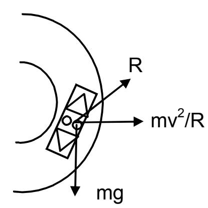
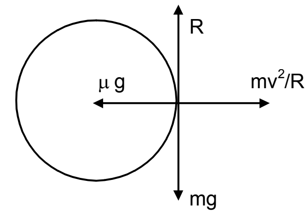
SOLUTIONS TO CONCEPTS circular motion;;
CHAPTER 7

1. Distance between Earth & Moon
$r = 3.85 \times 10^5$ km $= 3.85 \times 10^8$m
$T = 27.3$ days $= 24 \times 3600 \times (27.3)$ sec $= 2.36 \times 10^6$ sec
$$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 3.85 \times 10^8}{2.36 \times 10^6} = 1025.42\text{m/sec}$$
$$a = \frac{v^2}{r} = \frac{(1025.42)^2}{3.85 \times 10^8} = 0.00273\text{m/sec}^2 = 2.73 \times 10^{-3}\text{m/sec}^2$$
2. Diameter of earth = 12800km
Radius $R = 6400$km $= 64 \times 10^5$ m
$$V = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times 64 \times 10^5}{24 \times 3600}\text{ m/sec} = 465.185$$
$$a = \frac{V^2}{R} = \frac{(46.5185)^2}{64 \times 10^5} = 0.0338\text{m/sec}^2$$
3. $V = 2t$, $r = 1$cm
a) Radial acceleration at t = 1 sec.
$$a = \frac{v^2}{r} = \frac{2^2}{1} = 4\text{cm/sec}^2$$
b) Tangential acceleration at t = 1sec.
$$a = \frac{dv}{dt} = \frac{d}{dt}(2t) = 2\text{cm/sec}^2$$
c) Magnitude of acceleration at t = 1sec
$$a = \sqrt{4^2 + 2^2} = \sqrt{20}\text{ cm/sec}^2$$
4. Given that m = 150kg,
v= 36km/hr = 10m/sec, r = 30m
Horizontal force needed is $\frac{mv^2}{r} = \frac{150 \times (10)^2}{30} = \frac{150 \times 100}{30} = 500\text{N}$
5. in the diagram
$R \cos \theta = mg$ ..(i)
$R \sin \theta = \frac{mv^2}{r}$ ..(ii)



Dividing equation (i) with equation (ii)
$$\text{Tan } \theta = \frac{mv^2}{rmg} = \frac{v^2}{rg}$$
v = 36km/hr = 10m/sec, r = 30m
$$\text{Tan } \theta = \frac{v^2}{rg} = \frac{100}{30 \times 10} = (1/3)$$
$\Rightarrow \theta = \tan^{-1}(1/3)$
6. Radius of Park = r = 10m
speed of vehicle = 18km/hr = 5 m/sec
Angle of banking $\tan\theta = \frac{v^2}{rg}$
$$\Rightarrow \theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{25}{100} = \tan^{-1}(1/4)$$

7. The road is horizontal (no banking)



$$\frac{mv^2}{R} = \mu N$$

and N = mg

So $\frac{mv^2}{R} = \mu\, mg$ v = 5m/sec, R = 10m

$$\Rightarrow \frac{25}{10} = \mu g \Rightarrow \mu = \frac{25}{100} = 0.25$$

8. Angle of banking = θ = 30°

Radius = r = 50m

$$\tan\theta = \frac{v^2}{rg} \Rightarrow \tan 30° = \frac{v^2}{rg}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{v^2}{rg} \Rightarrow v^2 = \frac{rg}{\sqrt{3}} = \frac{50\times10}{\sqrt{3}}$$

$$\Rightarrow v = \sqrt{\frac{500}{\sqrt{3}}} = 17\text{m/sec.}$$

9. Electron revolves around the proton in a circle having proton at the centre.

Centripetal force is provided by coulomb attraction.

r = 5.3 →t 10^{-11}m m = mass of electron = 9.1 × 10^{-3}kg.

charge of electron = 1.6 × 10^{-19}c.

$$\frac{mv^2}{r} = k\frac{q^2}{r^2} \Rightarrow v^2 = \frac{kq^2}{rm} = \frac{9\times10^9\times1.6\times1.6\times10^{-38}}{5.3\times10^{-11}\times9.1\times10^{-31}} = \frac{23.04}{48.23}\times10^{13}$$

$$\Rightarrow v^2 = 0.477 \times 10^{13} = 4.7 \times 10^{12}$$

$$\Rightarrow v = \sqrt{4.7\times10^{12}} = 2.2 \times 10^6 \text{ m/sec}$$

10. At the highest point of a vertical circle

$$\frac{mv^2}{R} = mg$$

$$\Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$$

11. A celling fan has a diameter = 120cm.

∴Radius = r = 60cm = 0/6m

Mass of particle on the outer end of a blade is 1g.

n = 1500 rev/min = 25 rev/sec

$\omega = 2\,\pi\, n = 2\,\pi \times 25 = 157.14$

Force of the particle on the blade = $Mr\omega^2$ = (0.001) × 0.6 × (157.14) = 14.8N

The fan runs at a full speed in circular path. This exerts the force on the particle (inertia). The particle also exerts a force of 14.8N on the blade along its surface.

12. A mosquito is sitting on an L.P. record disc & rotating on a turn table at $33\frac{1}{3}$ rpm.

$$n = 33\frac{1}{3} \text{ rpm} = \frac{100}{3\times60} \text{ rps}$$

$$\therefore \omega = 2\,\pi\, n = 2\,\pi \times \frac{100}{180} = \frac{10\pi}{9} \text{ rad/sec}$$

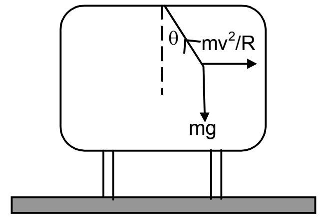
r = 10cm =0.1m, g = 10m/sec^2

$$\mu mg \ge mr\omega^2 \Rightarrow \mu = \frac{r\omega^2}{g} \ge \frac{0.1\times\left(\frac{10\pi}{9}\right)^2}{10}$$

$$\Rightarrow \mu \ge \frac{\pi^2}{81}$$

13. A pendulum is suspended from the ceiling of a car taking a turn

$r = 10\text{m}$, $v = 36\text{km/hr} = 10\text{ m/sec}$, $g = 10\text{m/sec}^2$



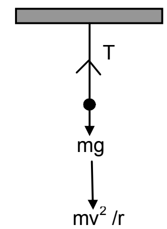
From the figure $T \sin\theta = \dfrac{mv^2}{r}$ ..(i)

$T \cos\theta = mg$ ..(ii)

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{mv^2}{rmg} \Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\frac{100}{10\times10} = \tan^{-1}(1) \Rightarrow \theta = 45°$$

14. At the lowest pt.



$$T = mg + \frac{mv^2}{r}$$

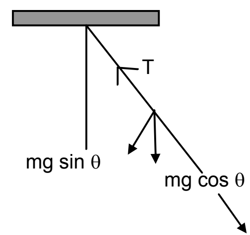
Here $m = 100\text{g} = 1/10\text{ kg}$, $r = 1\text{m}$, $v = 1.4\text{ m/sec}$

$$T = mg + \frac{mv^2}{r} = \frac{1}{10}\times9.8\times\frac{(1.4)^2}{10} = 0.98 + 0.196 = 1.176 = 1.2\text{ N}$$

15. Bob has a velocity 1.4m/sec, when the string makes an angle of 0.2 radian.

$m = 100\text{g} = 0.1\text{kg}$, $r = 1\text{m}$, $v = 1.4\text{m/sec.}$

From the diagram,



$$T - mg\cos\theta = \frac{mv^2}{R}$$

$$\Rightarrow T = \frac{mv^2}{R} + mg\cos\theta$$

$$\Rightarrow T = \frac{0.1\times(1.4)^2}{1} + (0.1)\times9.8\times\left(1-\frac{\theta^2}{2}\right)$$

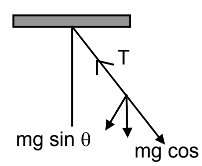
$$\Rightarrow T = 0.196 + 9.8 \times \left(1-\frac{(.2)^2}{2}\right) \qquad (\because \cos\theta = 1-\frac{\theta^2}{2} \text{ for small } \theta)$$

$$\Rightarrow T = 0.196 + (0.98) \times (0.98) = 0.196 + 0.964 = 1.156\text{N} \approx 1.16\text{ N}$$

16. At the extreme position, velocity of the pendulum is zero.

So there is no centrifugal force.

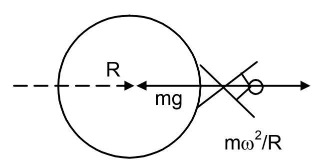
So $T = mg\cos\theta_o$



17. a) Net force on the spring balance.

$R = mg - m\omega^2 r$

So, fraction less than the true weight (3mg) is



$$= \frac{mg-(mg-m\omega^2 r)}{mg} = \frac{\omega^2}{g} = \left(\frac{2\pi}{24\times3600}\right)^2 \times \frac{6400\times10^3}{10} = 3.5 \times 10^{-3}$$

b) When the balance reading is half the true weight,

$$\frac{mg-(mg-m\omega^2 r)}{mg} = 1/2$$

$$\omega^2 r = g/2 \Rightarrow \omega = \sqrt{\frac{g}{2r}} = \sqrt{\frac{10}{2\times6400\times10^3}}\text{ rad/sec}$$

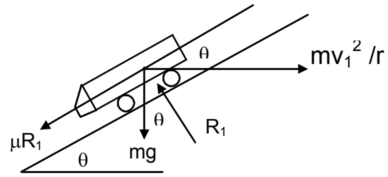
$\therefore$ Duration of the day is

$$T = \frac{2\pi}{\omega} = 2\pi\times\sqrt{\frac{2\times6400\times10^3}{9.8}}\text{ sec} = 2\pi\times\sqrt{\frac{64\times10^6}{49}}\text{ sec} = \frac{2\pi\times8000}{7\times3600}\text{ hr} = 2\text{hr}$$

18. Given, v = 36km/hr = 10m/s, r = 20m, μ = 0.4

The road is banked with an angle,

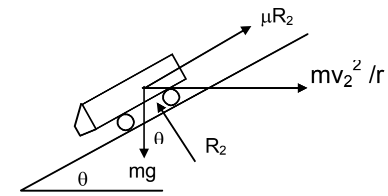
$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{100}{20 \times 10}\right) = \tan^{-1}\left(\frac{1}{2}\right)$ or $\tan\theta = 0.5$



When the car travels at max. speed so that it slips upward, μR_1 acts downward as shown in Fig.1

So, $R_1 - mg\cos\theta - \frac{mv_1^2}{r}\sin\theta = 0$ ..(i)

And $\mu R_1 + mg\sin\theta - \frac{mv_1^2}{r}\cos\theta = 0$ ..(ii)



Solving the equation we get,

$V_1 = \sqrt{rg\frac{\tan\theta - \mu}{1 + \mu\tan\theta}} = \sqrt{20 \times 10 \times \frac{0.1}{1.2}}$ = 4.082 m/s = 14.7 km/hr

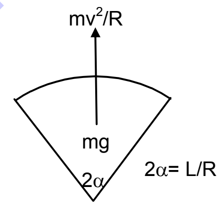
So, the possible speeds are between 14.7 km/hr and 54km/hr.

19. R = radius of the bridge

L = total length of the over bridge

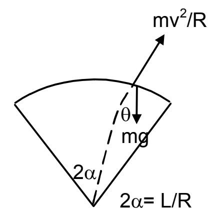
a) At the highest pt.

$mg = \frac{mv^2}{R} \Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$



b) Given, $v = \frac{1}{\sqrt{2}}\sqrt{Rg}$

suppose it loses contact at B. So, at B, $mg\cos\theta = \frac{mv^2}{R}$



$\Rightarrow v^2 = Rg\cos\theta$

$\Rightarrow \left(\sqrt{\frac{Rv}{2}}\right)^2 = Rg\cos\theta \Rightarrow \frac{Rg}{2} = Rg\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = 60° = \pi/3$

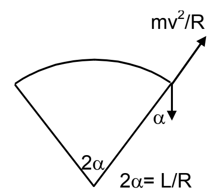
$\theta = \frac{\ell}{r} \rightarrow \ell = r\theta = \frac{\pi R}{3}$

So, it will lose contact at distance $\frac{\pi R}{3}$ from highest point

c) Let the uniform speed on the bridge be v.

The chances of losing contact is maximum at the end of the bridge for which $\alpha = \frac{L}{2R}$.

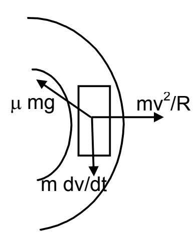
So, $\frac{mv^2}{R} = mg\cos\alpha \Rightarrow v = \sqrt{gR\cos\left(\frac{L}{2R}\right)}$

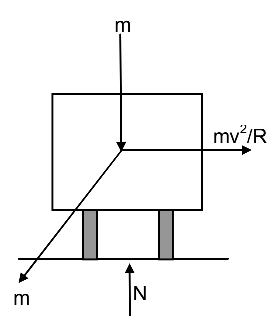


20. Since the motion is nonuniform, the acceleration has both radial & tangential component

$a_r = \frac{v^2}{r}$

$a_t = \frac{dv}{dt} = a$





Resultant magnitude = $\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$

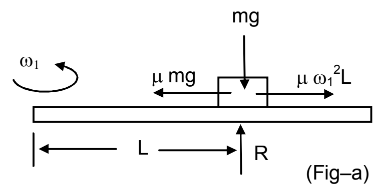
Now $\mu N = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu\, mg = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu^2 g^2 = \left(\frac{v^4}{r2}\right) + a^2$

$\Rightarrow v^4 = (\mu^2 g^2 - a^2)\, r^2 \Rightarrow v = [(\mu^2 g^2 - a^2)\, r^2]^{1/4}$

21. a) When the ruler makes uniform circular motion in the horizontal plane, (fig–a)

$\mu\ mg = m\omega_1^2 L$

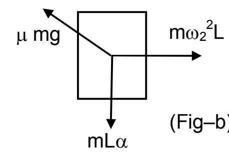
$\omega_1 = \sqrt{\dfrac{\mu g}{L}}$



(Fig–a)

b) When the ruler makes uniformly accelerated circular motion,(fig–b)

$\mu\ mg = \sqrt{(m\omega_2^2 L)^2 + (mL\alpha)^2} \Rightarrow \omega_2^4 + \alpha^2 = \dfrac{\mu^2 g^2}{L^2} \Rightarrow \omega_2 = \left[\left(\dfrac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$



(Fig–b)

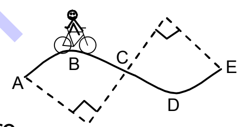
(When viewed from top)

22. Radius of the curves = 100m

Weight = 100kg

Velocity = 18km/hr = 5m/sec

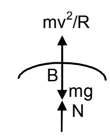
a) at B mg – $\dfrac{mv^2}{R}$ = N ⇒ N = (100 × 10) – $\dfrac{100 \times 25}{100}$ = 1000 – 25 = 975N



At d, N = mg + $\dfrac{mv^2}{R}$ = 1000 + 25 = 1025 N

b) At B & D the cycle has no tendency to slide. So at B & D, frictional force is zero.

At 'C', mg sin θ = F ⇒ F = 1000 × $\dfrac{1}{\sqrt{2}}$ = 707N



c) (i) Before 'C' mg cos θ – N = $\dfrac{mv^2}{R}$ ⇒ N = mg cos θ – $\dfrac{mv^2}{R}$ = 707 – 25 = 683N

(ii) N – mg cos θ = $\dfrac{mv^2}{R}$ ⇒ N = $\dfrac{mv^2}{R}$ + mg cos θ = 25 + 707 = 732N

d) To find out the minimum desired coeff. of friction, we have to consider a point just before C. (where N is minimum)

Now, μ N = mg sin θ ⇒ μ × 682 = 707

So, μ = 1.037

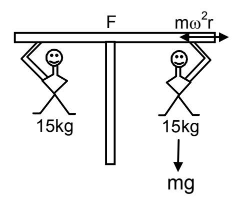
23. d = 3m ⇒ R = 1.5m

R = distance from the centre to one of the kids

N = 20 rev per min = 20/60 = 1/3 rev per sec

ω = 2πr = 2π/3

m = 15kg



∴ Frictional force F = $m r\omega^2 = 15 \times (1.5) \times \dfrac{(2\pi)^2}{9} = 5 \times (0.5) \times 4\pi^2 = 10\pi^2$

∴ Frictional force on one of the kids is $10\pi^2$

24. If the bowl rotates at maximum angular speed, the block tends to slip upwards. So, the frictional force acts downward.

Here, r = R sin θ

From FBD –1

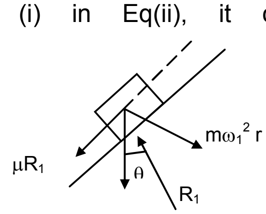
$R_1 - mg\cos\theta - m\omega_1^2 (R\sin\theta)\sin\theta = 0$ ..(i) [because r = R sin θ]

and $\mu R_1\ mg\sin\theta - m\omega_1^2 (R\sin\theta)\cos\theta = 0$ ..(ii)

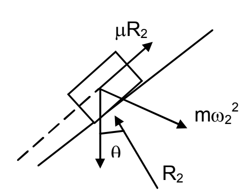
Substituting the value of R_1 from Eq (i) in Eq(ii), it can be found out that

$\omega_1 = \left[\dfrac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{1/2}$

Again, for minimum speed, the frictional force μR_2 acts upward. From FBD–2, it can be proved that,



(FBD – 1)



(FBD – 2)

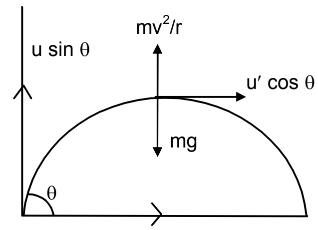
$$\omega_2 = \left[\frac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$$

$\therefore$ the range of speed is between ω_1 and ω_2

25. Particle is projected with speed 'u' at an angle θ. At the highest pt. the vertical component of velocity is '0'

So, at that point, velocity = u cos θ

centripetal force = $m\, u^2 \cos^2\left(\frac{\theta}{r}\right)$

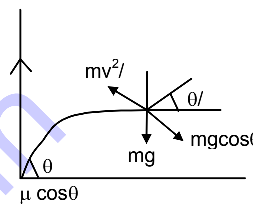


At highest pt.

$$mg = \frac{mv^2}{r} \Rightarrow r = \frac{u^2\cos^2\theta}{g}$$

26. Let 'u' the velocity at the pt where it makes an angle $\theta/2$ with horizontal. The horizontal component remains unchanged

So, $v\cos\theta/2 = \omega\cos\theta \Rightarrow v = \dfrac{u\cos\theta}{\cos\left(\frac{\theta}{2}\right)}$ ...(i)



From figure

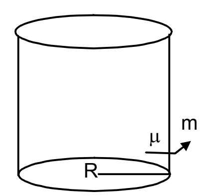
$$mg\cos(\theta/2) = \frac{mv^2}{r} \Rightarrow r = \frac{v^2}{g\cos(\theta/2)}$$

putting the value of 'v' from equn(i)

$$r = \frac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$$

27. A block of mass 'm' moves on a horizontal circle against the wall of a cylindrical room of radius 'R' Friction coefficient between wall & the block is μ.

a) Normal reaction by the wall on the block is = $\dfrac{mv^2}{R}$

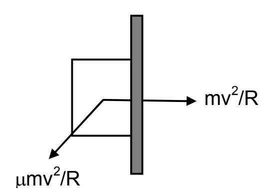


b) $\therefore$ Frictional force by wall = $\dfrac{\mu mv^2}{R}$

c) $\dfrac{\mu mv^2}{R} = ma \Rightarrow a = -\dfrac{\mu v^2}{R}$ (Deceleration)

d) Now, $\dfrac{dv}{dt} = v\dfrac{dv}{ds} = -\dfrac{\mu v^2}{R} \Rightarrow ds = -\dfrac{R}{\mu}\dfrac{dv}{v}$

$\Rightarrow s = -\dfrac{R\mu}{} \ln V + c$



At s = 0, v = v_0

Therefore, $c = \dfrac{R}{\mu}\ln V_0$

so, $s = -\dfrac{R}{\mu}\ln\dfrac{v}{v_0} \Rightarrow \dfrac{v}{v_0} = e^{-\mu s/R}$

For, one rotation $s = 2\pi R$, so $v = v_0 e^{-2\pi\mu}$

28. The cabin rotates with angular velocity ω & radius R

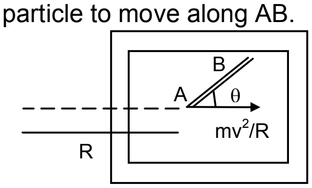
$\therefore$ The particle experiences a force $mR\omega^2$.

The component of $mR\omega^2$ along the groove provides the required force to the particle to move along AB.

$\therefore mR\omega^2\cos\theta = ma \Rightarrow a = R\omega^2\cos\theta$

length of groove = L

$L = ut + ½ at^2 \Rightarrow L = ½ R\omega^2\cos\theta\, t^2$



$$\Rightarrow t^2 = \frac{2L}{R\omega^2\cos\theta} = \Rightarrow t = 1\sqrt{\frac{2L}{R\omega^2\cos\theta}}$$

29. v = Velocity of car = 36km/hr = 10 m/s
r = Radius of circular path = 50m
m = mass of small body = 100g = 0.1kg.
μ = Friction coefficient between plate & body = 0.58
a) The normal contact force exerted by the plate on the block

$$N = \frac{mv^2}{r} = \frac{0.1\times 100}{50} = 0.2N$$

b) The plate is turned so the angle between the normal to the plate & the radius of the road slowly increases

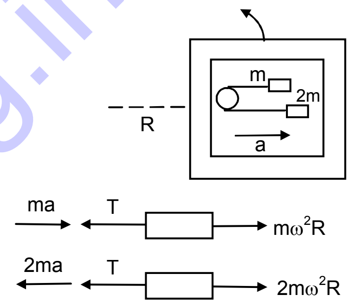
$$N = \frac{mv^2}{r}\cos\theta \qquad ..(i)$$

$$\mu N = \frac{mv^2}{r}\sin\theta \qquad ..(ii)$$

Putting value of N from (i)

$$\mu\frac{mv^2}{r}\cos\theta = \frac{mv^2}{r}\sin\theta \Rightarrow \mu = \tan\theta \Rightarrow \theta = \tan^{-1}\mu = \tan^{-1}(0.58) = 30°$$

30. Let the bigger mass accelerates towards right with 'a'.
From the free body diagrams,



$T - ma - m\omega^2R = 0 \qquad ...(i)$
$T + 2ma - 2m\omega^2R = 0 \qquad ...(ii)$
Eq (i) – Eq (ii) $\Rightarrow 3ma = m\omega^2R$

$$\Rightarrow a = \frac{m\omega^2R}{3}$$

Substituting the value of a in Equation (i), we get $T = 4/3\, m\omega^2R$.

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