

SOLUTIONS TO CONCEPTS CHAPTER 17

1. Given that, $400 \text{ nm} < \lambda < 700 \text{ nm}$.

$$\frac{1}{700 \text{ nm}} < \frac{1}{\lambda} < \frac{1}{400 \text{ nm}}$$

$$\Rightarrow \frac{1}{7 \times 10^{-7}} < \frac{1}{\lambda} < \frac{1}{4 \times 10^{-7}} \Rightarrow \frac{3 \times 10^8}{7 \times 10^{-7}} < \frac{c}{\lambda} < \frac{3 \times 10^8}{4 \times 10^{-7}} \quad (\text{Where, } c = \text{speed of light} = 3 \times 10^8 \text{ m/s})$$

$$\Rightarrow 4.3 \times 10^{14} < c/\lambda < 7.5 \times 10^{14}$$

$$\Rightarrow 4.3 \times 10^{14} \text{ Hz} < f < 7.5 \times 10^{14} \text{ Hz}$$

2. Given that, for sodium light, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

a) $f_a = \frac{c}{\lambda} = \frac{3 \times 10^8}{589 \times 10^{-9}} = 5.09 \times 10^{14} \text{ sec}^{-1}$ [$\therefore f = \frac{c}{\lambda}$]

b) $\frac{\mu_a}{\mu_w} = \frac{\lambda_w}{\lambda_a} \Rightarrow \frac{1}{1.33} = \frac{\lambda_w}{589 \times 10^{-9}} \Rightarrow \lambda_w = 443 \text{ nm}$

c) $f_w = f_a = 5.09 \times 10^{14} \text{ sec}^{-1}$ [Frequency does not change]

d) $\frac{\mu_a}{\mu_w} = \frac{v_w}{v_a} \Rightarrow v_w = \frac{\mu_a v_a}{\mu_w} = \frac{3 \times 10^8}{1.33} = 2.25 \times 10^8 \text{ m/sec}$.

3. We know that, $\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

$$\text{So, } \frac{1472}{1} = \frac{3 \times 10^8}{v_{400}} \Rightarrow v_{400} = 2.04 \times 10^8 \text{ m / sec.}$$

[because, for air, $\mu = 1$ and $v = 3 \times 10^8 \text{ m/s}$]

$$\text{Again, } \frac{1452}{1} = \frac{3 \times 10^8}{v_{760}} \Rightarrow v_{760} = 2.07 \times 10^8 \text{ m / sec.}$$

4. $\mu_t = \frac{1 \times 3 \times 10^8}{(2.4) \times 10^8} = 1.25$ [since, $\mu = \frac{\text{velocity of light in vaccum}}{\text{velocity of light in the given medium}}$]

5. Given that, $d = 1 \text{ cm} = 10^{-2} \text{ m}$, $\lambda = 5 \times 10^{-7} \text{ m}$ and $D = 1 \text{ m}$

- a) Separation between two consecutive maxima is equal to fringe width.

$$\text{So, } \beta = \frac{\lambda D}{d} = \frac{5 \times 10^{-7} \times 1}{10^{-2}} \text{ m} = 5 \times 10^{-5} \text{ m} = 0.05 \text{ mm.}$$

- b) When, $\beta = 1 \text{ mm} = 10^{-3} \text{ m}$

$$10^{-3} \text{ m} = \frac{5 \times 10^{-7} \times 1}{D} \Rightarrow D = 5 \times 10^{-4} \text{ m} = 0.50 \text{ mm.}$$

6. Given that, $\beta = 1 \text{ mm} = 10^{-3} \text{ m}$, $D = 2.1 \text{ m}$ and $d = 1 \text{ mm} = 10^{-3} \text{ m}$

$$\text{So, } 10^{-3} \text{ m} = \frac{25 \times \lambda}{10^{-3}} \Rightarrow \lambda = 4 \times 10^{-7} \text{ m} = 400 \text{ nm.}$$

7. Given that, $d = 1 \text{ mm} = 10^{-3} \text{ m}$, $D = 1 \text{ m}$.

$$\text{So, fringe width} = \frac{D\lambda}{d} = 0.5 \text{ mm.}$$

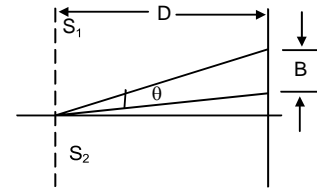
- a) So, distance of centre of first minimum from centre of central maximum = $0.5/2 \text{ mm} = 0.25 \text{ mm}$

- b) No. of fringes = $10 / 0.5 = 20$.

8. Given that, $d = 0.8 \text{ mm} = 0.8 \times 10^{-3} \text{ m}$, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$ and $D = 2 \text{ m}$.

$$\text{So, } \beta = \frac{D\lambda}{d} = \frac{589 \times 10^{-9} \times 2}{0.8 \times 10^{-3}} = 1.47 \times 10^{-3} \text{ m} = 147 \text{ nm.}$$

9. Given that, $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$ and $d = 2 \times 10^{-3} \text{ m}$
 As shown in the figure, angular separation $\theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$



$$\text{So, } \theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{2 \times 10^{-3}} = 250 \times 10^{-6}$$

$$= 25 \times 10^{-5} \text{ radian} = 0.014 \text{ degree.}$$

10. We know that, the first maximum (next to central maximum) occurs at $y = \frac{\lambda D}{d}$

Given that, $\lambda_1 = 480 \text{ nm}$, $\lambda_2 = 600 \text{ nm}$, $D = 150 \text{ cm} = 1.5 \text{ m}$ and $d = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}$

$$\text{So, } y_1 = \frac{D\lambda_1}{d} = \frac{1.5 \times 480 \times 10^{-9}}{0.25 \times 10^{-3}} = 2.88 \text{ mm}$$

$$y_2 = \frac{1.5 \times 600 \times 10^{-9}}{0.25 \times 10^{-3}} = 3.6 \text{ mm.}$$

So, the separation between these two bright fringes is given by,

$$\therefore \text{separation} = y_2 - y_1 = 3.60 - 2.88 = 0.72 \text{ mm.}$$

11. Let m^{th} bright fringe of violet light overlaps with n^{th} bright fringe of red light.

$$\therefore \frac{m \times 400 \text{ nm} \times D}{d} = \frac{n \times 700 \text{ nm} \times D}{d} \Rightarrow \frac{m}{n} = \frac{7}{4}$$

$\Rightarrow 7^{\text{th}}$ bright fringe of violet light overlaps with 4^{th} bright fringe of red light (minimum). Also, it can be seen that 14^{th} violet fringe will overlap 8^{th} red fringe.

Because, $m/n = 7/4 = 14/8$.

12. Let, t = thickness of the plate

Given, optical path difference = $(\mu - 1)t = \lambda/2$

$$\Rightarrow t = \frac{\lambda}{2(\mu - 1)}$$

13. a) Change in the optical path = $\mu t - t = (\mu - 1)t$

b) To have a dark fringe at the centre the pattern should shift by one half of a fringe.

$$\Rightarrow (\mu - 1)t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{2(\mu - 1)}$$

14. Given that, $\mu = 1.45$, $t = 0.02 \text{ mm} = 0.02 \times 10^{-3} \text{ m}$ and $\lambda = 620 \text{ nm} = 620 \times 10^{-9} \text{ m}$

We know, when the transparent paper is pasted in one of the slits, the optical path changes by $(\mu - 1)t$.

Again, for shift of one fringe, the optical path should be changed by λ .

So, no. of fringes crossing through the centre is given by,

$$n = \frac{(\mu - 1)t}{\lambda} = \frac{0.45 \times 0.02 \times 10^{-3}}{620 \times 10^{-9}} = 14.5$$

15. In the given Young's double slit experiment,

$\mu = 1.6$, $t = 1.964 \text{ micron} = 1.964 \times 10^{-6} \text{ m}$

We know, number of fringes shifted = $\frac{(\mu - 1)t}{\lambda}$

So, the corresponding shift = No. of fringes shifted \times fringe width

$$= \frac{(\mu - 1)t}{\lambda} \times \frac{\lambda D}{d} = \frac{(\mu - 1)tD}{d} \quad \dots (1)$$

Again, when the distance between the screen and the slits is doubled,

$$\text{Fringe width} = \frac{\lambda(2D)}{d} \quad \dots (2)$$

$$\text{From (1) and (2), } \frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$$

$$\Rightarrow \lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6 - 1) \times (1.964) \times 10^{-6}}{2} = 589.2 \times 10^{-9} = 589.2 \text{ nm.}$$

16. Given that, $t_1 = t_2 = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$, $\mu_m = 1.58$ and $\mu_p = 1.55$,
 $\lambda = 590 \text{ nm} = 590 \times 10^{-9} \text{ m}$, $d = 0.12 \text{ cm} = 12 \times 10^{-4} \text{ m}$, $D = 1 \text{ m}$

a) Fringe width = $\frac{D\lambda}{d} = \frac{1 \times 590 \times 10^{-9}}{12 \times 10^{-4}} = 4.91 \times 10^{-4} \text{ m}$.

- b) When both the strips are fitted, the optical path changes by

$$\Delta x = (\mu_m - 1)t_1 - (\mu_p - 1)t_2 = (\mu_m - \mu_p)t$$

$$= (1.58 - 1.55) \times (0.5)(10^{-3}) = 0.015 \times 10^{-3} \text{ m}.$$

So, No. of fringes shifted = $\frac{0.015 \times 10^{-3}}{590 \times 10^{-9}} = 25.43$.

⇒ There are 25 fringes and 0.43 th of a fringe.

⇒ There are 13 bright fringes and 12 dark fringes and 0.43 th of a dark fringe.

So, position of first maximum on both sides will be given by

$$\therefore x = 0.43 \times 4.91 \times 10^{-4} = 0.021 \text{ cm}$$

$$x' = (1 - 0.43) \times 4.91 \times 10^{-4} = 0.028 \text{ cm (since, fringe width} = 4.91 \times 10^{-4} \text{ m)}$$

17. The change in path difference due to the two slabs is $(\mu_1 - \mu_2)t$ (as in problem no. 16).

For having a minimum at P_0 , the path difference should change by $\lambda/2$.

So, $\Rightarrow \lambda/2 = (\mu_1 - \mu_2)t \Rightarrow t = \frac{\lambda}{2(\mu_1 - \mu_2)}$.

18. Given that, $t = 0.02 \text{ mm} = 0.02 \times 10^{-3} \text{ m}$, $\mu_1 = 1.45$, $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$

a) Let, $I_1 =$ Intensity of source without paper = I

b) Then $I_2 =$ Intensity of source with paper = $(4/9)I$

$$\Rightarrow \frac{I_1}{I_2} = \frac{9}{4} \Rightarrow \frac{r_1}{r_2} = \frac{3}{2} \text{ [because } I \propto r^2]$$

where, r_1 and r_2 are corresponding amplitudes.

So, $\frac{I_{\max}}{I_{\min}} = \frac{(r_1 + r_2)^2}{(r_1 - r_2)^2} = 25 : 1$

b) No. of fringes that will cross the origin is given by,

$$n = \frac{(\mu - 1)t}{\lambda} = \frac{(1.45 - 1) \times 0.02 \times 10^{-3}}{600 \times 10^{-9}} = 15.$$

19. Given that, $d = 0.28 \text{ mm} = 0.28 \times 10^{-3} \text{ m}$, $D = 48 \text{ cm} = 0.48 \text{ m}$, $\lambda_a = 700 \text{ nm}$ in vacuum

Let, $\lambda_w =$ wavelength of red light in water

Since, the fringe width of the pattern is given by,

$$\beta = \frac{\lambda_w D}{d} = \frac{525 \times 10^{-9} \times 0.48}{0.28 \times 10^{-3}} = 9 \times 10^{-4} \text{ m} = 0.90 \text{ mm}.$$

20. It can be seen from the figure that the wavefronts reaching O from S_1 and S_2 will have a path difference of $S_2 X$.

In the $\Delta S_1 S_2 X$,

$$\sin \theta = \frac{S_2 X}{S_1 S_2}$$

So, path difference = $S_2 X = S_1 S_2 \sin \theta = d \sin \theta = d \times \lambda/2d = \lambda/2$

As the path difference is an odd multiple of $\lambda/2$, there will be a dark fringe at point P_0 .

21. a) Since, there is a phase difference of π between direct light and reflecting light, the intensity just above the mirror will be zero.

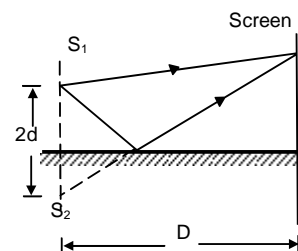
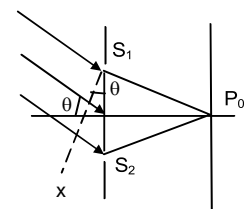
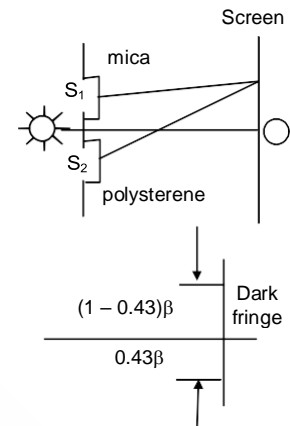
b) Here, $2d =$ equivalent slit separation

$D =$ Distance between slit and screen.

We know for bright fringe, $\Delta x = \frac{y \times 2d}{D} = n\lambda$

But as there is a phase reversal of $\lambda/2$.

$$\Rightarrow \frac{y \times 2d}{D} + \frac{\lambda}{2} = n\lambda \Rightarrow \frac{y \times 2d}{D} = n\lambda - \frac{\lambda}{2} \Rightarrow y = \frac{\lambda D}{4d}$$



22. Given that, $D = 1 \text{ m}$, $\lambda = 700 \text{ nm} = 700 \times 10^{-9} \text{ m}$

Since, $a = 2 \text{ mm}$, $d = 2a = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$ (Lloyd's mirror experiment)

$$\text{Fringe width} = \frac{\lambda D}{d} = \frac{700 \times 10^{-9} \text{ m} \times 1 \text{ m}}{2 \times 10^{-3} \text{ m}} = 0.35 \text{ mm}.$$

23. Given that, the mirror reflects 64% of energy (intensity) of the light.

$$\text{So, } \frac{I_1}{I_2} = 0.64 = \frac{16}{25} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5}$$

$$\text{So, } \frac{I_{\max}}{I_{\min}} = \frac{(r_1 + r_2)^2}{(r_1 - r_2)^2} = 81 : 1.$$

24. It can be seen from the figure that, the apparent distance of the screen from the slits is,

$$D = 2D_1 + D_2$$

$$\text{So, Fringe width} = \frac{D\lambda}{d} = \frac{(2D_1 + D_2)\lambda}{d}$$

25. Given that, $\lambda = (400 \text{ nm to } 700 \text{ nm})$, $d = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$,

$D = 50 \text{ cm} = 0.5 \text{ m}$ and on the screen $y_n = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

a) We know that for zero intensity (dark fringe)

$$y_n = \left(\frac{2n+1}{2} \right) \frac{\lambda_n D}{d} \text{ where } n = 0, 1, 2, \dots$$

$$\Rightarrow \lambda_n = \frac{2}{(2n+1)} \frac{y_n d}{D} = \frac{2}{(2n+1)} \times \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} \Rightarrow \frac{2}{(2n+1)} \times 10^{-6} \text{ m} = \frac{2}{(2n+1)} \times 10^3 \text{ nm}$$

$$\text{If } n = 1, \lambda_1 = (2/3) \times 1000 = 667 \text{ nm}$$

$$\text{If } n = 1, \lambda_2 = (2/5) \times 1000 = 400 \text{ nm}$$

So, the light waves of wavelengths 400 nm and 667 nm will be absent from the out coming light.

b) For strong intensity (bright fringes) at the hole

$$y_n = \frac{n\lambda_n D}{d} \Rightarrow \lambda_n = \frac{y_n d}{nD}$$

$$\text{When, } n = 1, \lambda_1 = \frac{y_n d}{D} = \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} = 10^{-6} \text{ m} = 1000 \text{ nm}.$$

1000 nm is not present in the range 400 nm – 700 nm

$$\text{Again, where } n = 2, \lambda_2 = \frac{y_n d}{2D} = 500 \text{ nm}$$

So, the only wavelength which will have strong intensity is 500 nm.

26. From the diagram, it can be seen that at point O.

$$\text{Path difference} = (AB + BO) - (AC + CO)$$

$$= 2(AB - AC) \quad [\text{Since, } AB = BO \text{ and } AC = CO] = 2(\sqrt{d^2 + D^2} - D)$$

For dark fringe, path difference should be odd multiple of $\lambda/2$.

$$\text{So, } 2(\sqrt{d^2 + D^2} - D) = (2n + 1)(\lambda/2)$$

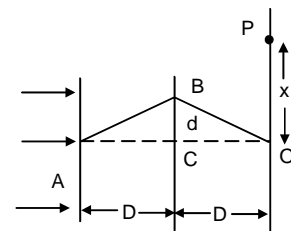
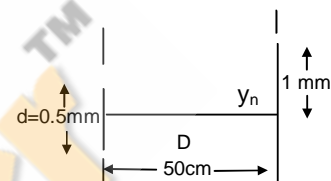
$$\Rightarrow \sqrt{d^2 + D^2} = D + (2n + 1) \lambda/4$$

$$\Rightarrow D^2 + d^2 = D^2 + (2n+1)^2 \lambda^2/16 + (2n + 1) \lambda D/2$$

Neglecting, $(2n+1)^2 \lambda^2/16$, as it is very small

$$\text{We get, } d = \sqrt{(2n+1) \frac{\lambda D}{2}}$$

$$\text{For minimum 'd', putting } n = 0 \Rightarrow d_{\min} = \sqrt{\frac{\lambda D}{2}}.$$



27. For minimum intensity

$$\therefore S_1P - S_2P = x = (2n + 1) \lambda/2$$

From the figure, we get

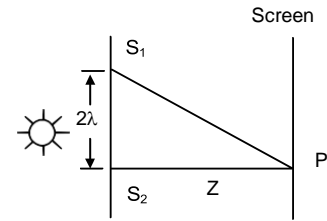
$$\Rightarrow \sqrt{Z^2 + (2\lambda)^2} - Z = (2n + 1) \frac{\lambda}{2}$$

$$\Rightarrow Z^2 + 4\lambda^2 = Z^2 + (2n + 1)^2 \frac{\lambda^2}{4} + Z(2n + 1)\lambda$$

$$\Rightarrow Z = \frac{4\lambda^2 - (2n + 1)^2(\lambda^2/4)}{(2n + 1)\lambda} = \frac{16\lambda^2 - (2n + 1)^2\lambda^2}{4(2n + 1)\lambda} \dots(1)$$

Putting, $n = 0 \Rightarrow Z = 15\lambda/4$ $n = -1 \Rightarrow Z = -15\lambda/4$
 $n = 1 \Rightarrow Z = 7\lambda/12$ $n = 2 \Rightarrow Z = -9\lambda/20$

$\therefore Z = 7\lambda/12$ is the smallest distance for which there will be minimum intensity.



28. Since S_1, S_2 are in same phase, at O there will be maximum intensity.

Given that, there will be a maximum intensity at P.

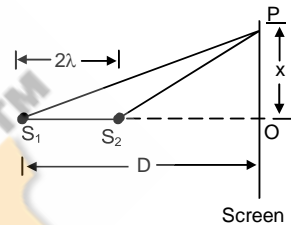
$$\Rightarrow \text{path difference} = \Delta x = n\lambda$$

From the figure,

$$(S_1P)^2 - (S_2P)^2 = (D^2 + X^2)^2 - (D^2 - 2\lambda)^2 + X^2)^2$$

$$= 4\lambda D - 4\lambda^2 = 4\lambda D \quad (\lambda^2 \text{ is so small and can be neglected})$$

$$\Rightarrow S_1P - S_2P = \frac{4\lambda D}{2\sqrt{X^2 + D^2}} = n\lambda$$



$$\Rightarrow \frac{2D}{\sqrt{X^2 + D^2}} = n$$

$$\Rightarrow n^2 (X^2 + D^2) = 4D^2 = \Delta X = \frac{D}{n} 4\sqrt{n^2 - 1}$$

when $n = 1, x = \sqrt{3} D$ (1st order)
 $n = 2, x = 0$ (2nd order)

\therefore When $X = \sqrt{3} D$, at P there will be maximum intensity.

29. As shown in the figure,

$$(S_1P)^2 = (PX)^2 + (S_1X)^2 \dots(1)$$

$$(S_2P)^2 = (PX)^2 + (S_2X)^2 \dots(2)$$

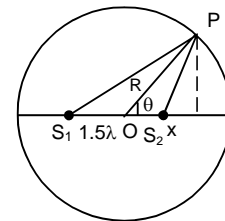
From (1) and (2),

$$(S_1P)^2 - (S_2P)^2 = (S_1X)^2 - (S_2X)^2$$

$$= (1.5\lambda + R \cos \theta)^2 - (R \cos \theta - 1.5\lambda)^2$$

$$= 6\lambda R \cos \theta$$

$$\Rightarrow (S_1P - S_2P) = \frac{6\lambda R \cos \theta}{2R} = 3\lambda \cos \theta.$$



For constructive interference,

$$(S_1P - S_2P)^2 = x = 3\lambda \cos \theta = n\lambda$$

$$\Rightarrow \cos \theta = n/3 \Rightarrow \theta = \cos^{-1}(n/3), \text{ where } n = 0, 1, 2, \dots$$

$$\Rightarrow \theta = 0^\circ, 48.2^\circ, 70.5^\circ, 90^\circ \text{ and similar points in other quadrants.}$$

30. a) As shown in the figure, $BP_0 - AP_0 = \lambda/3$

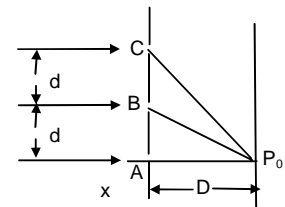
$$\Rightarrow \sqrt{D^2 + d^2} - D = \lambda/3$$

$$\Rightarrow D^2 + d^2 = D^2 + (\lambda^2/9) + (2\lambda D)/3$$

$$\Rightarrow d = \sqrt{(2\lambda D)/3} \quad (\text{neglecting the term } \lambda^2/9 \text{ as it is very small})$$

b) To find the intensity at P_0 , we have to consider the interference of light waves coming from all the three slits.

$$\text{Here, } CP_0 - AP_0 = \sqrt{D^2 + 4d^2} - D$$



$$\begin{aligned}
 &= \sqrt{D^2 + \frac{8\lambda D}{3}} - D = D \left\{ 1 + \frac{8\lambda}{3D} \right\}^{1/2} - D \\
 &= D \left\{ 1 + \frac{8\lambda}{3D \times 2} \right\} - D = \frac{4\lambda}{3} \quad \text{[using binomial expansion]}
 \end{aligned}$$

So, the corresponding phase difference between waves from C and A is,

$$\phi_c = \frac{2\pi x}{\lambda} = \frac{2\pi \times 4\lambda}{3\lambda} = \frac{8\pi}{3} = \left(2\pi + \frac{2\pi}{3} \right) = \frac{2\pi}{3} \quad \dots(1)$$

$$\text{Again, } \phi_B = \frac{2\pi x}{3\lambda} = \frac{2\pi}{3} \quad \dots(2)$$

So, it can be said that light from B and C are in same phase as they have some phase difference with respect to A.

$$\begin{aligned}
 \text{So, } R &= \sqrt{(2r)^2 + r^2 + 2 \times 2r \times r \cos(2\pi/3)} && \text{(using vector method)} \\
 &= \sqrt{4r^2 + r^2 - 2r^2} = \sqrt{3}r
 \end{aligned}$$

$$\therefore I_{P_0} = K(\sqrt{3}r)^2 = 3Kr^2 = 3I$$

As, the resulting amplitude is $\sqrt{3}$ times, the intensity will be three times the intensity due to individual slits.

31. Given that, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$, $\lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$, $I_{\max} = 0.20 \text{ W/m}^2$, $D = 2 \text{ m}$

For the point, $y = 0.5 \text{ cm}$

$$\text{We know, path difference} = x = \frac{yd}{D} = \frac{0.5 \times 10^{-2} \times 2 \times 10^{-3}}{2} = 5 \times 10^{-6} \text{ m}$$

So, the corresponding phase difference is,

$$\phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 5 \times 10^{-6}}{6 \times 10^{-7}} \Rightarrow \frac{50\pi}{3} = 16\pi + \frac{2\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$$

So, the amplitude of the resulting wave at the point $y = 0.5 \text{ cm}$ is,

$$A = \sqrt{r^2 + r^2 + 2r^2 \cos(2\pi/3)} = \sqrt{r^2 + r^2 - r^2} = r$$

$$\text{Since, } \frac{I}{I_{\max}} = \frac{A^2}{(2r)^2} \quad \text{[since, maximum amplitude} = 2r]$$

$$\begin{aligned}
 \Rightarrow \frac{I}{0.2} &= \frac{r^2}{4r^2} = \frac{r^2}{4r^2} \\
 \Rightarrow I &= \frac{0.2}{4} = 0.05 \text{ W/m}^2.
 \end{aligned}$$

32. i) When intensity is half the maximum $\frac{I}{I_{\max}} = \frac{1}{2}$

$$\Rightarrow \frac{4a^2 \cos^2(\phi/2)}{4a^2} = \frac{1}{2}$$

$$\Rightarrow \cos^2(\phi/2) = 1/2 \Rightarrow \cos(\phi/2) = 1/\sqrt{2}$$

$$\Rightarrow \phi/2 = \pi/4 \Rightarrow \phi = \pi/2$$

$$\Rightarrow \text{Path difference, } x = \lambda/4$$

$$\Rightarrow y = xD/d = \lambda D/4d$$

ii) When intensity is $1/4^{\text{th}}$ of the maximum $\frac{I}{I_{\max}} = \frac{1}{4}$

$$\Rightarrow \frac{4a^2 \cos^2(\phi/2)}{4a^2} = \frac{1}{4}$$

$$\Rightarrow \cos^2(\phi/2) = 1/4 \Rightarrow \cos(\phi/2) = 1/2$$

$$\Rightarrow \phi/2 = \pi/3 \Rightarrow \phi = 2\pi/3$$

$$\Rightarrow \text{Path difference, } x = \lambda/3$$

$$\Rightarrow y = xD/d = \lambda D/3d$$

33. Given that, $D = 1 \text{ m}$, $d = 1 \text{ mm} = 10^{-3} \text{ m}$, $\lambda = 500 \text{ nm} = 5 \times 10^{-7} \text{ m}$

For intensity to be half the maximum intensity.

$$y = \frac{\lambda D}{4d} \quad (\text{As in problem no. 32})$$

$$\Rightarrow y = \frac{5 \times 10^{-7} \times 1}{4 \times 10^{-3}} \Rightarrow y = 1.25 \times 10^{-4} \text{ m.}$$

34. The line width of a bright fringe is sometimes defined as the separation between the points on the two sides of the central line where the intensity falls to half the maximum.

We know that, for intensity to be half the maximum

$$y = \pm \frac{\lambda D}{4d}$$

$$\therefore \text{Line width} = \frac{\lambda D}{4d} + \frac{\lambda D}{4d} = \frac{\lambda D}{2d}$$

35. i) When, $z = \lambda D/2d$, at S_4 , minimum intensity occurs (dark fringe)

\Rightarrow Amplitude = 0,

At S_3 , path difference = 0

\Rightarrow Maximum intensity occurs.

\Rightarrow Amplitude = $2r$.

So, on Σ_2 screen,

$$\frac{I_{\max}}{I_{\min}} = \frac{(2r + 0)^2}{(2r - 0)^2} = 1$$

- ii) When, $z = \lambda D/4d$, at S_4 , minimum intensity occurs. (dark fringe)

\Rightarrow Amplitude = 0.

At S_3 , path difference = 0

\Rightarrow Maximum intensity occurs.

\Rightarrow Amplitude = $2r$.

So, on Σ_2 screen,

$$\frac{I_{\max}}{I_{\min}} = \frac{(2r + 2r)^2}{(2r - 0)^2} = \infty$$

- iii) When, $z = \lambda D/4d$, At S_4 , intensity = $I_{\max} / 2$

\Rightarrow Amplitude = $\sqrt{2}r$.

\therefore At S_3 , intensity is maximum.

\Rightarrow Amplitude = $2r$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(2r + \sqrt{2}r)^2}{(2r - \sqrt{2}r)^2} = 34.$$

36. a) When, $z = D\lambda/d$

So, $OS_3 = OS_4 = D\lambda/2d \Rightarrow$ Dark fringe at S_3 and S_4 .

\Rightarrow At S_3 , intensity at $S_3 = 0 \Rightarrow I_1 = 0$

At S_4 , intensity at $S_4 = 0 \Rightarrow I_2 = 0$

At P, path difference = 0 \Rightarrow Phase difference = 0.

$\Rightarrow I = I_1 + I_2 + \sqrt{I_1 I_2} \cos 0^\circ = 0 + 0 + 0 = 0 \Rightarrow$ Intensity at P = 0.

- b) Given that, when $z = D\lambda/2d$, intensity at P = I

Here, $OS_3 = OS_4 = y = D\lambda/4d$

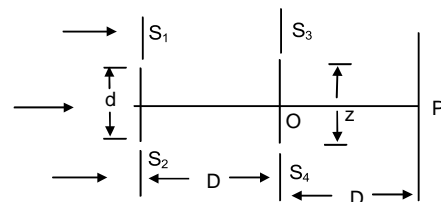
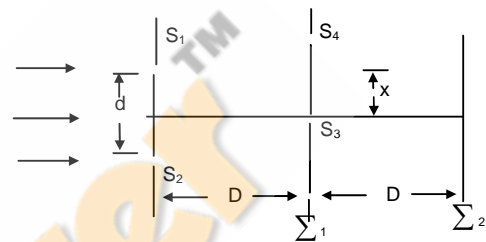
$$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{D\lambda}{4d} \times \frac{d}{D} = \frac{\pi}{2}. \quad [\text{Since, } x = \text{path difference} = yd/D]$$

Let, intensity at S_3 and $S_4 = I'$

\therefore At P, phase difference = 0

So, $I' + I' + 2I' \cos 0^\circ = I$.

$\Rightarrow 4I' = I \Rightarrow I' = I/4$.



$$\text{When, } z = \frac{3D\lambda}{2d}, \Rightarrow y = \frac{3D\lambda}{4d}$$

$$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{3D\lambda}{4d} \times \frac{d}{D} = \frac{3\pi}{2}$$

Let, I'' be the intensity at S_3 and S_4 when, $\phi = 3\pi/2$

Now comparing,

$$\frac{I'}{I} = \frac{a^2 + a^2 + 2a^2 \cos(3\pi/2)}{a^2 + a^2 + 2a^2 \cos \pi/2} = \frac{2a^2}{2a^2} = 1 \Rightarrow I'' = I' = I/4.$$

$$\therefore \text{Intensity at P} = I/4 + I/4 + 2 \times (I/4) \cos 0^\circ = I/2 + I/2 = I.$$

c) When $z = 2D\lambda/d$

$$\Rightarrow y = OS_3 = OS_4 = D\lambda/d$$

$$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{D\lambda}{d} \times \frac{d}{D} = 2\pi.$$

Let, $I''' =$ intensity at S_3 and S_4 when, $\phi = 2\pi$.

$$I''' = \frac{a^2 + a^2 + 2a^2 \cos 2\pi}{2} = \frac{4a^2}{2} = 2a^2$$

$$\Rightarrow I''' = \frac{a^2 + a^2 + 2a^2 \cos \pi/2}{2} = \frac{2a^2}{2} = a^2$$

$$\text{At P, } I_{\text{resultant}} = I/2 + I/2 + 2(I/2) \cos 0^\circ = I + I = 2I.$$

So, the resultant intensity at P will be $2I$.

37. Given $d = 0.0011 \times 10^{-3} \text{ m}$

For minimum reflection of light, $2\mu d = n\lambda$

$$\Rightarrow \mu = \frac{n\lambda}{2d} = \frac{2n\lambda}{4d} = \frac{580 \times 10^{-9} \times 2n}{4 \times 11 \times 10^{-7}} = \frac{5.8}{44} (2n) = 0.132 (2n)$$

Given that, μ has a value in between 1.2 and 1.5.

$$\Rightarrow \text{When, } n = 5, \mu = 0.132 \times 10 = 1.32.$$

38. Given that, $\lambda = 560 \times 10^{-9} \text{ m}$, $\mu = 1.4$.

$$\text{For strong reflection, } 2\mu d = (2n + 1)\lambda/2 \Rightarrow d = \frac{(2n + 1)\lambda}{4\mu}$$

For minimum thickness, putting $n = 0$.

$$\Rightarrow d = \frac{\lambda}{4\mu} \Rightarrow d = \frac{560 \times 10^{-9}}{14} = 10^{-7} \text{ m} = 100 \text{ nm}.$$

39. For strong transmission, $2\mu d = n\lambda \Rightarrow \lambda = \frac{2\mu d}{n}$

Given that, $\mu = 1.33$, $d = 1 \times 10^{-4} \text{ cm} = 1 \times 10^{-6} \text{ m}$.

$$\Rightarrow \lambda = \frac{2 \times 1.33 \times 1 \times 10^{-6}}{n} = \frac{2660 \times 10^{-9}}{n} \text{ m}$$

when, $n = 4, \lambda_1 = 665 \text{ nm}$

$n = 5, \lambda_2 = 532 \text{ nm}$

$n = 6, \lambda_3 = 443 \text{ nm}$

40. For the thin oil film,

$d = 1 \times 10^{-4} \text{ cm} = 10^{-6} \text{ m}$, $\mu_{\text{oil}} = 1.25$ and $\mu_x = 1.50$

$$\lambda = \frac{2\mu d}{(n+1/2)} = \frac{2 \times 10^{-6} \times 1.25 \times 2}{2n+1} = \frac{5 \times 10^{-6} \text{ m}}{2n+1}$$

$$\Rightarrow \lambda = \frac{5000 \text{ nm}}{2n+1}$$

For the wavelengths in the region (400 nm – 750 nm)

$$\text{When, } n = 3, \lambda = \frac{5000}{2 \times 3 + 1} = \frac{5000}{7} = 714.3 \text{ nm}$$

$$\text{When, } n = 4, \lambda = \frac{5000}{2 \times 4 + 1} = \frac{5000}{9} = 555.6 \text{ nm}$$

$$\text{When, } n = 5, \lambda = \frac{5000}{2 \times 5 + 1} = \frac{5000}{11} = 454.5 \text{ nm}$$

41. For first minimum diffraction, $b \sin \theta = \lambda$

Here, $\theta = 30^\circ$, $b = 5 \text{ cm}$

$$\therefore \lambda = 5 \times \sin 30^\circ = 5/2 = 2.5 \text{ cm.}$$

42. $\lambda = 560 \text{ nm} = 560 \times 10^{-9} \text{ m}$, $b = 0.20 \text{ mm} = 2 \times 10^{-4} \text{ m}$, $D = 2 \text{ m}$

$$\text{Since, } R = 1.22 \frac{\lambda D}{b} = 1.22 \times \frac{560 \times 10^{-9} \times 2}{2 \times 10^{-4}} = 6.832 \times 10^{-3} \text{ m} = 0.683 \text{ cm.}$$

So, Diameter = $2R = 1.37 \text{ cm}$.

43. $\lambda = 620 \text{ nm} = 620 \times 10^{-9} \text{ m}$,

$D = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$, $b = 8 \text{ cm} = 8 \times 10^{-2} \text{ m}$

$$\therefore R = 1.22 \times \frac{620 \times 10^{-4} \times 20 \times 10^{-2}}{8 \times 10^{-2}} = 1891 \times 10^{-7} = 1.9 \times 10^{-6} \text{ m}$$

So, diameter = $2R = 3.8 \times 10^{-6} \text{ m}$

