Class - 12 (Section A)

CHAPTER-11 APPLICATION OF DERIVATIVES

EXERCISE 11 (a)

Ans-1.

(i) $y = \frac{4}{x}$;

Diff both sides w.r.t x, we have

$$\frac{dy}{dx} = -\frac{4}{x^2}$$

So

slop of tangent to given curve at (2,2)

$$= \left(\frac{dy}{dx}\right)_{(2,2)} = \frac{-4}{2^2}$$

$$= -1$$

(ii)

$y = 2x^2 - 1$

diff both sides w.r.t. x, we have

$$\frac{dy}{dx} = 4x$$

So

slope of tangent to given curve at

$$x = 1 = \left(\frac{dy}{dx}\right)_{x=1}$$

$$= 4 \times 1 = 4$$

iii)

$y = 2x - x^2$

Diff both sides w.r.t. x; we have

$$\frac{dy}{dx} = 2 - 2x$$

That slope of tangent to given curve at $x = 1$

$$= \left(\frac{dy}{dx}\right)_{x=1} = 2 - 2$$

$$= 0$$

(iv)

$y = x^2 - \sin x$

Diff both sides w.r.t. x; we have

$$\frac{dy}{dx} = 2x - \cos x$$

So

slope of tangent to given curve at $x = 0$

$$= \left(\frac{dy}{dx}\right)_{x=0} = 2 \times 0 - 1 = -1$$

(v)

$f(x) = 9 \sin x + \sin 3x$

Diff both sides w.r.t. x; we have

$f'(x) = 9 \cos x + 3 \cos 3x$

That slope of tangent to given curve at $x = \pi/3$

$$= (f'(x))_{x=\pi/3} = 9 \cos \frac{\pi}{3} + 3 \cos \pi$$

$$= \frac{9}{2} - 3 = \frac{3}{2}$$

Ans-2.

$y = 3x^2$ --- (i)

when $x = 2$

so from (i);

$y = 3 \times 2^2 = 12$

So the point on given curve (i) be (2,12)

Diff both sides of equation (i) w.r.t. x; we have

so $\frac{dy}{dx} = 6x$

That, $\left(\frac{dy}{dx}\right)_{(2,12)} = 6 \times 2 = 12$

That slope of normal to given curve (i) at (2,12)

$$= \frac{-1}{\left(\frac{dy}{dx}\right)_{(2,12)}} = -\frac{1}{12}$$

Ans-3.

(i) $y = 2x^2 - 3x - 1$

so $\frac{dy}{dx} = 4x - 3$

so slope of tangent to given curve at

$(1,2) = \left(\frac{dy}{dx}\right)_{(1,2)} = 4 - 3 = 1$

& Slope of normal to given curve at (1,2)

$$= \left(\frac{-1}{\frac{dy}{dx}}\right)_{(1,2)}$$

$$= -\frac{1}{1}$$

$$= -1$$

That equation of tangent to given curve at point (1,2) is given by $y-2 = 1(x-1)$

$\Rightarrow x-y+1 = 0$ & the equation of normal to given curve at point (1,2) be given by $y-2 = -1(x-1)$

$= x+y-3=0.$

(ii) $x = \cos t$ ---- (i)

& $y = \sin t$ ---- (ii)

Diff equation (i) & (ii) w.r.t. t; we have

$\frac{dx}{dy} = -\sin t$. $\frac{dy}{dt} = \cos t$

so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\cot t$

That slope of tangent to give curve at $t = \pi/4$

$$= \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = -\cot \pi/4 = -1$$

& slope of normal to given curve $t = \pi/4$ is

given by $= \dfrac{-1}{\left(\frac{dy}{dx}\right)_{t=\pi/4}}$

$$= \frac{-1}{-1} = 1$$

That, the equation of tangent to given curve at $t = \pi/4$ is given by

$$y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right)$$

[$\because$ coordinates of point are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$]

$$= x + y - \sqrt{2} = 0$$

The equation of normal to given curve at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is given by

$$y - \frac{1}{\sqrt{2}} = 1\left(x - \frac{1}{\sqrt{2}}\right)$$

i.e $y = x$.

(iii) $y = x^2 + 4x + 1$

at $x = 3$; $y = 3^2 + 12 + 1$

$= 9 + 13$

$= 22$

That the point of contact be $(3, 22)$

Diff (i) both sides w.r.t x; we have

$$\frac{dy}{dx} = 2x + 4$$

So

$$\left(\frac{dy}{dx}\right)_{(3,22)}$$

$= 6 + 4$

$= 10$

That the required eqn. of tangent to curve at $(3, 22)$ is

$$y - 22 = \left(\frac{dy}{dx}\right)_{(3,22)} (x - 3)$$

$\Rightarrow$ $y - 22 = 10(x - 3)$

$\Rightarrow$ $10x - y - 8 = 0$

and the required eqn of normal at $(3, 22)$ is given by

$$y - 22 = \frac{-1}{\left(\frac{dy}{dx}\right)_{(3,22)}} (x - 3)$$

$\Rightarrow x + 10y - 223 = 0$

(iv)

$$y^2 = \frac{x^3}{4-x}$$

Diff both sides w.r.t. x; we have

$$2y\frac{dy}{dx} = \frac{(4-x)\,3x^2 + x^3}{(4-x)^2}$$

$$= \frac{12x^2 - 2x^3}{(4-x)^2}$$

So

$$\left(\frac{dy}{dx}\right)_{(2,-2)} = \frac{6\times 2^2 - 1\times 2^3}{-2\,(4-2)^2}$$

$$= \frac{24-8}{-8}$$

$$= \frac{16}{-8}$$

$$= -2$$

That the required eqn. of tangent to given curve at $(2,-2)$ is

$$y-(-2) = \left(\frac{dy}{dx}\right)_{(2,-2)}(x-2)$$

$$\Rightarrow y - y_0 = -\frac{a^2 y_0}{b^2 x_0}(x - x_0)$$

$$\Rightarrow b^2 y x_0 - b^2 x_0 y_0 = -a^2 x y_0 + a^2 x_0 y_0$$

$$\Rightarrow a^2 x y_0 + b^2 y x_0 = (b^2 + a^2) x_0 y_0$$

$$\Rightarrow \frac{a^2 x}{x_0} + \frac{b^2 y}{y_0} = b^2 + a^2$$

Ans - 4.

$$y^2 = 4ax$$

Diff both sides of equation (i) w.r.t x:

$$2y \frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$$

So slope of tangent to give curve at $(at^2, 2at)$

$$= \left(\frac{dy}{dx}\right)_{(at^2, 2at)} = \frac{2a}{2at} = \frac{1}{t}$$

& slope of normal to given curve at $(at^2, 2at)$

$$= \frac{-1}{\left(\frac{dy}{dx}\right)_{(at^2, 2at)}} = \frac{-1}{1/t} = -t$$

That eqn. of tangent to Given curve at $(at^2, 2at)$ be given by

$$y - 2at = \frac{1}{t}(x - at^2)$$

$$ty - 2at^2 = x - at^2$$

$$\Rightarrow x - ty + at^2 = 0.$$

The equation of normal to given curve at $(at^2, 2at)$ be given by $y - 2at = -t(x - at^2)$

$$\Rightarrow tx + y - 2at - at^3 = 0$$

Ans-5. $y = (x^2 - 1)(x - 2)$ --- (i)

i.e. $y = 0$

So from (i); we have

$$0 = (x^2 - 1)(x - 2)$$

$$\Rightarrow x = 2, \pm 1$$

So point on given curve (i) are $(2,0)$ & $(1,0)$; $(-1,0)$

Diff both sides of equation (i) w.r.t x; we have

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x^2 - x + 2) = 3x^2 - 4x - 1$$

That slope of tangent to given curve at (2,0)

$$= \left(\frac{dy}{dx}\right)_{(2,0)} = 12-8-1 = 3$$

and corresponding eqn. of tangent to given curve at (2,0) be given by $y-0 = 3(x-2)$

$$\Rightarrow 3x - y - 6 = 0$$

the slope tangent to given curve at (1,0)

$$= \left(\frac{dy}{dx}\right)_{(1,0)}$$

$$= 3-4-1$$

$$= -2$$

corresponding tangent at point (1,0) be given by

$$y-0 = -2(x-1)$$

$$\Rightarrow 2x + y - 2 = 0$$

so

slope of tangent to given curve at

$$(-1,0) = \left(\frac{dy}{dx}\right)_{(-1,0)}$$

$$= 3+4-1$$

$$= 6$$

Given curve at (-1,0) be given by $y-0 = 6(x+1)$

Ans - 6.

$y = 2x^2 - 6x - 4$

Point on curve (i) be (x_1, y_1) diff eqn (i) w.r.t x, we have

$$\frac{dy}{dx} = 4x - 6$$

$\therefore$ slope of tangent to given curve (i) at (x_1, y_1)

$$= \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$$

$= 4x_1 - 6$ since the tangent is parallel to x-axis

So slope the tangent given curve at $(x_1, y_1)|_{(x_1, y_1)}$

$= 0$

$\Rightarrow 4x_1 = 6$

$= x_1 = \frac{3}{2}$

Also the point (x_1, y_1) lies on curve (i)

so $y_1 = 2x_1^2 - 6x_1 - 4$

$= 2\left(\frac{3}{2}\right)^2 - 6 \times \frac{3}{2} - 4$

$= \frac{9}{2} - 9 - 4 = \frac{9 - 26}{2} = \frac{-17}{2}$

That required point on given curve be

$$\left(\frac{3}{2}, \frac{-17}{2}\right).$$

Ans. 17.

$$y = 12x - x^3 \quad \text{--- (i)}$$

given curve be (x_1, y_1)

so $y_1 = 12x - x_1^3$ --- (ii)

diff eqn. (i) both sides w.r.t x; we have

$$\frac{dy}{dx} = 12 - 3x^2$$

so slope of tangent to given curve (i) at

$$(x_1, y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$$

$$= 12 - 3x_1^2$$

It is given slope of tangent given cure at (x_1, y_1) be 0.

so $12 - 3x_1^2 = 0$

$\Rightarrow x_1^2 = 4$

$\Rightarrow x_1 = \pm 2$

when $x_1 = 2$

So from (ii);

$$y_1 = 24 - 2^3 = 24 - 8$$

$$= 16$$

when $x_1 = -2$

so from (ii); $y_1 = -24 - (-2)^3$

$$= -24 + 8 = -16$$

hence the required points on given curve are $(2, 16)$ and $(-2, 16)$

Ans-8.

$$x^2 + y^2 = 25 \quad \text{---(i)}$$

Given curve (i) be (x_1, y_1)

$$x_1^2 + y_1^2 = 25 \quad \text{---(ii)}$$

diff both sides of eqn. (i) w.r.t x; we have

$$2x + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

So $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{-x_1}{y_1}$

So

Slope of tangent to given curve at (x_1, y_1)

$$= \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \left(-\frac{x_1}{y_1}\right)$$

Since the tangent is parallel to x-axis

So $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0$

$$\Rightarrow \frac{-x_1}{y_1} = 0$$

$$\Rightarrow x_1 = 0$$

So from (ii) $y_1^2 = 25 \Rightarrow y_1 = \pm 5$

That required point on curve are $(0, \pm 5)$.

(ii) Since the tangent to given curve is parallel to y-axis

So $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} \to \infty \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0$

$\Rightarrow$ from (ii) ; $x_1^2 = 25$

$$\Rightarrow x_1 = \pm 5$$

Given curve are $(\pm 5, 0)$

Ans-9.

$y^2 = 4x$ ...(i)

let (x_1, y_1) be any point on given curve

So $y_1^2 = 4x_1$ ...(ii)

Diff (i) w.r.t. x; we have

$$2y \frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

So

slope of tangent to given curve at (x_1, y_1)

$$= \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{2}{y_1}$$

given eqn. of straight line $2x - y + 4 = 0$ ...(iii)

So

slope of line $= \frac{-2}{-1} = 2$

Given that tangent is parallel to line (iii)

That, $\frac{dy}{dx} = 2$

$\Rightarrow y_1 = 1$

So

from (ii); $4x_1 = 1$

$\Rightarrow x_1 = \frac{1}{4}$

Hence, the required point on given curve be

$\left(\frac{1}{4}, 1\right)$.

Ans-10.

$y = 2x^2+7 \quad \cdots (i)$

$p(x_1, y_1)$ be any point on curve (i)

$\therefore \; y_1 = x_1^2 + 7 \quad --(ii)$

Diff eqn (i) w.r.t x; we have

$$\frac{dy}{dx} = 4x$$

so

slope of tangent to curve (i) at (x_1, y_1)

$$= \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 4x_1$$

Given eqn. of straight line be

$$4x - y + 3 = 0$$

So slope of line (3) $= \frac{-4}{-1} = 4$

Given that tangent is parallel to line (3)

So

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 4$$

$$\Rightarrow 4x_1 = 4$$

$$\Rightarrow x_1 = 1$$

$\therefore$ from (2) ; $y_1 = 2 \times 1^2 + 7 = 9$

∴ That required point on given curve be (1, 9)

Given curve at (1, 9) be given by

$y - 9 = 4(x - 1)$

$\Rightarrow 4x - y + 5 = 0$

Ans-11.

$y = x^3 + 2x - 6$ --- (1)

$\therefore$ slope of tangent $= \frac{dy}{dx} = 3x^2 + 2$

Then slope of Normal $= \frac{1}{3x^2+2}$

But the normal is ∥ to line $x + 14y + 4 = 0$.

whose slope is $-\frac{1}{14}$

That both slopes must be equal.

$$\therefore -\frac{1}{3x^2+2} = -\frac{1}{14}$$

$$\Rightarrow 3x^2 + 2 = 14$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

when $x = 2$; from (1), we have

$y = 8 + 4 + 6 = 18$

when $x = -2$; from (1), we get

$y = -8 - 4 + 6 = -6$

Hence point of contact are $(2, 18)$ and $(-2, -6)$

So eqn. of Normal at $(2, 18)$ is given by

$y - 18 = -\frac{1}{14}(x - 2)$

$\Rightarrow x + 14y - 254 = 0$

So eqn. of Normal at $(-2, -6)$ is given by

$y + 6 = -\frac{1}{14}(x + 2)$

$\Rightarrow x + 14y + 86 = 0$

Ans-12.

$y = (x-3)^2$ --- (1)

Given curve (1) be (x_1, y_1)

So $\frac{dy}{dx} = 2(x-3)$

That slope of tangent to given curve at

$(x_1, y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2(x_1 - 3)$

Slope of chord joining $(3, 0)$ and $(4, 1)$

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{4-3} = 1$$

Given chord so their slopes must be equal.

So
$$2(x_1 - 3) = 1$$

$$\Rightarrow x_1 = \frac{7}{2}$$

Also point (x_1, y_1) lies on given curve (1)

So $y_1 = (x_1 - 3)^2$

$$\Rightarrow y_1 = \left(\frac{7}{2} - 3\right)^2 = \frac{1}{4}$$

Given curve be $\left(\frac{7}{2}, \frac{1}{4}\right)$

Ans - 13.

$$y = x + \frac{1}{x} \quad --(1)$$

Point on curve (1) be (x_1, y_1)

So $y_1 = x_1 + \frac{1}{x_1}$

Diff eqn (1) w.r.t x; we have

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}$$

$\therefore$ Slope of tangent to given curve (1) at (x_1, y_1)

$$= \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 1 - \frac{1}{x_1^2}$$

Since the tangent is parallel to x-axis

So $$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0$$

$$y = 1 - \frac{1}{x_1^2} = 0$$

$$= x_1^2 = 1$$

$$\Rightarrow x_1 = \pm 1$$

$x_1 = 1$ so from (2); we have

$$y_1 = 1 + 1 = 2$$

$x_1 - 1$ $\therefore$ from (2) ; we have

$$y_1 = -1 - 1 = -2$$

That the required point on given curve are $(1, 2)$ and $(-1, -2)$

Ans-14.

$y = \cot^2 x - 2\cot x + 2$ ---(1)

at $x = \frac{\pi}{4}$ $\therefore$ from (1); we have

$y = \cot^2 \frac{\pi}{4} - 2 \cot \frac{\pi}{4} + 2 = 1 - 2 + 2 = 1$

Hence the required point on given curve be

$\left(\frac{\pi}{4}, 1\right)$

Diff eqn (1) both sides w.r.t x; we have

$\frac{dx}{dy} = 2\cot x \; (-\operatorname{cosec}^2 x) + 2 \operatorname{cosec}^2 x$

at $x = \frac{\pi}{4}$:

$\frac{dy}{dx} = -2 \cot \frac{\pi}{4} \operatorname{cosec}^2 \frac{\pi}{4} + 2 \operatorname{cosec}^2 \frac{\pi}{4}$

$= -2 \times 1 \times (\sqrt{2})^2 + 2 (\sqrt{2})^2 = 0$

That eqn. of tangent to given curve at

$\left(\frac{\pi}{4}, 1\right)$ is given by $y - 1 = 0 \left(x - \frac{\pi}{4}\right)$

Ans - 15.

$$y^2 = ax^3 + b \quad \text{---(1)}$$

Diff both sides of eqn (1) w.r.t x; we have

$$2y\frac{dy}{dx} = 3ax^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3ax^2}{2y}$$

$\therefore$ Slope of tangent at $(2,3) = \left(\frac{dy}{dx}\right)_{(2,3)}$

$$= \frac{3a \times 4}{6} = 2a$$

also eqn of tangent to given curve be

$$y = 4x - 5$$

$\therefore$ Slope given tangent $= \frac{-4}{-1} = 4$

Now $\left(\frac{dy}{dx}\right)_{(2,3)} = 4$

$$\Rightarrow 2a = 4 \Rightarrow a = 2$$

Given point $(2,3)$ lies in eqn. (1)

$$\therefore 9 = 8a + b \quad \text{---(2)}$$

$$\Rightarrow 9 = 16 + b$$

$$\Rightarrow b = -7 \quad [\because a = 2]$$

Hence $a = 2$ and $b = -7$