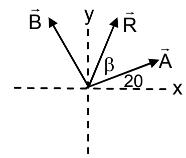
SOLUTIONS TO CONCEPTS
CHAPTER – 2

1. As shown in the figure,



The angle between $\vec{A}$ and $\vec{B}$ = 110° – 20° = 90°

$|\vec{A}|$ = 3 and $|\vec{B}|$ = 4m

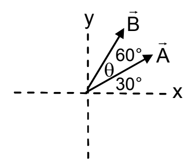
Resultant R = $\sqrt{A^2 + B^2 + 2AB\cos\theta}$ = 5 m

Let β be the angle between $\vec{R}$ and $\vec{A}$

β = $\tan^{-1}\left(\dfrac{4\sin 90^\circ}{3 + 4\cos 90^\circ}\right)$ = $\tan^{-1}$ (4/3) = 53°

$\therefore$ Resultant vector makes angle (53° + 20°) = 73° with x-axis.

2. Angle between $\vec{A}$ and $\vec{B}$ is θ = 60° – 30° =30°



$|\vec{A}|$ and $|\vec{B}|$ = 10 unit

R = $\sqrt{10^2 + 10^2 + 2.10.10.\cos 30^\circ}$ = 19.3

β be the angle between $\vec{R}$ and $\vec{A}$

β = $\tan^{-1}\left(\dfrac{10\sin 30^\circ}{10 + 10\cos 30^\circ}\right) = \tan^{-1}\left(\dfrac{1}{2+\sqrt{3}}\right)$ = $\tan^{-1}$ (0.26795) = 15°

$\therefore$ Resultant makes 15° + 30° = 45° angle with x-axis.

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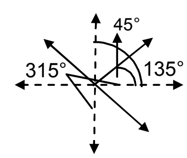
3. x component of $\vec{A}$ = 100 cos 45° = $100/\sqrt{2}$ unit

x component of $\vec{B}$ = 100 cos 135° = $100/\sqrt{2}$

x component of $\vec{C}$ = 100 cos 315° = $100/\sqrt{2}$



Resultant x component = $100/\sqrt{2}$ – $100/\sqrt{2}$ + $100/\sqrt{2}$ = $100/\sqrt{2}$

y component of $\vec{A}$ = 100 sin 45° = $100/\sqrt{2}$ unit

y component of $\vec{B}$ = 100 sin 135° = $100/\sqrt{2}$

y component of $\vec{C}$ = 100 sin 315° = $-100/\sqrt{2}$

Resultant y component = $100/\sqrt{2}$ + $100/\sqrt{2}$ – $100/\sqrt{2}$ = $100/\sqrt{2}$

Resultant = 100

Tan α = $\dfrac{\text{y component}}{\text{x component}}$ = 1

$\Rightarrow \alpha = \tan^{-1}$ (1) = 45°

The resultant is 100 unit at 45° with x-axis.

4. $\vec{a} = 4\vec{i} + 3\vec{j}$, $\vec{b} = 3\vec{i} + 4\vec{j}$

a) $|\vec{a}| = \sqrt{4^2 + 3^2}$ = 5

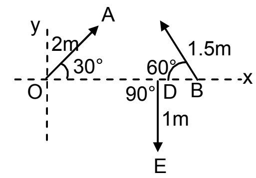
b) $|\vec{b}| = \sqrt{9+16}$ = 5

c) $|\vec{a} + \vec{b}| = |7\vec{i} + 7\vec{j}| = 7\sqrt{2}$

d) $\vec{a} - \vec{b} = (-3+4)\hat{i} + (-4+3)\hat{j} = \hat{i} - \hat{j}$

$|\vec{a} - \vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$.

5. x component of $\overrightarrow{OA}$ = 2cos30° = $\sqrt{3}$



x component of $\overrightarrow{BC}$ = 1.5 cos 120° = –0.75

x component of $\overrightarrow{DE}$ = 1 cos 270° = 0

y component of $\overrightarrow{OA}$ = 2 sin 30° = 1

y component of $\overrightarrow{BC}$ = 1.5 sin 120° = 1.3

y component of $\overrightarrow{DE}$ = 1 sin 270° = –1

R_x = x component of resultant = $\sqrt{3} - 0.75 + 0 = 0.98$ m

R_y = resultant y component = 1 + 1.3 – 1 = 1.3 m

So, R = Resultant = 1.6 m

If it makes and angle α with positive x-axis

Tan α = $\dfrac{\text{y component}}{\text{x component}}$ = 1.32

$\Rightarrow \alpha = \tan^{-1} 1.32$

6. $|\vec{a}|$ = 3m $|\vec{b}|$ = 4

a) If R = 1 unit $\Rightarrow \sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 1

θ = 180°

b) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 5

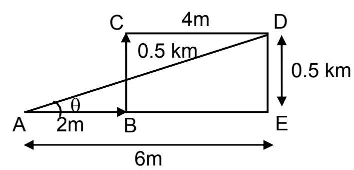
θ = 90°

c) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 7

θ = 0°

Angle between them is 0°.

7. $\overrightarrow{AD} = 2\hat{i} + 0.5\hat{J} + 4\hat{K} = 6\hat{i} + 0.5\hat{j}$



AD = $\sqrt{AE^2 + DE^2}$ = 6.02 KM

Tan θ = DE / AE = 1/12

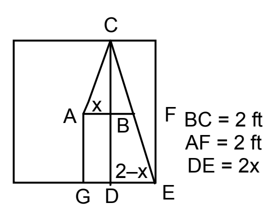
$\theta = \tan^{-1}$ (1/12)

The displacement of the car is 6.02 km along the distance $\tan^{-1}$ (1/12) with positive x-axis.

8. In ΔABC, tanθ = x/2 and in ΔDCE, tanθ = (2 – x)/4 tan θ = (x/2) = (2 – x)/4 = 4x

$\Rightarrow$ 4 – 2x = 4x

$\Rightarrow$ 6x = 4 $\Rightarrow$ x = 2/3 ft



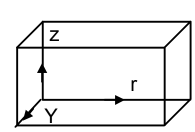
a) In ΔABC, AC = $\sqrt{AB^2 + BC^2} = \dfrac{2}{3}\sqrt{10}$ ft

b) In ΔCDE, DE = 1 – (2/3) = 4/3 ft

CD = 4 ft. So, CE = $\sqrt{CD^2 + DE^2} = \dfrac{4}{3}\sqrt{10}$ ft

c) In ΔAGE, AE = $\sqrt{AG^2 + GE^2} = 2\sqrt{2}$ ft.

9. Here the displacement vector $\vec{r} = 7\hat{i} + 4\hat{j} + 3\hat{k}$



a) magnitude of displacement = $\sqrt{74}$ ft

b) the components of the displacement vector are 7 ft, 4 ft and 3 ft.

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10. $\vec{a}$ is a vector of magnitude 4.5 unit due north.

 a) $3|\vec{a}| = 3 \times 4.5 = 13.5$

 $3\vec{a}$ is along north having magnitude 13.5 units.

 b) $-4|\vec{a}| = -4 \times 1.5 = -6$ unit

 $-4\vec{a}$ is a vector of magnitude 6 unit due south.

11. $|\vec{a}| = 2$ m, $|\vec{b}| = 3$ m

 angle between them $\theta = 60°$

 a) $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos 60° = 2 \times 3 \times 1/2 = 3\ \text{m}^2$

 b) $|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\sin 60° = 2 \times 3 \times \sqrt{3/2} = 3\sqrt{3}\ \text{m}^2$.

12. We know that according to polygon law of vector addition, the resultant of these six vectors is zero.

 Here A = B = C = D = E = F (magnitude)

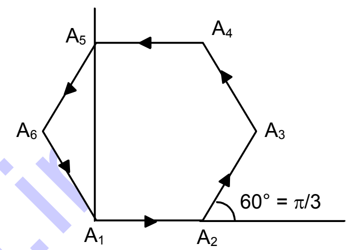
 So, Rx = A cosθ + A cos $\pi/3$ + A cos $2\pi/3$ + A cos $3\pi/3$ + A cos $4\pi/4$ + A cos $5\pi/5$ = 0

 [As resultant is zero. X component of resultant R_x = 0]

 = cos θ + cos $\pi/3$ + cos $2\pi/3$ + cos $3\pi/3$ + cos $4\pi/3$ + cos $5\pi/3$ = 0

 Note : Similarly it can be proved that,

 sin θ + sin $\pi/3$ + sin $2\pi/3$ + sin $3\pi/3$ + sin $4\pi/3$ + sin $5\pi/3$ = 0



13. $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$; $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$

 $$\vec{a}\cdot\vec{b} = ab\cos\theta \Rightarrow \theta = \cos^{-1}\frac{\vec{a}\cdot\vec{b}}{ab}$$

 $$\Rightarrow \cos^{-1}\frac{2\times3+3\times4+4\times5}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+4^2+5^2}} = \cos^{-1}\left(\frac{38}{\sqrt{1450}}\right)$$

14. $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$ (claim)

 As, $\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$

 AB sin θ $\hat{n}$ is a vector which is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, this implies that it is also perpendicular to $\vec{A}$. As dot product of two perpendicular vector is zero.

 Thus $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$.

15. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

 $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 3 & 2 \end{vmatrix} \Rightarrow \hat{i}(6-12) - \hat{j}(4-16) + \hat{k}(6-12) = -6\hat{i} + 12\hat{j} - 6\hat{k}.$$

16. Given that $\vec{A}$, $\vec{B}$ and $\vec{C}$ are mutually perpendicular

 $\vec{A} \times \vec{B}$ is a vector which direction is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

 Also $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$

 $\therefore$ Angle between $\vec{C}$ and $\vec{A} \times \vec{B}$ is 0° or 180° (fig.1)

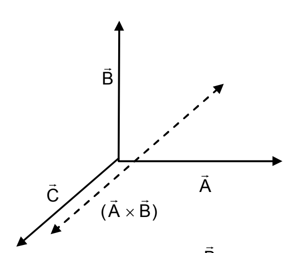
 So, $\vec{C} \times (\vec{A} \times \vec{B}) = 0$

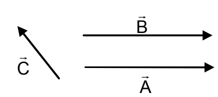
 The converse is not true.

 For example, if two of the vector are parallel, (fig.2), then also

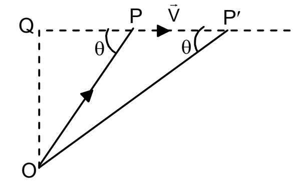
 $\vec{C} \times (\vec{A} \times \vec{B}) = 0$

 So, they need not be mutually perpendicular.





17. The particle moves on the straight line PP' at speed v.



From the figure,

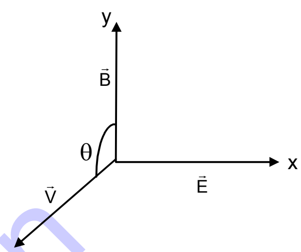
$$\overrightarrow{OP} \times v = (OP)v\sin\theta\hat{n} = v(OP)\sin\theta\ \hat{n} = v(OQ)\hat{n}$$

It can be seen from the figure, OQ = OP sin θ = OP' sin θ'

So, whatever may be the position of the particle, the magnitude and direction of $\overrightarrow{OP} \times \vec{v}$ remain constant.

$\therefore$ $\overrightarrow{OP} \times \vec{v}$ is independent of the position P.

18. Give $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0$



$$\Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$$

So, the direction of $\vec{v} \times \vec{B}$ should be opposite to the direction of $\vec{E}$. Hence, $\vec{v}$ should be in the positive yz-plane.

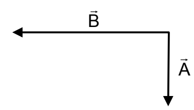
Again, E = vB sin θ ⇒ v = $\dfrac{E}{B\sin\theta}$

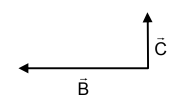
For v to be minimum, θ = 90° and so v_{min} = F/B

So, the particle must be projected at a minimum speed of E/B along +ve z-axis (θ = 90°) as shown in the figure, so that the force is zero.

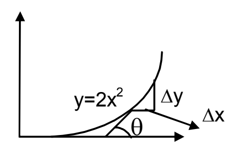
19. For example, as shown in the figure,

$\vec{A} \perp \vec{B}$	$\vec{B}$ along west
$\vec{B} \perp \vec{C}$	$\vec{A}$ along south
	$\vec{C}$ along north
$\vec{A}\cdot\vec{B}$ = 0	$\therefore$ $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{C}$
$\vec{B}\cdot\vec{C}$ = 0	But $\vec{B} \neq \vec{C}$





20. The graph y = $2x^2$ should be drawn by the student on a graph paper for exact results.



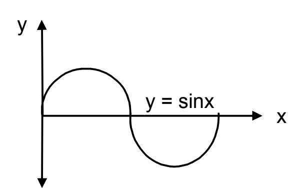
To find slope at any point, draw a tangent at the point and extend the line to meet x-axis. Then find tan θ as shown in the figure.

It can be checked that,

Slope = tan θ = $\dfrac{dy}{dx} = \dfrac{d}{dx}(2x^2)$ = 4x

Where x = the x-coordinate of the point where the slope is to be measured.

21. y = sinx



So, y + Δy = sin (x + Δx)

Δy = sin (x + Δx) – sin x

$$= \left(\frac{\pi}{3} + \frac{\pi}{100}\right) - \sin\frac{\pi}{3} = 0.0157.$$

22. Given that, i = $i_0 e^{-t/RC}$

$\therefore$ Rate of change of current = $\dfrac{di}{dt} = \dfrac{d}{dt} i_0 e^{-i/RC} = i_0 \dfrac{d}{dt} e^{-t/RC} = \dfrac{-i_0}{RC} \times e^{-t/RC}$

When a) t = 0, $\dfrac{di}{dt} = \dfrac{-i}{RC}$

b) when t = RC, $\dfrac{di}{dt} = \dfrac{-i}{RCe}$

c) when t = 10 RC, $\dfrac{di}{dt} = \dfrac{-i_0}{RCe^{10}}$

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23. Equation $i = i_0 e^{-t/RC}$

$i_0 = 2A$, $R = 6 \times 10^{-5}\ \Omega$, $C = 0.0500 \times 10^{-6}\ F = 5 \times 10^{-7}\ F$

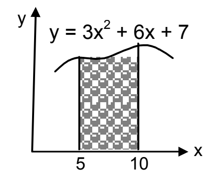
a) $i = 2 \times e^{\left(\frac{-0.3}{6 \times 0^3 \times 5 \times 10^{-7}}\right)} = 2 \times e^{\left(\frac{-0.3}{0.3}\right)} = \frac{2}{e}\text{amp}$.

b) $\frac{di}{dt} = \frac{-i_0}{RC} e^{-t/RC}$ when t = 0.3 sec $\Rightarrow \frac{di}{dt} = -\frac{2}{0.30} e^{(-0.3/0.3)} = \frac{-20}{3e}\text{Amp/sec}$

c) At t = 0.31 sec, $i = 2e^{(-0.3/0.3)} = \frac{5.8}{3e}\text{Amp}$.

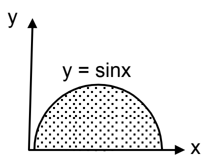
24. $y = 3x^2 + 6x + 7$

$\therefore$ Area bounded by the curve, x axis with coordinates with x = 5 and x = 10 is given by,



Area = $\int_0^y dy = \int_5^{10} (3x^2 + 6x + 7)dx = 3\left.\frac{x^3}{3}\right]_5^{10} + 5\left.\frac{x^2}{3}\right]_5^{10} + 7x]_5^{10}$ = 1135 sq.units.

25. Area = $\int_0^y dy = \int_0^{\pi} \sin x dx = -[\cos x]_0^{\pi} = 2$

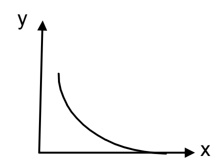


26. The given function is $y = e^{-x}$

When x = 0, $y = e^{-0} = 1$

x increases, y value deceases and only at $x = \infty$, y = 0.

So, the required area can be found out by integrating the function from 0 to ∞.

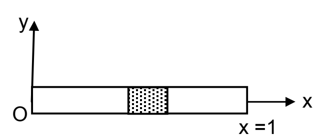


So, Area = $\int_0^{\infty} e^{-x}dx = -[e^{-x}]_0^{\infty} = 1$.

27. $\rho = \frac{\text{mass}}{\text{length}} = a + bx$

a) S.I. unit of 'a' = kg/m and SI unit of 'b' = kg/m^2 (from principle of homogeneity of dimensions)

b) Let us consider a small element of length 'dx' at a distance x from the origin as shown in the figure.



$\therefore$ dm = mass of the element = ρ dx = (a + bx) dx

So, mass of the rod = m = $\int dm = \int_0^L (a + bx)dx = \left[ax + \frac{bx^2}{2}\right]_0^L = aL + \frac{bL^2}{2}$

28. $\frac{dp}{dt}$ = (10 N) + (2 N/S)t

momentum is zero at t = 0

$\therefore$ momentum at t = 10 sec will be

dp = [(10 N) + 2Ns t]dt

$\int_0^p dp = \int_0^{10} 10dt + \int_0^{10} (2tdt) = 10t]_0^{10} + 2\left.\frac{t^2}{2}\right]_0^{10}$ = 200 kg m/s.

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29. The change in a function of y and the independent variable x are related as $\frac{dy}{dx} = x^2$.

$\Rightarrow dy = x^2\,dx$

Taking integration of both sides,

$\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + c$

$\therefore$ y as a function of x is represented by $y = \frac{x^3}{3} + c$.

30. The number significant digits

a) 1001 No.of significant digits = 4

b) 100.1 No.of significant digits = 4

c) 100.10 No.of significant digits = 5

d) 0.001001 No.of significant digits = 4

31. The metre scale is graduated at every millimeter.

1 m = 100 mm

The minimum no.of significant digit may be 1 (e.g. for measurements like 5 mm, 7 mm etc) and the maximum no.of significant digits may be 4 (e.g.1000 mm)

So, the no.of significant digits may be 1, 2, 3 or 4.

32. a) In the value 3472, after the digit 4, 7 is present. Its value is greater than 5.

So, the next two digits are neglected and the value of 4 is increased by 1.

$\therefore$ value becomes 3500

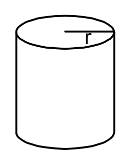
b) value = 84

c) 2.6

d) value is 28.

33. Given that, for the cylinder

Length = l = 4.54 cm, radius = r = 1.75 cm



Volume = $\pi r^2 l = \pi \times (4.54) \times (1.75)^2$

Since, the minimum no.of significant digits on a particular term is 3, the result should have 3 significant digits and others rounded off.

So, volume $V = \pi r^2 l = (3.14) \times (1.75) \times (1.75) \times (4.54) = 43.6577\ cm^3$

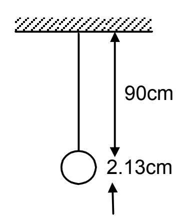
Since, it is to be rounded off to 3 significant digits, $V = 43.7\ cm^3$.

34. We know that,

Average thickness = $\frac{2.17 + 2.17 + 2.18}{3}$ = 2.1733 mm

Rounding off to 3 significant digits, average thickness = 2.17 mm.

35. As shown in the figure,



Actual effective length = (90.0 + 2.13) cm

But, in the measurement 90.0 cm, the no. of significant digits is only 2.

So, the addition must be done by considering only 2 significant digits of each measurement.

So, effective length = 90.0 + 2.1 = 92.1 cm.

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