

SOLUTIONS TO CONCEPTS circular motion;; CHAPTER 7

1. Distance between Earth & Moon

$$r = 3.85 \times 10^5 \text{ km} = 3.85 \times 10^8 \text{ m}$$

$$T = 27.3 \text{ days} = 24 \times 3600 \times (27.3) \text{ sec} = 2.36 \times 10^6 \text{ sec}$$

$$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 3.85 \times 10^8}{2.36 \times 10^6} = 1025.42 \text{ m/sec}$$

$$a = \frac{v^2}{r} = \frac{(1025.42)^2}{3.85 \times 10^8} = 0.00273 \text{ m/sec}^2 = 2.73 \times 10^{-3} \text{ m/sec}^2$$

2. Diameter of earth = 12800 km

$$\text{Radius } R = 6400 \text{ km} = 64 \times 10^5 \text{ m}$$

$$V = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times 64 \times 10^5}{24 \times 3600} \text{ m/sec} = 465.185$$

$$a = \frac{V^2}{R} = \frac{(465.185)^2}{64 \times 10^5} = 0.0338 \text{ m/sec}^2$$

3. $V = 2t$, $r = 1 \text{ cm}$

a) Radial acceleration at $t = 1 \text{ sec}$.

$$a = \frac{v^2}{r} = \frac{2^2}{1} = 4 \text{ cm/sec}^2$$

b) Tangential acceleration at $t = 1 \text{ sec}$.

$$a = \frac{dv}{dt} = \frac{d}{dt}(2t) = 2 \text{ cm/sec}^2$$

c) Magnitude of acceleration at $t = 1 \text{ sec}$

$$a = \sqrt{4^2 + 2^2} = \sqrt{20} \text{ cm/sec}^2$$

4. Given that $m = 150 \text{ kg}$,

$$v = 36 \text{ km/hr} = 10 \text{ m/sec}, \quad r = 30 \text{ m}$$

$$\text{Horizontal force needed is } \frac{mv^2}{r} = \frac{150 \times (10)^2}{30} = \frac{150 \times 100}{30} = 500 \text{ N}$$

5. in the diagram

$$R \cos \theta = mg \quad \dots(i)$$

$$R \sin \theta = \frac{mv^2}{r} \quad \dots(ii)$$

Dividing equation (i) with equation (ii)

$$\text{Tan } \theta = \frac{mv^2}{rmg} = \frac{v^2}{rg}$$

$$v = 36 \text{ km/hr} = 10 \text{ m/sec}, \quad r = 30 \text{ m}$$

$$\text{Tan } \theta = \frac{v^2}{rg} = \frac{100}{30 \times 10} = (1/3)$$

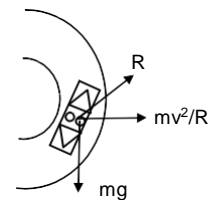
$$\Rightarrow \theta = \tan^{-1}(1/3)$$

6. Radius of Park = $r = 10 \text{ m}$

$$\text{speed of vehicle} = 18 \text{ km/hr} = 5 \text{ m/sec}$$

$$\text{Angle of banking } \tan \theta = \frac{v^2}{rg}$$

$$\Rightarrow \theta = \tan^{-1} \frac{v^2}{rg} = \tan^{-1} \frac{25}{100} = \tan^{-1}(1/4)$$



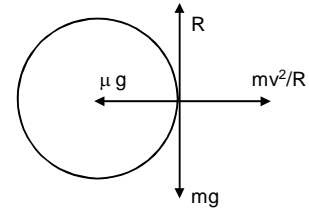
7. The road is horizontal (no banking)

$$\frac{mv^2}{R} = \mu N$$

$$\text{and } N = mg$$

$$\text{So } \frac{mv^2}{R} = \mu mg \quad v = 5\text{m/sec}, \quad R = 10\text{m}$$

$$\Rightarrow \frac{25}{10} = \mu g \Rightarrow \mu = \frac{25}{100} = 0.25$$



8. Angle of banking = $\theta = 30^\circ$

$$\text{Radius} = r = 50\text{m}$$

$$\tan \theta = \frac{v^2}{rg} \Rightarrow \tan 30^\circ = \frac{v^2}{rg}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{v^2}{rg} \Rightarrow v^2 = \frac{rg}{\sqrt{3}} = \frac{50 \times 10}{\sqrt{3}}$$

$$\Rightarrow v = \sqrt{\frac{500}{\sqrt{3}}} = 17\text{m/sec.}$$

9. Electron revolves around the proton in a circle having proton at the centre.

Centripetal force is provided by coulomb attraction.

$$r = 5.3 \times 10^{-11}\text{m} \quad m = \text{mass of electron} = 9.1 \times 10^{-31}\text{kg.}$$

$$\text{charge of electron} = 1.6 \times 10^{-19}\text{C.}$$

$$\frac{mv^2}{r} = k \frac{q^2}{r^2} \Rightarrow v = \frac{kq^2}{rm} = \frac{9 \times 10^9 \times 1.6 \times 1.6 \times 10^{-38}}{5.3 \times 10^{-11} \times 9.1 \times 10^{-31}} = \frac{23.04}{48.23} \times 10^{13}$$

$$\Rightarrow v^2 = 0.477 \times 10^{13} = 4.7 \times 10^{12}$$

$$\Rightarrow v = \sqrt{4.7 \times 10^{12}} = 2.2 \times 10^6 \text{m/sec}$$

10. At the highest point of a vertical circle

$$\frac{mv^2}{R} = mg$$

$$\Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$$

11. A ceiling fan has a diameter = 120cm.

$$\therefore \text{Radius} = r = 60\text{cm} = 0.6\text{m}$$

Mass of particle on the outer end of a blade is 1g.

$$n = 1500 \text{ rev/min} = 25 \text{ rev/sec}$$

$$\omega = 2\pi n = 2\pi \times 25 = 157.14$$

$$\text{Force of the particle on the blade} = Mr\omega^2 = (0.001) \times 0.6 \times (157.14)^2 = 14.8\text{N}$$

The fan runs at a full speed in circular path. This exerts the force on the particle (inertia). The particle also exerts a force of 14.8N on the blade along its surface.

12. A mosquito is sitting on an L.P. record disc & rotating on a turn table at $33\frac{1}{3}$ rpm.

$$n = 33\frac{1}{3} \text{ rpm} = \frac{100}{3 \times 60} \text{ rps}$$

$$\therefore \omega = 2\pi n = 2\pi \times \frac{100}{180} = \frac{10\pi}{9} \text{ rad/sec}$$

$$r = 10\text{cm} = 0.1\text{m}, \quad g = 10\text{m/sec}^2$$

$$\mu mg \geq mr\omega^2 \Rightarrow \mu = \frac{r\omega^2}{g} \geq \frac{0.1 \times \left(\frac{10\pi}{9}\right)^2}{10}$$

$$\Rightarrow \mu \geq \frac{\pi^2}{81}$$

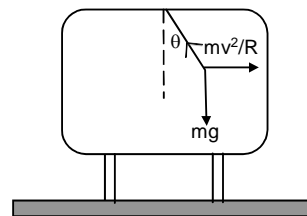
13. A pendulum is suspended from the ceiling of a car taking a turn
 $r = 10\text{m}$, $v = 36\text{km/hr} = 10 \text{ m/sec}$, $g = 10\text{m/sec}^2$

From the figure $T \sin \theta = \frac{mv^2}{r}$..(i)

$T \cos \theta = mg$..(ii)

$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{mv^2}{r \cdot mg} \Rightarrow \tan \theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$

$= \tan^{-1} \frac{100}{10 \times 10} = \tan^{-1}(1) \Rightarrow \theta = 45^\circ$



14. At the lowest pt.

$T = mg + \frac{mv^2}{r}$

Here $m = 100\text{g} = 1/10 \text{ kg}$, $r = 1\text{m}$, $v = 1.4 \text{ m/sec}$

$T = mg + \frac{mv^2}{r} = \frac{1}{10} \times 9.8 \times \frac{(1.4)^2}{10} = 0.98 + 0.196 = 1.176 = 1.2 \text{ N}$

15. Bob has a velocity 1.4m/sec, when the string makes an angle of 0.2 radian.

$m = 100\text{g} = 0.1\text{kg}$, $r = 1\text{m}$, $v = 1.4\text{m/sec}$.

From the diagram,

$T - mg \cos \theta = \frac{mv^2}{R}$

$\Rightarrow T = \frac{mv^2}{R} + mg \cos \theta$

$\Rightarrow T = \frac{0.1 \times (1.4)^2}{1} + (0.1) \times 9.8 \times \left(1 - \frac{\theta^2}{2} \right)$

$\Rightarrow T = 0.196 + 9.8 \times \left(1 - \frac{(0.2)^2}{2} \right)$ ($\therefore \cos \theta = 1 - \frac{\theta^2}{2}$ for small θ)

$\Rightarrow T = 0.196 + (0.98) \times (0.98) = 0.196 + 0.964 = 1.156\text{N} \approx 1.16 \text{ N}$

16. At the extreme position, velocity of the pendulum is zero.

So there is no centrifugal force.

So $T = mg \cos \theta$.

17. a) Net force on the spring balance.

$R = mg - m\omega^2 r$

So, fraction less than the true weight (3mg) is

$= \frac{mg - (mg - m\omega^2 r)}{mg} = \frac{\omega^2}{g} = \left(\frac{2\pi}{24 \times 3600} \right)^2 \times \frac{6400 \times 10^3}{10} = 3.5 \times 10^{-3}$

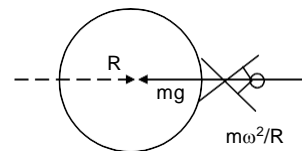
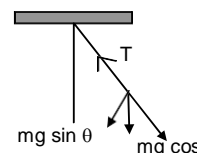
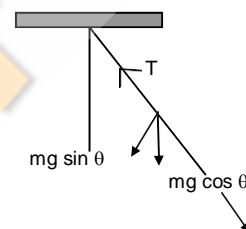
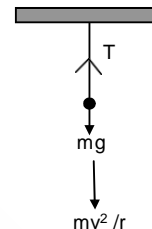
- b) When the balance reading is half the true weight,

$\frac{mg - (mg - m\omega^2 r)}{mg} = 1/2$

$\omega^2 r = g/2 \Rightarrow \omega = \sqrt{\frac{g}{2r}} = \sqrt{\frac{10}{2 \times 6400 \times 10^3}} \text{ rad/sec}$

\therefore Duration of the day is

$T = \frac{2\pi}{\omega} = 2\pi \times \sqrt{\frac{2 \times 6400 \times 10^3}{9.8}} \text{ sec} = 2\pi \times \sqrt{\frac{64 \times 10^6}{49}} \text{ sec} = \frac{2\pi \times 8000}{7 \times 3600} \text{ hr} = 2\text{hr}$



18. Given, $v = 36\text{km/hr} = 10\text{m/s}$, $r = 20\text{m}$, $\mu = 0.4$
 The road is banked with an angle,
 $\theta = \tan^{-1} \left(\frac{v^2}{rg} \right) = \tan^{-1} \left(\frac{100}{20 \times 10} \right) = \tan^{-1} \left(\frac{1}{2} \right)$ or $\tan \theta = 0.5$

When the car travels at max. speed so that it slips upward, μR_1 acts downward as shown in Fig.1

So, $R_1 - mg \cos \theta - \frac{mv_1^2}{r} \sin \theta = 0$..(i)

And $\mu R_1 + mg \sin \theta - \frac{mv_1^2}{r} \cos \theta = 0$..(ii)

Solving the equation we get,

$$v_1 = \sqrt{\frac{rg \tan \theta - \mu}{1 + \mu \tan \theta}} = \sqrt{\frac{20 \times 10 \times 0.1}{1.2}} = 4.082 \text{ m/s} = 14.7 \text{ km/hr}$$

So, the possible speeds are between 14.7 km/hr and 54km/hr.

19. R = radius of the bridge

L = total length of the over bridge

a) At the highest pt.

$$mg = \frac{mv^2}{R} \Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$$

b) Given, $v = \frac{1}{\sqrt{2}} \sqrt{Rg}$

suppose it loses contact at B. So, at B, $mg \cos \theta = \frac{mv^2}{R}$

$$\Rightarrow v^2 = Rg \cos \theta$$

$$\Rightarrow \left(\frac{v}{\sqrt{2}} \right)^2 = Rg \cos \theta \Rightarrow \frac{Rg}{2} = Rg \cos \theta \Rightarrow \cos \theta = 1/2 \Rightarrow \theta = 60^\circ = \pi/3$$

$$\theta = \frac{\ell}{r} \rightarrow \ell = r\theta = \frac{\pi R}{3}$$

So, it will lose contact at distance $\frac{\pi R}{3}$ from highest point

c) Let the uniform speed on the bridge be v.

The chances of losing contact is maximum at the end of the bridge for which $\alpha = \frac{L}{2R}$.

$$\text{So, } \frac{mv^2}{R} = mg \cos \alpha \Rightarrow v = \sqrt{gR \cos \left(\frac{L}{2R} \right)}$$

20. Since the motion is nonuniform, the acceleration has both radial & tangential component

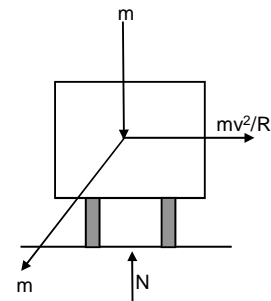
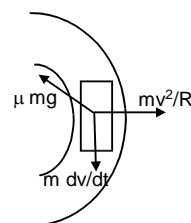
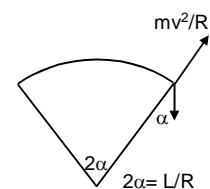
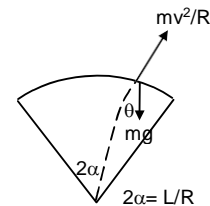
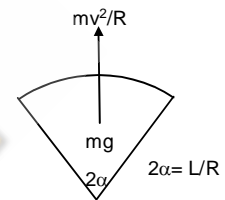
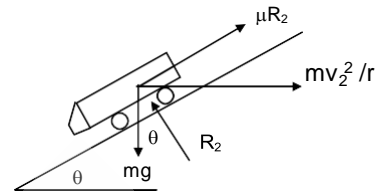
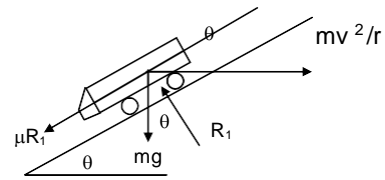
$$a_r = \frac{v^2}{r}$$

$$a_t = \frac{dv}{dt} = a$$

$$\text{Resultant magnitude} = \sqrt{\left(\frac{v^2}{r} \right)^2 + a^2}$$

$$\text{Now } \mu N = m \sqrt{\left(\frac{v^2}{r} \right)^2 + a^2} \Rightarrow \mu mg = m \sqrt{\left(\frac{v^2}{r} \right)^2 + a^2} \Rightarrow \mu^2 g^2 = \left(\frac{v^4}{r^2} + a^2 \right)$$

$$\Rightarrow v^4 = (\mu^2 g^2 - a^2) r^2 \Rightarrow v = [(\mu^2 g^2 - a^2) r^2]^{1/4}$$



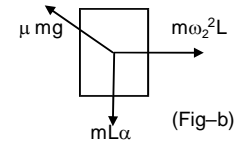
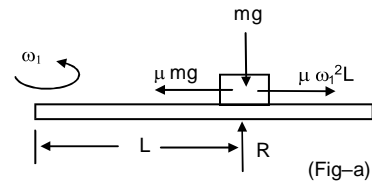
21. a) When the ruler makes uniform circular motion in the horizontal plane, (fig-a)

$$\mu mg = m\omega_1^2 L$$

$$\omega_1 = \sqrt{\frac{\mu g}{L}}$$

- b) When the ruler makes uniformly accelerated circular motion, (fig-b)

$$\mu mg = \sqrt{(m\omega_2^2 L)^2 + (mL\alpha)^2} \Rightarrow \omega_2^4 + \alpha^2 = \frac{\mu^2 g^2}{L^2} \Rightarrow \omega_2 = \left[\frac{(\mu g)^2}{L^2} - \alpha^2 \right]^{1/4}$$



(When viewed from top)

22. Radius of the curves = 100m

Weight = 100kg

Velocity = 18km/hr = 5m/sec

a) at B $mg - \frac{mv^2}{R} = N \Rightarrow N = (100 \times 10) - \frac{100 \times 25}{100} = 1000 - 25 = 975\text{N}$

At d, $N = mg + \frac{mv^2}{R} = 1000 + 25 = 1025\text{N}$

- b) At B & D the cycle has no tendency to slide. So at B & D, frictional force is zero.

At 'C', $mg \sin \theta = F \Rightarrow F = 1000 \times \frac{1}{\sqrt{2}} = 707\text{N}$

c) (i) Before 'C' $mg \cos \theta - N = \frac{mv^2}{R} \Rightarrow N = mg \cos \theta - \frac{mv^2}{R} = 707 - 25 = 683\text{N}$

(ii) $N - mg \cos \theta = \frac{mv^2}{R} \Rightarrow N = \frac{mv^2}{R} + mg \cos \theta = 25 + 707 = 732\text{N}$

- d) To find out the minimum desired coeff. of friction, we have to consider a point just before C. (where N is minimum)

Now, $\mu N = mg \sin \theta \Rightarrow \mu \times 682 = 707$

So, $\mu = 1.037$

23. d = 3m $\Rightarrow R = 1.5\text{m}$

R = distance from the centre to one of the kids

N = 20 rev per min = 20/60 = 1/3 rev per sec

$$\omega = 2\pi r = 2\pi/3$$

m = 15kg

\therefore Frictional force $F = m\omega^2 r = 15 \times (1.5) \times \frac{(2\pi)^2}{9} = 5 \times (0.5) \times 4\pi^2 = 10\pi^2$

\therefore Frictional force on one of the kids is $10\pi^2$

24. If the bowl rotates at maximum angular speed, the block tends to slip upwards. So, the frictional force acts downward.

Here, $r = R \sin \theta$

From FBD -1

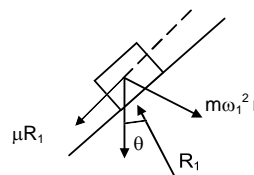
$$R_1 - mg \cos \theta - m\omega_1^2 (R \sin \theta) \sin \theta = 0 \quad \dots(i) \text{ [because } r = R \sin \theta \text{]}$$

$$\text{and } \mu R_1 - mg \sin \theta - m\omega_1^2 (R \sin \theta) \cos \theta = 0 \quad \dots(ii)$$

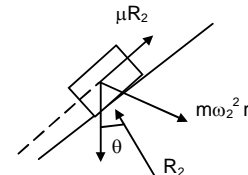
Substituting the value of R_1 from Eq (i) in Eq(ii), it can be found out that

$$\omega_1 = \left[\frac{g(\sin \theta + \mu \cos \theta)}{R \sin \theta (\cos \theta - \mu \sin \theta)} \right]^{1/2}$$

Again, for minimum speed, the frictional force μR_2 acts upward. From FBD-2, it can be proved that,



(FBD - 1)



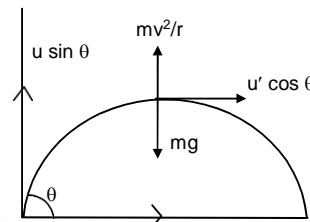
(FBD - 2)

$$\omega_2 = \left[\frac{g(\sin\theta - \mu \cos\theta)}{R \sin\theta(\cos\theta + \mu \sin\theta)} \right]^{1/2}$$

∴ the range of speed is between ω_1 and ω_2

25. Particle is projected with speed 'u' at an angle θ . At the highest pt. the vertical component of velocity is '0'

So, at that point, velocity = $u \cos \theta$
centripetal force = $m u^2 \cos^2 \theta / r$

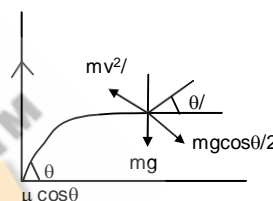


At highest pt.

$$mg = \frac{mv^2}{r} \Rightarrow r = \frac{u^2 \cos^2 \theta}{g}$$

26. Let 'u' the velocity at the pt where it makes an angle $\theta/2$ with horizontal. The horizontal component remains unchanged

So, $v \cos \theta/2 = u \cos \theta \Rightarrow v = \frac{u \cos \theta}{\cos(\theta/2)}$... (i)



From figure

$$mg \cos(\theta/2) = \frac{mv^2}{r} \Rightarrow r = \frac{v^2}{g \cos(\theta/2)}$$

putting the value of 'v' from equn(i)

$$r = \frac{u^2 \cos^2 \theta}{g \cos^3(\theta/2)}$$

27. A block of mass 'm' moves on a horizontal circle against the wall of a cylindrical room of radius 'R'. Friction coefficient between wall & the block is μ .

a) Normal reaction by the wall on the block is = $\frac{mv^2}{R}$

b) ∴ Frictional force by wall = $\frac{\mu mv^2}{R}$

c) $\frac{\mu mv^2}{R} = ma \Rightarrow a = -\frac{\mu v^2}{R}$ (Deceleration)

d) Now, $\frac{dv}{dt} = v \frac{dv}{ds} = -\frac{\mu v^2}{R} \Rightarrow ds = -\frac{R dv}{\mu v}$

$$\Rightarrow s = -\frac{R\mu}{\mu} \ln v + c$$

At $s = 0$, $v = v_0$

Therefore, $c = \frac{R}{\mu} \ln v_0$

so, $s = -\frac{R}{\mu} \ln \frac{v}{v_0} \Rightarrow \frac{v}{v_0} = e^{-\mu s/R}$

For, one rotation $s = 2\pi R$, so $v = v_0 e^{-2\pi\mu}$

28. The cabin rotates with angular velocity ω & radius R

∴ The particle experiences a force $mR\omega^2$.

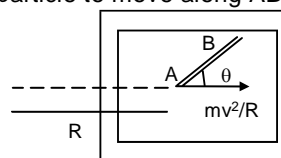
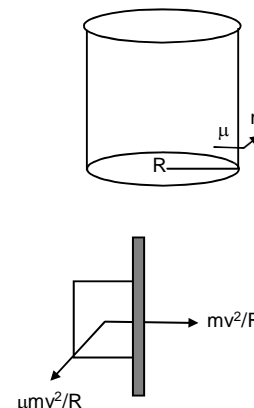
The component of $mR\omega^2$ along the groove provides the required force to the particle to move along AB.

∴ $mR\omega^2 \cos \theta = ma \Rightarrow a = R\omega^2 \cos \theta$

length of groove = L

$$L = ut + \frac{1}{2} at^2 \Rightarrow L = \frac{1}{2} R\omega^2 \cos \theta t^2$$

$$\Rightarrow t^2 = \frac{2L}{R\omega^2 \cos \theta} \Rightarrow t = \sqrt{\frac{2L}{R\omega^2 \cos \theta}}$$



29. $v =$ Velocity of car = 36km/hr = 10 m/s
 $r =$ Radius of circular path = 50m
 $m =$ mass of small body = 100g = 0.1kg.
 $\mu =$ Friction coefficient between plate & body = 0.58
 a) The normal contact force exerted by the plate on the block

$$N = \frac{mv^2}{r} = \frac{0.1 \times 100}{50} = 0.2\text{N}$$

- b) The plate is turned so the angle between the normal to the plate & the radius of the road slowly increases

$$N = \frac{mv^2}{r} \cos \theta \quad \dots(i)$$

$$\mu N = \frac{mv^2}{r} \sin \theta \quad \dots(ii)$$

Putting value of N from (i)

$$\mu \frac{mv^2}{r} \cos \theta = \frac{mv^2}{r} \sin \theta \Rightarrow \mu = \tan \theta \Rightarrow \theta = \tan^{-1} \mu = \tan^{-1}(0.58) = 30^\circ$$

30. Let the bigger mass accelerates towards right with 'a'.

From the free body diagrams,

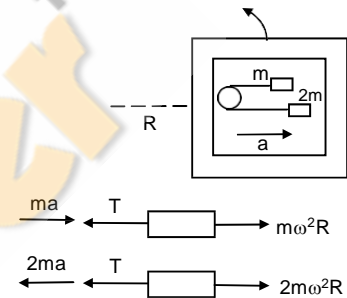
$$T - ma - m\omega^2 R = 0 \quad \dots(i)$$

$$T + 2ma - 2m\omega^2 R = 0 \quad \dots(ii)$$

$$\text{Eq (i) - Eq (ii)} \Rightarrow 3ma = m\omega^2 R$$

$$\Rightarrow a = \frac{m\omega^2 R}{3}$$

Substituting the value of a in Equation (i), we get $T = 4/3 m\omega^2 R$.



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