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Class-12 SECTION-A

Chapter-2 FUNCTIONS

Exercise-2(a)

Ans-1

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$f : A \rightarrow B$ Defined by $f(x) = x + 2 \; \forall x \in A$

$f(1) = 1 + 2 = 3;$

$f(2) = 2 + 2 = 4;$

$f(3) = 3 + 2 = 5;$

$f(4) = 4 + 2 = 6;$

Different elements in A have different in B

So f is one-one

$2 \in B$ and let $x \in A$ s.t $f(x) = 2$

$= x + 2 = 2$

$= x = 0 \notin A$

So element 2 in B has no pre-image 0 in A.

That, f is into

Hence f is 1-1

So Ans. (c) proved

Ans - 2

$f: R \rightarrow R$ defined by $f(x) = ax + b, a, b \in R, a \neq 0$.

$\forall x, y \in R$ s.t $f(x) = f(y)$

$\Rightarrow ax + b = ay + b$

$\Rightarrow ax = ay$

$\Rightarrow x = y \ (\because a \neq 0)$

That, f is one - one

$y \in R$ be any arbitrary element

Let $y = ax + b$ is $x = \frac{y-b}{a} \in R$, as

$y \in R$ a, $b \in R$

$\forall y \in R \ \exists \ x = \frac{y-b}{a} \in R$,

$f(x) = f\left(\frac{y-b}{a}\right) = a\left(\frac{y-b}{a}\right) + b = y$

Hence f is onto.

That f is one - one and onto and hence f is bijective.

Ans - 3.

$f: N \rightarrow (N)$ Defined

$f(x) = 2x \ \forall x \in N$

$\forall x, y \in N$ s.t $f(x) = f(y)$

$\Rightarrow 2x = 2y \Rightarrow x = y$

That f is one - one

$3 \in N$

$\therefore$ Let $x \in N$

$f(x) = 3$

$= 2x = 3 =$

$= x = \frac{3}{2} \notin N$

Hence 3 has no pre-image in N

That f is into.

Hence f is one-one, into

Ans - 4

$f: R \to R$ defined

$f(x) = 3x \ \forall x \in R$

$\forall x, y \in R$

$f(x) = f(y)$

$= 3x = 3y$

$= x = y$

So f is one - one.

Let $y \in R$ be any arbitary element

Let $y = 3x$

$x = \frac{y}{3}$

$y \in R = \frac{y}{3} \in R$

$= x \in R$

$\forall y \in R \ \exists \ x \in R$

$f(x) = f\left(\frac{y}{3}\right) = 3 \times \frac{y}{3} = y$

So f is onto

That f is one - one

So Ans (a) Proved

Ans - 5.

R → R defined

$f(x) = x^3 \quad \forall x \in R$

$\forall x, y \in R$

$f(x) = f(y)$

$\Rightarrow x^3 = y^3$

$\Rightarrow (x-y)(x^2 + xy + y^2) = 0$

$\Rightarrow x - y = 0$

[Since $x^2 + xy + y^2 \neq 0 \quad \forall x, y \in R$]

$\Rightarrow x = y$

So f is one - one

Let $y \in R$

Let $y = x^3 = f(x) \Rightarrow x = \sqrt[3]{y}$

$y \in R \Rightarrow \sqrt[3]{y} \in R \Rightarrow x \in R$

That $\forall y \in R \; \exists \; x \in R$

s.t $y = f(x)$

So f is onto

That f is one - one and onto

So f is bijective

Ans - 6

(a) $f : N \to N$

$f(x) = x^2$

$\forall x \in N$

Injectivity : $\forall x, y \in N$ s.t $f(x) = f(y)$

$= x^2 = y^2$

$= (x - y)(x + y) = 0$

$= x - y = 0$

$[\text{Since } x + y \neq 0 \ \forall x, y \in N]$

$= x = y$

So f is one - one or injective

Surjectivity : Now $2 \in N$

Let $x \in N$

$f(x) = 2$

$= x^2 = 2$

$= x = \pm \sqrt{2} \notin N$

Hene $2 \in N$

So f is not surjective

That f is injective but not surjective

(b) $f : Z \to Z$ defined

$f(x) = x^2 \ \forall x \in Z$

element 1 and $-1 \in Z$

Same Image $1 \in Z$

So f is many-one

That f is not injective

further $2 \in Z$

Let $x \in Z$

$f(x) = 2$

$= x^2 = 2$

$= x = \pm\sqrt{2} \notin Z$

Hence 2 has no pre Image

So f is not surjective

That, f is neither surjective nor injective

(C)

$f : R \to R$ defined

$f(x) = x^2 \ \forall \ x \in R$

since $f(1) = 1^2 = 1$; $f(-1) = (-1)^2 = 1$

So different element 1 and -1 have same Image $1 \in Z$

So f is many-one and hence f is not injective

since $-1 \in R$

Let $x \in R$

Such that $f(x) = -1$

$= x^2 = -1$

Real value of $x \in R$

That f is not surjective

Hence, f is neither injective nor surjective

(d)

$f: N \to N$ defined

$f(x) = x^3 \ \forall x \in N$

one-one: $\forall x, y \in N$

$f(x) = f(y)$

$= x^3 = y^3$

$= (x-y)(x^2 + xy + y^2) = 0$

$= x - y = 0$

$[\because x^2 + xy + y^2 \neq 0 \ \forall x, y \in N]$

$= x = y$

So f is one-one injective

Now $2 \in N$

Let $x \in N$

$f(x) = 2$

$= x^3 = 2$

$= x = \pm \sqrt[3]{2} \in N$

That, $2 \in N$

So f is not onto surjective

Hence f is Injective but not surjective.

(e)

$f: Z \to Z$ defined

$f(x) = x^3$

Injectivity : $\forall x, y \in Z$

$f(x) = f(y)$

$\Rightarrow x^3 = y^3$

$\Rightarrow x^3 - y^3 = 0$

$\Rightarrow (x-y)(x^2 + xy + y^2) = 0$

$\Rightarrow x - y = 0$

$[\because x^2 + xy + y^2 \neq 0 \ \forall x, y \in Z$

$\Rightarrow x = y$

So f is one – one i.e injective

Surjectivity : since $2 \in Z$

let $x \in Z$

$f(x) = 2$

$\Rightarrow x^3 = 2$

$\Rightarrow x = \sqrt[3]{2} \notin Z$

Hence 2 has no pre image in (Z)

So f is not surjective

Ans - 7.

$f: W \rightarrow W$ defined

$$f(n) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-1, & \text{if } n \text{ is odd} \end{cases}$$

Case - I :

$\forall\ n, y \in W$ and n, y are even

s.t $f(n) = f(y)$

$= n+1 = y+1 = n = y$

Case - II :

$\forall\ n, y \in W$ and n, y both are odd

$f(n) = f(y)$

$\Rightarrow n-1 = y-1$

$= n = y$

Case - III :

If n is odd and y is even

$f(n) = n-1$ & $f(y) = n+1$

$n \neq y = f(n) \neq f(y)$

so f is 1-1

Case - IV :

if n is even y is odd

so $f(n) = n+1$ and $f(y) = y-1$

so $n \neq y = f(n) \neq f(y)$

$\therefore f$ is one - one

if odd natural Number ∃ ano even natural Number

$n-1 \in N$

s.t $f(n-1) = n-1+1 = n$

so f is onto

That f is one-one

Hence f is bijective function

Ans - 8.

$f: R \to R$ defined

$f(x) = \cos x \ \forall x \in R$

one-one : $\forall x, y \in R$ s.t $f(x) = f(y)$

$\Rightarrow \cos x = \cos y$

$\Rightarrow x = 2n\pi \pm y \ \forall n \in I$

$\Rightarrow x \neq y$

$f(0) = \cos 0 = 1$; $f(2\pi) = \cos 2\pi = 1$

0 and $2\pi \in R$ same Image $1 \in R$ hence

not one-one

onto : since $2 \in R$

Let $x \in R$

$f(x) = 2$

$\Rightarrow \cos x = 2$

since $|\cos x| \leq 1$

That 2 has no pre Image in R domain of f

so f is not onto

Hence f is neither one-one nor onto

Ans - 9.

$f : [0, \infty) \to [0, \infty)$

defined by $f(x) = \frac{2x}{1+2x}$

one - one : $\forall x, y \in [0, \infty)$

$f(x) = f(y)$

$= \frac{2x}{1+2x} = \frac{2y}{1+2y}$

$= 2x(1+2y) = 2y(1+2x)$

$= 2x + 4xy = 2y + 4xy$

$= 2x = 2y \Rightarrow x = y$

So f is one - one

Let $y \in [0, \infty)$ be any arbitrary element

That $f(x) = y = \frac{2x}{1+2x} = y$.

$= 2x = y + 2xy$.

$= 2x(1-y) = y = x = \frac{y}{2(1-y)}$

$y = 1 \in [0, \infty)$

So $1 \in [0, \infty)$ has no in $[0, \infty)$ (domain of f)

So f is not onto

Hence f is not onto.

Hence f is one - one but not onto

So Ans. (b) Proved

Ans - 10.

$f: R \to R$ function defined

$f(x) = 2x^3 - 5 \quad \forall x \in R$

$\forall x, y \in R$

$f(x) = f(y)$

$= 2x^3 - 5 = 2y^3 - 5$

$= x^3 = y^3$

$= (x-y)(x^2 + xy + y^2) = 0$

$= x - y = 0$

$= x = y$

$[\because x^2 + xy + x^2 + xy + \frac{y^2}{4} + \frac{3}{4}y^2$

$= \left(x + \frac{y}{2}\right)^2 + \frac{3}{4}y^2 \neq 0]$

Let $y \in R$

That $f(x) = y$

$= y = 2x^3 - 5$

$= x = \left(\frac{y+5}{2}\right)^{\frac{1}{3}}$

Since $y \in R = \left(\frac{y+5}{2}\right)^{\frac{1}{3}} \in R = x \in R$

That, $\forall y \in R \; \exists \; x \in R$

$f(x) = f\left(\left(\frac{y+5}{2}\right)^{\frac{1}{3}}\right)$

$= 2\left(\left(\frac{y+5}{2}\right)^{\frac{1}{3}}\right)^3 - 5$

$= 2\left(\frac{y+5}{2}\right) - 5 = y$

So f is onto

So f is one - one onto hence f is bijective

Ans - 11 -

$f : R \to R$ defined

$f(x) = x^5 \ \forall x \in R$

$\forall x, y \in R$

s.t $f(x) = f(y) = x^5 + y^5$

$= x^5 - y^5 = 0$

$= (x-y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4) = 0$

$= x - y = 0$

$[\because x^4 + x^3y + x^2y^2 + xy^3 + y^4 \neq 0 \ \forall x, y \in R]$

$= x = y$

So f is one-one

Let $y \in R$ be any orbitrary element Then $fx = y$

$= x^5 = y \Rightarrow x = (y)^{1/5}$

$y \in R \Rightarrow y^{1/5} \in R \Rightarrow x \in R$

$\forall y \in R \ \exists \ x \in R$

$f(x) = f(y^{1/5}) = (y^{1/5})^5 = y$

So f is onto

That, f is one-one and onto and hence f is bijective function.

Ans -12.

$f : N \to (N)$ defined

by $f(n) = \begin{cases} n^2 & n \text{ is odd} \\ 2n+1 & n \text{ is even} \end{cases}$

$f(3) = 3^2 = 9$

$f(4) = 2 \times 4 + 1 = 9$

That Element 3 and 4 has same Imge 9.

So diffrent Element in N has same image $9 \in N$

So f is many one.

That, f is not injective

Since $2 \in N$

Let $n \in N$ (d

if x is odd then $f(n) = n^2 = 2$

$\Rightarrow n = \sqrt{2} . \in N$

if n is even then $f(n) = 2$

$\Rightarrow 2n + 1 = 2 \Rightarrow n = \frac{1}{2} \in N$

both case $2 \in N$

pre Image in (N)

So f is not onto i.e. surjective

Hence, f is neither injective nor surjective