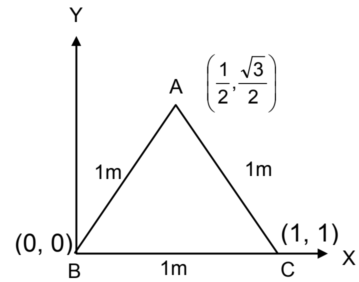
SOLUTIONS TO CONCEPTS
CHAPTER 9

1. $m_1 = 1kg$, $m_2 = 2kg$, $m_3 = 3kg$,
$x_1 = 0$, $x_2 = 1$, $x_3 = 1/2$
$y_1 = 0$, $y_2 = 0$, $y_3 = \sqrt{3}/2$
The position of centre of mass is

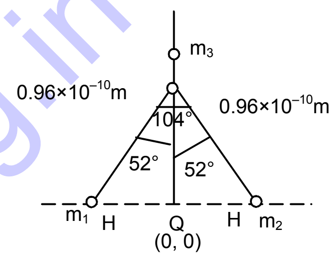


$$C.M = \left(\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}, \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}\right)$$

$$= \left(\frac{(1\times 0)+(2\times 1)+(3\times 1/2)}{1+2+3}, \frac{(1\times 0)+(2\times 0)+(3\times(\sqrt{3}/2))}{1+2+3}\right)$$

$$= \left(\frac{7}{12}, \frac{3\sqrt{3}}{12}\right) \text{ from the point B.}$$

2. Let θ be the origin of the system
In the above figure
$m_1 = 1gm$, $x_1 = -(0.96\times10^{-10})\sin 52°$ $y_1 = 0$
$m_2 = 1gm$, $x_2 = -(0.96\times10^{-10})\sin 52°$ $y_2 = 0$
$x_3 = 0$ $y_3 = (0.96 \times 10^{-10}) \cos 52°$
The position of centre of mass

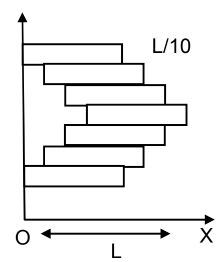


$$\left(\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}, \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}\right)$$

$$= \left(\frac{-(0.96\times10^{-10})\times\sin 52 + (0.96\times10^{-10})\sin 52 + 16\times 0}{1+1+16}, \frac{0+0+16y_3}{18}\right)$$

$$= \left(0, \ (8/9)0.96\times10^{-10}\cos 52^o\right)$$

3. Let 'O' (0,0) be the origin of the system.
Each brick is mass 'M' & length 'L'.
Each brick is displaced w.r.t. one in contact by 'L/10'
$\therefore$ The X coordinate of the centre of mass

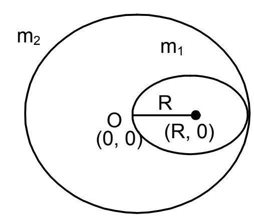


$$\overline{X}_{cm} = \frac{m\left(\frac{L}{2}\right) + m\left(\frac{L}{2}+\frac{L}{10}\right) + m\left(\frac{L}{2}+\frac{2L}{10}\right) + m\left(\frac{L}{2}+\frac{3L}{10}\right) + m\left(\frac{L}{2}+\frac{3L}{10}-\frac{L}{10}\right) + m\left(\frac{L}{2}+\frac{L}{10}\right) + m\left(\frac{L}{2}\right)}{7m}$$

$$= \frac{\frac{L}{2}+\frac{L}{2}+\frac{L}{10}+\frac{L}{2}+\frac{L}{5}+\frac{L}{2}+\frac{3L}{10}+\frac{L}{2}+\frac{L}{5}+\frac{L}{2}+\frac{L}{10}+\frac{L}{2}}{7}$$

$$= \frac{\frac{7L}{2}+\frac{5L}{10}+\frac{2L}{5}}{7} = \frac{35L+5L+4L}{10\times 7} = \frac{44L}{70} = \frac{11}{35}L$$

4. Let the centre of the bigger disc be the origin.
$2R$ = Radius of bigger disc
R = Radius of smaller disc
$m_1 = \pi R^2 \times T \times \rho$
$m_2 = \pi(2R)^2 \, I \, T \times \rho$
where T = Thickness of the two discs
ρ = Density of the two discs
$\therefore$ The position of the centre of mass



$$\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}, \frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$$

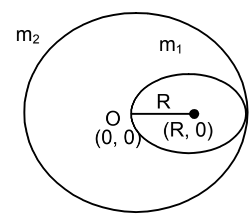
$x_1 = R \qquad y_1 = 0$

$x_2 = 0 \qquad y_2 = 0$

$$\left(\frac{\pi R^2T\rho R+0}{\pi R^2T\rho+\pi(2R)^2T\rho}, \frac{0}{m_1+m_2}\right) = \left(\frac{\pi R^2T\rho R}{5\pi R^2T\rho}, 0\right) = \left(\frac{R}{5}, 0\right)$$

At R/5 from the centre of bigger disc towards the centre of smaller disc.

5. Let '0' be the origin of the system.
   R = radius of the smaller disc
   2R = radius of the bigger disc
   The smaller disc is cut out from the bigger disc
   As from the figure



$m_1 = \pi R^2T\rho \qquad x_1 = R \qquad y_1 = 0$

$m_2 = \pi(2R)^2T\rho \qquad x_2 = 0 \qquad y_2 = 0$

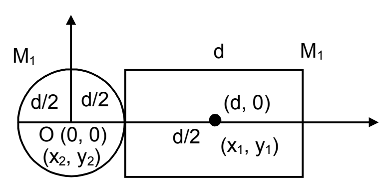
The position of C.M. = $\left(\frac{-\pi R^2T\rho R+0}{-\pi R^2T\rho+\pi(2R)^2T\rho R}, \frac{0+0}{m_1+m_2}\right)$

$$= \left(\frac{-\pi R^2T\rho R}{3\pi R^2T\rho}, 0\right) = \left(-\frac{R}{3}, 0\right)$$

C.M. is at R/3 from the centre of bigger disc away from centre of the hole.

6. Let m be the mass per unit area.
   $\therefore$ Mass of the square plate = $M_1 = d^2m$

   Mass of the circular disc = $M_2 = \frac{\pi d^2}{4}m$



Let the centre of the circular disc be the origin of the system.
$\therefore$ Position of centre of mass

$$= \left(\frac{d^2md+\pi(d^2/4)m\times 0}{d^2m+\pi(d^2/4)m}, \frac{0+0}{M_1+M_2}\right) = \left(\frac{d^3m}{d^2m\left(1+\frac{\pi}{4}\right)}, 0\right) = \left(\frac{4d}{\pi+4}, 0\right)$$

The new centre of mass is $\left(\frac{4d}{\pi+4}\right)$ right of the centre of circular disc.

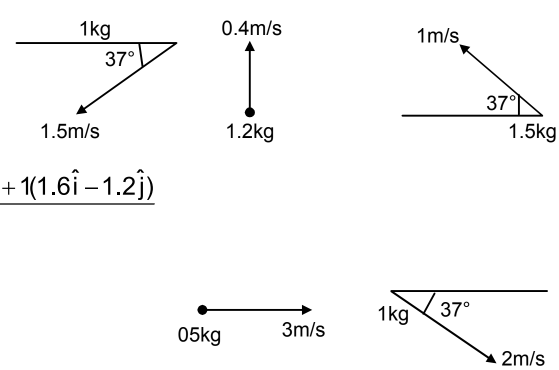
7. $m_1 = 1\text{kg}. \qquad \vec{v}_1 = -1.5\cos 37\ \hat{i} - 1.55\sin 37\ \hat{j} = -1.2\hat{i} - 0.9\hat{j}$

   $m_2 = 1.2\text{kg}. \qquad \vec{v}_2 = 0.4\hat{j}$

   $m_3 = 1.5\text{kg} \qquad \vec{v}_3 = -0.8\hat{i} + 0.6\hat{j}$

   $m_4 = 0.5\text{kg} \qquad \vec{v}_4 = 3\hat{i}$

   $m_5 = 1\text{kg} \qquad \vec{v}_5 = 1.6\hat{i} - 1.2\hat{j}$



So, $\vec{v}_c = \frac{m_1\vec{v}_1+m_2\vec{v}_2+m_3\vec{v}_3+m_4\vec{v}_4+m_5\vec{v}_5}{m_1+m_2+m_3+m_4+m_5}$

$$= \frac{1(-1.2\hat{i}-0.9\hat{j})+1.2(0.4\hat{j})+1.5(-0.8\hat{i}+0.6\hat{j})+0.5(3\hat{i})+1(1.6\hat{i}-1.2\hat{j})}{5.2}$$

$$= \frac{-1.2\hat{i}-0.9\hat{j}+4.8\hat{j}-1.2\hat{i}+.90\hat{j}+1.5\hat{i}+1.6\hat{i}-1.2\hat{j}}{5.2}$$

$$= \frac{0.7\hat{i}}{5.2} - \frac{0.72\hat{j}}{5.2}$$

8. Two masses m_1 & m_2 are placed on the X-axis
   m_1 = 10 kg, m_2 = 20kg.
   The first mass is displaced by a distance of 2 cm

   $$\therefore \overline{X}_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{10 \times 2 + 20x_2}{30}$$

   $$\Rightarrow 0 = \frac{20 + 20x_2}{30} \Rightarrow 20 + 20x_2 = 0$$

   $\Rightarrow 20 = -20x_2 \Rightarrow x_2 = -1.$

   $\therefore$ The 2^{nd} mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

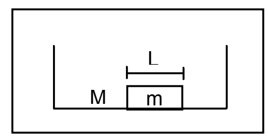
9. Two masses m_1 & m_2 are kept in a vertical line
   m_1 = 10kg, m_2 = 30kg
   The first block is raised through a height of 7 cm.
   The centre of mass is raised by 1 cm.

   $$\therefore 1 = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{10 \times 7 + 30y_2}{40}$$

   $$\Rightarrow 1 = \frac{70 + 30y_2}{40} \Rightarrow 70 + 30y_2 = 40 \Rightarrow 30y_2 = -30 \Rightarrow y_2 = -1.$$

   The 30 kg body should be displaced 1cm downward inorder to raise the centre of mass through 1 cm.

10. As the hall is gravity free, after the ice melts, it would tend to acquire a spherical shape. But, there is no external force acting on the system. So, the centre of mass of the system would not move.

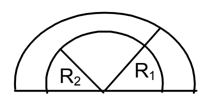


11. The centre of mass of the blate will be on the symmetrical axis.

   $$\Rightarrow \overline{y}_{cm} = \frac{\left(\frac{\pi R_2^2}{2}\right)\left(\frac{4R_2}{3\pi}\right) - \left(\frac{\pi R_1^2}{2}\right)\left(\frac{4R_1}{3\pi}\right)}{\frac{\pi R_2^2}{2} - \frac{\pi R_1^2}{2}}$$

   $$= \frac{(2/3)R_2^3 - (2/3)R_1^3}{\pi/2(R_2^2 - R_1^2)} = \frac{4}{3\pi}\frac{(R_2 - R_1)(R_2^2 + R_1^2 + R_1R_2)}{(R_2 - R_1)(R_2 + R_1)}$$

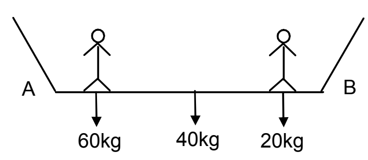
   $$= \frac{4}{3\pi}\frac{(R_2^2 + R_1^2 + R_1R_2)}{R_1 + R_2}$$ above the centre.



12. m_1 = 60kg, m_2 = 40kg , m_3 = 50kg,
   Let A be the origin of the system.
   Initially Mr. Verma & Mr. Mathur are at extreme position of the boat.
   $\therefore$ The centre of mass will be at a distance

   $$= \frac{60 \times 0 + 40 \times 2 + 50 \times 4}{150} = \frac{280}{150} = 1.87\text{m from 'A'}$$

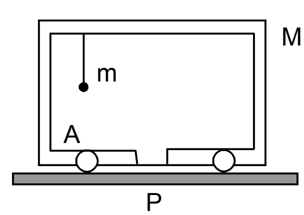
   When they come to the mid point of the boat the CM lies at 2m from 'A'.
   $\therefore$ The shift in CM = 2 – 1.87 = 0.13m towards right.
   But as there is no external force in longitudinal direction their CM would not shift.
   So, the boat moves 0.13m or 13 cm towards right.



13. Let the bob fall at A,. The mass of bob = m.
   The mass of cart = M.
   Initially their centre of mass will be at

   $$\frac{m \times L + M \times 0}{M + m} = \left(\frac{m}{M + m}\right)L$$

   Distance from P
   When, the bob falls in the slot the CM is at a distance 'O' from P.



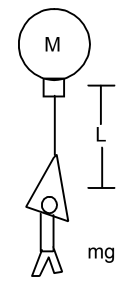
Shift in CM = $0 - \frac{mL}{M+m} = -\frac{mL}{M+m}$ towards left

$= \frac{mL}{M+m}$ towards right.

But there is no external force in horizontal direction.

So the cart displaces a distance $\frac{mL}{M+m}$ towards right.

14. Initially the monkey & balloon are at rest.
So the CM is at 'P'
When the monkey descends through a distance 'L'
The CM will shift



$t_o = \frac{m \times L + M \times 0}{M+m} = \frac{mL}{M+m}$ from P

So, the balloon descends through a distance $\frac{mL}{M+m}$

15. Let the mass of the to particles be m_1 & m_2 respectively
m_1 = 1kg, m_2 = 4kg
$\therefore$ According to question
$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$

$$\Rightarrow \frac{m_1}{m_2} = \frac{v_2^2}{v_1^2} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

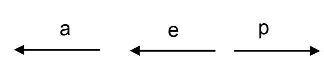
Now, $\frac{m_1 v_1}{m_2 v_2} = \frac{m_1}{m_2} \times \sqrt{\frac{m_2}{m_1}} = \frac{\sqrt{m_1}}{\sqrt{m_2}} = \frac{\sqrt{1}}{\sqrt{4}} = 1/2$

$\Rightarrow \frac{m_1 v_1}{m_2 v_2} = 1 : 2$

16. As uranium 238 nucleus emits a α-particle with a speed of 1.4×10^7 m/sec. Let v_2 be the speed of the residual nucleus thorium 234.
$\therefore m_1 v_1 = m_2 v_2$
$\Rightarrow 4 \times 1.4 \times 10^7 = 234 \times v_2$
$\Rightarrow v_2 = \frac{4 \times 1.4 \times 10^7}{234} = 2.4 \times 10^5$ m/sec.

17. $m_1 v_1 = m_2 v_2$
$\Rightarrow 50 \times 1.8 = 6 \times 10^{24} \times v_2$
$\Rightarrow v_2 = \frac{50 \times 1.8}{6 \times 10^{24}} = 1.5 \times 10^{-23}$ m/sec
so, the earth will recoil at a speed of 1.5×10^{-23} m/sec.

18. Mass of proton = 1.67×10^{-27}
Let 'V_p' be the velocity of proton



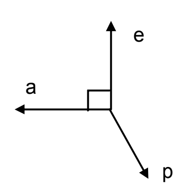
Given momentum of electron = 1.4×10^{-26} kg m/sec
Given momentum of antineutrino = 6.4×10^{-27} kg m/sec

a) The electron & the antineutrino are ejected in the same direction. As the total momentum is conserved the proton should be ejected in the opposite direction.

$1.67 \times 10^{-27} \times V_p = 1.4 \times 10^{-26} + 6.4 \times 10^{-27} = 20.4 \times 10^{-27}$
$\Rightarrow V_p = (20.4 / 1.67) = 12.2$ m/sec in the opposite direction.

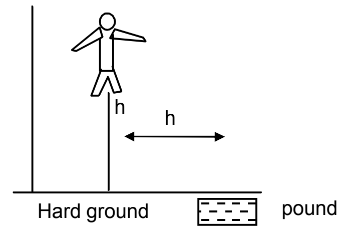
b) The electron & antineutrino are ejected $\perp^r$ to each other.

Total momentum of electron and antineutrino,



$= \sqrt{(14)^2 + (6.4)^2} \times 10^{-27}$ kg m/s = 15.4×10^{-27} kg m/s
Since, $1.67 \times 10^{-27} V_p = 15.4 \times 10^{-27}$ kg m/s
So $V_p = 9.2$ m/s

19. Mass of man = M, Initial velocity = 0
Mass of bad = m
Let the throws the bag towards left with a velocity v towards left. So, there is no external force in the horizontal direction.
The momentum will be conserved. Let he goes right with a velocity



$$mv = MV \Rightarrow V = \frac{mv}{M} \Rightarrow v = \frac{MV}{m} \quad ..(i)$$

Let the total time he will take to reach ground = $\sqrt{2H/g}$ = t_1

Let the total time he will take to reach the height h = t_2 = $\sqrt{2(H-h)/g}$

Then the time of his flying = $t_1 - t_2$ = $\sqrt{2H/g} - \sqrt{2(H-h)/g} = \sqrt{2/g}\left(\sqrt{H} - \sqrt{H-h}\right)$

Within this time he reaches the ground in the pond covering a horizontal distance x

$\Rightarrow x = V \times t \Rightarrow V = x / t$

$$\therefore v = \frac{M}{m}\frac{x}{t} = \frac{M}{m} \times \frac{\sqrt{g}}{\sqrt{2}\left(\sqrt{H} - \sqrt{H-h}\right)}$$

As there is no external force in horizontal direction, the x-coordinate of CM will remain at that position.

$$\Rightarrow 0 = \frac{M \times (x) + m \times x_1}{M + m} \Rightarrow x_1 = -\frac{M}{m}x$$

$\therefore$ The bag will reach the bottom at a distance (M/m) x towards left of the line it falls.

20. Mass = 50g = 0.05kg

$v = 2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$

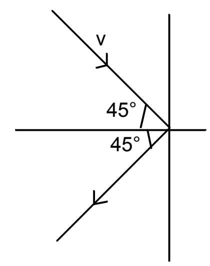
$v_1 = -2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$

a) change in momentum = $m\vec{v} - m\vec{v}_1$

$= 0.05\,(2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}) - 0.05\,(-2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j})$

$= 0.1\cos 45°\,\hat{i} - 0.1\sin 45°\,\hat{j} + 0.1\cos 45°\,\hat{i} + 0.1\sin 45°\,\hat{j}$

$= 0.2\cos 45°\,\hat{i}$



$$\therefore \text{magnitude} = \sqrt{\left(\frac{0.2}{\sqrt{2}}\right)^2} = \frac{0.2}{\sqrt{2}} = 0.14 \text{ kg m/s}$$

c) The change in magnitude of the momentum of the ball

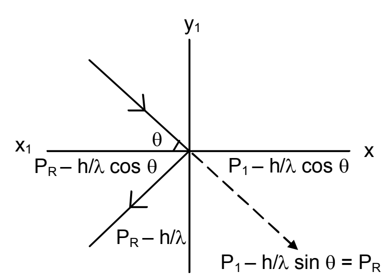
$-\left|\vec{P}_i\right| - \left|\vec{P}_f\right| = 2 \times 0.5 - 2 \times 0.5 = 0.$

21. $\vec{P}_{incidence} = (h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$

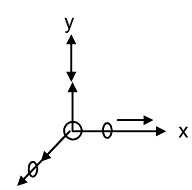
$P_{Reflected} = -(h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$

The change in momentum will be only in the x-axis direction. i.e.

$|\Delta P| = (h/\lambda)\cos\theta - ((h/\lambda)\cos\theta) = (2h/\lambda)\cos\theta$



22. As the block is exploded only due to its internal energy. So net external force during this process is 0. So the centre mass will not change.
Let the body while exploded was at the origin of the co-ordinate system.
If the two bodies of equal mass is moving at a speed of 10m/s in + x & +y axis direction respectively,



$\sqrt{10^2 + 10^2 + 2 10.10\cos 90°} = 10\sqrt{2}$ m/s 45° w.r.t. + x axis

If the centre mass is at rest, then the third mass which have equal mass with other two, will move in the opposite direction (i.e. 135° w.r.t. + x- axis) of the resultant at the same velocity.

23. Since the spaceship is removed from any material object & totally isolated from surrounding, the missions by astronauts couldn't slip away from the spaceship. So the total mass of the spaceship remain unchanged and also its velocity.

24. d = 1cm, v = 20 m/s, u = 0, ρ = 900 kg/m^3 = 0.9gm/cm^3
volume = $(4/3)\pi r^3 = (4/3)\ \pi\ (0.5)^3 = 0.5238\text{cm}^3$
$\therefore$ mass = vρ = 0.5238 × 0.9 = 0.4714258gm
$\therefore$ mass of 2000 hailstone = 2000 × 0.4714 = 947.857
$\therefore$ Rate of change in momentum per unit area = 947.857 × 2000 = 19N/m^3
$\therefore$ Total force exerted = 19 × 100 = 1900 N.

25. A ball of mass m is dropped onto a floor from a certain height let 'h'.
$\therefore v_1 = \sqrt{2gh}$, $v_1 = 0$, $v_2 = -\sqrt{2gh}$ & $v_2 = 0$
$\therefore$ Rate of change of velocity :-
$$F = \frac{m \times 2\sqrt{2gh}}{t}$$
$\therefore v = \sqrt{2gh}$, s = h, v = 0
$\Rightarrow$ v = u + at
$$\Rightarrow \sqrt{2gh} = g\,t \Rightarrow t = \sqrt{\frac{2h}{g}}$$
$$\therefore \text{Total time } 2\sqrt{\frac{2h}{t}}$$
$$\therefore F = \frac{m \times 2\sqrt{2gh}}{2\sqrt{\frac{2h}{g}}} = mg$$

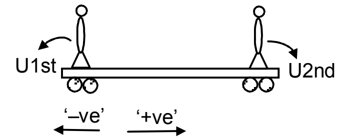
26. A railroad car of mass M is at rest on frictionless rails when a man of mass m starts moving on the car towards the engine. The car recoils with a speed v backward on the rails.
Let the mass is moving with a velocity x w.r.t. the engine.
$\therefore$The velocity of the mass w.r.t earth is (x – v) towards right
V_{cm} = 0 (Initially at rest)
$\therefore$ 0 = –Mv + m(x – v)
$$\Rightarrow Mv = m(x - v) \Rightarrow mx = Mv + mv \Rightarrow x = \left(\frac{M+m}{m}\right)v \Rightarrow x = \left(1 + \frac{M}{m}\right)v$$

27. A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is 50m where m is the mass of one shell. The muzzle velocity of the shells is 200m/s.
Initial, V_{cm} = 0.
$$\therefore 0 = 49\ m \times V + m \times 200 \Rightarrow V = \frac{-200}{49}\ \text{m/s}$$
$$\therefore \frac{200}{49}\ \text{m/s towards left.}$$
When another shell is fired, then the velocity of the car, with respect to the platform is,
$$\Rightarrow V` = \frac{200}{49}\ \text{m/s towards left.}$$
When another shell is fired, then the velocity of the car, with respect to the platform is,
$$\Rightarrow v` = \frac{200}{48}\ \text{m/s towards left}$$
$$\therefore \text{Velocity of the car w.r.t the earth is } \left(\frac{200}{49} + \frac{200}{48}\right) \text{m/s towards left.}$$

28. Two persons each of mass m are standing at the two extremes of a railroad car of mass m resting on a smooth track.
Case – I
Let the velocity of the railroad car w.r.t the earth is V after the jump of the left man.
$\therefore$ 0 = – mu + (M + m) V

$\Rightarrow$ V = $\dfrac{mu}{M+m}$ towards right

Case – II



When the man on the right jumps, the velocity of it w.r.t the car is u.

$\therefore$ **0 = mu – Mv'**

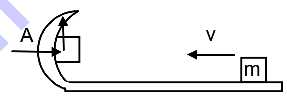
$\Rightarrow$ **v′ =** $\dfrac{mu}{M}$

(V′ is the change is velocity of the platform when platform itself is taken as reference assuming the car to be at rest)

$\therefore$ So, net velocity towards left (i.e. the velocity of the car w.r.t. the earth)

$= \dfrac{mv}{M} - \dfrac{mv}{M+m} = \dfrac{mMu + m^2v - Mmu}{M(M+m)} = \dfrac{m^2v}{M(M+m)}$

29. A small block of mass m which is started with a velocity V on the horizontal part of the bigger block of mass M placed on a horizontal floor.
Since the small body of mass m is started with a velocity V in the horizontal direction, so the total initial momentum at the initial position in the horizontal direction will remain same as the total final momentum at the point A on the bigger block in the horizontal direction.
From L.C.K. m:



mv + M×O = (m + M) v $\Rightarrow$ v′ = $\dfrac{mv}{M+m}$

30. Mass of the boggli = 200kg, V_B = 10 km/hour.
$\therefore$ Mass of the boy = 2.5kg & V_{Boy} = 4km/hour.
If we take the boy & boggle as a system then total momentum before the process of sitting will remain constant after the process of sitting.
$\therefore m_b V_b = m_{boy}V_{boy} = (m_b + m_{boy})\, v$
$\Rightarrow$ 200 × 10 + 25 × 4 = (200 +25) × v
$\Rightarrow$ v = $\dfrac{2100}{225} = \dfrac{28}{3}$ = 9.3 m/sec

31. Mass of the ball = m_1 = 0.5kg, velocity of the ball = 5m/s
Mass of the another ball m_2 = 1kg
Let it's velocity = v′ m/s
Using law of conservation of momentum,
0.5 × 5 + 1 × v′ = 0 $\Rightarrow$ v′ = – 2.5
$\therefore$ Velocity of second ball is 2.5 m/s opposite to the direction of motion of 1^{st} ball.

32. Mass of the man = m_1 = 60kg
Speed of the man = v_1 = 10m/s
Mass of the skater = m_2 = 40kg
let its velocity = v′
$\therefore$ 60 × 10 + 0 = 100 × v′ $\Rightarrow$ v′ = 6m/s
loss in K.E.= $(1/2)60 \times (10)^2 - (1/2) \times 100 \times 36$ = 1200 J

33. Using law of conservation of momentum.
$m_1u_1 + m_2u_2 = m_1v(t) + m_2v'$
Where v′ = speed of 2^{nd} particle during collision.
$\Rightarrow m_1u_1 + m_2u_2 = m_1u_1 + m_1 + (t/\Delta t)(v_1 - u_1) + m_2v'$
$\Rightarrow \dfrac{m_2u_2}{m^2} - \dfrac{m_1}{m2}\dfrac{t}{\Delta t}(v_1 - u_1)v'$
$\therefore v' = u_2 - \dfrac{m_1}{m_2}\dfrac{t}{\Delta t}(v_1 - u)$

34. Mass of the bullet = m and speed = v
Mass of the ball = M
m′ = frictional mass from the ball.

Using law of conservation of momentum,
$mv + 0 = (m' + m)\, v' + (M - m')\, v_1$
where v' = final velocity of the bullet + frictional mass

$\Rightarrow v' = \dfrac{mv - (M + m')V_1}{m + m'}$

35. Mass of 1st ball = m and speed = v
Mass of 2nd ball = m
Let final velocities of 1st and 2nd ball are v_1 and v_2 respectively
Using law of conservation of momentum,
$m(v_1 + v_2) = mv.$
$\Rightarrow v_1 + v_2 = v$ …(1)
Also
$v_1 - v_2 = ev$ …(2)
Given that final K.E. = ¾ Initial K.E.
$\Rightarrow \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 = \frac{3}{4} \times \frac{1}{2} mv^2$
$\Rightarrow v_1^2 + v_2^2 = \frac{3}{4} v^2$

$\Rightarrow \dfrac{(v_1 + v_2)^2 + (v_1 - v_2)^2}{2} = \dfrac{3}{4} v^2$

$\Rightarrow \dfrac{(1 + e^2)v^2}{2} = \dfrac{3}{4} v^2 \Rightarrow 1 + e^2 = \dfrac{3}{2} \Rightarrow e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

36. Mass of block = 2kg and speed = 2m/s
Mass of 2nd block = 2kg.
Let final velocity of 2nd block = v
using law of conservation of momentum.
$2 \times 2 = (2 + 2)\, v \Rightarrow v = 1 m/s$
$\therefore$ Loss in K.E. in inelastic collision
$= (1/2) \times 2 \times (2)^2\, v - (1/2)\,(2 + 2) \times (1)^2 = 4 - 2 = 2$ J

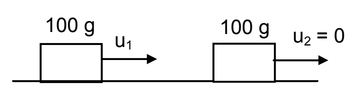
b) Actual loss = $\dfrac{\text{Maximum loss}}{2}$ = 1J

$(1/2) \times 2 \times 2^2 - (1/2)\, 2 \times v_1^2 + (1/2) \times 2 \times v_2^2 = 1$
$\Rightarrow 4 - (v_1^2 + v_2^2) = 1$

$\Rightarrow 4 - \dfrac{(1 + e^2) \times 4}{2} = 1$

$\Rightarrow 2(1 + e^2) = 3 \Rightarrow 1 + e^2 = \dfrac{3}{2} \Rightarrow e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

37. Final K.E. = 0.2J
Initial K.E. = $\frac{1}{2} mV_1^2 + 0 = \frac{1}{2} \times 0.1\, u^2 = 0.05\, u^2$
$mv_1 = mv_2' = mu$
Where v_1 and v_2 are final velocities of 1st and 2nd block respectively.



$\Rightarrow v_1 + v_2 = u$ …(1)
$(v_1 - v_2) + \ell\, (a_1 - u_2) = 0 \Rightarrow \ell a = v_2 - v_1$ ..(2)
$u_2 = 0,\quad u_1 = u.$
Adding Eq.(1) and Eq.(2)
$2v_2 = (1 + \ell)u \Rightarrow v_2 = (u/2)(1 + \ell)$

$\therefore v_1 = u - \dfrac{u}{2} - \dfrac{u}{2}\ell$

$v_1 = \dfrac{u}{2}(1 - \ell)$

Given $(1/2)mv_1^2 + (1/2)mv_2^2 = 0.2$
$\Rightarrow v_1^2 + v_2^2 = 4$

$\Rightarrow \frac{u^2}{4}(1-\ell)^2 + \frac{u^2}{4}(1+\ell)^2 = 4 \qquad \Rightarrow \frac{u^2}{2}(1+\ell^2) = 4 \qquad \Rightarrow u^2 = \frac{8}{1+\ell^2}$

For maximum value of u, denominator should be minimum,

$\Rightarrow \ell = 0.$

$\Rightarrow u^2 = 8 \Rightarrow u = 2\sqrt{2}$ m/s

For minimum value of u, denominator should be maximum,

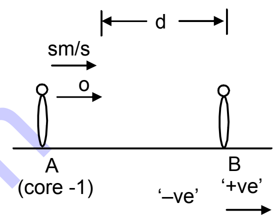
$\Rightarrow \ell = 1$

$u^2 = 4 \Rightarrow u = 2$ m/s

38. Two friends A & B (each 40kg) are sitting on a frictionless platform some distance d apart A rolls a ball of mass 4kg on the platform towards B, which B catches. Then B rolls the ball towards A and A catches it. The ball keeps on moving back & forth between A and B. The ball has a fixed velocity 5m/s.

a) Case – I :– Total momentum of the man A & the ball will remain constant

$\therefore 0 = 4 \times 5 - 40 \times v \qquad \Rightarrow v = 0.5$ m/s towards left



b) Case – II :– When B catches the ball, the momentum between the B & the ball will remain constant.

$\Rightarrow 4 \times 5 = 44v \Rightarrow v = (20/44)$ m/s

Case – III :– When B throws the ball, then applying L.C.L.M

$\Rightarrow 44 \times (20/44) = -4 \times 5 + 40 \times v \qquad \Rightarrow v = 1$m/s (towards right)

Case – IV :– When a Catches the ball, the applying L.C.L.M.

$\Rightarrow -4 \times 5 + (-0.5) \times 40 = -44v \qquad \Rightarrow v = \frac{10}{11}$ m/s towards left.

c) Case – V :– When A throws the ball, then applying L.C.L.M.

$\Rightarrow 44 \times (10/11) = 4 \times 5 - 40 \times V \qquad \Rightarrow V = 60/40 = 3/2$ m/s towards left.

Case – VI :– When B receives the ball, then applying L.C.L.M

$\Rightarrow 40 \times 1 + 4 \times 5 = 44 \times v \qquad \Rightarrow v = 60/44$ m/s towards right.

Case – VII :– When B throws the ball, then applying L.C.L.M.

$\Rightarrow 44 \times (66/44) = -4 \times 5 + 40 \times V \qquad \Rightarrow V = 80/40 = 2$ m/s towards right.

Case – VIII :– When A catches the ball, then applying L.C.L.M

$\Rightarrow -4 \times 5 - 40 \times (3/2) = -44v \qquad \Rightarrow v = (80/44) = (20/11)$ m/s towards left.

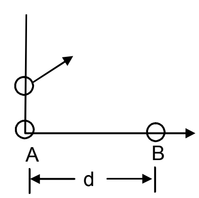
Similarly after 5 round trips

The velocity of A will be (50/11) & velocity of B will be 5 m/s.

d) Since after 6 round trip, the velocity of A is 60/11 i.e.

> 5m/s. So, it can't catch the ball. So it can only roll the ball six.

e) Let the ball & the body A at the initial position be at origin.



$\therefore X_C = \frac{40 \times 0 + 4 \times 0 + 40 \times d}{40 + 40 + 4} = \frac{10}{11}d$

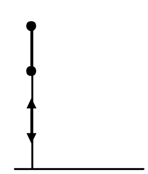
39. $u = \sqrt{2gh}$ = velocity on the ground when ball approaches the ground.

$\Rightarrow u = \sqrt{2 \times 9.8 \times 2}$

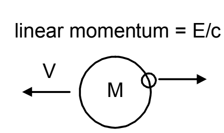
v = velocity of ball when it separates from the ground.

$\vec{v} + \ell\vec{u} = 0$

$\Rightarrow \ell\vec{u} = -\vec{v} \quad \Rightarrow \ell = \frac{\sqrt{2 \times 9.8 \times 1.5}}{\sqrt{2 \times 9.8 \times 2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$



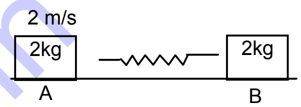
40. K.E. of Nucleus = $(1/2)mv^2 = (1/2)\,m\left(\frac{E}{mc}\right)^2 = \frac{E^2}{2mc^2}$



Energy limited by Gamma photon = E.

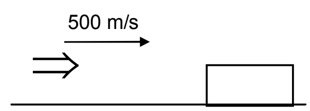
Decrease in internal energy = $E + \frac{E^2}{2mc^2}$

41. Mass of each block M_A and M_B = 2kg.
Initial velocity of the 1st block, (V) = 1m/s
V_A = 1 m/s, V_B = 0m/s
Spring constant of the spring = 100 N/m.
The block A strikes the spring with a velocity 1m/s/
After the collision, it's velocity decreases continuously and at a instant the whole system (Block A + the compound spring + Block B) move together with a common velocity.
Let that velocity be V.
Using conservation of energy, $(1/2)\, M_AV_A^2 + (1/2)M_BV_B^2 = (1/2)M_Av^2 + (1/2)M_Bv^2 + (1/2)kx^2$.
$(1/2) \times 2(1)^2 + 0 = (1/2) \times 2\times v^2 + (1/2) \times 2 \times v^2 + (1/2)\, x^2 \times 100$
(Where x = max. compression of spring)
$\Rightarrow 1 = 2v^2 + 50x^2$ …(1)
As there is no external force in the horizontal direction, the momentum should be conserved.
$\Rightarrow M_AV_A + M_BV_B = (M_A + M_B)V$.
$\Rightarrow 2 \times 1 = 4 \times v$
$\Rightarrow V = (1/2)$ m/s. …(2)
Putting in eq.(1)



$1 = 2 \times (1/4) + 50x+2+$
$\Rightarrow (1/2) = 50x^2$
$\Rightarrow x^2 = 1/100\text{m}^2$
$\Rightarrow x = (1/10)\text{m} = 0.1\text{m} = 10\text{cm}$.

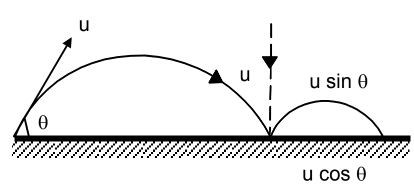
42. Mass of bullet m = 0.02kg.
Initial velocity of bullet V_1 = 500m/s
Mass of block, M = 10kg.
Initial velocity of block u_2 = 0.
Final velocity of bullet = 100 m/s = v.



Let the final velocity of block when the bullet emerges out, if block = v′.
$mv_1 + Mu_2 = mv + Mv'$
$\Rightarrow 0.02 \times 500 = 0.02 \times 100 + 10 \times v'$
$\Rightarrow v' = 0.8\text{m/s}$
After moving a distance 0.2 m it stops.
$\Rightarrow$ change in K.E. = Work done
$\Rightarrow 0 - (1/2) \times 10\times (0.8)^2 = -\mu \times 10 \times 10 \times 0.2 \Rightarrow \mu = 0.16$

43. The projected velocity = u.
The angle of projection = θ.
When the projectile hits the ground for the 1st time, the velocity would be the same i.e. u.
Here the component of velocity parallel to ground, u cos θ should remain constant. But the vertical component of the projectile undergoes a change after the collision.

$\Rightarrow e = \dfrac{u\sin\theta}{v} \Rightarrow v = eu\sin\theta$.



Now for the 2nd projectile motion,
U = velocity of projection = $\sqrt{(u\cos\theta)^2 + (eu\sin\theta)^2}$

and Angle of projection = $\alpha = \tan^{-1}\left(\dfrac{eu\sin\theta}{a\cos\theta}\right) = \tan^{-1}(e\tan\theta)$

or $\tan\alpha = e\tan\theta$ …(2)

Because, $y = x\tan\alpha - \dfrac{gx^2\sec^2\alpha}{2u^2}$ …(3)

Here, y = 0, $\tan\alpha = e\tan\theta$, $\sec^2\alpha = 1 + e^2\tan^2\theta$
And $u^2 = u^2\cos^2\theta + e^2\sin^2\theta$
Putting the above values in the equation (3),

$$x\,e\tan\theta = \frac{gx^2(1+e^2\tan^2\theta)}{2u^2(\cos^2\theta+e^2\sin^2\theta)}$$

$$\Rightarrow x = \frac{2eu^2\tan\theta(\cos^2\theta+e^2\sin^2\theta)}{g(1+e^2\tan^2\theta)}$$

$$\Rightarrow x = \frac{2eu^2\tan\theta - \cos^2\theta}{g} = \frac{eu^2\sin 2\theta}{g}$$

$\Rightarrow$ So, from the starting point O, it will fall at a distance

$$= \frac{u^2\sin 2\theta}{g} + \frac{eu^2\sin 2\theta}{g} = \frac{u^2\sin 2\theta}{g}(1+e)$$

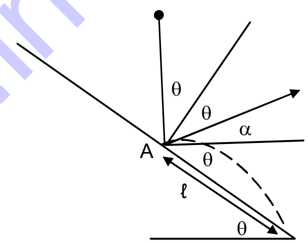
44. Angle inclination of the plane = θ

M the body falls through a height of h,

The striking velocity of the projectile with the indined plane $v = \sqrt{2gh}$

Now, the projectile makes on angle $(90° - 2\theta)$

Velocity of projection = $u = \sqrt{2gh}$



Let AB = L.

So, $x = \ell\cos\theta$, $y = -\ell\sin\theta$

From equation of trajectory,

$$y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2u^2}$$

$$-\ell\sin\theta = \ell\cos\theta \,.\, \tan(90° - 2\theta) - \frac{g\times\ell^2\cos^2\theta\sec^2(90°-2\theta)}{2\times 2gh}$$

$$\Rightarrow -\ell\sin\theta = \ell\cos\theta \,.\, \cot 2\theta - \frac{g\ell^2\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4gh}$$

$$\text{So, } \frac{\ell\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4h} = \sin\theta + \cos\theta\cot 2\theta$$

$$\Rightarrow \ell = \frac{4h}{\cos^2\theta\,\mathrm{cosec}^2 2\theta}(\sin\theta + \cos\theta\cot 2\theta) = \frac{4h\times\sin^2 2\theta}{\cos^2\theta}\left(\sin\theta+\cos\theta\times\frac{\cos 2\theta}{\sin 2\theta}\right)$$

$$= \frac{4h\times 4\sin^2\theta\cos^2\theta}{\cos^2\theta}\left(\frac{\sin\theta\times\sin 2\theta+\cos\theta\cos 2\theta}{\sin 2\theta}\right) = 16\,h\sin^2\theta\times\frac{\cos\theta}{2\sin\theta\cos\theta} = 8h\sin\theta$$

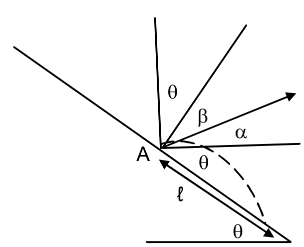
45. h = 5m, $\theta = 45°$, $e = (3/4)$

Here the velocity with which it would strike = $v = \sqrt{2g\times 5}$ = 10m/sec

After collision, let it make an angle β with horizontal. The horizontal component of velocity 10 cos 45° will remain unchanged and the velocity in the perpendicular direction to the plane after wllisine.

$\Rightarrow V_y = e \times 10\sin 45°$

$$= (3/4)\times 10\times\frac{1}{\sqrt{2}} = (3.75)\sqrt{2}\text{ m/sec}$$



$V_x = 10\cos 45° = 5\sqrt{2}$ m/sec

$$\text{So, } u = \sqrt{V_x^{\,2}+V_y^{\,2}} = \sqrt{50+28.125} = \sqrt{78.125} = 8.83\text{ m/sec}$$

$$\text{Angle of reflection from the wall } \beta = \tan^{-1}\left(\frac{3.75\sqrt{2}}{5\sqrt{2}}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 37°$$

$\Rightarrow$ Angle of projection $\alpha = 90 - (\theta + \beta) = 90 - (45° + 37°) = 8°$

Let the distance where it falls = L

$\Rightarrow x = L\cos\theta$, $y = -L\sin\theta$

Angle of projection $(\alpha) = -8°$

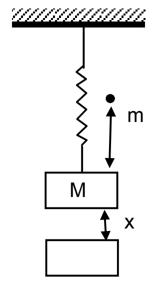
Using equation of trajectory, y = x tan α – $\dfrac{gx^2 \sec^2 \alpha}{2u^2}$

$\Rightarrow$ – ℓ sin θ = ℓ cos θ × tan 8° – $\dfrac{g}{2} \times \dfrac{\ell \cos^2\theta \sec^2 8°}{u^2}$

$\Rightarrow$ – sin 45° = cos 45° – tan 8° – $\dfrac{10\cos^2 45° \sec 8°}{(8.83)^2}(\ell)$

Solving the above equation we get,
ℓ = 18.5 m.

46. Mass of block

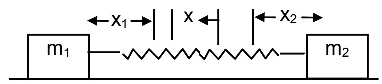


Block of the particle = m = 120gm = 0.12kg.
In the equilibrium condition, the spring is stretched by a distance x = 1.00 cm = 0.01m.
$\Rightarrow$ 0.2 × g = K. x.
$\Rightarrow$ 2 = K × 0.01 $\Rightarrow$ K = 200 N/m.
The velocity with which the particle m will strike M is given by u
= $\sqrt{2 \times 10 \times 0.45}$ = $\sqrt{9}$ = 3 m/sec.
So, after the collision, the velocity of the particle and the block is
V = $\dfrac{0.12 \times 3}{0.32}$ = $\dfrac{9}{8}$ m/sec.
Let the spring be stretched through an extra deflection of δ.
0 –(1/2) × 0.32 × (81/64) = 0.32 × 10 × δ – (1/2 × 200 × $(\delta + 0.1)^2$ – (1/2) × 200 × $(0.01)^2$
Solving the above equation we get
δ = 0.045 = 4.5cm

47. Mass of bullet = 25g = 0.025kg.
Mass of pendulum = 5kg.
The vertical displacement h = 10cm = 0.1m
Let it strike the pendulum with a velocity u.
Let the final velocity be v.
$\Rightarrow$ mu = (M + m)v.
$\Rightarrow$ v = $\dfrac{m}{(M+m)}u$ = $\dfrac{0.025}{5.025} \times u$ = $\dfrac{u}{201}$
Using conservation of energy.
0 – (1/2) (M + m). V^2 = – (M + m) g × h $\Rightarrow$ $\dfrac{u^2}{(201)^2}$ = 2 × 10 × 0.1 = 2
$\Rightarrow$ u = 201 × $\sqrt{2}$ = 280 m/sec.

48. Mass of bullet = M = 20gm = 0.02kg.
Mass of wooden block M = 500gm = 0.5kg
Velocity of the bullet with which it strikes u = 300 m/sec.
Let the bullet emerges out with velocity V and the velocity of block = V′
As per law of conservation of momentum.
mu = Mv′+ mv ….(1)
Again applying work – energy principle for the block after the collision,
0 – (1/2) M × V'^2 = – Mgh (where h = 0.2m)
$\Rightarrow V'^2$ = 2gh
V′ = $\sqrt{2gh}$ = $\sqrt{20 \times 0.2}$ = 2m/sec
Substituting the value of V′ in the equation (1), we get\
0.02 × 300 = 0.5 × 2 + 0.2 × v
$\Rightarrow$ V = $\dfrac{6.1}{0.02}$ = 250m/sec.

49. Mass of the two blocks are m_1 , m_2.
Initially the spring is stretched by x_0
Spring constant K.
For the blocks to come to rest again,
Let the distance travelled by m_1 & m_2
Be x_1 and x_2 towards right and left respectively.
As o external forc acts in horizontal direction,



$m_1x_1 = m_2x_2$ ...(1)
Again, the energy would be conserved in the spring.
$\Rightarrow (1/2)\ k \times x^2 = (1/2)\ k\ (x_1 + x_2 - x_0)^2$
$\Rightarrow x_o = x_1 + x_2 - x_0$
$\Rightarrow x_1 + x_2 = 2x_0$ ...(2)

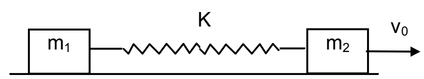
$\Rightarrow x_1 = 2x_0 - x_2$ similarly $x_1 = \left(\dfrac{2m_2}{m_1+m_2}\right)x_0$

$\Rightarrow m_1(2x_0 - x_2) = m_2x_2$ $\quad \Rightarrow 2m_1x_0 - m_1x_2 = m_2x_2$ $\quad \Rightarrow x_2 = \left(\dfrac{2m_1}{m_1+m_2}\right)x_0$

50. a) $\therefore$ Velocity of centre of mass = $\dfrac{m_2 \times v_0 + m_1 \times 0}{m_1+m_2} = \dfrac{m_2v_0}{m_1+m_2}$

b) The spring will attain maximum elongation when both velocity of two blocks will attain the velocity of centre of mass.
d) $x \rightarrow$ maximum elongation of spring.
Change of kinetic energy = Potential stored in spring.

$\Rightarrow (1/2)\ m_2\ v_0^2 - (1/2)\ (m_1 + m_2)\ (\left(\dfrac{m_2v_0}{m_1+m_2}\right)^2 = (1/2)\ kx^2$



$\Rightarrow m_2\ v_0^2\left(1 - \dfrac{m_2}{m_1+m_2}\right) = kx^2$ $\qquad \Rightarrow x = \left(\dfrac{m_1m_2}{m_1+m_2}\right)^{1/2} \times v_0$

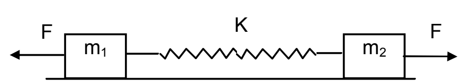
51. If both the blocks are pulled by some force, they suddenly move with some acceleration and instantaneously stop at same position where the elongation of spring is maximum.
$\therefore$ Let x_1, $x_2 \rightarrow$ extension by block m_1 and m_2
Total work done = $Fx_1 + Fx_2$ ...(1)
$\therefore$ Increase the potential energy of spring = $(1/2)\ K\ (x_1 + x_2)^2$ ...(2)
Equating (1) and (2)

$F(x_1 + x_2) = (1/2)\ K\ (x_1 + x_2)^2 \Rightarrow (x_1 + x_2) = \dfrac{2F}{K}$

Since the net external force on the two blocks is zero thus same force act on opposite direction.
$\therefore m_1x_1 = m_2x_2$ ...(3)

And $(x_1 + x_2) = \dfrac{2F}{K}$

$\therefore x_2 = \dfrac{m_1}{m_2} \times 1$



Substituting $\dfrac{m_1}{m_2} \times 1 + x_1 = \dfrac{2F}{K}$

$\Rightarrow x_1\left(1 + \dfrac{m_1}{m_2}\right) = \dfrac{2F}{K}$ $\qquad \Rightarrow x_1 = \dfrac{2F}{K}\dfrac{m_2}{m_1+m_2}$

Similarly $x_2 = \dfrac{2F}{K}\dfrac{m_1}{m_1+m_2}$

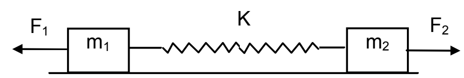
52. Acceleration of mass $m_1 = \dfrac{F_1 - F_2}{m_1 + m_2}$

Similarly Acceleration of mass $m_2 = \dfrac{F_2 - F_1}{m_1 + m_2}$

Due to F_1 and F_2 block of mass m_1 and m_2 will experience different acceleration and experience an inertia force.

$\therefore$ Net force on $m_1 = F_1 - m_1 a$

$= F_1 - m_1 \times \dfrac{F_1 - F_2}{m_1 + m_2} = \dfrac{m_1F_1 + m_2F_1 - m_1F_1 + F_2m_1}{m_1 + m_2} = \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2}$



Similarly Net force on $m_2 = F_2 - m_2 a$

$= F_2 - m_2 \times \dfrac{F_2 - F_1}{m_1 + m_2} = \dfrac{m_1F_2 + m_2F_2 - m_2F_2 + F_1m_2}{m_1 + m_2} = \dfrac{m_1F_2 + m_2F_2}{m_1 + m_2}$

$\therefore$ If m_1 displaces by a distance x_1 and x_2 by m_2 the maximum extension of the spring is $x_1 + m_2$.

$\therefore$ Work done by the blocks = energy stored in the spring.,

$\Rightarrow \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_1 + \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_2 = (1/2)\, K\, (x_1 + x_2)^2$

$\Rightarrow x_1 + x_2 = \dfrac{2}{K} \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2}$

53. Mass of the man (M_m) is 50 kg.

Mass of the pillow (M_p) is 5 kg.

When the pillow is pushed by the man, the pillow will go down while the man goes up. It becomes the external force on the system which is zero.

$\Rightarrow$ acceleration of centre of mass is zero

$\Rightarrow$ velocity of centre of mass is constant

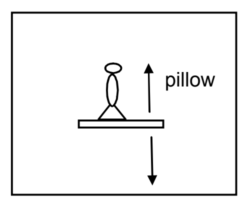
$\therefore$ As the initial velocity of the system is zero.

$\therefore M_m \times V_m = M_p \times V_p$ …(1)

Given the velocity of pillow is 80 ft/s.

Which is relative velocity of pillow w.r.t. man.

$\vec{V}_{p/m} = \vec{V}_p - \vec{V}_m = V_p - (-V_m) = V_p + V_m \Rightarrow V_p = V_{p/m} - V_m$



Putting in equation (1)

$M_m \times V_m = M_p\, (V_{p/m} - V_m)$

$\Rightarrow 50 \times V_m = 5 \times (8 - V_m)$

$\Rightarrow 10 \times V_m = 8 - V_m \Rightarrow V_m = \dfrac{8}{11} = 0.727$m/s

$\therefore$ Absolute velocity of pillow = 8 – 0.727 = 7.2 ft/sec.

$\therefore$ Time taken to reach the floor $= \dfrac{S}{v} = \dfrac{8}{7.2} = 1.1$ sec.

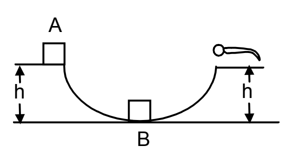
As the mass of wall >>> then pillow

The velocity of block before the collision = velocity after the collision.

$\Rightarrow$ Times of ascent = 1.11 sec.

$\therefore$ Total time taken = 1.11 + 1.11 = 2.22 sec.

54. Let the velocity of A = u_1.



Let the final velocity when reaching at B becomes collision = v_1.

$\therefore (1/2)\, mv_1^2 - (1/2)mu_1^2 = mgh$

$\Rightarrow v_1^2 - u_1^2 = 2\,gh$ $\Rightarrow v_1 = \sqrt{2gh - u_1^2}$ …(1)

When the block B reached at the upper man's head, the velocity of B is just zero.

For B, block

$\therefore (1/2) \times 2m \times 0^2 - (1/2) \times 2m \times v^2 = mgh$ $\Rightarrow v = \sqrt{2gh}$

$\therefore$ Before collision velocity of $u_A = v_1$, $u_B = 0$.

After collision velocity of $v_A = v$ (say) $v_B = \sqrt{2gh}$

Since it is an elastic collision the momentum and K.E. should be coserved.

$\therefore$ m × v_1 + 2m × 0 = m × v + 2m × $\sqrt{2gh}$

$\Rightarrow v_1 - v = 2\sqrt{2gh}$

Also, $(1/2) \times m \times v_1^2 + (1/2) \text{ I } 2m \times 0^2 = (1/2) \times m \times v^2 + (1/2) \times 2m \times \left(\sqrt{2gh}\right)^2$

$\Rightarrow v_1^2 - v^2 = 2 \times \sqrt{2gh} \times \sqrt{2gh}$ …(2)

Dividing (1) by (2)

$\frac{(v_1+v)(v_1-v)}{(v_1+v)} = \frac{2\times\sqrt{2gh}\times\sqrt{2gh}}{2\times\sqrt{2gh}} \Rightarrow v_1 + v = \sqrt{2gh}$ …(3)

Adding (1) and (3)

$2v_1 = 3\sqrt{2gh} \Rightarrow v_1 = \left(\frac{3}{2}\right)\sqrt{2gh}$

But $v_1 = \sqrt{2gh + u^2} = \left(\frac{3}{2}\right)\sqrt{2gh}$

$\Rightarrow 2gh + u^2 = \frac{9}{4}\times 2gh$

$\Rightarrow u = 2.5\sqrt{2gh}$

So the block will travel with a velocity greater than $2.5\sqrt{2gh}$ so awake the man by B.

55. Mass of block = 490 gm.

Mass of bullet = 10 gm.

Since the bullet embedded inside the block, it is an plastic collision.

Initial velocity of bullet $v_1 = 50\sqrt{7}$ m/s.

Velocity of the block is $v_2 = 0$.

Let Final velocity of both = v.

$\therefore 10 \times 10^{-3} \times 50 \times \sqrt{7} + 10^{-3} \times 190 \text{ I } 0 = (490 + 10) \times 10^{-3} \times V_A$

$\Rightarrow V_A = \sqrt{7}$ m/s.

When the block losses the contact at 'D' the component mg will act on it.

$\frac{m(V_B)^2}{r} = mg \sin\theta \Rightarrow (V_B)^2 = gr \sin\theta$ …(1)

Puttin work energy principle

$(1/2)\, m \times (V_B)^2 - (1/2) \times m \times (V_A)^2 = -mg\,(0.2 + 0.2 \sin\theta)$

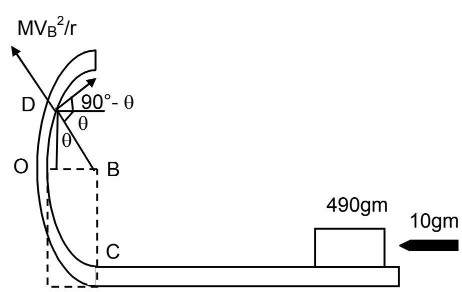
$\Rightarrow (1/2) \times gr \sin\theta - (1/2) \times \left(\sqrt{7}\right)^2 = -mg\,(0.2 + 0.2 \sin\theta)$

$\Rightarrow 3.5 - (1/2) \times 9.8 \times 0.2 \times \sin\theta = 9.8 \times 0.2\,(1 + \sin\theta)$

$\Rightarrow 3.5 - 0.98 \sin\theta = 1.96 + 1.96 \sin\theta$

$\Rightarrow \sin\theta = (1/2) \Rightarrow \theta = 30°$

$\therefore$ Angle of projection = 90° - 30° = 60°.



$\therefore$ time of reaching the ground $= \sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{2\times(0.2+0.2\times\sin 30°)}{9.8}} = 0.247$ sec.

$\therefore$ Distance travelled in horizontal direction.

$s = V\cos\theta \times t = \sqrt{gr\sin\theta}\times t = \sqrt{9.8\times 2\times(1/2)}\times 0.247 = 0.196\text{m}$

$\therefore$ Total distance = (0.2 – 0.2 cos 30°) + 0.196 = 0.22m.

56. Let the velocity of m reaching at lower end = V_1

From work energy principle.

$\therefore$ (1/2) × m × V_1^2 – (1/2) × m × 0^2 = mg ℓ

$\Rightarrow v_1 = \sqrt{2g\ell}$.

Similarly velocity of heavy block will be $v_2 = \sqrt{2gh}$.

$\therefore$ $v_1 = V_2$ = u(say)

Let the final velocity of m and 2m v_1 and v_2 respectively.

According to law of conservation of momentum.

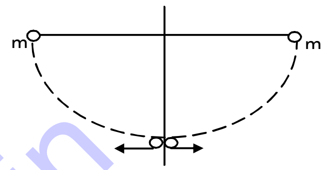
m × x_1 + 2m × V_2 = $mv_1 + 2mv_2$

$\Rightarrow$ m × u – 2 m u = $mv_1 + 2mv_2$

$\Rightarrow v_1 + 2v_2 = -u$ …(1)

Again, $v_1 - v_2 = -(V_1 - V_2)$

$\Rightarrow v_1 - v_2 = -[u - (-v)] = -2V$ …(2)



Subtracting.

$$3v_2 = u \Rightarrow v_2 = \frac{u}{3} = \frac{\sqrt{2g\ell}}{3}$$

Substituting in (2)

$$v_1 - v_2 = -2u \Rightarrow v_1 = -2u + v_2 = -2u + \frac{u}{3} = -\frac{5}{3}u = -\frac{5}{3}\times\sqrt{2g\ell} = -\frac{\sqrt{50g\ell}}{3}$$

b) Putting the work energy principle

(1/2) × 2m × 0^2 – (1/2) × 2m × $(v_2)^2$ = – 2m × g × h

[h $\rightarrow$ height gone by heavy ball]

$$\Rightarrow (1/2)\ \frac{2g}{9} = \ell \times h \qquad \Rightarrow h = \frac{\ell}{9}$$

Similarly, (1/2) × m × 0^2 – (1/2) × m × v_1^2 = m × g × h_2

[height reached by small ball]

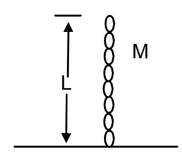
$$\Rightarrow (1/2) \times \frac{50g\ell}{9} = g \times h_2 \qquad \Rightarrow h_2 = \frac{25\ell}{9}$$

Someh_2 is more than 2ℓ, the velocity at height point will not be zero. And the 'm' will rise by a distance 2ℓ.

57. Let us consider a small element at a distance 'x' from the floor of length 'dy' .

So, $dm = \frac{M}{L}dx$

So, the velocity with which the element will strike the floor is, $v = \sqrt{2gx}$



(Initial position)

$\therefore$ So, the momentum transferred to the floor is,

$M = (dm)v = \frac{M}{L}\times dx\times\sqrt{2gx}$ [because the element comes to rest]

So, the force exerted on the floor change in momentum is given by,

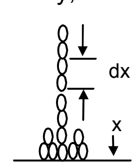
$$F_1 = \frac{dM}{dt} = \frac{M}{L}\times\frac{dx}{dt}\times\sqrt{2gx}$$

Because, $v = \frac{dx}{dt} = \sqrt{2gx}$ (for the chain element)

$$F_1 = \frac{M}{L}\times\sqrt{2gx}\times\sqrt{2gx} = \frac{M}{L}\times 2gx = \frac{2Mgx}{L}$$

Again, the force exerted due to 'x' length of the chain on the floor due to its own weight is given by,

$$W = \frac{M}{L}(x)\times g = \frac{Mgx}{L}$$



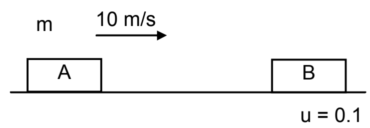
So, the total forced exerted is given by,

$$F = F_1 + W = \frac{2Mgx}{L} + \frac{Mgx}{L} = \frac{3Mgx}{L}$$

58. $V_1 = 10$ m/s $\quad V_2 = 0$

V_1, $v_2 \rightarrow$ velocity of ACB after collision.

a) If the edlision is perfectly elastic.



$mV_1 + mV_2 = mv_1 + mv_2$

$\Rightarrow 10 + 0 = v_1 + v_2$

$\Rightarrow v_1 + v_2 = 10 \quad \ldots(1)$

Again, $v_1 - v_2 = -(u_1 - v_2) = -(10 - 0) = -10 \quad \ldots(2)$

Subtracting (2) from (1)

$2v_2 = 20 \Rightarrow v_2 = 10$ m/s.

The deacceleration of B = μg

Putting work energy principle

$\therefore (1/2) \times m \times 0^2 - (1/2) \times m \times v_2^2 = - m \times a \times h$

$\Rightarrow -(1/2) \times 10^2 = - \mu\, g \times h \qquad \Rightarrow h = \dfrac{100}{2 \times 0.1 \times 10} = 50\text{m}$

b) If the collision perfectly in elastic.

$m \times u_1 + m \times u_2 = (m + m) \times v$

$\Rightarrow m \times 10 + m \times 0 = 2m \times v \qquad \Rightarrow v = \dfrac{10}{2} = 5$ m/s.

The two blocks will move together sticking to each other.

$\therefore$ Putting work energy principle.

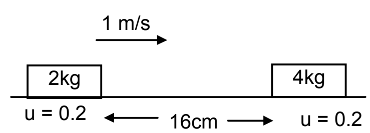
$(1/2) \times 2m \times 0^2 - (1/2) \times 2m \times v^2 = 2m \times \mu\, g \times s$

$\Rightarrow \dfrac{5^2}{0.1 \times 10 \times 2} = s \qquad \Rightarrow s = 12.5$ m.

59. Let velocity of 2kg block on reaching the 4kg block before collision $= u_1$.

Given, $V_2 = 0$ (velocity of 4kg block).

$\therefore$ From work energy principle,



$(1/2)\, m \times u_1^2 - (1/2)\, m \times 1^2 = - m \times ug \times s$

$\Rightarrow \dfrac{u_1^2 - 1}{2} = -2 \times 5 \qquad \Rightarrow -16 = \dfrac{u_1^2 - 1}{4}$

$\Rightarrow 64 \times 10^{-2} = u_1^2 - 1 \qquad \Rightarrow u_1 = 6$m/s

Since it is a perfectly elastic collision.

Let V_1, $V_2 \rightarrow$ velocity of 2kg & 4kg block after collision.

$m_1V_1 + m_2V_2 = m_1v_1 + m_2v_2$

$\Rightarrow 2 \times 0.6 + 4 \times 0 = 2v_1 + 4\, v_2 \qquad \Rightarrow v_1 + 2v_2 = 0.6 \quad \ldots(1)$

Again, $V_1 - V_2 = -(u_1 - u_2) = -(0.6 - 0) = -0.6 \quad \ldots(2)$

Subtracting (2) from (1)

$3v_2 = 1.2 \qquad \Rightarrow v_2 = 0.4$ m/s.

$\therefore v_1 = -0.6 + 0.4 = -0.2$ m/s

$\therefore$ Putting work energy principle for 1^{st} 2kg block when come to rest.

$(1/2) \times 2 \times 0^2 - (1/2) \times 2 \times (0.2)^2 = -2 \times 0.2 \times 10 \times s$

$\Rightarrow (1/2) \times 2 \times 0.2 \times 0.2 = 2 \times 0.2 \times 10 \times s \qquad \Rightarrow S_1 = 1$cm.

Putting work energy principle for 4kg block.

$(1/2) \times 4 \times 0^2 - (1/2) \times 4 \times (0.4)^2 = -4 \times 0.2 \times 10 \times s$

$\Rightarrow 2 \times 0.4 \times 0.4 = 4 \times 0.2 \times 10 \times s \qquad \Rightarrow S_2 = 4$ cm.

Distance between 2kg & 4kg block = $S_1 + S_2 = 1 + 4 = 5$ cm.

60. The block 'm' will slide down the inclined plane of mass M with acceleration a_1 g sin α (relative) to the inclined plane.

The horizontal component of a_1 will be, $a_x = g \sin\alpha \cos\alpha$, for which the block M will accelerate towards left. Let, the acceleration be a_2.

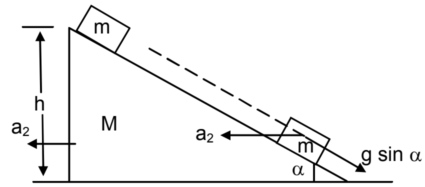
According to the concept of centre of mass, (in the horizontal direction external force is zero).

$ma_x = (M + m)\, a_2$

$\Rightarrow a_2 = \dfrac{ma_x}{M+m} = \dfrac{mg\sin\alpha\cos\alpha}{M+m}$ …(1)

So, the absolute (Resultant) acceleration of 'm' on the block 'M' along the direction of the incline will be,

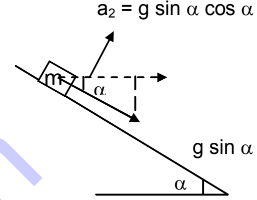
$a = g \sin \alpha - a_2 \cos \alpha$



$= g \sin \alpha - \dfrac{mg\sin\alpha\cos^2\alpha}{M+m} = g \sin \alpha \left[1 - \dfrac{m\cos^2\alpha}{M+m}\right]$

$= g \sin \alpha \left[\dfrac{M+m-m\cos^2\alpha}{M+m}\right]$

So, $a = g \sin \alpha \left[\dfrac{M+m\sin^2\alpha}{M+m}\right]$ …(2)



Let, the time taken by the block 'm' to reach the bottom end be 't'.

Now, $S = ut + (1/2)\, at^2$

$\Rightarrow \dfrac{h}{\sin\alpha} = (1/2)\, at^2$ $\quad\Rightarrow t = \sqrt{\dfrac{2}{a\sin\alpha}}$

So, the velocity of the bigger block after time 't' will be.

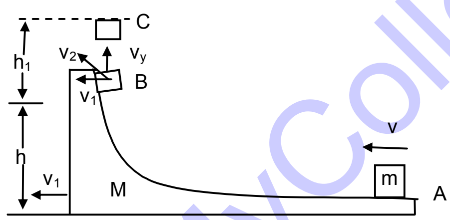
$V_m = u + a_2 t = \dfrac{mg\sin\alpha\cos\alpha}{M+m}\sqrt{\dfrac{2h}{a\sin\alpha}} = \sqrt{\dfrac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2 a\sin\alpha}}$

Now, subtracting the value of a from equation (2) we get,

$V_M = \left[\dfrac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2\sin\alpha} \times \dfrac{(M+m)}{g\sin\alpha(M+m\sin^2\alpha)}\right]^{1/2}$

or $V_M = \left[\dfrac{2m^2g^2h\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}\right]^{1/2}$

61.



The mass 'm' is given a velocity 'v' over the larger mass M.

a) When the smaller block is travelling on the vertical part, let the velocity of the bigger block be v_1 towards left.

From law of conservation of momentum, (in the horizontal direction)

$mv = (M + m)\, v_1$

$\Rightarrow v_1 = \dfrac{mv}{M+m}$

b) When the smaller block breaks off, let its resultant velocity is v_2.

From law of conservation of energy,

$(1/2)\, mv^2 = (1/2)\, Mv_1^2 + (1/2)\, mv_2^2 + mgh$

$\Rightarrow v_2^2 = v^2 - \dfrac{M}{m} v_1^2 - 2gh$ ..(1)

$\Rightarrow v_2^2 = v^2\left[1 - \dfrac{M}{m} \times \dfrac{m^2}{(M+m)^2}\right] - 2gh$

$\Rightarrow v_2 = \left[\dfrac{(m^2+Mm+m^2)}{(M+m)^2} v^2 - 2gh\right]^{1/2}$

e) Now, the vertical component of the velocity v_2 of mass 'm' is given by,

$v_y^2 = v_2^2 - v_1^2$

$$= \frac{(M^2 + Mm + m^2)}{(M+m)^2} v^2 - 2gh - \frac{m^2 v^2}{(M+m)^2}$$

$[\therefore v_1 = \frac{mv}{M+v}]$

$$\Rightarrow v_y^2 = \frac{M^2 + Mm + m^2 - m^2}{(M+m)^2} v^2 - 2gh$$

$$\Rightarrow v_y^2 = \frac{Mv^2}{(M+m)} - 2gh \qquad \ldots(2)$$

To find the maximum height (from the ground), let us assume the body rises to a height 'h', over and above 'h'.

Now, $(1/2)mv_y^2 = mgh_1 \Rightarrow h_1 = \frac{v_y^2}{2g}$ …(3)

So, Total height $= h + h_1 = h + \frac{v_y^2}{2g} = h + \frac{mv^2}{(M+m)2g} - h$

[from equation (2) and (3)]

$$\Rightarrow H = \frac{mv^2}{(M+m)2g}$$

d) Because, the smaller mass has also got a horizontal component of velocity 'v_1' at the time it breaks off from 'M' (which has a velocity v_1), the block 'm' will again land on the block 'M' (bigger one).

Let us find out the time of flight of block 'm' after it breaks off.

During the upward motion (BC),

$0 = v_y - gt_1$

$$\Rightarrow t_1 = \frac{v_y}{g} = \frac{1}{g}\left[\frac{Mv^2}{(M+m)} - 2gh\right]^{1/2} \qquad \ldots(4) \text{ [from equation (2)]}$$

So, the time for which the smaller block was in its flight is given by,

$$T = 2t_1 = \frac{2}{g}\left[\frac{Mv^2 - 2(M+m)gh}{(M+m)}\right]^{1/2}$$

So, the distance travelled by the bigger block during this time is,

$$S = v_1 T = \frac{mv}{M+m} \times \frac{2}{g} \frac{[Mv^2 - 2(M+m)gh]^{1/2}}{(M+m)^{1/2}}$$

$$\text{or } S = \frac{2mv[Mv^2 - 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$$

62. Given h < < < R.

$G_{mass} = 6 \text{ I } 10^{24}$ kg.

$M_b = 3 \times 10^{24}$ kg.

Let $V_e \rightarrow$ Velocity of earth

$V_b \rightarrow$ velocity of the block.

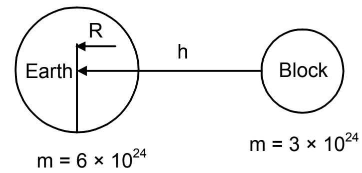
The two blocks are attracted by gravitational force of attraction. The gravitation potential energy stored will be the K.E. of two blocks.

$$\overline{G}^{pim}\left[\frac{1}{R+(h/2)} - \frac{1}{R+h}\right] = (1/2)\, m_e \times v_e^2 + (1/2)\, m_b \times v_b^2$$

Again as the an internal force acts.

$$M_e V_e = m_b V_b \qquad \Rightarrow V_e = \frac{m_b V_b}{M_e} \qquad \ldots(2)$$

Putting in equation (1)



$$G_{me} \times m_b \left[\frac{2}{2R+h} - \frac{1}{R+h}\right]$$

$$= (1/2) \times M_e \times \frac{{m_b}^2 {v_b}^2}{{M_e}^2} \times {v_e}^2 + (1/2)\, M_b \times {V_b}^2$$

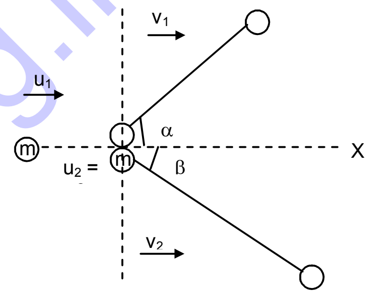
$$= (1/2) \times m_b \times {V_b}^2 \left(\frac{M_b}{M_e} + 1\right)$$

$$\Rightarrow GM\left[\frac{2R+2h-2R-h}{(2R+h)(R+h)}\right] = (1/2) \times {V_b}^2 \times \left(\frac{3\times10^{24}}{6\times10^{24}}+1\right) \qquad \Rightarrow \left[\frac{GM\times h}{2R^2+3Rh+h^2}\right] = (1/2) \times {V_b}^2 \times (3/2)$$

As h < < < R, if can be neglected

$$\Rightarrow \frac{GM\times h}{2R^2} = (1/2) \times {V_b}^2 \times (3/2) \qquad \Rightarrow V_b = \sqrt{\frac{2gh}{3}}$$

63. Since it is not an head on collision, the two bodies move in different dimensions. Let V_1, $V_2 \rightarrow$ velocities of the bodies vector collision. Since, the collision is elastic. Applying law of conservation of momentum on X-direction.



$mu_1 + mxo = mv_1 \cos\alpha + mv_2 \cos\beta$

$\Rightarrow v_1 \cos a + v_2 \cos b = u_1$ …(1)

Putting law of conservation of momentum in y direction.

$0 = mv_1 \sin\alpha - mv_2 \sin\beta$

$\Rightarrow v_1 \sin\alpha = v_2 \sin\beta$ …(2)

Again ½ m ${u_1}^2$ + 0 = ½ m ${v_1}^2$ + ½ m x ${v_2}^2$

$\Rightarrow {u_1}^2 = {v_1}^2 + {v_2}^2$ …(3)

Squaring equation(1)

${u_1}^2 = {v_1}^2 \cos^2\alpha + {v_2}^2 \cos^2\beta + 2\, v_1 v_2 \cos\alpha \cos\beta$

Equating (1) & (3)

${v_1}^2 + {v_2}^2 = {v_1}^2 \cos^2\alpha + {v_2}^2 \cos^2\beta + 2\, v_1 v_2 \cos\alpha \cos\beta$

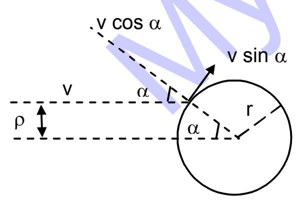
$\Rightarrow {v_1}^2 \sin^2\alpha + {v_2}^2 \sin^2\beta = 2\, v_1 v_2 \cos\alpha \cos\beta$

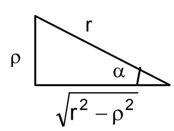
$$\Rightarrow 2{v_1}^2 \sin^2\alpha = 2 \times v_1 \times \frac{v_1 \sin\alpha}{\sin\beta} \times \cos\alpha \cos\beta$$

$\Rightarrow \sin\alpha \sin\beta = \cos\alpha \cos\beta$ $\qquad \Rightarrow \cos\alpha \cos\beta - \sin\alpha \sin\beta = 0$

$\Rightarrow \cos(\alpha + \beta) = 0 = \cos 90°$ $\qquad \Rightarrow (\alpha + \beta) = 90°$

64.





Let the mass of both the particle and the spherical body be 'm'. The particle velocity 'v' has two components, v cos α normal to the sphere and v sin α tangential to the sphere.

After the collision, they will exchange their velocities. So, the spherical body will have a velocity v cos α and the particle will not have any component of velocity in this direction.

[The collision will due to the component v cos α in the normal direction. But, the tangential velocity, of the particle v sin α will be unaffected]

$$\text{So, velocity of the sphere} = v\cos\alpha = \frac{v}{r}\sqrt{r^2-\rho^2} \quad \text{[from (fig-2)]}$$

$$\text{And velocity of the particle} = v\sin\alpha = \frac{v\rho}{r}$$

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