# SOLUTIONS TO CONCEPTS
## CHAPTER – 1

1. a) Linear momentum : mv $= [MLT^{-1}]$

   b) Frequency : $\frac{1}{T} = [M^0L^0T^{-1}]$

   c) Pressure : $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

2. a) Angular speed $\omega = \theta/t = [M^0L^0T^{-1}]$

   b) Angular acceleration $\alpha = \frac{\omega}{t} = \frac{M^0L^0T^{-2}}{T} = [M^0L^0T^{-2}]$

   c) Torque $\tau = F\,r = [MLT^{-2}]\,[L] = [ML^2T^{-2}]$

   d) Moment of inertia $= Mr^2 = [M]\,[L^2] = [ML^2T^0]$

3. a) Electric field $E = F/q = \frac{MLT^{-2}}{[IT]} = [MLT^{-3}I^{-1}]$

   b) Magnetic field $B = \frac{F}{qv} = \frac{MLT^{-2}}{[IT][LT^{-1}]} = [MT^{-2}I^{-1}]$

   c) Magnetic permeability $\mu_0 = \frac{B \times 2\pi a}{I} = \frac{MT^{-2}I^{-1}] \times [L]}{[I]} = [MLT^{-2}I^{-2}]$

4. a) Electric dipole moment $P = ql = [IT] \times [L] = [LTI]$

   b) Magnetic dipole moment $M = IA = [I]\,[L^2]\,[L^2I]$

5. $E = h\nu$ where E = energy and $\nu$ = frequency.

   $h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]}[ML^2T^{-1}]$

6. a) Specific heat capacity $= C = \frac{Q}{m\Delta T} = \frac{[ML^2T^{-2}]}{[M][K]} = [L^2T^{-2}K^{-1}]$

   b) Coefficient of linear expansion $= \alpha = \frac{L_1 - L_2}{L_0\Delta T} = \frac{[L]}{[L][R]} = [K^{-1}]$

   c) Gas constant $= R = \frac{PV}{nT} = \frac{[ML^{-1}T^{-2}][L^3]}{[(mol)][K]} = [ML^2T^{-2}K^{-1}(mol)^{-1}]$

7. Taking force, length and time as fundamental quantity

   a) Density $= \frac{m}{V} = \frac{(\text{force/acceleration})}{\text{Volume}} = \frac{[F/LT^{-2}]}{[L^2]} = \frac{F}{L^4T^{-2}} = [FL^{-4}T^2]$

   b) Pressure $= F/A = F/L^2 = [FL^{-2}]$

   c) Momentum = mv (Force / acceleration) × Velocity $= [F / LT^{-2}] \times [LT^{-1}] = [FT]$

   d) Energy $= \frac{1}{2}mv^2 = \frac{\text{Force}}{\text{acceleration}} \times (\text{velocity})^2$

   $= \left[\frac{F}{LT^{-2}}\right] \times [LT^{-1}]^2 = \left[\frac{F}{LT^{-2}}\right] \times [L^2T^{-2}] = [FL]$

8. $g = 10\,\frac{\text{metre}}{\text{sec}^2} = 36 \times 10^5\ \text{cm/min}^2$

9. The average speed of a snail is 0.02 mile/hr

   Converting to S.I. units, $\frac{0.02 \times 1.6 \times 1000}{3600}$ m/sec [1 mile = 1.6 km = 1600 m] $= 0.0089\ ms^{-1}$

   The average speed of leopard = 70 miles/hr

   In SI units = 70 miles/hour $= \frac{70 \times 1.6 \times 1000}{3600} = 31$ m/s

10. Height h = 75 cm, Density of mercury = 13600 kg/m$^3$, g = 9.8 ms$^{-2}$ then
    Pressure = hfg = $10 \times 10^4$ N/m$^2$ (approximately)
    In C.G.S. Units, P = $10 \times 10^5$ dyne/cm$^2$
11. In S.I. unit 100 watt = 100 Joule/sec
    In C.G.S. Unit = $10^9$ erg/sec
12. 1 micro century = $10^4 \times 100$ years = $10^{-4} \times 365 \times 24 \times 60$ min
    So, 100 min = $10^5$ / 52560 = 1.9 microcentury
13. Surface tension of water = 72 dyne/cm
    In S.I. Unit, 72 dyne/cm = 0.072 N/m
14. $K = kI^a \omega^b$ where k = Kinetic energy of rotating body and k = dimensionless constant
    Dimensions of left side are,
    $K = [ML^2T^{-2}]$
    Dimensions of right side are,
    $I^a = [ML^2]^a$, $\omega^b = [T^{-1}]^b$
    According to principle of homogeneity of dimension,
    $[ML^2T^{-2}] = [ML^2T^{-2}]\,[T^{-1}]^b$
    Equating the dimension of both sides,
    $2 = 2a$ and $-2 = -b \Rightarrow a = 1$ and $b = 2$
15. Let energy $E \propto M^aC^b$ where M = Mass, C = speed of light
    $\Rightarrow E = KM^aC^b$ (K = proportionality constant)
    Dimension of left side
    $E = [ML^2T^{-2}]$
    Dimension of right side
    $M^a = [M]^a$, $[C]^b = [LT^{-1}]^b$
    $\therefore [ML^2T^{-2}] = [M]^a[LT^{-1}]^b$
    $\Rightarrow a = 1;\ b = 2$
    So, the relation is $E = KMC^2$
16. Dimensional formulae of R = $[ML^2T^{-3}I^{-2}]$
    Dimensional formulae of V = $[ML^2T^3I^{-1}]$
    Dimensional formulae of I = [I]
    $\therefore [ML^2T^3I^{-1}] = [ML^2T^{-3}I^{-2}]\,[I]$
    $\Rightarrow V = IR$
17. Frequency $f = KL^aF^bM^c$ M = Mass/unit length, L = length, F = tension (force)
    Dimension of f = $[T^{-1}]$
    Dimension of right side,
    $L^a = [L^a]$, $F^b = [MLT^{-2}]^b$, $M^c = [ML^{-1}]^c$
    $\therefore [T^{-1}] = K[L]^a\,[MLT^{-2}]^b\,[ML^{-1}]^c$
    $M^0L^0T^{-1} = KM^{b+c}\,L^{a+b-c}\,T^{-2b}$
    Equating the dimensions of both sides,
    $\therefore b + c = 0$ …(1)
    $-c + a + b = 0$ …(2)
    $-2b = -1$ …(3)
    Solving the equations we get,
    $a = -1$, $b = 1/2$ and $c = -1/2$
    $\therefore$ So, frequency $f = KL^{-1}F^{1/2}M^{-1/2} = \frac{K}{L}F^{1/2}M^{-1/2} = \frac{K}{L} = \sqrt{\frac{F}{M}}$

18. a) h = $\frac{2SCos\theta}{\rho rg}$

LHS = [L]

Surface tension = S = F/l = $\frac{MLT^{-2}}{L} = [MT^{-2}]$

Density = $\rho$ = M/V = $[ML^{-3}T^{0}]$

Radius = r = [L], g = $[LT^{-2}]$

RHS = $\frac{2S\cos\theta}{\rho rg} = \frac{[MT^{-2}]}{[ML^{-3}T^{0}][L][LT^{-2}]} = [M^{0}L^{1}T^{0}] = [L]$

LHS = RHS

So, the relation is correct

b) v = $\sqrt{\frac{p}{\rho}}$ where v = velocity

LHS = Dimension of v = $[LT^{-1}]$

Dimension of p = F/A = $[ML^{-1}T^{-2}]$

Dimension of $\rho$ = m/V = $[ML^{-3}]$

RHS = $\sqrt{\frac{p}{\rho}} = \sqrt{\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}} = [L^{2}T^{-2}]^{1/2}$ = $[LT^{-1}]$

So, the relation is correct.

c) V = $(\pi pr^{4}t) / (8\eta l)$

LHS = Dimension of V = $[L^{3}]$

Dimension of p = $[ML^{-1}T^{-2}]$, $r^4 = [L^4]$, t = [T]

Coefficient of viscosity = $[ML^{-1}T^{-1}]$

RHS = $\frac{\pi pr^{4}t}{8\eta l} = \frac{[ML^{-1}T^{-2}][L^{4}][T]}{[ML^{-1}T^{-1}][L]}$

So, the relation is correct.

d) v = $\frac{1}{2\pi}\sqrt{(mgl/I)}$

LHS = dimension of v = $[T^{-1}]$

RHS = $\sqrt{(mgl/I)} = \sqrt{\frac{[M][LT^{-2}][L]}{[ML^{2}]}}$ = $[T^{-1}]$

LHS = RHS

So, the relation is correct.

19. Dimension of the left side = $\int \frac{dx}{\sqrt{(a^2 - x^2)}} = \int \frac{L}{\sqrt{(L^2 - L^2)}}$ = $[L^0]$

Dimension of the right side = $\frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right)$ = $[L^{-1}]$

So, the dimension of $\int \frac{dx}{\sqrt{(a^2 - x^2)}} \neq \frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right)$

So, the equation is dimensionally incorrect.

20. Important Dimensions and Units :

| Physical quantity | Dimension | SI unit |
|---|---|---|
| Force (F) | $[M^1L^1T^{-2}]$ | newton |
| Work (W) | $[M^1L^2T^{-2}]$ | joule |
| Power (P) | $[M^1L^2T^{-3}]$ | watt |
| Gravitational constant (G) | $[M^{-1}L^3T^{-2}]$ | $N\text{-}m^2/kg^2$ |
| Angular velocity ($\omega$) | $[T^{-1}]$ | radian/s |
| Angular momentum (L) | $[M^1L^2T^{-1}]$ | $kg\text{-}m^2/s$ |
| Moment of inertia (I) | $[M^1L^2]$ | $kg\text{-}m^2$ |
| Torque ($\tau$) | $[M^1L^2T^{-2}]$ | N-m |
| Young's modulus (Y) | $[M^1L^{-1}T^{-2}]$ | $N/m^2$ |
| Surface Tension (S) | $[M^1T^{-2}]$ | N/m |
| Coefficient of viscosity ($\eta$) | $[M^1L^{-1}T^{-1}]$ | $N\text{-}s/m^2$ |
| Pressure (p) | $[M^1L^{-1}T^{-2}]$ | $N/m^2$ (Pascal) |
| Intensity of wave (I) | $[M^1T^{-3}]$ | $watt/m^2$ |
| Specific heat capacity (c) | $[L^2T^{-2}K^{-1}]$ | J/kg-K |
| Stefan's constant ($\sigma$) | $[M^1T^{-3}K^{-4}]$ | $watt/m^2\text{-}k^4$ |
| Thermal conductivity (k) | $[M^1L^1T^{-3}K^{-1}]$ | watt/m-K |
| Current density (j) | $[I^1L^{-2}]$ | $ampere/m^2$ |
| Electrical conductivity ($\sigma$) | $[I^2T^3M^{-1}L^{-3}]$ | $\Omega^{-1}\ m^{-1}$ |
| Electric dipole moment (p) | $[L^1I^1T^1]$ | C-m |
| Electric field (E) | $[M^1L^1I^{-1}T^{-3}]$ | V/m |
| Electrical potential (V) | $[M^1L^2I^{-1}T^{-3}]$ | volt |
| Electric flux ($\Psi$) | $[M^1T^3I^{-1}L^{-3}]$ | volt/m |
| Capacitance (C) | $[I^2T^4M^{-1}L^{-2}]$ | farad (F) |
| Permittivity ($\varepsilon$) | $[I^2T^4M^{-1}L^{-3}]$ | $C^2/N\text{-}m^2$ |
| Permeability ($\mu$) | $[M^1L^1I^{-2}T^{-3}]$ | $Newton/A^2$ |
| Magnetic dipole moment (M) | $[I^1L^2]$ | N-m/T |
| Magnetic flux ($\phi$) | $[M^1L^2I^{-1}T^{-2}]$ | Weber (Wb) |
| Magnetic field (B) | $[M^1I^{-1}T^{-2}]$ | tesla |
| Inductance (L) | $[M^1L^2I^{-2}T^{-2}]$ | henry |
| Resistance (R) | $[M^1L^2I^{-2}T^{-3}]$ | ohm ($\Omega$) |

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