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Class - 12

CHAPTER-23-THREE DIMENSIONAL GEOMETRY

EXERCISE 23 (a)

Ans-1. The direction ratio of line are $<1, -2, -2>$

So direction cosines of line be

$$< \frac{1}{\sqrt{1^2+(-2)^2+(-2)^2}}, \frac{-2}{\sqrt{1^2+(-2)^2+(-2)^2}}, \frac{-2}{\sqrt{1^2+(-2)^2+(-2)^2}} >$$

i.e. $< \frac{1}{3}, \frac{-2}{3}, \frac{-2}{3} >$

Ans-2. a, β and γ are angles which a line makes with axes

So direction cosines of line are $< \cos a, \cos\beta, \cos\gamma >$

So

$\cos^2 a + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2 a + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2 a + \sin^2\beta + \sin^2\gamma = 3 - 1 = 2$

Ans-3. Direction cosines of line be $< \cos 45°, \cos 60°, \cos 120° >$

$< \frac{1}{\sqrt{2}}, \frac{1}{2}, \cos(180° - 60°) >$ i.e $< \frac{1}{\sqrt{2}}, \frac{1}{2}, -\frac{1}{2} >$

$l = \frac{1}{\sqrt{2}}$, $m = \frac{1}{2}$ and $n = -\frac{1}{2}$

So $l^2 + m^2 + n^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$

Yes a line can hence direction angles 45° 60° and 120°

Ans- 4.

- $l = m = n = 1$

So $l^2 + m^2 + n^2 = 1 + 1 = 3 \neq 1$

That, $\langle 1, 1, 1 \rangle$ can't be the dirction cosines of straight line

Ans-5.

we know that direction ratios of the line Joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are $\langle x_2 - x_1, y_2 - y_1, z_1 \rangle$

(i) Direction ratios of line AB are $\langle 4-0, 8-0 \rangle$

i.e, $\langle 4, 8, -8 \rangle$

i.e $\langle 1, 2, -2 \rangle$

So Direction cosines of line AB are

$$\left\langle \frac{4}{\sqrt{4^2+8^2+(-8)^2}}, \frac{8}{\sqrt{4^2+8^2+(-8)^2}}, \frac{-8}{\sqrt{4^2+8^2+(-8)^2}} \right\rangle$$

i.e $\left\langle \frac{4}{12}, \frac{8}{12}, \frac{-8}{12} \right\rangle$

i.e $\left\langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \right\rangle$.

(ii) D' ratios of line AB are $\langle -1-1, 0-3, -1-5 \rangle$

i.e $\langle -2, -3, -6 \rangle$

i.e $\langle 2, 3, 6 \rangle$

So D' cosines of line AB are

$$\left\langle \frac{2}{\sqrt{2^2+3^2+6^2}}, \frac{3}{\sqrt{2^2+3^2+6^2}}, \frac{6}{\sqrt{2^2+3^2+6^2}} \right\rangle$$

i.e. $\left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right\rangle$.

(iii) D' ratios of line AB are $\langle 1-5, -6-6, 3+3 \rangle$

i.e $\langle -4, -12, 6 \rangle$

i.e $\langle 2, 6, -3 \rangle$

So D' cosines of line AB are

$$\left\langle \frac{2}{\sqrt{2^2+6^2+(-3)^2}}, \frac{6}{\sqrt{2^2+6^2+(-3)^2}}, \frac{-3}{\sqrt{2^2+6^2+(-3)^2}} \right\rangle$$

i.e. $\left\langle \frac{2}{7}, \frac{6}{7}, \frac{-3}{7} \right\rangle$

(iv) D' ratios of line AB are $\langle -2-4, 1-2, 3+6 \rangle$

i.e $\langle -6, -1, 9 \rangle$

i.e $\langle 6, 1, -9 \rangle$

So D' cosines of line are

Ans-6.

(i) Direction ratios of line AB are $\langle 4-1, 0-2, 4-3 \rangle$

i.e. $\langle 3, -2, 1 \rangle$

Direction ratios of line BC are $\langle -2-4, 4-0, 2-4 \rangle$

i.e $\langle 3, -2, 1 \rangle$

That line AB is parallel to line BC and the point B is common in both lines.

So points A, B and C line on same line

So points A, B and C are collinear.

(iii) Direction ratios of line AB are $\langle 3+2, -6-4, -8-7 \rangle$

i.e. $\langle 5, -10, -15 \rangle$

i.e. $\langle 1, -2, -3 \rangle$

more $\frac{1}{-2} = \frac{-2}{4} = \frac{-3}{6}$

i.e direction ratios of both lines are proportion and hence line AB and BC are parallel and the points B in common to both lines

So A, B and C are collinear.

Ans-7.

θ the angle made by the line with z-axis

so direction cosines of given line be $\langle \cos\frac{\pi}{4}, \cos\frac{\pi}{4}, \cos\theta \rangle$

i.e. $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \cos\theta \rangle$

So $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\theta = 1$ or $\cos^2\theta = 0$ or $\cos\theta = 0$

$= \quad \theta = \frac{\pi}{4}$

Ans-8.

θ be the angle made by the line OP with z-axis

so direction cosines of line OP are

$\langle \cos 120^\circ, \cos 60^\circ, \cos\theta \rangle$

i.e. $\langle -\frac{1}{2}, \frac{1}{2}, \cos\theta \rangle$

since $l^2 + m^2 + n^2 = 1 \Rightarrow \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2 = \cos^2\frac{\pi}{4}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

Since $0 \le \theta \le \pi$

So $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Ans-9.

θ be the angle between given vectors whose direction ratios are $\langle 2, 3, -6 \rangle$ and $\langle 3, -4, 5 \rangle$

so $a_1 = 2$; b_1 3; $c_1 = -6$ and $a_2 = 3$; $b_2 = -4$;

$c_2 = 5$

So $\cos\theta = \frac{(a_1a_2 + b_1b_2 + c_1c_2)}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$$= \frac{2(3) + 3(-4) - 6(5)}{\sqrt{2^2 + 3^2 + (-6)^2}\sqrt{3^2 + (-4)^2 + 5^2}}$$

$$\Rightarrow \cos\theta = \frac{-36}{7 \times 5\sqrt{2}} = -\frac{18\sqrt{2}}{35}$$

Ans-10.

(i) Since a, β, γ are the angles the line makes with the axies.

So direction cosines of line are $<\cos a, \cos\beta, \cos\gamma>$

So $\cos^2 a + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \left(\frac{14}{15}\right)^2 + \left(-\frac{1}{3}\right)^2 + \cos^2\gamma = 1$$

$$\Rightarrow \frac{196}{225} + \frac{1}{9} + \cos^2\gamma = 1$$

$$\Rightarrow \frac{221}{225} + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\gamma = 1 - \frac{221}{225} = \frac{4}{225} = \left[\frac{2}{15}\right]^2$$

$$\Rightarrow \cos^2\gamma = \pm\frac{2}{15}$$

(ii) $a = 60°$, $\beta = 135°$

So $\cos^2\gamma = 1 - \cos^2 60° - \cos^2 135° = 1 - \left[\frac{1}{2}\right]^2 - \left[\frac{1}{\sqrt{2}}\right]^2$

$$= 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$$

$\Rightarrow \quad \cos^2 y = \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad \cos y = \pm \frac{1}{2}$

Ans-11.

D' ratios of line OA are $\langle 2-0, 3-0, 4-0 \rangle$

i.e $\langle 2, 3, 4 \rangle$

D' ratios of line OB are $\langle 1-0, -2-0, 1-0 \rangle$

i.e $\langle 1, -2, 1 \rangle$

Here $a_1 a_2 + b_1 b_2 + c_1 c_2 = 2(1) + 3(-2) + 4(1) = 0$

So line OA be $\perp$ to line OB.

Ans- 12.

Direction Ratios of the Joining A(1, 2, 3) and B(4, 5, 7)

are $\langle 4-1, 5-2, 7-3 \rangle$ i.e $\langle 3, 3, 4 \rangle$

and D' ratios of the Joing Join C(-4, 3, -6) and

D(2, 9, 2) are

$\langle 2+4, 9-3, 2+6 \rangle$ i.e $\langle 6, 6, 8 \rangle$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Since $\frac{3}{6} = \frac{3}{6} = \frac{4}{8} = \frac{1}{2}$

i.e D' ratios of both lines are proportional.

So line AB is parallel to line CD.

Ans - 13.

(i) Given direction ratios of lines are $\langle 5, -12, 13 \rangle$

and $\langle -3, 4, 5 \rangle$

$a_1 = 3 ; b_1 = -12 ; c_1 = 13$

$a_2 = -3 ; b_2 = 4 ; c_2 = 5$

Let θ be the angle between the lines.

Then $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \dfrac{5(-3) - 12(4) + 13(5)}{\sqrt{5^2 + (-12)^2 + 13^2}\sqrt{(-3)^2 + 4^2 + 5^2}}$

$= \dfrac{-13 - 48 + 65}{\sqrt{5 \times 169}\sqrt{25 \times 2}} = \dfrac{2}{5 \times 13 \times 2} = \dfrac{1}{65}$

So $\theta = \cos^{-1}\left(\dfrac{1}{65}\right)$

(ii) $a_1 = 1 ; b_1 = 1 ; c_1 = 2$

$a_2 = \sqrt{3} - 1 ; b_2 = -\sqrt{3} - 1 ; c_2 = 4$

So $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \dfrac{1(\sqrt{3} - 1) + 1(-\sqrt{3} - 1) + 2(4)}{\sqrt{1^2 + 1^2 + 2^2}\sqrt{(\sqrt{3} - 1)^2 (-\sqrt{3} - 1)^2 + 4^2}}$

$$= \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{6}\ \sqrt{3+1+3+1+16}}$$

$$= \frac{6}{\sqrt{144}}$$

$$= \frac{6}{12} = \frac{1}{2}$$

$$= \theta = \frac{\pi}{3}$$

Ans - 14.

So D' ratios of line PQ are $\langle -1-2, -3, 2-5 \rangle$

i.e. $\langle -3, 0, -3 \rangle$

i.e $\langle 1, 0, 1 \rangle$

So D' cosines of line PQ are $\langle \frac{1}{\sqrt{1+1}}, \frac{0}{\sqrt{1+1}}, \frac{1}{\sqrt{1+1}} \rangle$

i.e. $\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$

D' ratios of line QR are $\langle 3+1, 5-3, -2-2 \rangle$

i.e. $\langle 4, 2, -4 \rangle$ i.e. $\langle 2, 1, -2 \rangle$

So D' cosines of line QR are $\langle \frac{2}{\sqrt{4+1+4}}, \frac{1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} \rangle$

i.e, $\langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \rangle$

D' ratios of line PR are $\langle 3-2, 5-3, -2-5 \rangle$

i.e $\langle 1, 2, -7 \rangle$

So D' cosines of line PR are $\langle \frac{1}{\sqrt{1+4+49}}, \frac{2}{\sqrt{1+4+49}}, \frac{-7}{\sqrt{1+4+49}} \rangle$

i.e., $\left\langle \frac{1}{3\sqrt{6}}, \frac{2}{3\sqrt{6}}, \frac{-7}{3\sqrt{6}} \right\rangle$

Ans-15.

D' ratios of line PQ are $\langle 5-3, 4-2, -6+4 \rangle$

i.e. $\langle 2, 2, -2 \rangle$

and D' ratios of line QR are $\langle 9-5, 8-4, -10+6 \rangle$

i.e. $\langle 4, 4, -4 \rangle$

More

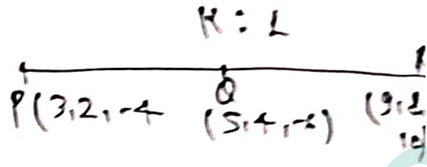
$$\frac{2}{4} = \frac{2}{4} = \frac{-2}{-4}$$

i.e. D' Ratios of both lines PQ and QR are proportional

so line PQ and QR are parallel and the point Q is common to both sides.

So coordinates of point Q are

$$\left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k-4}{k+1} \right)$$



Also coordinates of point Q be $(5, 4, -6)$

So $\frac{9k+3}{k+1} = 5 \Rightarrow 9k+3 = 5k+5$

$\Rightarrow 4k = 2 \Rightarrow k = \frac{1}{2}$

and $\frac{8k+2}{k+1} = 4 \Rightarrow 8k+2 = 4k+4$

$\Rightarrow 4k = 2 \Rightarrow k = \frac{1}{2}$

Also $\frac{-10K-4}{K+1} = -6 \Rightarrow -10K-4 = -6K-6$

$\Rightarrow 4K = 2 \quad \Rightarrow K = \frac{1}{2}$

Times required rastio be $K:1$

i.e $1:2$

Ans-16.

(i) D' ratios of line Joining the points $A(8,2,0)$ and $B(4,6,7)$ are $\langle 4-8, 6-2, -7-0\rangle$

i.e $\langle -4, 4, -7\rangle$

and D' ratios of line Joining the points $C(-3,1,2)$ and $D(-9,-2,4)$ and $\langle -9+3, -2-1, 4-2\rangle$

i.e $\langle -6, -3, 2\rangle$

let θ be the acute angle between the lines AB and CD

$$\cos\theta = \frac{|-4(-6)+4(-3)-7(2)|}{\sqrt{16+16+49}\sqrt{36+9+4}} = \frac{2}{9\times7} = \frac{2}{63}$$

So $\theta = \cos^{-1}\left(\frac{2}{63}\right)$

(ii) D' ratios of the line Joining the points $A(4,-2,3)$ and $B(6,1,7)$ are $\langle 6-4, 1+2, 7-3\rangle$

i.e $\langle 2, 3, 4\rangle$

and D' ratios of the line joining the points C(4, -2, 3)

and D(5, 4, -2) are < 5 - 4, 4 + 2, -2 - 3 >

i.e < 1, 6, -5 >

Let θ be the angle between the lines AB and CD.

So $\cos\theta = \dfrac{|2(1) + 3(6) + 4(-5)|}{\sqrt{4+9+16}\sqrt{1+36+25}} = 0$

$\Rightarrow \theta = \dfrac{\pi}{2}$

(iii) D' ratios of the line joining the points A (3, 1, -2)

and B (4, 0, -4) are < 4 - 3, 0 - 1, -4 + 2 >

i.e < 1, -1, -2 >

and D' ratios of the line joining the points

C(4, -3, 3) and D(6, -2, 2) are < 6 - 4, -2 + 3, 2 - 3 >

i.e < 2, 1, -1 >

Let θ be the angle between the lines AB and CD

Then $\cos\theta = \dfrac{|1(2) - 1(1) - 2(-1)|}{\sqrt{1+1+4}\sqrt{4+1+1}}$

$= \dfrac{3}{\sqrt{6}\sqrt{6}} = \dfrac{3}{6} = \dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

Ans-17. Let the required direction ratios of given line be $\langle a, b, c\rangle$ and given line $\perp$ to the lines having direction ratio are $\langle 1, -2, -2\rangle$ and $\langle 0, 2, 1\rangle$

$$a - 2b - 2c = 0$$
$$0a + 2b + c = 0$$

using cross multiplication method, we have

$$\frac{a}{-2+4} = \frac{b}{0-1} = \frac{c}{2-0}$$

i.e, $\frac{a}{2} = \frac{b}{-1} = \frac{c}{2}$

so D' Ratios of line are $\langle 2, -1, 2\rangle$

Time required direction cosines of line are

$$\left\langle \frac{2}{\sqrt{2^2+(-1)^2+(2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+2^2}}, \frac{2}{\sqrt{2^2+(-1)^2+2^2}} \right\rangle$$

i.e $\left\langle \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right\rangle$

Ans-18.

Ans - 18.

D' ratios of given lines are $\langle -3-2, 3-3, -2+4 \rangle$

and $\langle 3+1, 5-4, 1-2 \rangle$

i.e $\langle -5, 0, 2 \rangle$ and $\langle 4, 1, -1 \rangle$

Let the direction ratios of required line be $\langle a, b, c \rangle$ since the required line be $\perp$ to given lines.

Then $-5a + 0b + 2c = 0$ --- (1)

$4a + b - c = 0$ --- (2)

on solving (1) and eqn (2) by cross multiplication method, we have

$$\frac{a}{0-2} = \frac{b}{8-5} = \frac{c}{-5-0}$$

i.e $\frac{a}{-2} = \frac{b}{3} = \frac{c}{-5}$

So direction ratios of required line be

$\langle -2, 3, -5 \rangle$

Ans - 19.

D' ratios of the line AB through the points A (4, 1, 2) and B (5, x, 0) are < 5 - 4 , x - 1 , 0 - 2 >

i.e < 1, x - 1 , - 2 >

also direction ratio of line (1) through (1) through the points C (2, 1, 1) and D (3, 3, -1) are < 3 - 2, 3 - 1, -1 - 1 >

i.e < 1, 2 , -2 >

Since both lines are parallel

So direction ratios of both lines are proportional.

$\frac{1}{1} = \frac{x-1}{2} = \frac{-2}{2} \Rightarrow x - 1 = 2 \Rightarrow x = 3$

Ans - 20.

Now both lines AB and CD are perpendicular

So $1(1) + (x-1)2 + (-2)(-2) = 0$

$\Rightarrow 1 + 2x - 2 + 4 = 0$

$\Rightarrow 2x + 3 = 0$

$\Rightarrow x = -\frac{3}{2}$

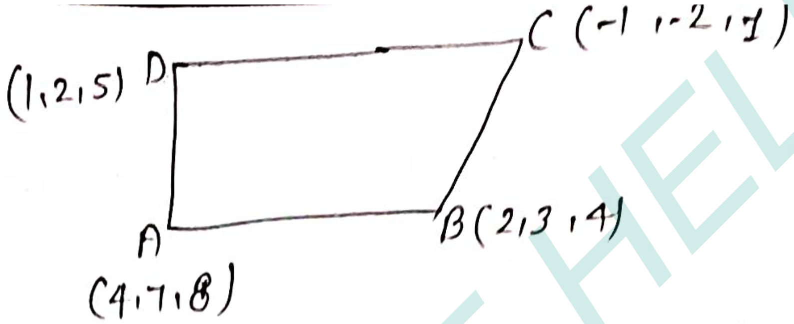
Ans - 21.

ABCD be the quadrilateral.

mid point of AC $= \left(\frac{4-1}{2}, \frac{7-2}{2}, \frac{8+1}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}, \frac{9}{2}\right)$

and mid point of BD $= \left(\frac{2+1}{2}, \frac{3+2}{2}, \frac{4+5}{2}\right)$

i.e $\left(\frac{3}{2}, \frac{5}{2}, \frac{9}{2}\right)$



That diagonals AC and BD bisect each other.

D' ratios of line AB are $\langle 2-4, 3-7, 4-8 \rangle$

ie $\langle -2, -4, -4 \rangle$

D' ratios of side DC are $\langle -1-1, -2-2, 1-5 \rangle$

i.e $\langle -2, -4, -4 \rangle$

So $AB \parallel DC$.

Similarly direction ratios of side AD are

$\langle 1-4, 2-7, 5-8 \rangle$

i.e $\langle -3, -5, -3 \rangle$

and direction ratios of side AD are $\langle 1-4, 2-7, 5-8 \rangle$

i.e $\langle -3, -5, -3 \rangle$

and direction ratios of side BC are $\langle -1-2, -2-3, 1-4 \rangle$

i.e $\langle -3, -5, -3 \rangle$

That $AD \parallel BC$

Hence A, B, C and D are the vertice of parallelogram

Ans-22.

A, B, C and D are the given vertices of quadrilateral.

Dr ratios of side AB are;

$\langle 7-5, -4+1, 7-1 \rangle$

i.e $\langle 2, -3, 6 \rangle$

Dr ratios of side AD are;

$\langle -1-5, -3+1, 4-1 \rangle$

i.e $\langle -6, -2, 3 \rangle$

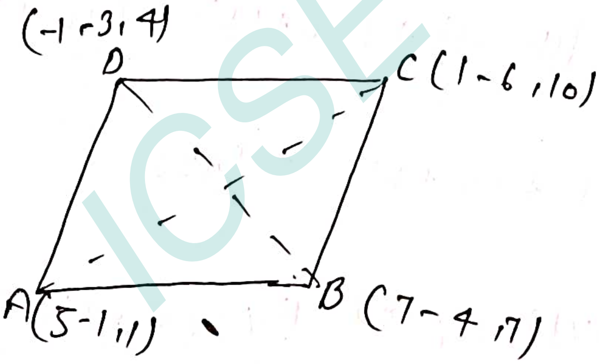
Here Dr Ratios of line AC are

$\langle 1-5, -6+1, 10-1 \rangle$

i.e $\langle -4, -5, 9 \rangle$

and Dr ratios of Diagonals BD are

$\langle -1-7, -3+4, 4-7 \rangle$ i.e $\langle -8, 1, -3 \rangle$



Here $a_1a_2 + b_1b_2 + c_1c_2 = (-4)(-8) + (-5)1 + 9(-3) = 0$

So both diagonals each other at right angle.

Also mid point of AC $= \left(\frac{5+1}{2}, \frac{-1-6}{2}, \frac{1+10}{2}\right)$

i.e. $\left(3, \frac{-7}{2}, \frac{11}{2}\right)$

and mid point of BD $= \left(\frac{7-1}{2}, \frac{-4-3}{2}, \frac{4+7}{2}\right)$

i.e $\left(3, \frac{-7}{2}, \frac{11}{2}\right)$

So both diagonals bisects each other at right angles.

$|AB| = \sqrt{(7-5)^2+(-4+1)^2+(7-1)^2}$

$= \sqrt{4+9+36} = 7$

$|AD| = \sqrt{(-1-5)^2+(-3+1)^2+(4-1)^2} = 7$

$|DC| = \sqrt{(7-1)^2+(-4+6)^2(10-7)^2} = \sqrt{36+4+9} = 7$

$|DC| = \sqrt{(1+1)^2+(-6+3)^2+(10-4)^2} = 7$

$|AB| = |BC| = |AD| = |DC|$

$|AC| = \sqrt{(1-5)^2+(-6+1)^2+(10-1)^2}$

$= \sqrt{16+25+81} = \sqrt{122}$

$|BD| = \sqrt{(-1-7)^2+(-3+4)^2+(4-7)^2} = \sqrt{64+1+9} = \sqrt{74}$

So $|AC| \neq |BD|$

Clearly A, B, C and D are the vertices of rhombus.

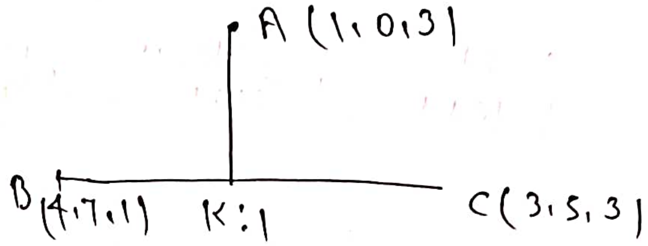
Ans 23.

D be the foot of $\perp$ drawn from A (1, 0, 3) to BC.

Point D divides the line BC in the ratio K : 1

So

coordinates of point D are $\left(\frac{3K+4}{K+1}, \frac{5K+7}{K+1}, \frac{3K+1}{K+1}\right)$



So direction ratios of line AD are

$$\left< \frac{3K+4}{K+1} - 1 \quad \frac{5K+7}{K+1} - 0 \quad \frac{3K+1}{K+1} - 3 \right>$$

i.e. $\left< \frac{2K+3}{K+1}, \frac{5K+7}{K+1}, \frac{-2}{K+1} \right>$

Also direction ratios of line BC are $<3-4, 5-7, 3-1>$

i.e $<-1, -2, 2>$

since line AD is $\perp$ to line BC.

So $\left(\frac{2K+3}{K+1}\right)(-1) + \left(\frac{5K+7}{K+1}\right)(-2) + \left(\frac{-2}{K+1}\right)2 = 0$

$\Rightarrow -2K - 3 - 10K - 14 - 4 = 0 \Rightarrow -12K - 21 = 0$

$$\Rightarrow K = -\frac{7}{4}$$

That required coordinates of foot of $\perp$ D are

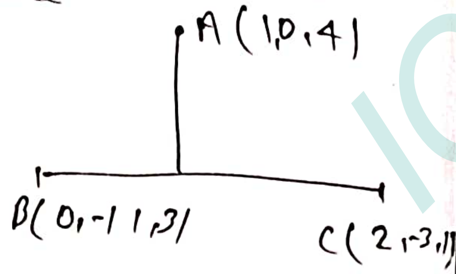
$$\frac{-\frac{21}{4}}{-\frac{7}{4}+1}, \frac{-\frac{35}{4}+7}{-\frac{7}{4}+1}, \frac{-\frac{21}{4}+1}{-\frac{7}{4}+1}\Bigg)$$

i.e. $\left(\frac{5}{2}, \frac{7}{3}, \frac{17}{3}\right)$

Ans-24.

Point D divides the line BC in ratios K : 1

Then coordinates of point D are



$$\left(\frac{2K}{K+1}, \frac{-3K-11}{K+1}, \frac{K+3}{K+1}\right)$$

So D' ratios of line AD are

$$\left\langle \frac{2K}{K+1}-1, \frac{-3K-11}{K+1}-0, \frac{K+3}{K+1}-4 \right\rangle$$

i.e $\left\langle \frac{K-1}{K+1}, \frac{-3K-11}{K+1}, \frac{-3K-1}{K+1} \right\rangle$

D' ratios of line BC are $\langle 2, -3+11, 1-3 \rangle$

i.e $\langle 2, 8, -2 \rangle$

since line AD is $\perp$ to line BC.

So $\left(\frac{K-1}{K+1}\right)2 + \left(\frac{-3K-11}{K+1}\right)8 + \left(\frac{-3K-1}{K+1}\right)(-2) = 0$

$\Rightarrow 2K - 2 - 24K - 88 + 6K + 2 = 0$

$\Rightarrow -16K = 88 \quad \Rightarrow K = \frac{88}{-16} = \frac{-11}{2}$

Then coordinates of foot of $\perp$ are

$$\left(\frac{\frac{-22}{2}}{\frac{-11}{2}+1}, \frac{\frac{+11}{2}}{\frac{-11}{2}+1}, \frac{\frac{-11}{2}+3}{\frac{-11}{2}+1} \right)$$

i.e $\left(\frac{22}{9}, \frac{-11}{9}, \frac{5}{9} \right)$

Ans-25.

D' ratios of side AB are

$< 6-1, 11+1, 2-2 >$

i.e $<5, 12, 0>$

D' ratios of line AC are

$<1-1, 2+1, 6-2>$

i.e, $<0, 3, 4>$

$a_1 = 5$; $b_1 = 12$; $c_1 = 0$

$a_2 = 0$; $b_2 = 3$; $c_2 = 4$

So $\cos A = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$$= \frac{5(0)+12(3)+0(4)}{\sqrt{5^2+12^2+0^2}\ \sqrt{0^2+3^2+4^2}}$$

$$= \frac{36}{13\times5} = \frac{36}{65}$$

Ans-26.

Here direction ratios of line AB are

$\langle -1-6, -7+6, 6-0 \rangle$

i.e $\langle -7, -1, 6 \rangle$

Direction ratios of line CD are

$\langle 2-3, -9+4, 2-4 \rangle$

i.e $\langle -1, -5, -2 \rangle$

$a_1 = -7;\ b_1 = -1;\ c_1 = -2$

$a_1 a_2 + b_1 b_2 + c_1 c_2 = (-7)(-1) + (-1)(-5)$

$+ 6(-2) = 7+5-12 = 0$

Ans-27.

a be the length of edge of the cube and let one corner of the cube be at $(0,0,0)$.

Then the diagonals of cube are OP, AR, BS and CQ

$\langle -a-0, a-0, a-0 \rangle$

i.e $\langle a, a, a \rangle$

$\langle 0-a, a-0, a-0 \rangle$

i.e $\langle -a, a, a \rangle$

Let θ be the angles between OP and AR

$$\cos\theta = \frac{a-(a)+a(a)+a(a)}{\sqrt{a^2+a^2+a^2}\sqrt{a^2+a^2+a^2}}$$

$$= \frac{a^2}{\sqrt{3}a\sqrt{3}a} = \frac{1}{3}$$

So $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

Similarly the angle between other pairs of diagonals be $\cos^{-1}\left(\frac{1}{3}\right)$.