Chapter-13 Indefinite Integral -1
(Standard Forms)

Exercise 13(a)

Sol1 ⇒ (i) $\int \sin 2x \, dx$

$\Rightarrow -\frac{\cos 2x}{2} + C.$

(ii) $\int 2 \sin 3x \, dx.$

$\Rightarrow -2\frac{\cos 3x}{3} + C.$

(iii) $\int \frac{1}{3} \cos 4x \, dx. = \frac{1}{3} \frac{\sin 4x}{4} + C.$

$\Rightarrow \frac{1}{12} \sin 4x + C.$

(iv) $\int \frac{\cos 5x}{2} \, dx$

$\Rightarrow \frac{\sin 5x}{10} + C.$

(v) $\int 8 \sec^2 8x \, dx.$

$\Rightarrow 8 \frac{\tan 8x}{8} + C.$

$\Rightarrow \tan 8x + C.$

(vi) $\int \operatorname{cosec}^2 2x = -\frac{\cot 2x}{2} + C$

(vii) $\int \sec 5x \tan 5x \, dx$

$\Rightarrow \frac{\sec 5x}{5} + C$

(viii) $\int -\operatorname{cosec} 3x \cot 3x \, dx$

$\Rightarrow \left(-\frac{\operatorname{cosec} 3x}{3}\right) + C$

$\Rightarrow \frac{\operatorname{cosec} 3x}{3} + C.$

Sol 2 ⇒ (i) $\int \cos(5-3x) dx$

$\Rightarrow \frac{\sin(5-3x)}{-3} + C$ $\left[\because \int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + C\right]$

(ii) $\int 2\sin\left(\frac{\pi}{2} - \frac{x}{2}\right) dx.$

$\Rightarrow \int 2\cos\frac{x}{2} dx$

$\Rightarrow 2 \frac{\sin\frac{x}{2}}{\frac{1}{2}} + C \Rightarrow 4\sin\frac{x}{2} dx + C.$

(iii) $\int \sin\left(\frac{3}{4}x + 5\right) dx$

$$\Rightarrow \frac{-\cos\left(\frac{3}{4}x + 5\right)}{3/4} + C$$

$$\Rightarrow -\frac{4}{3}\cos\left(\frac{3}{4}x + 5\right) + C.$$

(iv) $\int 4\sec^2(2x-4)\, dx$

$$\Rightarrow \frac{4\tan(2x-4)}{2} + C.$$

$$\Rightarrow 2\tan(2x-4) + C.$$

Sol 3 $\Rightarrow$ (i) $\int \sin^2 x\, dx$

$$\Rightarrow \int \frac{1-\cos 2x}{2}\, dx.$$

$$\Rightarrow \frac{1}{2}\left[\int 1\, dx - \int \cos 2x\, dx\right]$$

$$\Rightarrow \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C.$$

(ii) $\int \cos^2 x \, dx$

$\Rightarrow \int \frac{1+\cos 2x}{2} \, dx.$

$\Rightarrow \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + C.$

$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{4} + C.$

(iii) $\int \sin^3 x \, dx$

$\Rightarrow \int \frac{1}{4}\left[3\sin x - \sin 3x\right] dx.$

$\left[\text{Since } \sin 3\theta = 3\sin\theta - 4\sin^3\theta \right.$

$\left.\Rightarrow \sin^3\theta = \frac{1}{4}\left[3\sin\theta - \sin 3\theta\right]\right]$

$\Rightarrow \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right] + C.$

(iv) $\int \sin^2 mx \, dx$

$\Rightarrow \int \frac{1-\cos 2mx}{2} \, dx$

$\Rightarrow \frac{1}{2}\left[x - \frac{\sin 2mx}{2m}\right] + C.$

(v) $\int \sin^2 x \cos^2 x \, dx.$

$\Rightarrow \frac{1}{4} \int 4\sin^2 x \cos^2 x \, dx.$

$\Rightarrow \frac{1}{4} \int (2\sin x \cos x)^2 dx.$

$\Rightarrow \frac{1}{4} \int \sin^2 2x \, dx$

$\Rightarrow \frac{1}{4} \int \frac{1-\cos 4x}{2} dx.$

$\Rightarrow \frac{1}{8} \left[x - \frac{\sin 4x}{4}\right] + C.$

$\Rightarrow \frac{x}{8} - \frac{1}{32} \sin 4x + C.$

(vi) $\int \sin^3 x \cos^3 x \, dx$

$\Rightarrow \frac{1}{8} \int 8\sin^3 x \cos^3 x \, dx.$

$\Rightarrow \frac{1}{8} \int (2\sin x \cos x)^3 dx.$

$\Rightarrow \frac{1}{8} \int \sin^3 2x \, dx$

$\Rightarrow \frac{1}{8} \int \frac{1}{4} [3\sin 2x - \sin 6x] dx.$ $[\because \sin 3x = 3\sin x - 4\sin^3 x]$

$\Rightarrow \frac{1}{32} \left[-\frac{3\cos 2x}{2} + \frac{\cos 6x}{6}\right] + C$

$= -\frac{3\cos 2x}{64} + \frac{1}{192}\cos 6x + C.$

(vii) $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\,dx.$

$\Rightarrow \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x}\,dx.$

$\Rightarrow \int \frac{1}{\cos^2 x}\,dx$

$\Rightarrow \int \sec^2 x\,dx \Rightarrow \tan x + C.$

(viii) $\int \sin x \sec^2 x\,dx.$

$\Rightarrow \int \sin x \cdot \sec x \cdot \sec x \cdot dx.$

$\Rightarrow \int \frac{\sin x}{\cos x} \sec x \cdot dx.$

$\Rightarrow \int \tan x \sec x\,dx$

$\Rightarrow \sec x + C.$

(ix) $\int \sin x \cos x\,dx.$

$\Rightarrow \frac{1}{2} \int \sin 2x\,dx.$

$\Rightarrow \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + C.$

$= -\frac{\cos 2x}{4} + C.$

(x) $\int \frac{1}{\sin^2 x \cos^2 x} dx$

$\Rightarrow \int \frac{4 dx}{4 \sin^2 x \cos^2 x}$

$\Rightarrow \int \frac{4 dx}{(2 \sin x \cos x)^2}$

$\Rightarrow 4 \int \frac{dx}{\sin^2 2x}$

$\Rightarrow 4 \int \operatorname{cosec}^2 2x \, dx$

$\Rightarrow \frac{\cancel{4}^{2} (-\cot 2x)}{\cancel{2}} + C$

$\Rightarrow 2(-\cot 2x) + C.$

(xi) $\int \frac{\sec x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{\sec x (\sec x - \tan x) dx}{(\sec x + \tan x)(\sec x - \tan x)}$

$\Rightarrow \int \frac{\sec x (\sec x - \tan x) dx}{\sec^2 x - \tan^2 x}$

$\Rightarrow \int \frac{(\sec^2 x - \sec x \tan x)}{1} dx.$

$\Rightarrow \int \sec^2 x\,dx - \int \sec x \tan x\,dx.$

$\Rightarrow \tan x - \sec x + C.$

(xiii) $\int (3\operatorname{cosec}^2 x + 2\sin 3x)\,dx.$

$\Rightarrow 3\int \operatorname{cosec}^2 x + 2\int \sin 3x\,dx.$

$\Rightarrow 3(-\cot x) + 2\left(-\frac{\cos 3x}{3}\right) + C$

$\Rightarrow -3\cot x - \frac{2}{3}\cos 3x + C.$

Sol 14 $\Rightarrow$ (i) $\int \cos 4x \cos 3x\,dx.$

$\Rightarrow \frac{1}{2}\int 2\cos 4x \cos 3x\,dx.$

$\Rightarrow \frac{1}{2}\int [\cos(4x+3x) + \cos(4x-3x)]\,dx$

$\Rightarrow \frac{1}{2}\int [\cos 7x + \cos x]\,dx.$

$\Rightarrow \frac{1}{2}\left[\frac{\sin 7x}{7} + \sin x\right] + C.$

(ii) $\int \sin 4x \sin 8x.$

$\Rightarrow \frac{1}{2}\int 2\sin 8x \sin 4x\,dx.$

$\Rightarrow \frac{1}{2}\int [\cos(8x-4x) - \cos(8x+4x)]\,dx$

$\Rightarrow \frac{1}{2}\int[\cos 4x - \cos 12x]\,dx$

$\Rightarrow \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right] + C$

Sol 5 ⇒ (i) $\int \cos 2x \cos 4x \cos 6x\,dx$

$\Rightarrow \frac{1}{2}\int(2\cos 4x \cos 2x)\cos 6x\,dx$

$\Rightarrow \frac{1}{2}\int[\cos(4x+2x) + \cos(4x-2x)]\cos 6x\,dx$

$\Rightarrow \frac{1}{2}\int(\cos 6x + \cos 2x)\cos 6x\,dx$

$\Rightarrow \frac{1}{2}\int(\cos^2 6x + \cos 6x \cos 2x)\,dx$

$\Rightarrow \frac{1}{2}\int\left(\frac{1+\cos 12x}{2}\right)dx + \frac{1}{4}\int 2\cos 6x \cos 2x\,dx$

$\Rightarrow \frac{1}{4}\left[x + \frac{\sin 12x}{12}\right] + \frac{1}{4}\int(\cos 8x + \cos 4x)\,dx$

(ii) $\int \sin x \sin 2x \sin 3x \, dx.$

$$= \frac{1}{2}\int (2\sin 2x \sin x) \sin 3x \, dx$$

$$= \frac{1}{2}\int [\cos(2x-x) - \cos(2x+x)] \sin 3x \, dx$$

$$= \frac{1}{2}\int (\cancel{\sin}\cos x - \cos 3x) \sin 3x \, dx$$

$$= \frac{1}{2}\int (\sin 3x \cos x - \cos 3x \sin 3x) \, dx$$

$$= \frac{1}{4}\int [\sin 4x + \sin 2x - \sin 6x] \, dx$$

$$= \frac{1}{4}\left[-\frac{\cos 4x}{4} - \frac{\cos 2x}{2} + \frac{\cos 6x}{6}\right] + C$$

(iii) $\int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}} \, dx$

$$= \int \frac{\cos 2x \, dx}{\sqrt{2\cos^2 2x}} = \int \frac{\cos 2x}{\sqrt{2}\cos 2x} \, dx$$

$$= \int \frac{dx}{\sqrt{2}} = \frac{x}{\sqrt{2}} + C$$

(iv) $\int \cos^4 x \sin^4 x \, dx$

$$= \frac{1}{16}\int 16 \sin^4 x \cos^4 x \, dx$$

$$= \frac{1}{16}\int (2\sin x \cos x)^4 \, dx$$

$$= \frac{1}{16}\int \sin^4 2x \, dx$$

$$= \frac{1}{16}\int \left(\frac{1-\cos 4x}{2}\right)^2 dx$$

$$= \frac{1}{64}\int [1 + \cos^2 4x - 2\cos 4x]\, dx$$

$$= \frac{1}{64}\int \left[1 + \frac{1+\cos 8x}{2} - 2\cos 4x\right] dx$$

$$= \frac{1}{128}\int (3 + \cos 8x - 4\cos 4x)\, dx$$

$$= \frac{1}{128}\left[3x + \frac{\sin 8x}{8} - 4\frac{\sin 4x}{4}\right] + C$$

$$= \frac{1}{128}\left[3x + \frac{\sin 8x}{8} - \sin 4x\right] + C$$

(v) $\int \frac{7\cos^3 x + 4\sin^3 x}{3\sin^2 x \cos^2 x}\, dx$

$$= \int \left[\frac{7}{3}\frac{\cos x}{\sin^2 x} + \frac{8}{3}\frac{\sin x}{\cos^2 x}\right] dx$$

$$= \frac{7}{3}\int \cot x \operatorname{cosec} x\, dx + \frac{8}{3}\int \tan x \sec x\, dx$$

$$= -\frac{7}{3}\operatorname{cosec} x + \frac{8}{3}\sec x + C$$

(6) Sol:- (i) $\int \frac{1}{1+\cos x}\, dx = \int \frac{1}{2\cos^2 \frac{x}{2}}\, dx$

$$= \frac{1}{2}\int \sec^2 \frac{x}{2}\, dx = \frac{1}{2}\,\frac{\tan \frac{x}{2}}{\frac{1}{2}} + C$$

$$= \tan \frac{x}{2} + C$$

Sol:- (ii) $\int \frac{1}{1-\cos 2x}\,dx$

$= \int \frac{dx}{2\sin^2 x} = \frac{1}{2}\int \operatorname{cosec}^2 x\,dx$

$= \frac{-\cot x}{2} + C$

Sol:- (iii) $\int \frac{1}{1-\sin x}\,dx$

$= \int \frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}\,dx$

$= \int \frac{(1+\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1+\sin x)}{\cos^2 x}\,dx$

$= \int [\sec^2 x + \tan x \sec x]\,dx$

$= \tan x + \sec x + C$

Sol:- (iv) $= \int \frac{1-\cos 2x}{1+\cos 2x}\,dx$

$= \int \frac{2\sin^2 x}{2\cos^2 x}\,dx = \int \tan^2 x\,dx$

$= \int (\sec^2 x - 1)\,dx$

$= \int \sec^2 x\,dx - \int dx$

$= \tan x - x + C$

Sol:-(v) $\int\sqrt{1+\cos x}\,dx$

$= \int\sqrt{2\cos^2\frac{x}{2}}\,dx$

$= \sqrt{2}\int\cos\frac{x}{2}\,dx = \sqrt{2}\,\frac{\sin\frac{x}{2}}{\frac{1}{2}} + c$

$= 2\sqrt{2}\sin\frac{x}{2} + c$

Sol:-(vi) $\int\sqrt{1+\sin 2x}\,dx$

$= \int\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}\,dx$

$= \int\sqrt{(\sin x+\cos x)^2}\,dx$

$= \int(\sin x+\cos x)\,dx$

$= -\cos x + \sin x + c$

(vii) $\int\cos x\sqrt{1+\cos 2x}\,dx$

$= \int\cos x\sqrt{2\cos^2 x}\,dx$

$= \sqrt{2}\int\cos^2 x\,dx = \sqrt{2}\int\frac{1+\cos 2x}{2}\,dx$

$= \frac{1}{\sqrt{2}}\left[x+\frac{\sin 2x}{2}\right] + c$

(viii) $\int \sin x \sqrt{1+\cos 2x}\, dx$

$= \int \sin x \sqrt{2\sin^2 x}\, dx$

$= \sqrt{2} \int \sin^2 dx = \sqrt{2} \int \frac{1-\cos 2x}{2} dx$

$= \frac{1}{\sqrt{2}} \left[x - \frac{\sin 2x}{2} \right] + C$

(ix) $\int \frac{\cos x - \sin x}{\cos x + \sin x} (2 + 2\sin 2x)\, dx$

$= 2\int \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right) [\sin^2 x + \cos^2 x + 2\sin x \cos x] dx$

$= 2\int \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right) (\cos x + \sin x)^2\, dx = 2\int (\cos^2 x - \sin^2 x)\, dx$

$= 2\int \cos 2x\, dx = 2\frac{\sin 2x}{2} + C = \sin 2x + C$

(x) $\int \left[\frac{4 - 5\sin x}{\cos^2 x} + \frac{1}{\sin^2 x \cos^2 x} \right] dx$

$= \int 4\sec^2 dx - 5\int \tan x \sec x\, dx + \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cos^2 x} dx$

$= 4\tan x - 5\sec x + \int \sec^2 x\, dx + \int \operatorname{cosec}^2 x\, dx$

$= 4\tan x - 5\sec x + \tan x - \cot x + C = 5\tan x - 5\sec x - \cot x + C$

(xi) $\int \frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\,dx = \int \frac{(\sin x+\cos x)\,dx}{\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}}$

$= \int \frac{(\sin x+\cos x)\,dx}{\sqrt{(\sin x+\cos x)^2}} = \int \frac{(\sin x+\cos x)\,dx}{\sin x+\cos x} = \int dx = x+C$

Sol 7 ⇒ $\int \sqrt{\left(1+\sin \frac{x}{2}\right)}\,dx$

$= \int \sqrt{\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} + 2\sin \frac{x}{4}\cos \frac{x}{4}}\,dx$

$= \int \sqrt{\left(\sin \frac{x}{4} + \cos \frac{x}{4}\right)^2}\,dx$

$= \int \left(\sin \frac{x}{4} + \cos \frac{x}{4}\right)dx$

$= \frac{-\cos \frac{x}{4}}{\frac{1}{4}} + \frac{\sin \frac{x}{4}}{\frac{1}{4}} + C$

$\Rightarrow 4\left[-\cos \frac{x}{4} + \sin \frac{x}{4}\right] + C$

Sol 8 ⇒ $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos x}\,dx$

$\Rightarrow \int \frac{(\sin^2 x+\cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)}{\sin^2 x\cos^2 x}\,dx$

$$\Rightarrow \int \left(\frac{1 - 3\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$\Rightarrow \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cos^2 x}\, dx - \int 3\, dx$$

$$\Rightarrow \int \sec^2 dx + \int \operatorname{cosec}^2 x\, dx - 3x + C$$

$$\Rightarrow \tan x - \cot x - 3x + C$$

Sol 9 $\Rightarrow \int \sin^6 x\, dx$

$$= \int (\sin^3 x)^2 dx$$

$$\Rightarrow \int \left[\frac{3\sin x - \sin 3x}{4} \right]^2 dx$$

$$\Rightarrow \frac{1}{16} \int \left(9\sin^2 x + \sin^2 3x - 6\sin 3x \sin x \right) dx$$

$$\Rightarrow \frac{1}{16} \int \left[9\left(\frac{1 - \cos 2x}{2} \right) + \left(\frac{1 - \cos 6x}{2} \right) - 3(\cos 2x - \cos 4x) \right] dx$$

$$\Rightarrow \frac{1}{32} \int (10 - 9\cos 2x - \cos 6x - 6\cos 2x + 6\cos 4x)\, dx$$

$\Rightarrow \frac{1}{32}\int (10 - 15\cos 2x - \cos 6x + 6\cos 4x)\,dx.$

$\Rightarrow \frac{1}{32}\left(10x - \frac{15}{2}\sin 2x - \frac{\sin 6x}{6} + \frac{3}{2}\sin 4x\right) + C.$

Sol 10 $\Rightarrow \int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right) dx$

$\Rightarrow \int \tan^{-1}\left(\frac{2\sin x\cos x}{2\cos^2 x}\right) dx.$

$\Rightarrow \int \tan^{-1}(\tan x)\, dx$

$\Rightarrow \int x\, dx.$

$\Rightarrow \frac{x^2}{2} + C.$

Sol 11 $\Rightarrow \int \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan x}\right) dx.$

$\Rightarrow \int \cos^{-1}(\cos 2x)\, dx.$

$\Rightarrow \int 2x\, dx$

$\Rightarrow x^2 + C.$

Sol 12 => $\int \cos^{-1}(\sin x)\,dx$.

$\Rightarrow \int \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] dx$

$\Rightarrow \int \left(\frac{\pi}{2} - x\right) dx = \frac{\pi}{2}x - \frac{x^2}{2} + C.$

Sol 13 => Given $f'(x) = 3x^2 - \frac{2}{x^3}$;

by integrating both sides, we get.

$f(x) = \frac{3x^3}{3} - 2\frac{x^{-3+1}}{-3+1} + C$

$\Rightarrow f(x) = x^3 + \frac{1}{x^2} + C$ —— (1)

Since $f(1) = 0$

$\therefore$ from (1); $0 = 1 + \frac{1}{1} + C$

$\Rightarrow C = -2$

Hence eq(1) gives; $f(x) = x^3 + \frac{1}{x^2}$.

Sol 14 => Given $f'(x) = a\sin x + b\cos x$

since $f'(0) = 4$

$\therefore$ from (1); $4 = a \times 0 + b \times 1$

$\Rightarrow b = 4$

Also $f(x) = \int f'(x)\,dx + C$.

$\Rightarrow f(x) = \int (a \sin x + b \cos x)\, dx + C$

$\Rightarrow f(x) = -a \cos x + b \sin x + C$ —— (2)

Since $f(0) = 3$.

$\therefore$ from (2); $3 = -a \times 1 + b \times 0 + C$

$\Rightarrow 3 = -a + C$ —— (3)

Also $f(\pi/2) = 5$ i.e when $x = (\pi/2)$, $f(x) = 5$

$\therefore$ from (2); $5 = -a \times 0 + b \times 1 + C$

$\Rightarrow 5 = b + C \Rightarrow 5 = 4 + C$

$\Rightarrow C = 1$

Hence from eq(3); $3 = -a + 1$.

$\Rightarrow a = -2$

$\therefore$ from eq(2); we have

$f(x) = -2 \cos x + 4 \sin x + 1$