 3, 4\}$ and R be a relation on A is given by

$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$

Clearly $(1,1), (2,2), (3,3), (4,4) \notin R$

⇒ R is reflexive on A.

Since $(1, 2) \in R$ but $(2,1) \notin R$

∴ R is not symmetric on A.

Clearly,

$(1, 2), (2, 2) \in R \Rightarrow (1, 2) \in R$

$(1, 1), (1, 2) \in R \Rightarrow (1, 2) \in R$

$(1, 1), (1, 3) \in R \Rightarrow (1, 3) \in R$

$(1, 3), (3, 3) \in R \Rightarrow (1, 3) \in R$

$(1, 3), (3, 2) \in R \Rightarrow (1, 2) \in R$

$(3, 2), (2, 2) \in R \Rightarrow (3, 2) \in R$

$\forall$ a, b, c $\in$ A show that (a, b) $\in$ R, (b, c) $\in$ R

Then (a, c) $\in$ R

Then R is transitive on A.

Hence, R is reflexive and transitive but not a symmetric.

Sol 6 => Let A be the set of real numbers and $R = \{(a, b) : a \leq b\}$

Reflexive: $\forall$ $a \in R$, $a \leq a$

$\Rightarrow (a, a) \in R \Rightarrow R$ is reflexive.

Symmetric:

Let $(a, b) \in$ $\forall$ $a, b \in A$

$\Rightarrow a \leq b \Rightarrow b \not\leq a \Rightarrow (b, a) \notin R$

$\therefore$ R is not symmetric on A.

Transitive:

$\forall$ a, b, c $\in$ A show that (a, b) $\in$ R, (b, c) $\in$ R.

Since $(a, b) \in R \Rightarrow a \leq b$ — (1)

and $(b, c) \in R \Rightarrow b \leq c$ — (2)

$\Rightarrow a \leq c$ by using (1) and (2)

$\Rightarrow (a, c) \in R$

Thus, R is transitive on A.

Hence, R is reflexive and transitive but not symmetric.

Sol 7 ⇒ Given relation R on A set of real numbers is given by $R = \{(a,b) : a \leq b^2\}$

since $\frac{1}{2} \in A$ but $\frac{1}{2} \nleq \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

since $\frac{1}{4} < \frac{1}{2}$ $\therefore \left(\frac{1}{2}, \frac{1}{2}\right] \notin R$

Hence R is not reflexive.

since $(a,a) \notin R$ $\forall$ $a \notin A$

Symmetric :

let $(a,b) \in R$ $\forall$ $a,b \in A$ it need not imply that $(b,a) \in R$

e.g. $(1,2) \in R$ since $1 \leq 2^2 = 4$

but $(2,1) \notin R$ since $2 < 1^2 = 1$

Thus, R is not symmetric on A.

Transitive : $(a,b),(b,c) \in R$ it does not imply $(a,c) \in R$

e.g $(40,7) \in R$ since $40 \leq 7^2 = 49$

$(7,3) \in R$ since $7 \leq 3^2 = 9$

but $(40,3) \notin R$ since $40 \nleq 3^2 = 9$

Thus, R is not transitive on A.

Hence R is neither reflexive nor symmetric and transitive on A.

Q Sol 8 => Let S = set of all triangles in a plane and relation R on S is given by R = {(a, b), a is similar to b}

Reflexive :-

∀ a ∈ S since every triangle is similar to itself

Hence, (a, a) ∈ R

⇒ a is reflexive on S

Symmetric :-

∀ a, b ∈ S show that (a, b) ∈ S.

⇒ a is similar to triangle b.

Then the b is also similar to triangle a

⇒ (a, b) ∈ R

Thus, R is a symmetric relation.

Transitive :-

∀ a, b, c ∈ S show that (a, b), (b, c) ∈ R

Now (a, b) ∈ R ⇒ a is similar to b and (b, c) ∈ R ⇒ b is similar to c.

Thus a is similar to the c. ⇒ (a, c) ∈ R

Hence, R is transitive on S.

Hence, R is a symmetric, reflexive and transitive on S. Thus R be an equivalence on S.

Sol 9 ⇒ Let $R = \{(a,b) ;$ a is the square of $b, a, b \in N\}$

Reflexive :

Clearly 2 is not a square of 2

$\therefore (2,2) \notin R$ as $2 \in N$

Thus $(a,a) \notin R \ \forall \ a \in N$

Hence R is not a reflexive on N.

So R is not an equivalence relation on N.

Sol 10 ⇒ (a) Let $R_1 = \{(a,b) : a < b\}$

Clearly $a \nless a \Rightarrow (a,a) \notin R$,

Hence R_1 is not reflexive

let $(a,b) \in R_1 \Rightarrow a < b$

$\Rightarrow b \nless a$ (as $b < a$)

$\Rightarrow (b,a) \notin R_1$

Hence, R_1 is not symmetric.

let $(a,b), (b,c) \in R_1$

$\Rightarrow a < b$ and $b < c$

$\Rightarrow a < c \Rightarrow (a,c) \in R_1$

$\therefore R_1$ is transitive only

(b) Let $R_1 = \{(a, b) : a$ is the father of $b\}$

Since a cannot be the father of itself

Hence, $(a, a) \notin R_1 \ \forall a$

Thus. R_1 is not reflexive.

Let $(a, b) \in R_1 \Rightarrow a$ is a father of b

The the be is the son/doughter, of a.

$\Rightarrow (b, a) \notin R_1$

$\Rightarrow R_1$ is not symmetric.

Let $(a, b) \in R_1 \Rightarrow a$ is a father of b.

and $(b, c) \in R_1$

$\Rightarrow b$ is father of c.

Then a is grand father of c.

$\Rightarrow (a, c) \notin R_1$

$\therefore R_1$ is not a transitive relation.

Thus, R_1 is neither reflexive, nor symmetric and transitive and hence not an equivalence relation.

Hence, (None) Ans

(c) Let $R_1 = \{(a, b) ; a$ is parallel to $b ; a, b \in L\}$

Where L is the set of all straight lines.

Since every line is parallel to itself

Hence, $(a, a) \in R$

Thus R is reflexive on L.

Symmetric:

$\forall\ a, b \in L$ show that $(a,b) \in R_1$

$\Rightarrow$ a is parallel to b

$\Rightarrow$ line b is parallel to a

$\Rightarrow (b,a) \in R_1$

Hence R_1 is symmetric on L.

Transitive:-

$a, b, c \in L$ show that $(a,b) \in R_1$, $(b,c) \in R_1$

Now $(a,b) \in R_1 \Rightarrow$ a is parallel to b — (1)

$\Rightarrow (b,c) \in R_1$

$\Rightarrow$ b is parallel to c — (2)

$\Rightarrow$ a is parallel to c. — by using (1) and (2)

$\Rightarrow (a,c) \in R_1$

Thus R_1 is transitive on L.

Hence R_1 is symmetric, reflexive and transitive on L.

Thus, R_1 be an equivalence relation on L.

$\therefore$ Ans. (all, E)

(d) let $R_1 = \{(a,b) : a$ is a multiple of $b : a, b \in Z\}$

Clearly a is a multiple of a as $a = a$.

$\therefore (a,a) \in S \Rightarrow S$ is reflexive.

let $(a, b) \in \rho \Rightarrow a$ is a multiple b

$\Rightarrow a = \lambda b$ where $\lambda \in I$

Clearly b is a factor of a i.e. not a multiple of a $\therefore (b, a) \notin \rho$

Thus ρ is not a symmetric relation.

let $(a, b) \in \rho \Rightarrow (b, c) \in \rho$

Since $(a, b) \in \rho$

$\Rightarrow$ a is a multiple of b

$\Rightarrow a = \lambda b$ and $(b, c) \in \rho$

$\Rightarrow$ b is a multiple of c where $\lambda \in I$

$\Rightarrow b = \mu c$ where $\mu \in I$

$\Rightarrow a = \lambda \mu c = \lambda' c$ where $\lambda' = \lambda \mu \in I$

Thus a be a multiple of $c \Rightarrow (a, c) \in \rho$

Therefore ρ is transitive relation.

(e) let $R_1 = \{(a, b) : a$ is congruent to $b\}$

Since every triangle is congruent to itself

Hence, $(a, a) \in R_1$.

Thus, R_1 is a reflexive relation.

Symmetric: let $(a, b) \in R_1$

$\Rightarrow$ a is congruent to b

Then triangle b is congruent to triangle a.

$\Rightarrow R_1$ is a symmetric relation.

Transitive:

Let $(a,b) \in R_1$

$\Rightarrow$ a is congruent to b and $(b,c) \in R_1$

$\Rightarrow$ b is congruent to c.

Thus a is congruent to c $\Rightarrow (a,c) \in R_1$

$\therefore R_1$ is a transitive relation.

Hence, R_1 is reflexive, transitive and symmetric and thus R_1 be an equivalence relation set of all triangles in a plane.

$\therefore$ Ans (all, F)

Sol 11 $\Rightarrow$ let $R_1 = \{(a,b) : a$ is congruent to b$\}$

Since every triangle is congruent to itself

Hence $(a,a) \in R_1$

Thus, R_1 is reflexive relation.

Symmetric:

let $(a,b) \in R_1$

$\Rightarrow$ a is congruent to b

Then the Δb is congruent to Δa

$\Rightarrow R_1$ is a symmetric relation.

Transitive

let $(a,b) \in R_1$

$\Rightarrow$ a is congruent to b and $(b,c) \in R_1$

$\Rightarrow$ b is congruent to c

Thus a is congruent to c

$\Rightarrow (a,c) \in R_1$

$\therefore R_1$ is a transitive relation.

Hence, R_1 is reflexive, transitive and symmetric and thus R_1 be an equivalence relation set of all triangles in a plane. Hence, Ans (all, E)

Sol 12 $\Rightarrow$ Given R be relation in $N \times N$ defined by $(a,b) R (c,d)$ iff $a+d = b+c$

Reflexive :-

$\forall (a,b) \in N \times N$

Show that $(a,b) R (a,b)$

$\Rightarrow a+b = b+a$ which is true.

[since commutative law holds under addition for natural no.]

Thus R is reflexive on $N \times N$

Symmetric :- let $(a,b) R (c,d)$ $\forall a,b,c,d \in N$

$\Rightarrow a+d = b+c$

$\Rightarrow d+a = c+b$

$\Rightarrow c+b = d+a$

$\Rightarrow (c,d)\, R\, (a,b)$

Hence R is symmetric on $N \times N$.

Transitive :-

let $a, b, c, d, e, f \in N$

Show that $(a,b) R (c,d)$ and $(c,d) R (e,f)$

Since $(a,b) R (c,d)$

$\Rightarrow a+d = b+c$ —(1)

& $(c,d) R (e,f)$

$\Rightarrow c+f = d+e$ —(2)

on adding (1) and (2), we get

$a+d+c+f = b+c+d+e$

$\Rightarrow (a,b) R (e,f) \Rightarrow a+f = b+e$

Thus R is transitive on $N \times N$

Hence, R be an equivalence relation $N \times N$.

Sol_13 $\Rightarrow$ Given $A = \{1, 2, 3, 4, 5\}$.

and relation R on A be given

Reflexive : Now $(a,a) \in R$

$\Rightarrow |a-a|$ is even $\Rightarrow 0$ is even, which is true.

Hence, R is reflexive on A.

Symmetric :-

let $(a, b) \in R \ \forall \ a, b \in A$

$\Rightarrow |a-b|$ is even

$\Rightarrow |a-b| = 2m, m \in Z$

$\Rightarrow |-(b-a)| = 2m$

$\Rightarrow |b-a| = 2m \Rightarrow |b-a|$ be

$\Rightarrow (b, a) \in R$

Thus, R is symmetric on A

Transitive :- $\forall a, b, c \in A$ show that $(a, b), (b, c) \in R$

Now $(a, b) \in R \Rightarrow |a-b|$ is even.

$\Rightarrow a-b = \pm 2m$ —(1)

and $(b, c) \in R \Rightarrow |b-c|$ is even

$\Rightarrow b-c = \pm 2n$ —(2)

On adding (1) and (2), we get

$a-c = \pm 2(m+n) = \pm 2m'$

where $m+n = m' \in Z$

$\Rightarrow |a-c| = 2m' \Rightarrow (a, c) \in R$

Thus, R is transitive on A.

Hence, R is reflexive, symmetric and transitive on A.

Thus, R be an equivalence relation on A.

Sol 14 ⇒ Given I be the set of integers and R be the relation on I defined by aRb iff $a+b$ is an even integer $\forall a, b \in I$

Reflexive:

Now $aRa \Rightarrow a+a = 2a$; which is clearly an even integer $\forall a \in I$.

Symmetric:

$\forall a, b \in I$ show that $(a, b) \in R$.

$\Rightarrow a+b$ is an even integer

$\Rightarrow b+a$ is an even integer

$\Rightarrow (b, a) \in R$.

Hence, R is a symmetric on I.

Transitive:

$\forall a, b, c \in I$ show that $(a, b), (b, c) \in R$

Now $(a, b) \in R$

$\Rightarrow a+b$ is an even integer

$\Rightarrow a+b = 2m$,

where $m \in I$ and $(b, c) \in R$

$\Rightarrow b+c$ is an even integer

$\Rightarrow b+c = 2n$ where $n \in I$

$\therefore a+b+b+c = 2(m+n)$

$\Rightarrow a+c = 2(m+n-b) = 2m'$

where $m' = m + n - b$

Since m, n & $b \in I$

Thus, $a + c$ is an even integer

$\Rightarrow (a, c) \in R$

Hence, R be an transitive relation on I.

Thus, R is symmetric, transitive and reflexive relation on I. Hence R be an equivalence relation on I.

Sol 15 $\Rightarrow$ Let I be the set of all integers and be the relation on I

defined by

$R = \{(x, y) : x \in I, x - y$ is divisible by $11\}$

Reflexive: $\forall x \in I, x - x$ is divisible by 11

i.e. 0 is divisible by 11, which is true

$\Rightarrow (x, x) \in R$

Thus, R is reflexive on I.

Symmetric: $\forall x, y \in I$ show that $(x, y) \in R$

$\Rightarrow x - y$ is divisible by 11

$\Rightarrow x - y = 11k$ — where $k \in I$

$\Rightarrow y - x = -11k = 11k'$ — where $k' = -k \in I$

$\Rightarrow y - x$ is divisible by 11.

$\Rightarrow (y, x) \in R$

Hence R is symmetric on I.

Transitive:-

$\forall\ x, y, z \in I$ show that $(x, y)\ (y, z) \in R$

Now $(x, y) \in R \Rightarrow x - y$ is divisible by 11

$\Rightarrow x - y = 11K_1$, where $k_1 \in I$ ——— (1)

and $(y, z) \in R \Rightarrow y - z$ is divisible by 11

$\Rightarrow y - z = 11k_2$, where $k_2 \in I$ ——— (2)

Adding (1) & (2), we get

$x - y + y - z = 11(k_1 + k_2)$

$\Rightarrow x - z = 11k'$, where $k' = k_1 + k_2 \in I$

$\Rightarrow x - z$ is divisible by 11.

Hence $(x, z) \in I$

Thus, R is transitive on I

Hence R is symmetric, reflexive and transitive relation on I.

Thus, R be an equivalence relation on I.