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Class - 12.

CHAPTER - 16 DEFINITE INTEGRALS

Exercise 16 (a)

Ans 1.

(i) $\int_{\pi/4}^{\pi/2} \cot x\,dx = \log|\sin x| \Big]_{\pi/4}^{\pi/2}$

$= \log \sin\frac{\pi}{2} - \log \sin\frac{\pi}{4}$

$= \log 1 - \log \frac{1}{\sqrt{2}}$

$= 0 - \log 2^{-1/2} = \frac{1}{2}\log 2$

(ii) $\int_{\pi/6}^{\pi/3} \frac{dx}{\sin 2x}$

$= \int_{\pi/6}^{\pi/3} \operatorname{cosec} 2x\,dx$;

Put $2x = t \Rightarrow 2dx = dt$

when $x = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{3}$

$x = \frac{\pi}{3} \Rightarrow t = \frac{2\pi}{3}$

$= \frac{1}{2}\int_{\pi/3}^{2\pi/3} \operatorname{cosec} t\,dt = \frac{1}{2}\log\left|\tan\frac{t}{2}\right|\Big]_{\pi/3}^{2\pi/3}$

$= \frac{1}{2}\left[\log\left|\tan\frac{\pi}{3}\right| - \log\left|\tan\frac{\pi}{6}\right|\right]$

$$= \frac{1}{2}\left[\log\sqrt{3} - \log\frac{1}{\sqrt{3}}\right]$$

$$= \frac{1}{2}\left[\log\sqrt{3} - \log 1 + \log\sqrt{3}\right] = \log\sqrt{3}$$

(iii) $$\int_0^{\pi/4} (2\sec^2 x + x^3 + 2)\, dx$$

$$= \frac{1}{2}\left[2\tan x + \frac{x^4}{4} + 2x\right]_0^{\pi/4}$$

$$= \left(2\tan\frac{\pi}{4} + \left(\frac{\pi}{4}\right)^4 \frac{1}{4} + \frac{2\pi}{4}\right) - (2\times 0 - 0 - 0)$$

$$= 2 + \frac{\pi^4}{1024} + \frac{\pi}{2}$$

(iv) $$\int_0^{\pi} \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx$$

$$= \int_0^{\pi} -\cos\left(2\,\frac{x}{2}\right) dx$$

$[\because \cos 2\theta = \cos^2\theta - \sin^2\theta]$

$$= -\int_0^{\pi} \cos x\, dx = -\sin x\Big]_0^{\pi}$$

$$= -(0-0) = 0$$

Ans-2. (i) $$\int_0^1 \frac{dx}{2x-3} = \frac{\log|2x-3|}{2}\Bigg]_0^1$$

$$= \frac{1}{2}\left[\log|2-3| - \log|-3|\right]$$

$$= \frac{1}{2}\left[\log 1 - \log 3\right] = -\frac{1}{2}\log 3$$

(iii) $\int_1^3 \frac{dx}{7-2x} = \frac{\log|7-2x|}{-2}\Big]_1^3$

$= -\frac{1}{2}[\log|7-6| - \log|7-2|]$

$= -\frac{1}{2}[\log 1 - \log 5] = \frac{1}{2}\log 5$

Ans-3.

(i) $\int_0^{\pi/4} \cos^2 3x \, dx$

$= \int_0^{\pi/4} \frac{1+\cos 6x}{2} dx$

$= \frac{1}{2}\Big[x + \frac{\sin 6x}{6} \Big]_0^{\pi/4}$

$= \frac{1}{2}\left[\frac{\pi}{4} + \frac{1}{6}\sin\frac{3\pi}{2} - 0 - 0\right]$

$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{6}\right]$

(ii) $\int_0^{\pi/4} \tan^2 x \, dx$

$= \int_0^{\pi/4} (\sec^2 x - 1) dx = \tan x - x\Big]_0^{\pi/4}$

$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$

$= 1 - \frac{\pi}{4}$

(iii) $\int_{\pi/3}^{\pi/4} (\tan x + \cot x)^2 dx$

$= \int_{\pi/3}^{\pi/4} (\tan^2 x + \cot^2 x + 2) dx$

$= \int_{\pi/3}^{\pi/4} (\sec^2 x - 1 + \operatorname{cosec}^2 x - 1 + 2) dx$

$= \int_{\pi/3}^{\pi/4} (\sec^2 x + \operatorname{cosec}^2 x) dx$

$= \tan x - \cot x \Big]_{\pi/3}^{\pi/4}$

$= \left(\tan\frac{\pi}{4} - \cot\frac{\pi}{4}\right) - \left(\tan\frac{\pi}{3} - \cot\frac{\pi}{3}\right)$

$= (1-1) - \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$= -\left(\frac{3-1}{\sqrt{3}}\right) = \frac{-2}{\sqrt{3}}$

(iv) $\int_0^{\pi} \frac{dx}{1+\sin x}$

$= \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$

$= \int_0^{x} \frac{1-\sin x}{\cos^2 x} dx$

$$= \int_0^{\pi} [\sec^2 x - \tan x \sec x]\, dx$$

$$= \tan x - \sec x \Big]_0^{\pi}$$

$$= (\tan \pi - \sec \pi) - (\tan 0 - \sec 0)$$

$$= (0 - (-1)) - (0 - 1) = 1 + 1 = 2$$

$$\int_0^{\pi/4} \sin 3x \sin 2x\, dx$$

$$= \frac{1}{2} \int_0^{\pi/4} (2 \sin 3x \sin 2x)\, dx$$

$$= \frac{1}{2} \int_0^{\pi/4} [\cos x - \cos 5x]\, dx$$

$$= \frac{1}{2} \left[\sin x - \frac{\sin 5x}{5}\right]_0^{\pi/4}$$

$$= \frac{1}{2} \left[\left(\sin \frac{\pi}{4} - \frac{1}{5} \sin \frac{5\pi}{4}\right) - (0 - 0)\right]$$

$$= \frac{1}{2} \left[\sin \frac{\pi}{4} - \frac{1}{5} \sin\left(\pi + \frac{\pi}{4}\right)\right]$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{2}} + \frac{1}{5\sqrt{2}}\right]$$

$$= \frac{1}{2} \left[\frac{5+1}{5\sqrt{2}}\right] = \frac{3}{5\sqrt{2}}$$

(iv) $$\int_0^{\pi/4} \sqrt{1 - \sin 2x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\sin^2 x + \cos^2 x - 2 \sin x \cos x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{(\cos x - \sin x)^2}\, dx$$

$$= \int_0^{\pi/4} |\cos x - \sin x|\, dx$$

$$\left[\text{when } 0 < x < \frac{\pi}{4} \Rightarrow \cos x > \sin x\right.$$

$$\left.\Rightarrow \cos x - \sin x > 0\right]$$

$$= \int_0^{\pi/4} (\cos x - \sin x)\, dx$$

$$= \sin x + \cos x\Big]_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 - \cos 0)$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1) = \sqrt{2} - 1$$

Ans-4.

$$I = \int_0^{\pi} \frac{d\phi}{5+3\cos\phi}$$

$$\tan\frac{\phi}{2} = t \Rightarrow \sec^2\frac{\phi}{2}\,\frac{1}{2}\,d\phi = dt \Rightarrow d\phi = \frac{2\,dt}{1+t^2}$$

$$\& \cos\phi = \frac{1 - \tan^2\frac{\phi}{2}}{1 + \tan^2\frac{\phi}{2}} = \frac{1-t^2}{1+t^2}$$

$$\text{when } \phi = 0 \Rightarrow t = 0;$$

$$\phi = \pi \Rightarrow t = \frac{\pi}{2} \to \infty$$

$$= \int_0^{\infty} \frac{\frac{2dt}{1+t^2}}{5+3\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= 2\int_0^{\infty} \frac{dt}{5+5t^2+3-3t^2} = 2\int_0^{\infty} \frac{dt}{2t^2+8}$$

$$= \int_0^{\infty} \frac{dt}{t^2+2^2} = \frac{1}{2}\tan^{-1}\frac{t}{2}\Big]_0^{\infty}$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{4}$$

(iii) $I = \int_0^{\pi/4} 2\tan^3 x \, dx$

$$= \int_0^{\pi/4} 2\tan x \cdot \tan^2 x \, dx$$

$$= 2\int_0^{\pi/4} \tan x(\sec^2 x - 1)\, dx$$

$$= 2\int_0^{\pi/4} \tan x \cdot \sec^2 x \, dx - 2\int_0^{\pi/4} \tan x \, dx$$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

when $x = 0 \Rightarrow t = 0$;

$x = \frac{\pi}{4} \Rightarrow t = 1$

$$= 2\int_0^1 t \cdot dt - 2\int_0^{\pi/4} \tan x \, dx$$

$$= 2\times\frac{t^2}{2}\Big]_0^1 + 2\log|\cos x|\Big]_0^{\pi/4}$$

$$= (1-0) + 2[\log(\cos\pi/4 - \log|\cos 0|]$$

$$= 1 + 2\log\frac{1}{\sqrt{2}}$$

$$= 1 + 2\log 2^{-1/2}$$

$$= 1 - \log 2$$

Ans - 5.

(i) $$I = \int_0^2 \frac{x^4+1}{x^2+1}\,dx$$

$$= \int_0^2 \frac{x^4-1+2}{x^2+1}\,dx$$

$$= \int_0^2 \frac{x^4-1}{x^2+1}\,dx + 2\int_0^2 \frac{dx}{x^2+1}$$

$$= \int_0^2 (x^2-1)\,dx + 2\int_0^2 \frac{dx}{x^2+1^2}$$

$$= \frac{x^3}{3} - x\Big]_0^2 + 2\tan^{-1}x\Big]_0^2$$

$$= \left(\frac{8}{3} - 2\right) + 2(\tan^{-1}2 - \tan^{-1}0)$$

$$= \frac{2}{3} + 2\tan^{-1}2$$

(ii) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \tan^{-1} x \Big]_1^{\sqrt{3}}$

$= \tan^{-1}\sqrt{3} - \tan^{-1} 1$

$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

(iii) $I = \int_3^5 \frac{x^2\,dx}{x^2-4}$

$= \int_3^5 \frac{x^2-4+4}{x^2-4}\,dx = \int_3^5 \left[1 + \frac{4}{x^2-4}\right] dx$

$= \left[x - \frac{4}{2\times 2} \log\left|\frac{x-2}{x+2}\right|\right]_3^5$

$\left[\because \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right|\right]$

$= \left[x - \log\left|\frac{x-2}{x+2}\right|\right]_3^5$

$= \left[5 - \log\left|\frac{5-2}{5+2}\right| - 3 + \log\left|\frac{3-2}{3+2}\right|\right]$

$= \left[-2 - \log\left|\frac{3}{7}\right| + \log\left|\frac{1}{5}\right|\right]$

$= 2 + \log \frac{\frac{1}{5}}{\frac{3}{7}} = 2 + \log \frac{7}{15}$

(iv) $I = \int_0^a \frac{dx}{\sqrt{ax - x^2}}$

$$= \int_0^a \frac{dx}{\sqrt{-\left(x^2 - ax + \frac{a^2}{4} - \frac{a^2}{4}\right)}}$$

$$= \int_0^a \frac{dx}{\sqrt{\left(\frac{a}{2}\right)^2 - \left(x - \frac{a}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{x - a/2}{a/2}\right)\Big]_0^a$$

$$\left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}\right]$$

$$= \sin^{-1}\left(\frac{2x - a}{a}\right)\Big]_0^a$$

$$= \sin^{-1}(1) - \sin(-1)$$

$$= \sin^{-1} 1 + \sin^{-1} 1$$

$$[\because \sin^{-1}(-x) = -\sin^{-1} x]$$

$$= 2\sin^{-1} 1 = 2 \times \frac{\pi}{2} = \pi$$

Ans-6. $I = \int_1^3 \frac{1}{x^2(x+1)} dx$

Let $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$ --- (1)

multiplying both sides of eqn (1) by $x^2(x+1)$; we get

$1 = Ax(x+1) + B(x+1) + Cx^2$ --(2)

Putting $x=0$ in eqn (2); we have $1 = B$

Putting $x=-1$ in eqn (2); we have $1 = C$

coeff of x^2; $0 = A+C$

$\Rightarrow A = -C = -1$

so from (1); we have

$$I = \int_1^3 \frac{-1}{x} dx + \int_1^3 \frac{1}{x^2} dx + \int_1^3 \frac{1}{x+1} dx$$

$$= -\log|x|\Big]_1^3 - \frac{1}{x}\Big]_1^3 + \log|x+1|\Big]_1^3$$

$$= [\log 3 - \log 1] - \left(\frac{1}{3} - 1\right) + \log 4 - \log 2$$

$$= -\log 3 + \frac{2}{3} + \log \frac{4}{2}$$

$$= \log 2 - \log 3 + \frac{2}{3}$$

$$= \log \frac{2}{3} + \frac{2}{3}$$

(iii) Let $I = \int_2^3 \frac{dx}{\sqrt{5x-6-x^2}}$

$$= \int_2^3 \frac{dx}{\sqrt{-(x^2+6-5x)}}$$

$$= \int_2^3 \frac{dx}{\sqrt{-\left\{x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 6\right\}}}$$

$$= \int_2^3 \frac{dx}{\sqrt{-\left\{\left(x-\frac{5}{2}\right)^2-\frac{1}{4}\right.}}$$

$$= \int_2^3 \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{5}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{x-5/2}{1/2}\right)\Big]_2^3$$

$$= \sin^{-1}(2x-5)\Big]_2^3$$

$$= \sin^{-1}1 + \sin^{-1}1$$

$$= 2\sin^{-1}1 = 2 \times \frac{\pi}{2} = \pi$$

(iii) $$I = \int_1^3 \frac{\log x\,dx}{(1+x)^2} = \int_1^3 \log x \frac{1}{(1+x)^2}\,dx$$

$$= \log x \frac{(1+x)^{-2+1}}{-2+1}\Big]_1^3 + \int_1^3 \frac{1}{x}\frac{(1+x)^{-2+1}}{(-2+1)}\,dx$$

$$= -\frac{\log x}{x+1}\Big]_1^3 + \int_1^3 \frac{1}{x(1+x)}\,dx$$

$$= -\left[\frac{\log 3}{4} - \frac{\log 1}{2}\right] + \int_1^3 \frac{1}{x} - \frac{1}{1+x}\Big]\,dx$$

$$= -\frac{\log 3}{4}\left[\log(x) - \log|1+x|\right]_1^3$$

$$= -\frac{1}{4}\log 3 + \log 3 - \log 4 - \log 1 + \log 2$$

$$= \frac{3}{4}\log 3 - \log 2^2 + \log 2$$

$$= \frac{3}{4}\log 3 - 2\log 2 + \log 2$$

$$= \frac{3}{4}\log 3 - \log 2$$

Ans-7.

(i) $$\int_0^1 \overset{(1)}{x}\, \overset{(2)}{e^x}\, dx$$

$$= x e^x \Big]_0^1 - \int_0^1 1\cdot e^x\, dx = \left[x e^x - e^x\right]_0^1$$

$$= (x-1)e^x\Big]_0^1 = (1-1)e^1 - (0-1)e^0 = 1$$

(ii) $$\int_e^{e^2} \frac{dx}{x\log x}$$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

when $x = e \Rightarrow t = \log e = 1$

when $x = e^2 \Rightarrow t = \log e^2 = 2\log e = 2$

$$= \int_1^2 \frac{dt}{t} = \log(t)\Big]_1^2 = \log 2 - \log 1$$

$$= \log 2 - 0 = \log 2$$

(iii) $$I = \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x}\, dx$$

$$= \int_1^2 \frac{1}{x} e^{2x}\, dx - \int_1^2 \frac{1}{2x^2} e^{2x}\, dx$$

$$= \frac{1}{x}\,\frac{e^{2x}}{2}\Big]_1^2 - \int_1^2 -\frac{1}{x^2}\,\frac{e^{2x}}{2}\, dx - \int_1^2 \frac{1}{2x^2}\, dx$$

$$= \frac{1}{2x} e^{2x} \Big]_1^2 + \int_1^2 \frac{e^{2x}}{2x^2} dx - \int_1^2 \frac{1}{2x^2} e^{2x} dx$$

$$= \frac{1}{4} e^4 - \frac{1}{2} e^2$$

Ans-8.

(i) $$I = \int_0^{\pi/2} x^2 \sin x \, dx$$

$$= x^2 (-\cos x)\Big]_0^{\pi/2} + \int_0^{\pi/2} \overset{(1)}{2x} \overset{(2)}{\cos x} \, dx$$

$$= -\left[\left(\frac{\pi}{2}\right)^2 \cos\frac{\pi}{2} - 0^2 \cos 0\right] + 2\left[x \sin x + \cos x\right]_0^{\pi/2}$$

$$= -(0-0) + 2\left[\frac{\pi}{2} \sin\frac{\pi}{2} + \cos\frac{\pi}{2} - 0 - 1\right]$$

$$= 2\left[\frac{\pi}{2} \times 1 + 0 - 1\right] = \pi - 2$$

(ii) $$I = \int_0^{\pi} \theta \sin^2\theta \cos\theta \, d\theta$$

$$= \theta \frac{(\sin\theta)^3}{3}\Big]_0^{\pi} - \int \frac{\sin^3\theta}{3} d\theta$$

$$\left[\text{if } \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1};\ n \neq -1\right]$$

$$= \frac{1}{3}\left[\pi \sin^3\pi - 0\right] - \frac{1}{3} \int_0^{\pi} \left(\frac{3\sin\theta - \sin 3\theta}{4}\right) d\theta$$

$$= +\frac{1}{3}(\pi \times 0 - 0) - \frac{1}{12}\left[-3\cos\theta + \frac{\cos 3\theta}{3}\right]_0^{\pi}$$

$$= -\frac{1}{12}\left[-3\cos\pi + \frac{\cos 3\pi}{3} + 3\cos 0 - \frac{\cos 0}{3}\right]$$

$$= -\frac{1}{12}\left[-3(-1) - \frac{1}{3} + 3 - \frac{1}{3}\right]$$

$$= -\frac{1}{12}\left[6 - \frac{2}{3}\right] = -\frac{16}{36} = -\frac{4}{9}$$

Ans-9.

(i) $I = \int_0^1 \overset{(1)}{\sin^{-1}\sqrt{x}} \cdot \overset{(2)}{1}\, dx$

$$= \left(\sin^{-1}\sqrt{x}\,\right)x\Big]_0^1 - \int_0^1 \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}} \cdot x\, dx$$

$$= (\sin^{-1} 1 - \sin^{-1} 0) - \frac{1}{2}\int_0^1 \frac{x\, dx}{x - x^2}$$

$$= \frac{\pi}{2} + \frac{1}{4}\int_0^1 \frac{(-2x+1-1)\,dx}{\sqrt{x-x^2}}$$

$$= \frac{\pi}{2} + \frac{1}{4}\int_0^1 (x-x^2)^{-\frac{1}{2}}(-2x+1)\,dx$$

$$- \frac{1}{4}\int_0^1 \frac{dx}{\sqrt{-(x^2 - x + \frac{1}{4} - \frac{1}{4})}}$$

$$= \frac{\pi}{2} + \frac{1}{4}\,\frac{(x-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Big]_0^1 - \frac{1}{4}\int_0^1 \frac{dx}{\sqrt{(\frac{1}{2})^2 - (x-\frac{1}{2})^2}}$$

$$= \frac{\pi}{2} + \frac{1}{4}(0-0) - \frac{1}{4}\sin^{-1}(2x-1)\Big]_0^1$$

$$= \frac{\pi}{2} - \frac{1}{4}(2\sin^{-1}1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

(iii) $I = \int_0^1 \overset{(2)}{x^2}\, \overset{(1)}{\sin^{-1}} x\, dx$;

Put $\sin^{-1}x = \theta \Rightarrow x = \sin\theta$

$\Rightarrow dx = \cos\theta \, d\theta$

when $x = 0 \Rightarrow \theta = 0$;

when $x = 1 \Rightarrow \theta = \frac{\pi}{2}$

So $I = \int_0^{\pi/2} \sin^2\theta \cdot \theta \cos\theta \, d\theta$

$$= \int_0^{\pi/2} \theta \cdot (\sin^2\theta \cos\theta)\, d\theta$$

$$= \theta \frac{\sin^3\theta}{3}\Big]_0^{\pi/2} - \int_0^{\pi/2} \frac{\sin^3\theta}{3}\, d\theta$$

$$= \left\{\frac{\pi}{6}(1)^3 - 0\right\} - \frac{1}{3}\int_0^{\pi/2} \frac{3\sin\theta - \sin 3\theta}{4}\, d\theta$$

$$= \frac{\pi}{6} - \frac{1}{12}\left[-3\cos\theta + \frac{\cos 3\theta}{3}\right]_0^{\pi/2}$$

$$= \frac{\pi}{6} - \frac{1}{12}\left[0 + 0 + 3 - \frac{1}{3}\right] = \frac{\pi}{6} - \frac{2}{9}$$

Ans-10.

(i) $\int_0^a 3x^2\,dx = 8$

$\Rightarrow 3\frac{x^2}{3}\Big]_0^a = 8 \Rightarrow a^3 = 8$

$\Rightarrow (a-2)(a^2+2a+4) = 0$

$\Rightarrow a-2 = 0 \Rightarrow a = 2$

since $a^2+2a+4=0$ does not given any real values of a.

(ii) $\int_a^b x^3\,dx = 0$

$\Rightarrow \frac{x^4}{4}\Big]_a^b = 0 \Rightarrow \frac{1}{4}(b^4-a^4) = 0$

$\Rightarrow b^4 - a^4 = 0$

$\Rightarrow (b-a)(b+a)(b^2-a^2) = 0$ --(1)

& $\int_a^b x^2\,dx = \frac{2}{3}$

$\Rightarrow \frac{x^3}{3}\Big]_a^b = \frac{2}{3} \Rightarrow \frac{1}{3}(b^3-a^3) = \frac{2}{3}$

$\Rightarrow b^3 - a^3 = 2$

$\Rightarrow (b-a)(b^2+ab+a^2) = 2$ --(2)

from (1); $b-a = 0 \Rightarrow b = a$

so from (2); $0 = 2$, which is false.

when $b+a = 0 \Rightarrow b = -a$

so from (2); $(-2a)(a^2-a^2+a^2) = 2$

$\Rightarrow -2a^3 = 2$

$\Rightarrow a^3 = -1 \Rightarrow a = -1$

So $b = 1$

Also $b^2 + 2 = 0$ & $(b-a)(b^2 + ab + a^2) = 2$

does not given any real values of a and b.

(iii) Given $f(x) = a + bx + cx^2$ ---(i)

LHS $= \int_0^1 f(x)\,dx = \int_0^1 (a + bx + cx^2)\,dx = \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]$

$= a + \frac{b}{2} + \frac{c}{3}$

Putting $x = 0$ in eqn (1) ; $f(0) = a$

Putting $x = \frac{1}{2}$ in eqn (1) ;

$f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

Putting $x = 1$ in eqn (1) ;

$f(1) = a + b + c$

R.H.S. $= \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right]$

$= \frac{1}{6}\left[a + 4\left(a + \frac{b}{2} + \frac{c}{4}\right) + a + b + c\right]$

$= \frac{1}{6}\left[6a + 3b + c\right] = a + \frac{b}{2} + \frac{c}{4}$

That L.H.S = R.H.S

$\therefore$ Hence, $\int_0^1 f(x)\,dx = \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right]$

(iv) $\int_0^K \frac{dx}{2+8x^2} = \frac{\pi}{16}$

$\Rightarrow \frac{1}{8}\int_0^K \frac{dx}{x^2 + \left(\frac{1}{2}\right)^2} = \frac{\pi}{16}$

$\Rightarrow \frac{1}{8} \times \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x}{1/2}\right)\Big]_0^K = \frac{\pi}{16}$

$\Rightarrow \frac{1}{4}\left[\tan^{-1}(2K) - \tan^{-1} 0\right] = \frac{\pi}{16}$

$\Rightarrow \tan^{-1} 2K = \frac{\pi}{4} \Rightarrow 2K = \tan\frac{\pi}{4} = 1$