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Class - 12

# CHAPTER - 6 MATRICES

## EXERCISE 6 (a)



**Ans-1.** We know that a matrix of order $m \times n$ has $mn$ element.

Hence to find all possible order of matrix containing 8 elements, we will find ordered pair where product of whose components be equal to 8.

That, such ordered pairs are $(2,4), (4,2), (1,8), (8,1)$.

Hence required possible orders are $2 \times 4$, $4 \times 2$, $1 \times 8$ and $8 \times 1$.

Now if a matrix containing 5 elements such possiblee ordered pairs are $(1,5)$ & $(5,1)$.

Hence, required possible ordered pairs are $(1 \times 5)$ & $5 \times 1)$.

**Ans-2.** Some we know that a matrix of order $m \times n$ containing $mn$ elements.

(i) That a matrix of order $3 \times 3$ containing $3 \times 3$ i.e. 9 entries.

(ii) A $3 \times 4$ matrix contains $3 \times 4$
i.e. 12 elements.

(iii) A $m \times n$ matrix contains $mn$ elements.

(iv) A square matrix of order $n$ contains $n \times n$
i.e. $n^2$ elements.

Ans-3.

$a_{ij} = 4i - 3j$

So

$a_{11} = 4 - 3 = 1;$

$a_{12} = 4 \times 1 - 3 \times 2$
$= -2$

$a_{13} = 4 - 3 \times 3$
$= -5$

$a_{21} = 4 \times 2 - 3 \times 1$
$= 5$

$a_{22} = 4 \times 2 - 3 \times 2$
$= 2$

$a_{23} = 4 \times 2 - 3 \times 3$
$= -1$

$a_{31} = 4 \times 3 - 3 \times 1 = 9$

$a_{32} = 4 \times 3 - 3 \times 2 = 6$

$a_{33} = 4 \times 3 - 3 \times 3 = 3$

That required matrix

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & -2 & -5 \\ 5 & 2 & -1 \\ 9 & 6 & 3 \end{bmatrix}$$

Ans.4.

(i) $B = [b_{ij}]_{2\times 2}$ Here $1 \leq i, j \leq 2$

$$b_{ij} = \frac{(i-2j)^2}{2}$$

$$i = 1, j = 1;\ b_{11} = \frac{(1-2)^2}{2} = \frac{1}{2}$$

$$i = 1, j = 2;\ b_{12} = \frac{(1-4)^2}{2} = \frac{9}{2}$$

$$i = 2, j = 1;\ b_{21} = \frac{(2-2)^2}{2} = 0$$

$$i = 2, j = 2;\ b_{22} = \frac{(2-4)^2}{2} = 2$$

That

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 1/2 & 9/2 \\ 0 & 2 \end{bmatrix}$$

(iii)

$b_{ij} = \frac{1}{2}\left|-3i+j\right|$

$i=1, j=1;\ b_{11} = \frac{1}{2}\left|-3+1\right| = 1$

$i=1, j=2;\ b_{12} = \frac{1}{2}\left|-3+2\right| = \frac{1}{2}$

$i=1, j=1;\ b_{21} = \frac{1}{2}\left|-6+1\right| = \frac{5}{2}$

$i=2, j=2;\ b_{22} = \frac{1}{2}\left|-6+2\right| = \frac{4}{2} = 2$

That

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 5/2 & 2 \end{bmatrix}$$

Ans - 5.

(i) $A = [a_{ij}]_{3\times4};$

$1 \le i \le 3;\ 1 \le j \le 4;$

where

$a_{ij} = i - j$

$i=1, j=1;\ b_{11} = 1-1 = 0$

$i=1, j=2;\ b_{12} = 1-2 = -1$

$i=1, j=3;\ b_{13} = 1-3 = -2$

$i=1, j=4;\ b_{14} = 1-4 = -3$

Similarly

$a_{21} = 2-1 = 1; \quad a_{22} = 2-2 = 0;$

$a_{23} = 2-3 = -1; \quad a_{24} = 2-4 = -2;$

$a_{31} = 3-1 = 2; \quad a_{32} = 3-2 = 1;$

$a_{33} = 3-3 = 0; \quad a_{34} = 3-4 = -1$

So

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

(iii)

$A = [a_{ij}]_{3\times 4}$

where

$1 \le i \le 3; \; 1 \le i \le 4;$

Here,

$a_{ij} = ij$

$a_{11} = 1 \times 1 = 1;$

$a_{12} = 1 \times 2 = 2;$

$a_{13} = 1 \times 3 = 3;$

$a_{14} = 1 \times 4 = 4;$

$a_{21} = 2 \times 1 = 2;$

$a_{22} = 2 \times 2 = 4;$

$a_{23} = 2 \times 3 = 6;$

$a_{24} = 2 \times 4 = 8;$

$a_{31} = 3 \times 1 = 3;$

$a_{32} = 3 \times 2 = 6;$

$a_{33} = 3 \times 3 = 9;$

$a_{34} = 3 \times 4 = 12;$

So

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

(iii)

$A = [a_{ij}]_{3 \times 4};\ 1 \le i \le 3;\ 1 \le j \le 4$

$a_{ij} = \frac{i}{j}$

$a_{11} = \frac{1}{1} = 1;$ $a_{12} = \frac{1}{2};$

$a_{13} = \frac{1}{3};$ $a_{14} = \frac{1}{4};$

$a_{21} = \frac{2}{1} = 2;$ $a_{22} = \frac{2}{2} = 1;$

$a_{23} = \frac{2}{3};$ $a_{24} = \frac{2}{4} = \frac{1}{2};$

$a_{31} = \frac{3}{1} = 3;$ $a_{32} = \frac{3}{2};$

$a_{33} = \frac{3}{3} = 1;$ $a_{34} = \frac{3}{4}$

So

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 2 & 1 & 2/3 & 1/2 \\ 3 & 3/2 & 1 & 3/4 \end{bmatrix}_{3 \times 4}$$

Ans - 6.

(a) (i)

$A = [a_{ij}]_{2\times3}$;

$1 \le i \le 2;\ 1 \le j \le 3;$

Here,

$$a_{ij} = \frac{3i - j}{2}$$

$$a_{11} = \frac{3-1}{2} = 1;\quad a_{12} = \frac{3-2}{2} = \frac{1}{2};$$

$$a_{13} = \frac{3-3}{2} = 0;$$

$$a_{21} = \frac{6-1}{2} = \frac{5}{2};\quad a_{22} = \frac{6-2}{2} = 2$$

$$a_{23} = \frac{6-3}{2} = \frac{3}{2};$$

That

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 0 \\ 5/2 & 2 & 3/2 \end{bmatrix}$$

(ii)

$A = [a_{ij}]_{2\times3}$; $1 \le i \le 2;\ 1 \le j \le 3;$

$$a_{ij} = \frac{i + 3j}{2}$$

$a_{11} = \frac{1+3}{2} = 2$; $a_{12} = \frac{1+6}{2} = \frac{7}{2}$;

$a_{13} = \frac{1+9}{2} = 5$;

$a_{21} = \frac{2+3}{2} = \frac{5}{2}$; $a_{22} = \frac{2+6}{2} = 4$

$a_{23} = \frac{2+9}{2} = \frac{11}{2}$;

That,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 2 & 7/2 & 5 \\ 5/2 & 4 & 11/2 \end{bmatrix}$$

(b) (i)

$A = [a_{ij}]_{3\times2}$; $1 \le i \le 3$; $1 \le j \le 2$;

$$a_{ij} = \frac{i+3j}{2}$$

$a_{11} = \frac{1+3}{2} = 2$; $a_{12} = \frac{1+6}{2} = \frac{7}{2}$;

$a_{21} = \frac{2+3}{2} = \frac{5}{2}$; $a_{22} = \frac{2+6}{2} = 4$

$a_{31} = \frac{3+3}{2} = 3$; $a_{32} = \frac{3+6}{2} = \frac{9}{2}$.

That,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 9/2 \\ 5/2 & 4 \\ 3 & 9/2 \end{bmatrix}_{3\times2}$$

(ii) $A = [a_{ij}]_{3\times2}$;

$1 \leq i \leq 2; 1 \leq j \leq 3$;

$$a_{ij} = \frac{(i+2j)^2}{2}$$

$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$;

$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$;

$a_{21} = \frac{(2+2)^2}{2} = 8$;

$a_{22} = \frac{(2+4)^2}{2} = 18$ ;

$a_{31} = \frac{(3+2)^2}{2} = \frac{25}{2}$ ;

$a_{32} = \begin{bmatrix} 5 & -2 & 1 & 0 & 3 \\ 7 & 6 & 4 & 2 & -1 \\ 0 & 8 & 3 & 5 & 6 \end{bmatrix} = \frac{49}{2}$ ;

That

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 8 \\ 25/2 & 49/2 \end{bmatrix}$$

Ans-7.

$$\begin{bmatrix} 5 & -2 & 1 & 0 & 3 \\ 7 & 6 & 4 & 2 & -1 \\ 0 & 8 & 3 & 5 & 6 \end{bmatrix}$$

(i) Since given matrix contains 3 rows and 5 columns order of A be $3 \times 5$

(ii) we know that a matrix of order $m \times n$ contains $mn$ element.

That given matrix A contains $3 \times 5$ i.e. 15 elements.

(iii) The entries of second row of A are 7, 6, 4, 2 & -1.

(iv) The entries of third column of A are 1, 4 & 3.

(v) $a_{12} = -2$; $a_{23} = 4$; $a_{34} = 5$; $a_{15} = 3$;

(vi) $a_{ij} = 4 = a_{23}$

So

$i = 2$ & $j = 3$

Ans. 8.

(i)

$$\begin{bmatrix} x & y \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -1 & 5 \end{bmatrix}$$

So Their corresponding element are equal

So

$x = -2$ & $y = 0$

(ii) $[x \ 3] = [-1 \ y]$

So

$x = -1$ & $y = 3$

[since their corresponding entries are equal]

(iii) $\begin{bmatrix} x+1 \\ -3+y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

So

There corresponding entries are equal

So

$x+1 = -2$

$\Rightarrow x = -3$

&

$-3+y = 0$

$y = 3$

Ans- 9.

$$\begin{bmatrix} x+y & y-z \\ z-2x & y-x \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$$

That, There corresponding entries are equal.

So

$x+y = 3$

$y-z = -1$

$z-2x = 1$

$y-x = 1$

on adding (1) & (4); we have

$$2y = 4$$

$$\Rightarrow y = 2$$

So

from (1); $x = 3 - 2$

$$= 1$$

So

from (2); $z = y + 1$

$$= 2 + 1$$

$$= 3$$

That $x = 1$; $y = 2$ & $z = 3$

Ans-10.

(i)

$$\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

That their correspounding element are equal.

so

$$x - y = -1$$

$$2x - y = 0$$

$$2x + z = 5$$

$3x + w = 13$

eqn. (1) - eqn (2) gives:

$-x = -1$

$\Rightarrow x = 1$

So from (2); $2 - y = 0$

$\Rightarrow y = 2$

So from (3); $2 + z = 5$

$\Rightarrow z = 3$

from (4); $y = 2$; $z = 3$, $w = 4$

(ii)

$$\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}.$$

That their corresponding element are equal.

$a + b = 6$ ....(1)

$ab = 8$ ....(2)

Now.

$(a-b)^2 = (a+b)^2 - 4ab$

$= 6^2 - 4 \times 8 = 4$

$\Rightarrow a - b = \pm 2$

Case - I

$a - b = 2$

on eqn (1) & (3);

$2a = 8$

$\Rightarrow a = 4$

so

from (3); $b = 4 - 2$

$= 2$

Case - II

$a - b = -2$

eqn (1) & (4);

$a = 2$ and $b = 4$

Ans - 11

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Their corresponding entries are equal.

So

$x+y+z=9$ ---(1)

$x+z=5$ ---(2)

$y+z=7$ ---(3)

So from (1) & (2) ;

$5+y=9$

$\Rightarrow y=4$

So from (1) & (3) ;

$x+7=9$

$\Rightarrow x=2$

So from (1)

$z=9-6$

$=3$

Ans - 12.

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

That their corresponding element are equal.

So

$a - b = -1$ ---- (1)

$2a - b = 0$ ---- (2)

$2a + c = 5$ ---- (3)

$3c + d = 13$ ---- (4)

eqn (1) & eqn (2)

$a = 1$ ; $b = 2$

So

from (3);

$2 + c = 5$

$\Rightarrow c = 3$

So

from (4);

$9 + d = 13$

$\Rightarrow d = 4$

Hence

$a = 1$; $b = 2$; $c = 3$ & $d = 4$